Probabilistic Programming

Luc De Raedt and Angelika Kimmig

IJCAI 2015



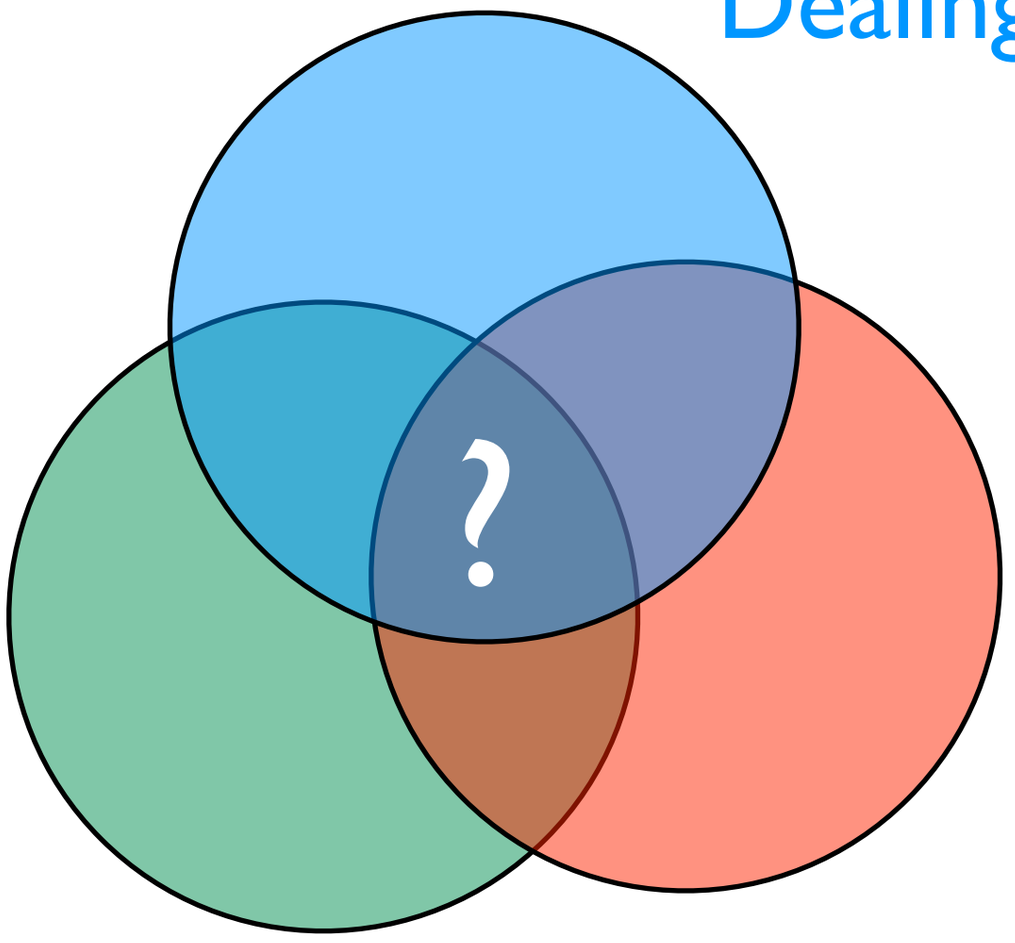


https://dtai.cs.kuleuven.be/problog/ijcai15-tutorial.html

A key question in AI:

Dealing with uncertainty

Reasoning with relational data



Learning

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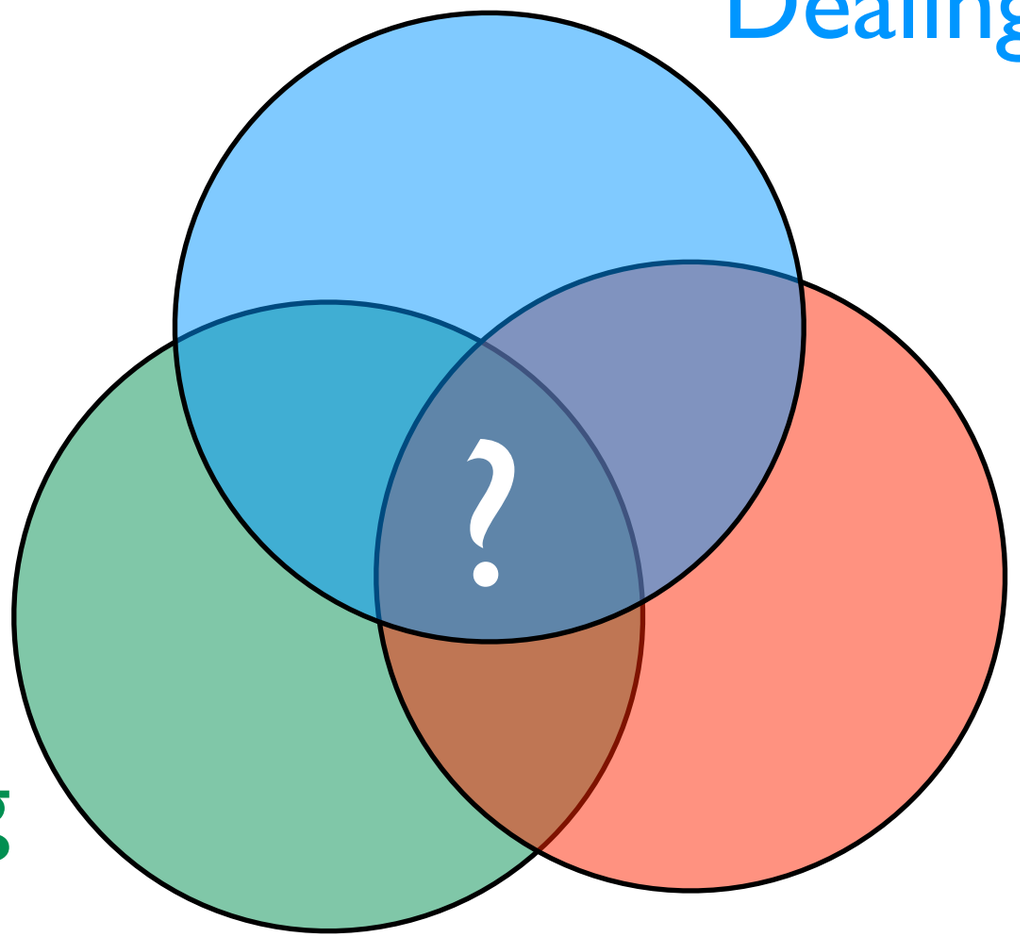
Reasoning with relational data

- logic
- databases
- programming
- ...



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- graphical models
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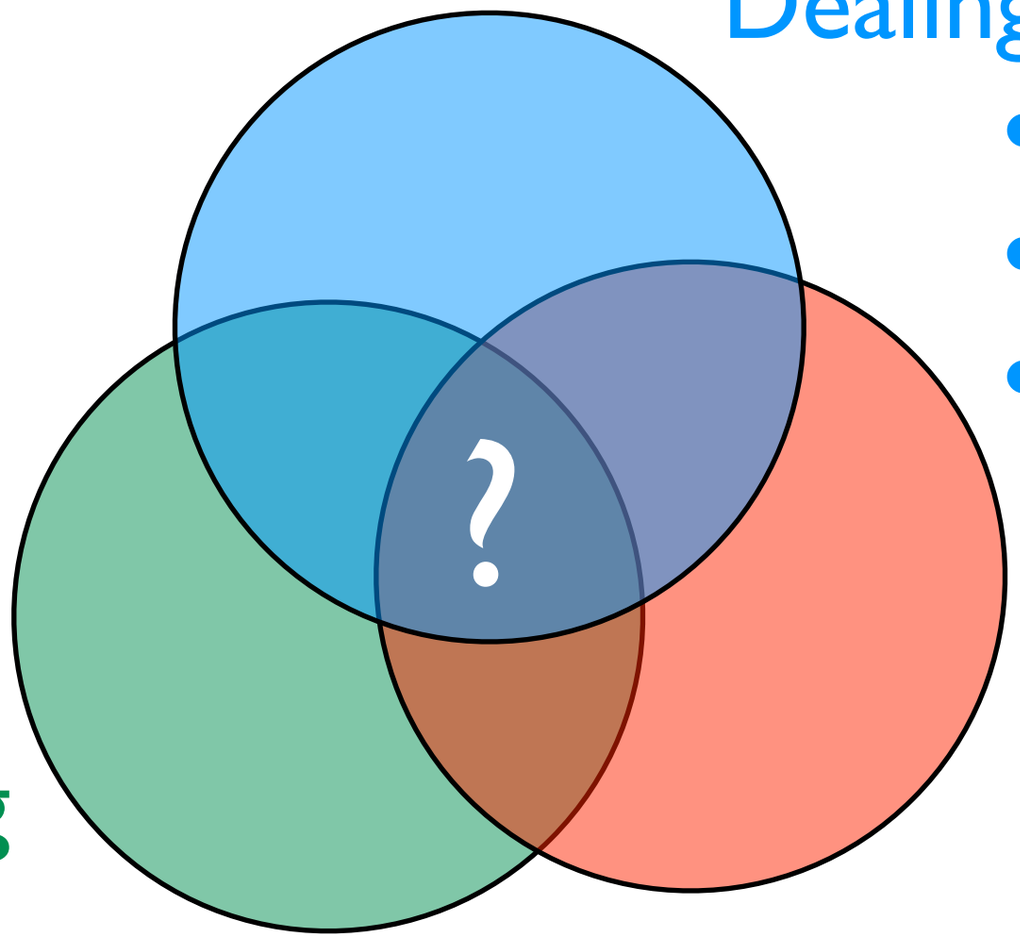

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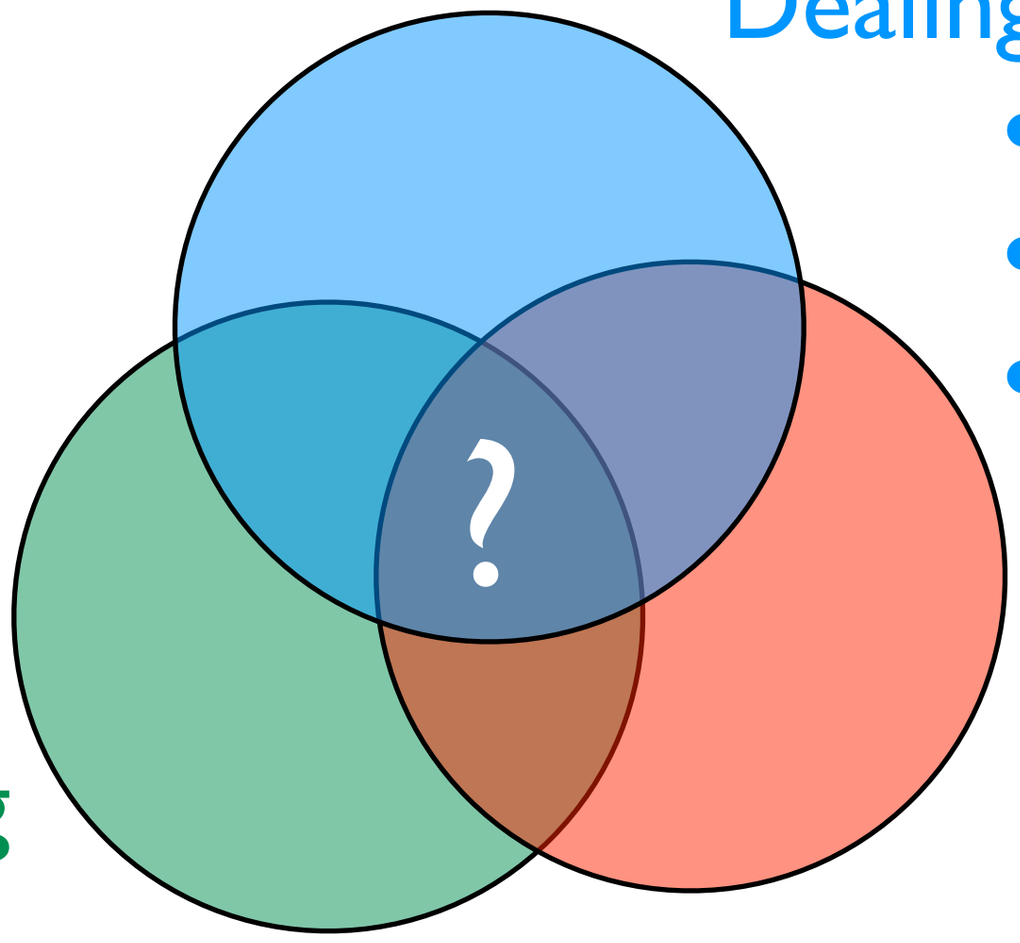

Learning

- parameters
- structure

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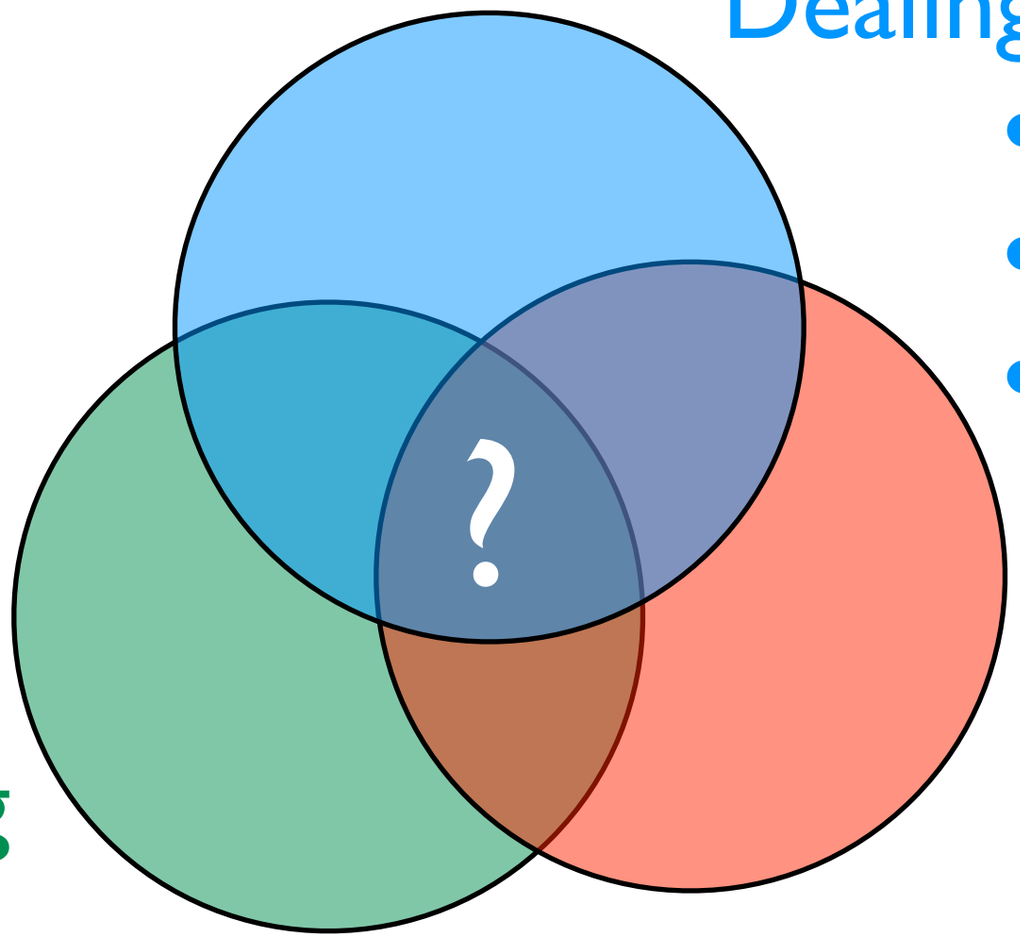

Dealing with uncertainty

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- graphical models
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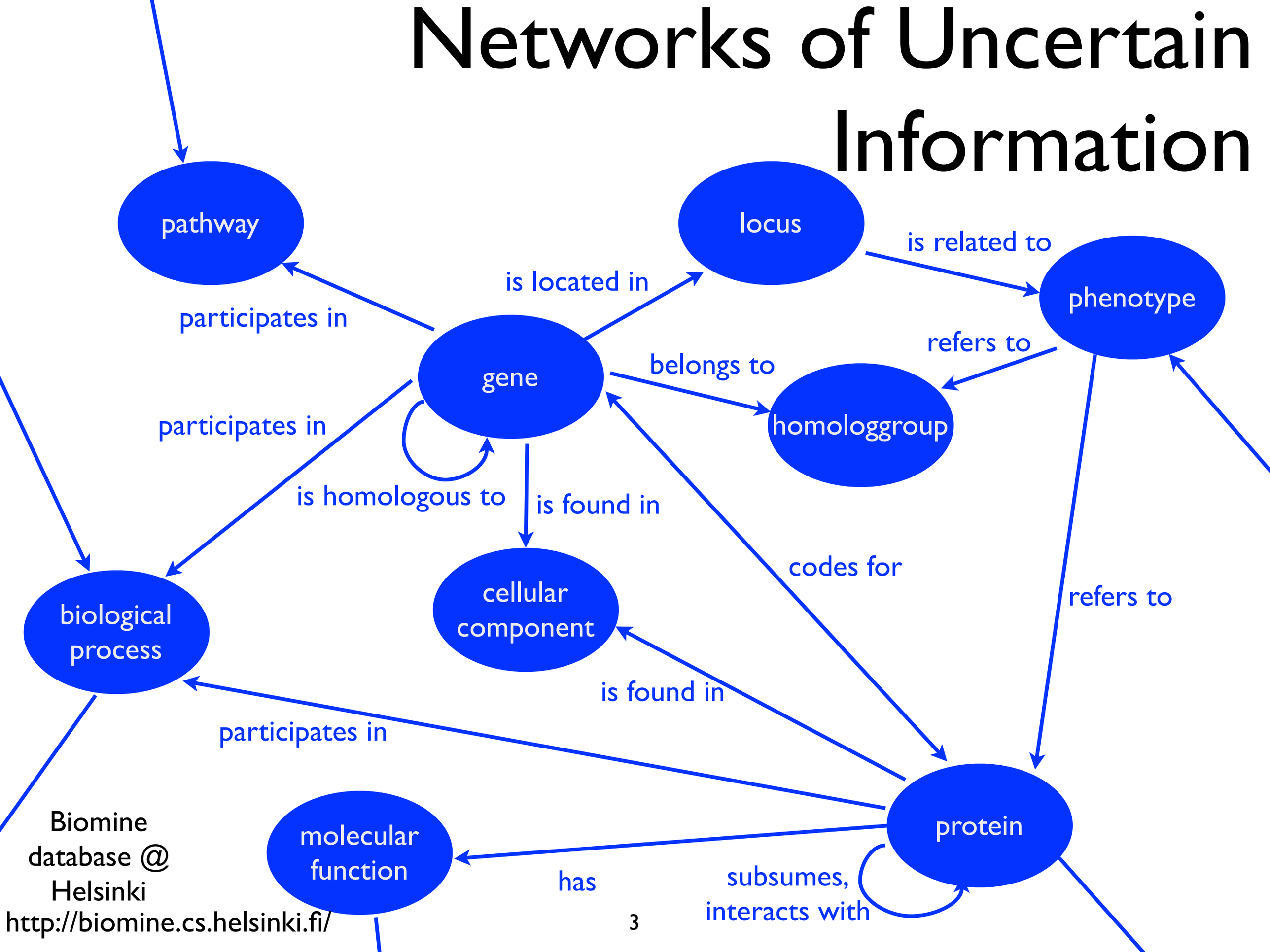
Learning

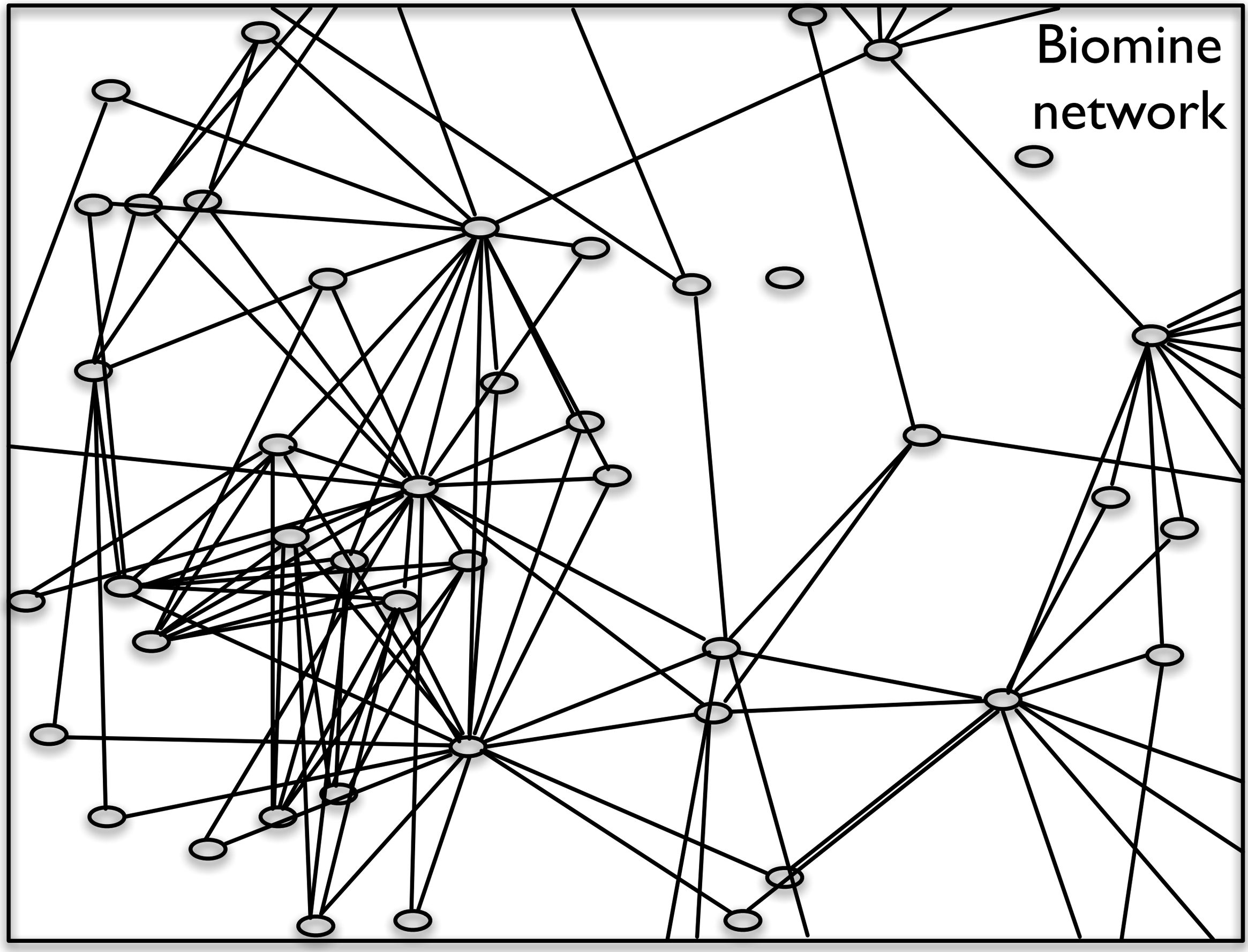
- parameters
- structure

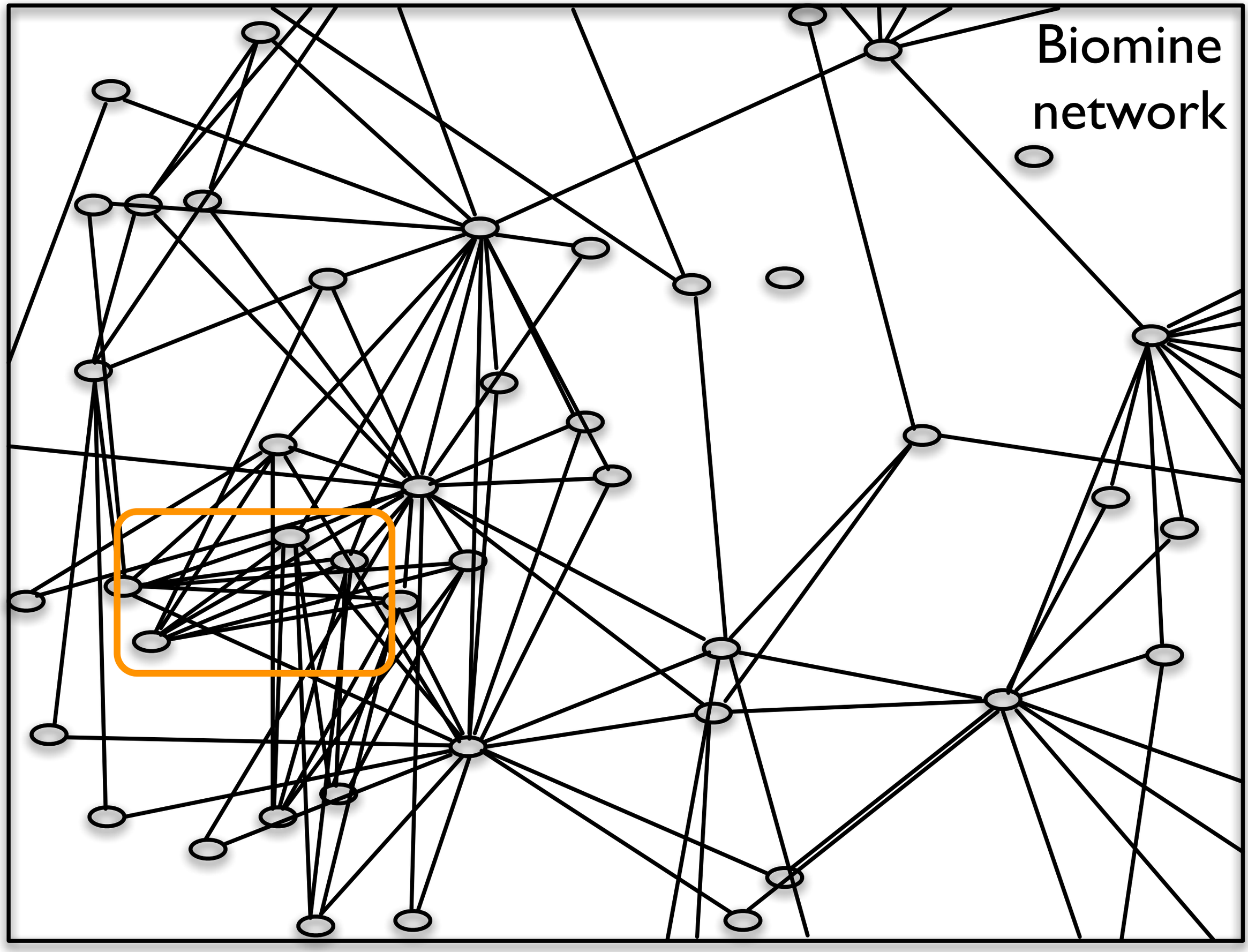
Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

Networks of Uncertain Information



Biomine database @ Helsinki
http://biomine.cs.helsinki.fi/


Biomine
network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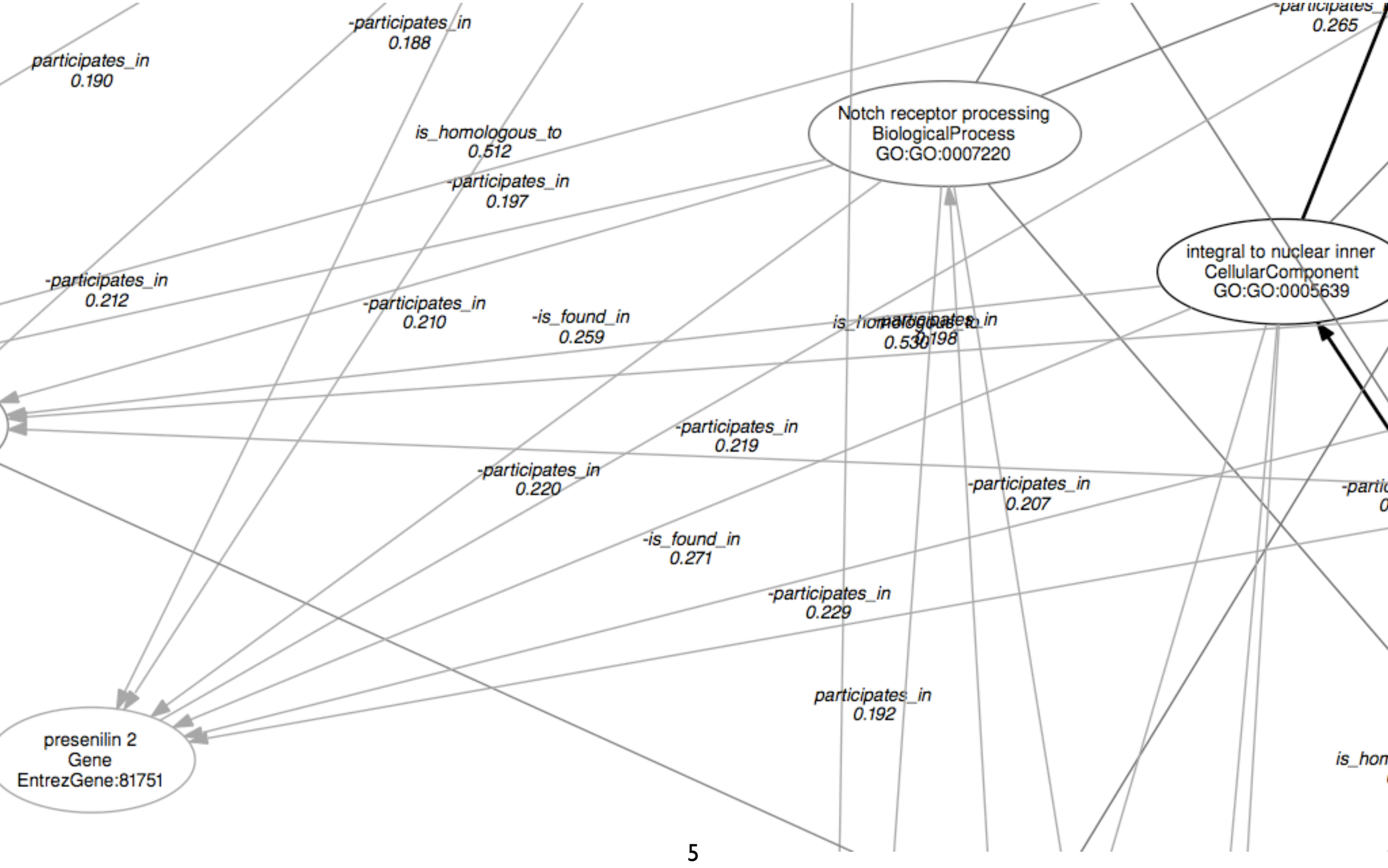
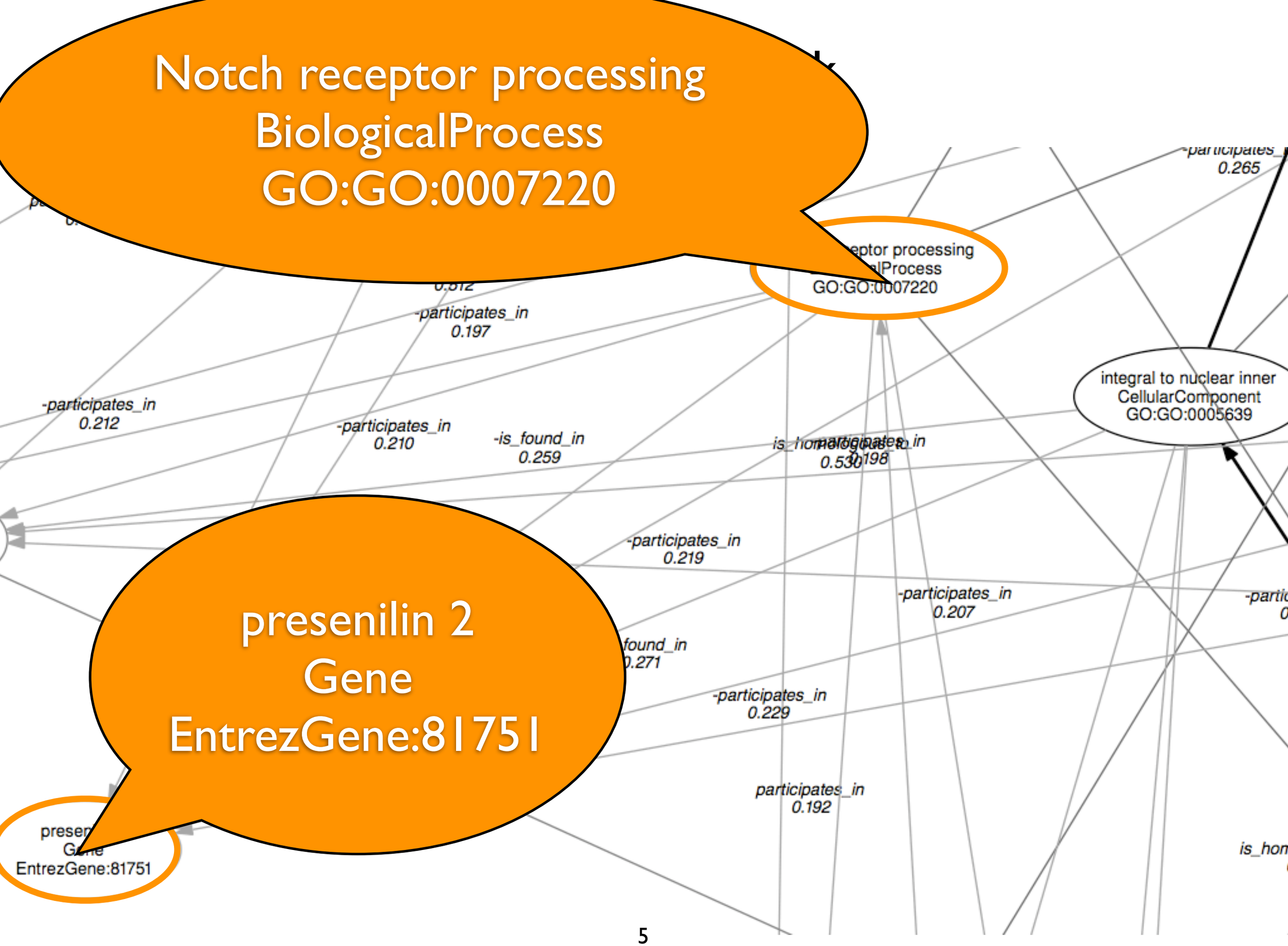
Biomine Network

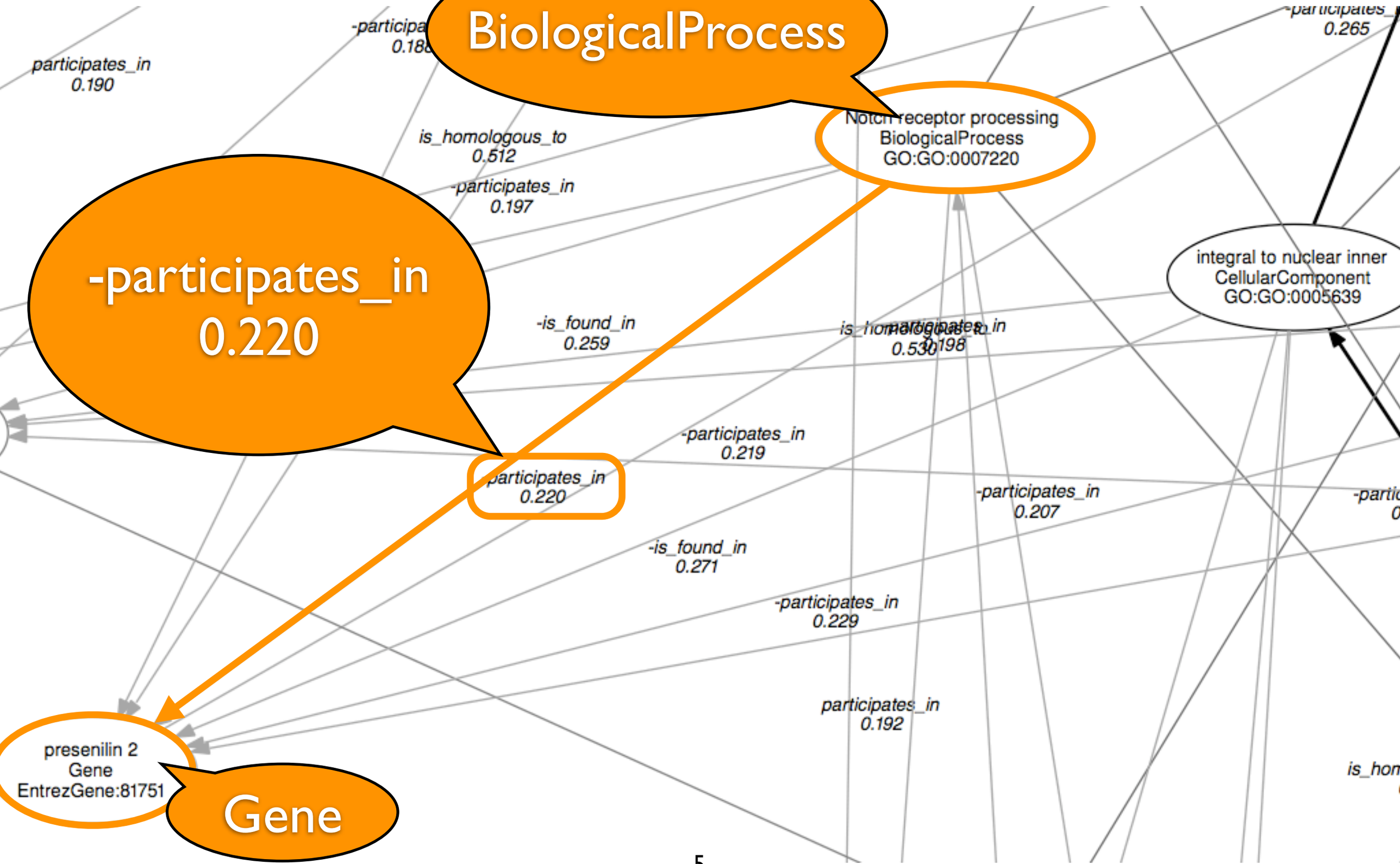


Notch receptor processing
BiologicalProcess
GO:GO:0007220

presenilin 2
Gene
EntrezGene:81751



Biomine Network



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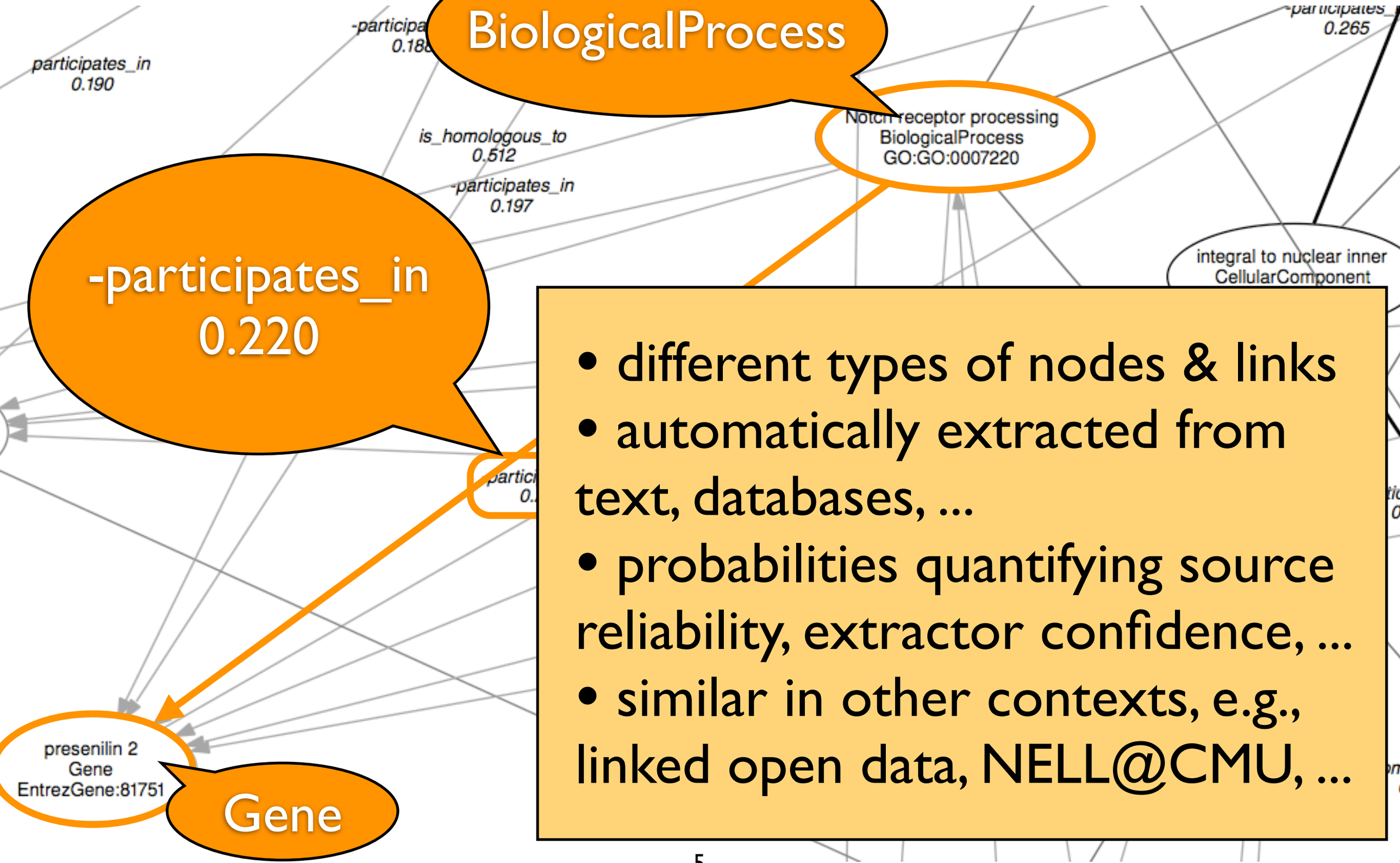


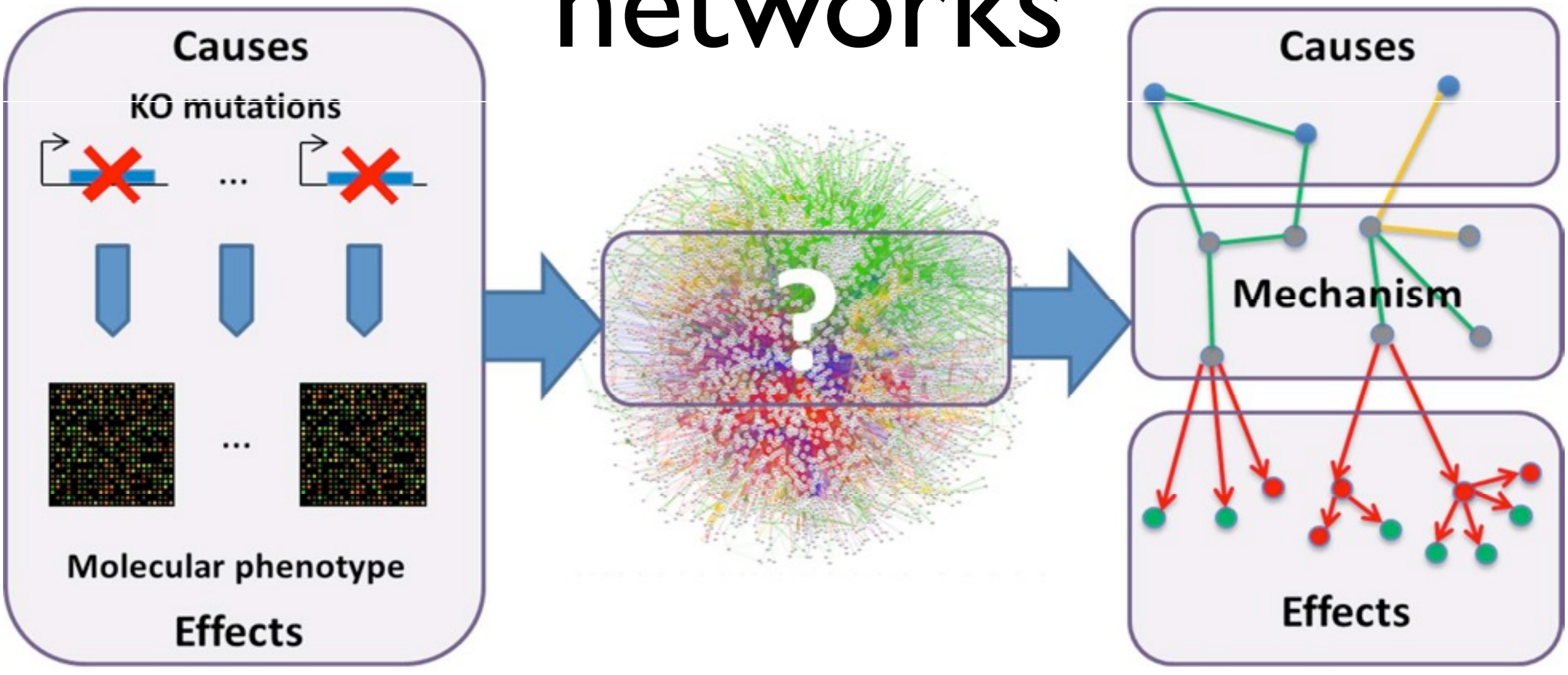
BiologicalProcess

-participates_in
0.220

Gene

- different types of nodes & links
- automatically extracted from text, databases, ...
- probabilities quantifying source reliability, extractor confidence, ...
- similar in other contexts, e.g., linked open data, NELL@CMU, ...

Molecular interaction networks



Can we find the mechanism connecting causes to effects?

[De Maeyer et al, Molecular Biosystems 13]

Example: Information Extraction

Recently-Learned Facts twitter Refresh

instance	iteration	date learned	confidence
kelly_andrews is a female	826	29-mar-2014	98.7
investment_next_year is an economic sector	829	10-apr-2014	95.3
shibenik is a geopolitical entity that is an organization	829	10-apr-2014	97.2
quality_web_design_work is a character trait	826	29-mar-2014	91.0
mercedes_benz_cls_by_carlsson is an automobile manufacturer	829	10-apr-2014	95.2
social_work is an academic program at the university rutgers_university	827	02-apr-2014	93.8
dante wrote the book the_divine_comedy	826	29-mar-2014	93.8
willie_aames was born in the city los_angeles	831	16-apr-2014	100.0
kitt_peak is a mountain in the state or province arizona	831	16-apr-2014	96.9
greenwich is a park in the city london	831	16-apr-2014	100.0

NELL: http://rtw.ml.cmu.edu/rtw/

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instances for many different relations

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instances for many different relations

degree of certainty

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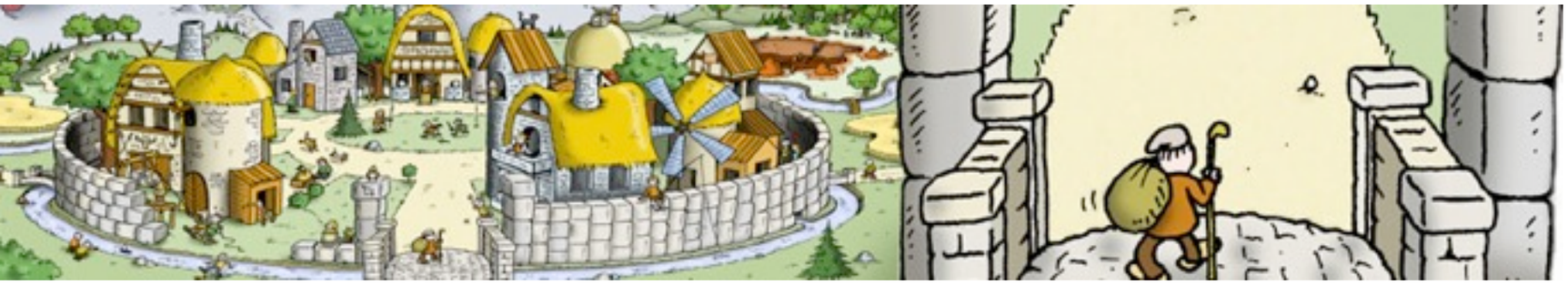
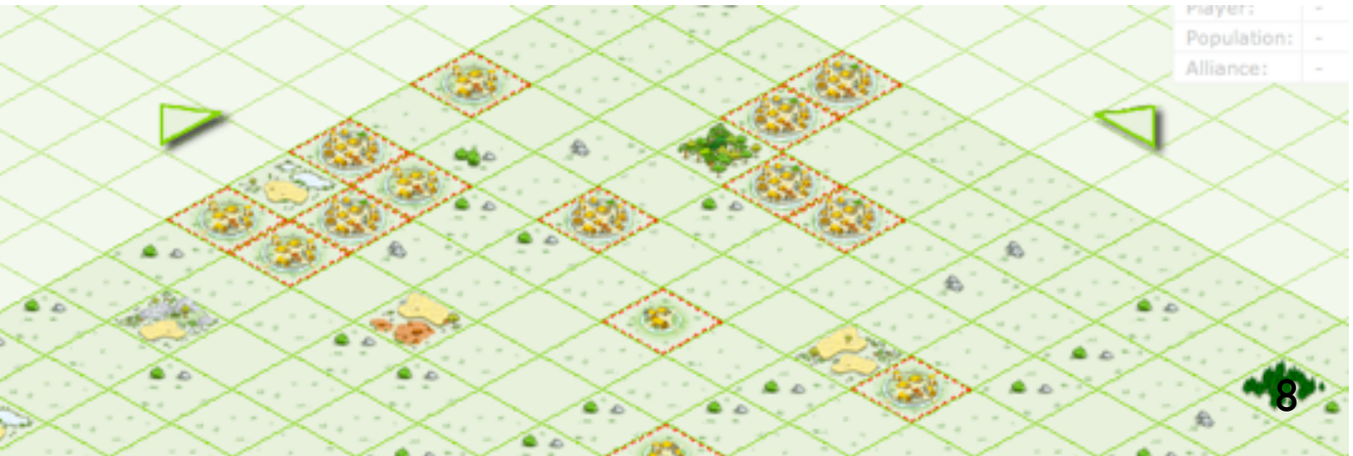
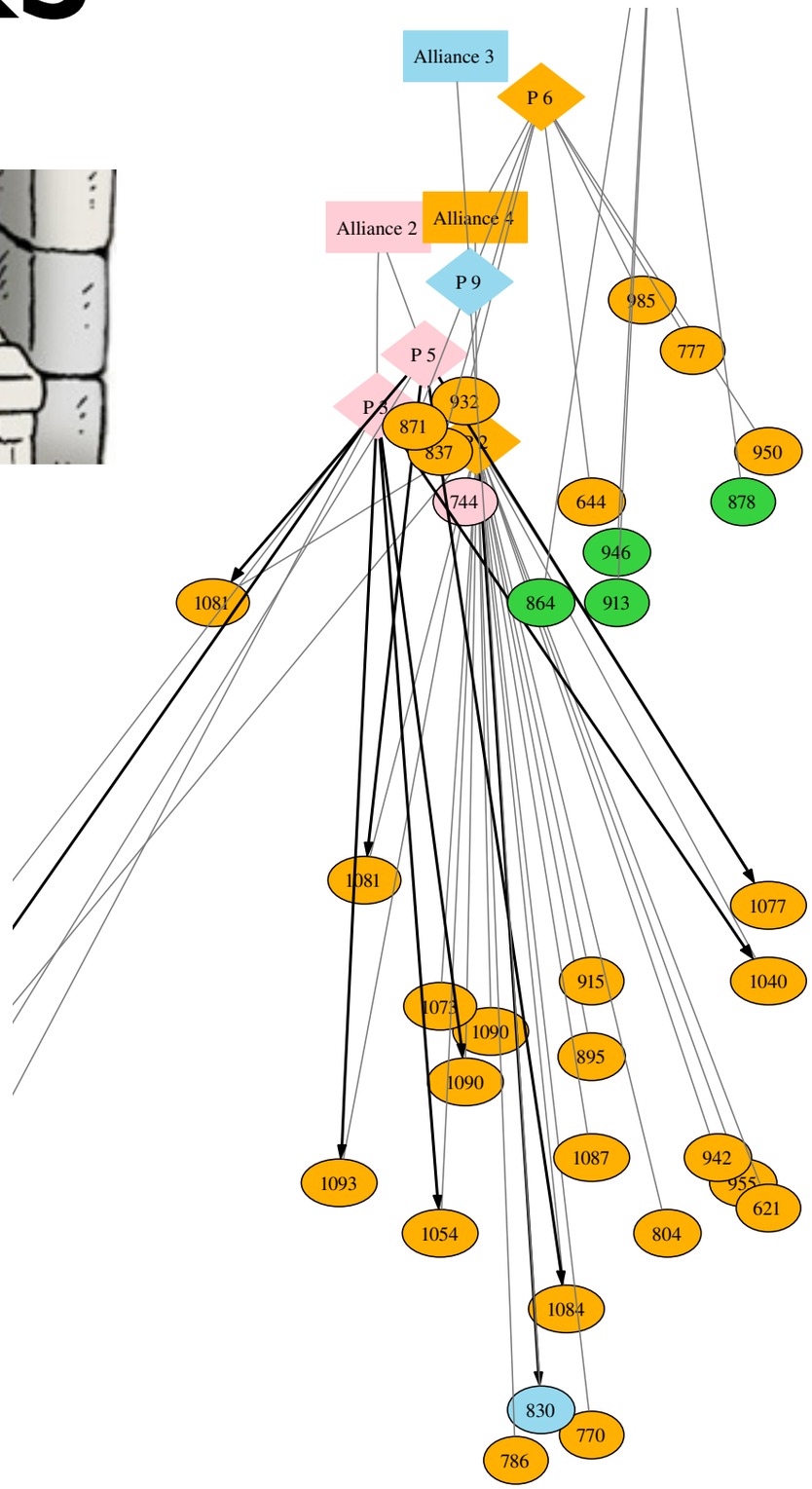
Dynamic networks



Travian: A massively multiplayer real-time strategy game

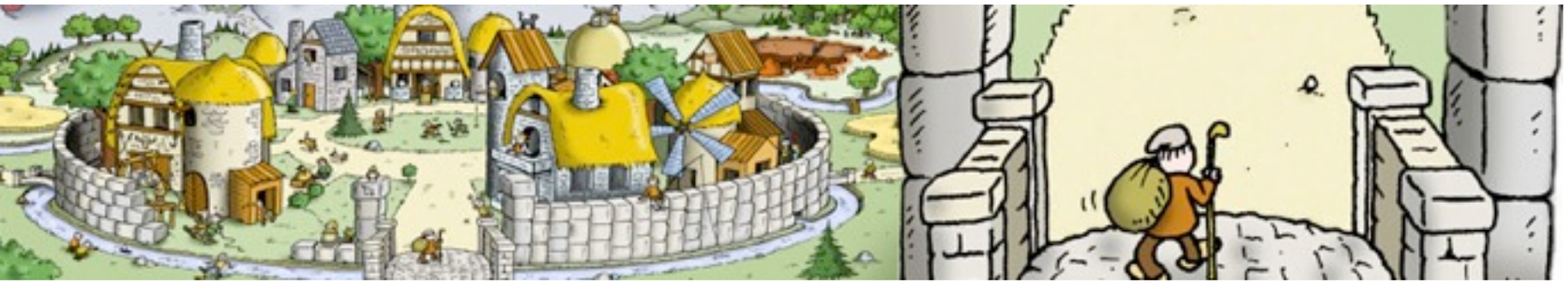
Can we build a model
of this world ?
Can we use it for playing
better ?





[Thon et al, MLJ 11]

Dynamic networks



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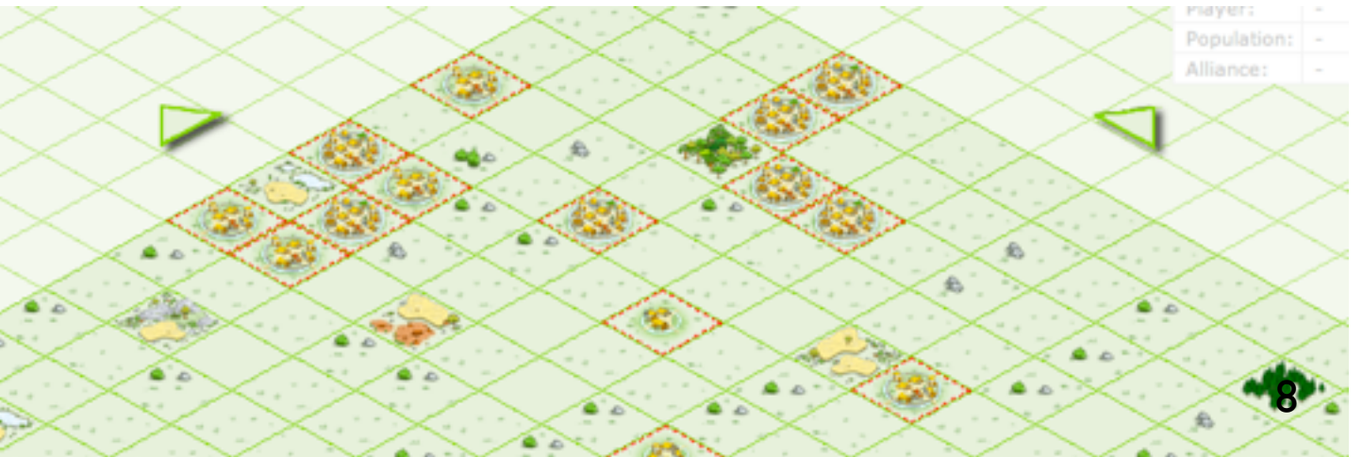

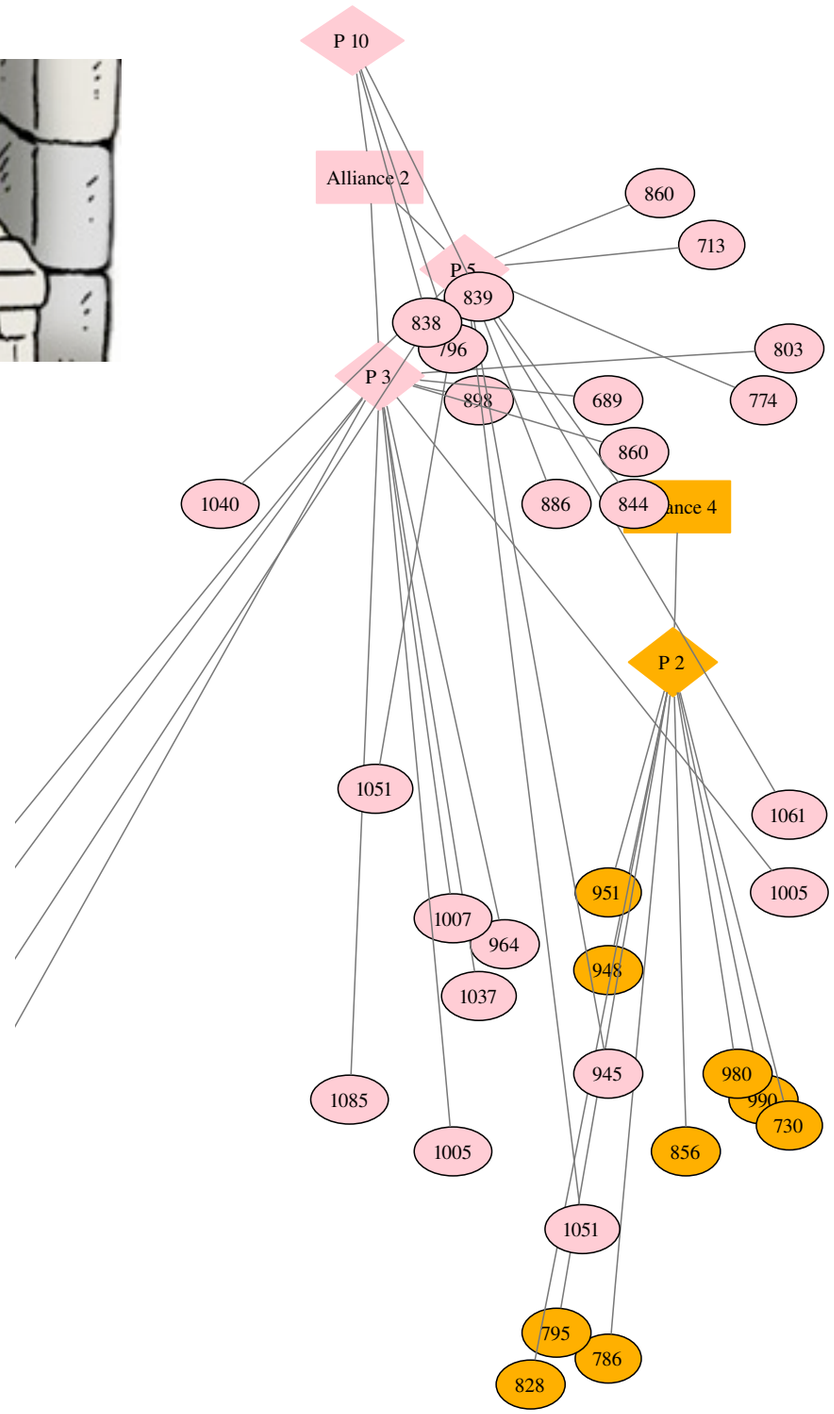

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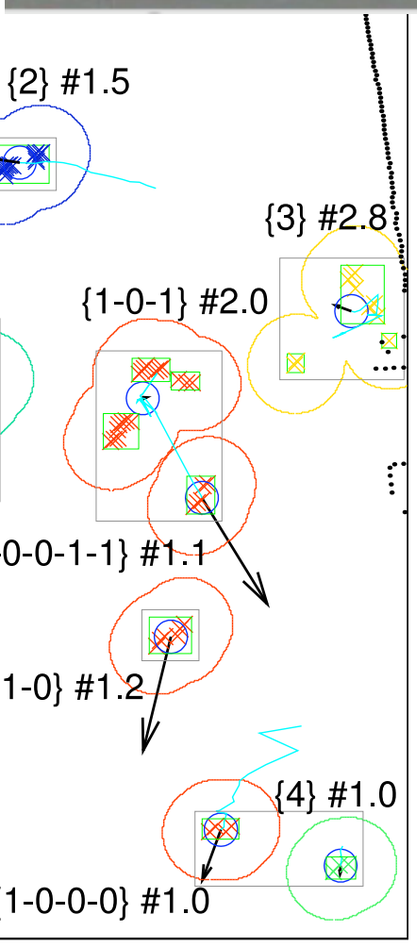
Analyzing Video Data







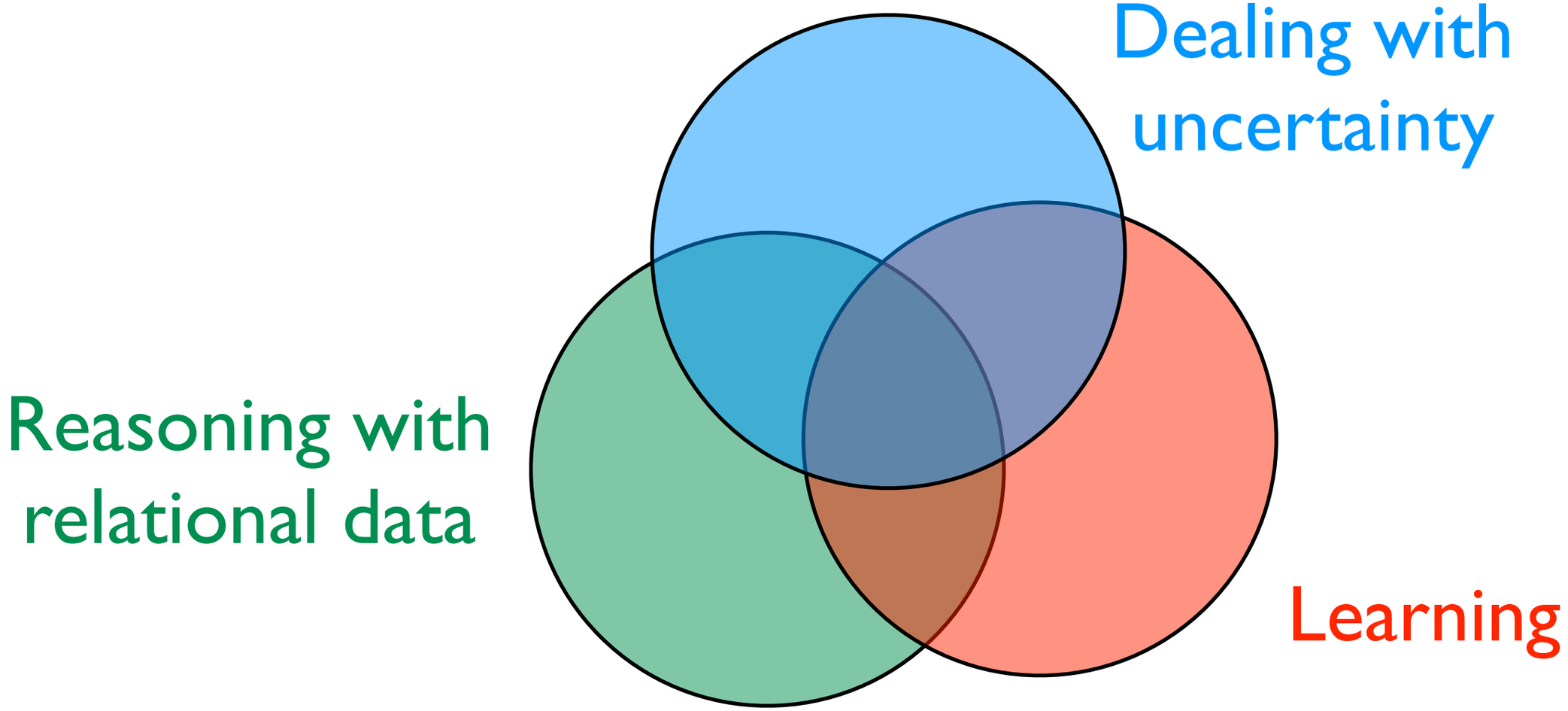
- Track people or objects over time? Even if temporarily hidden?
- Recognize activities?
- Infer object properties?





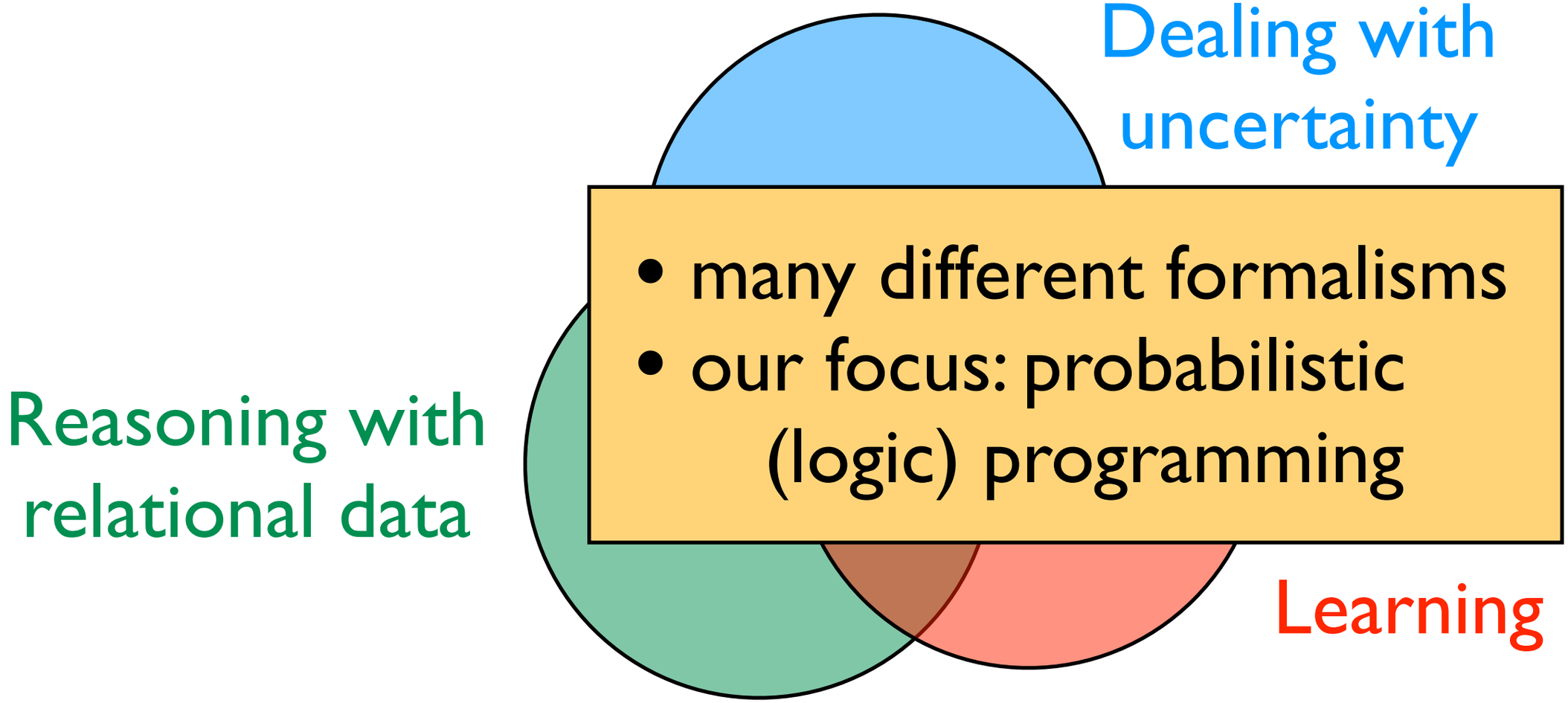
[Skarlatidis et al, TPLP 14;
Nitti et al, IROS 13, ICRA 14]

Common theme



Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

Common theme



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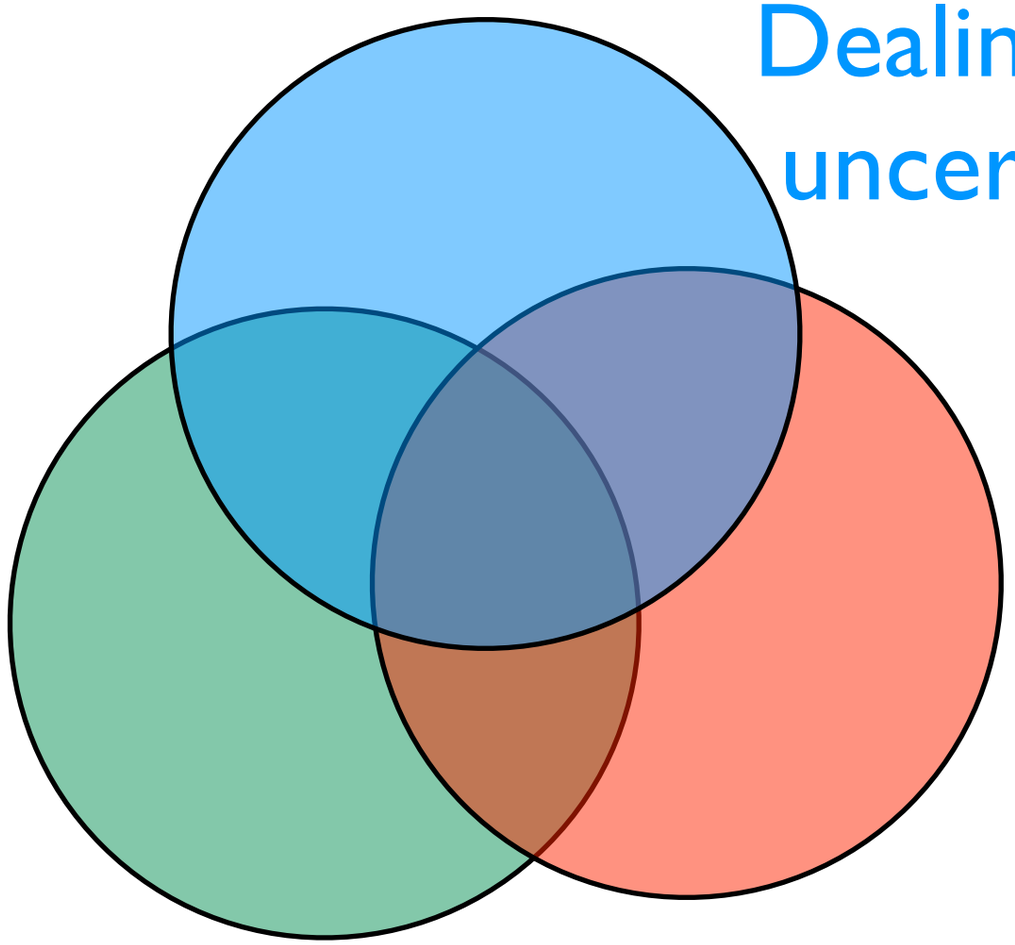
ProbLog

probabilistic Prolog



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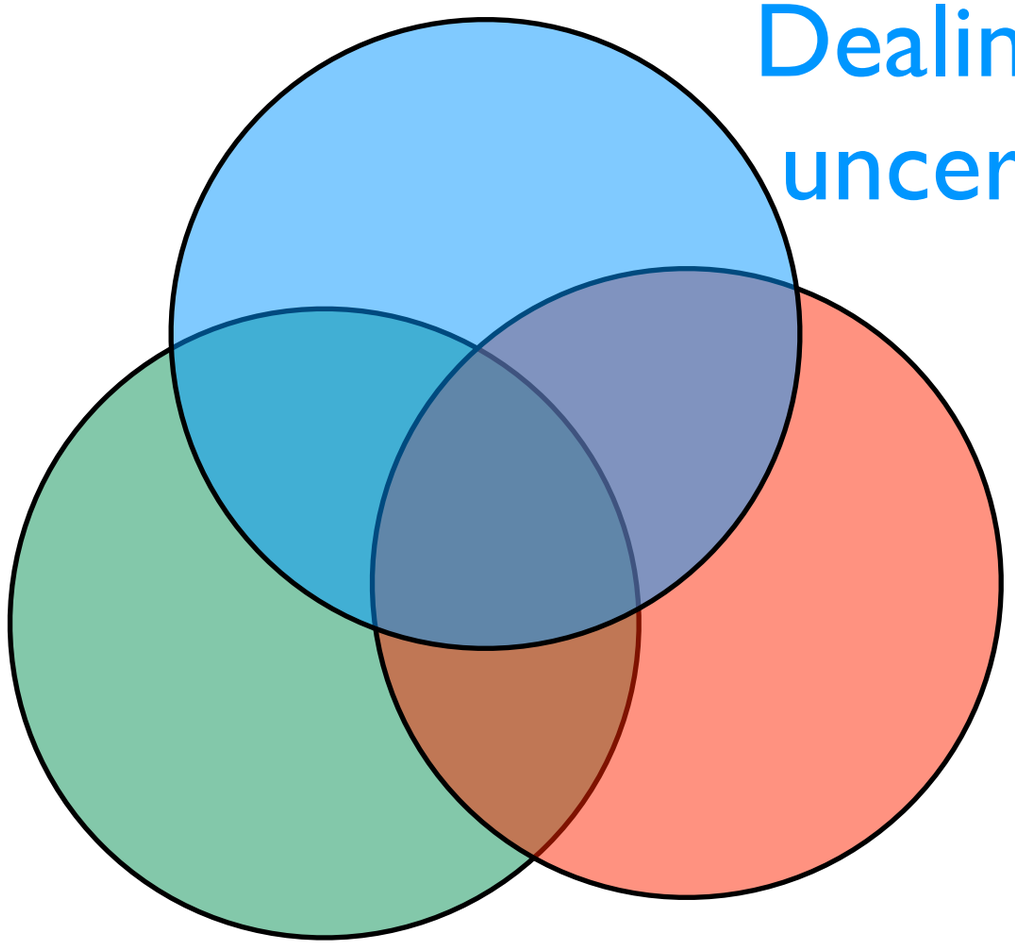


```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
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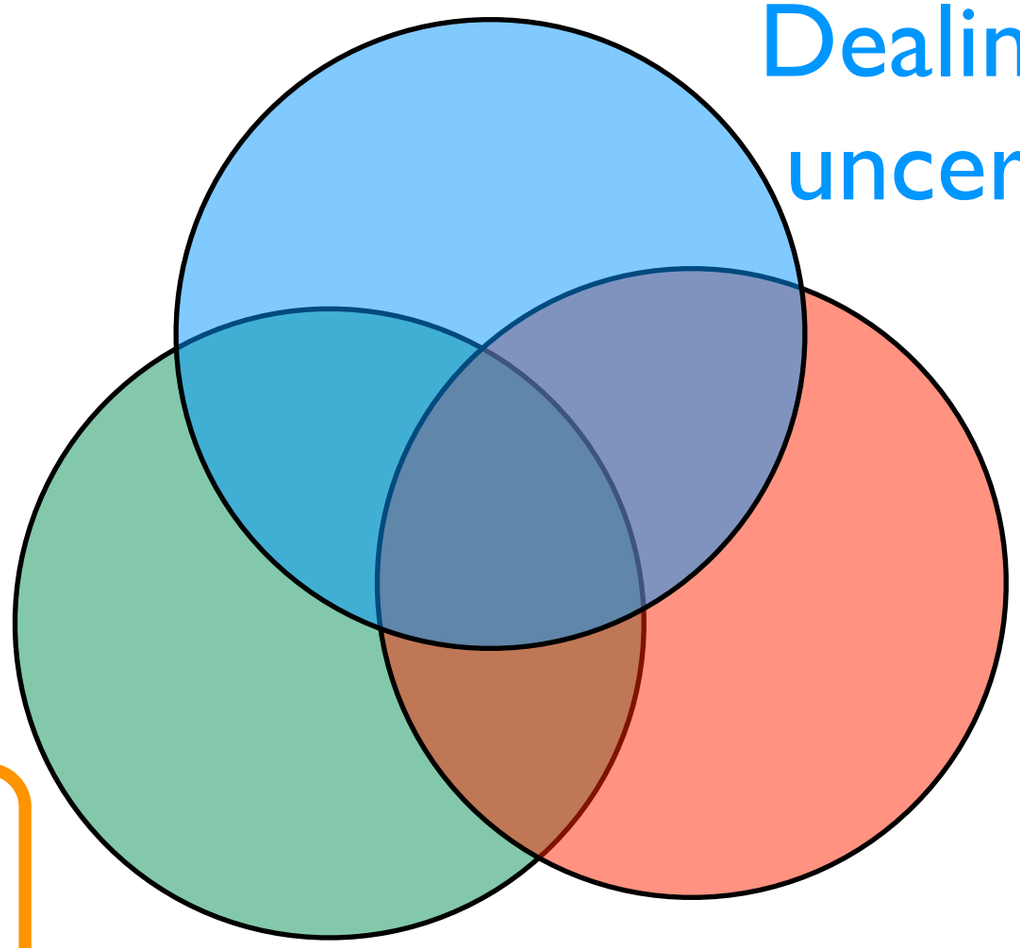


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one world

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ProbLog

probabilistic Prolog

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0.2::influences(bob,carl).
```
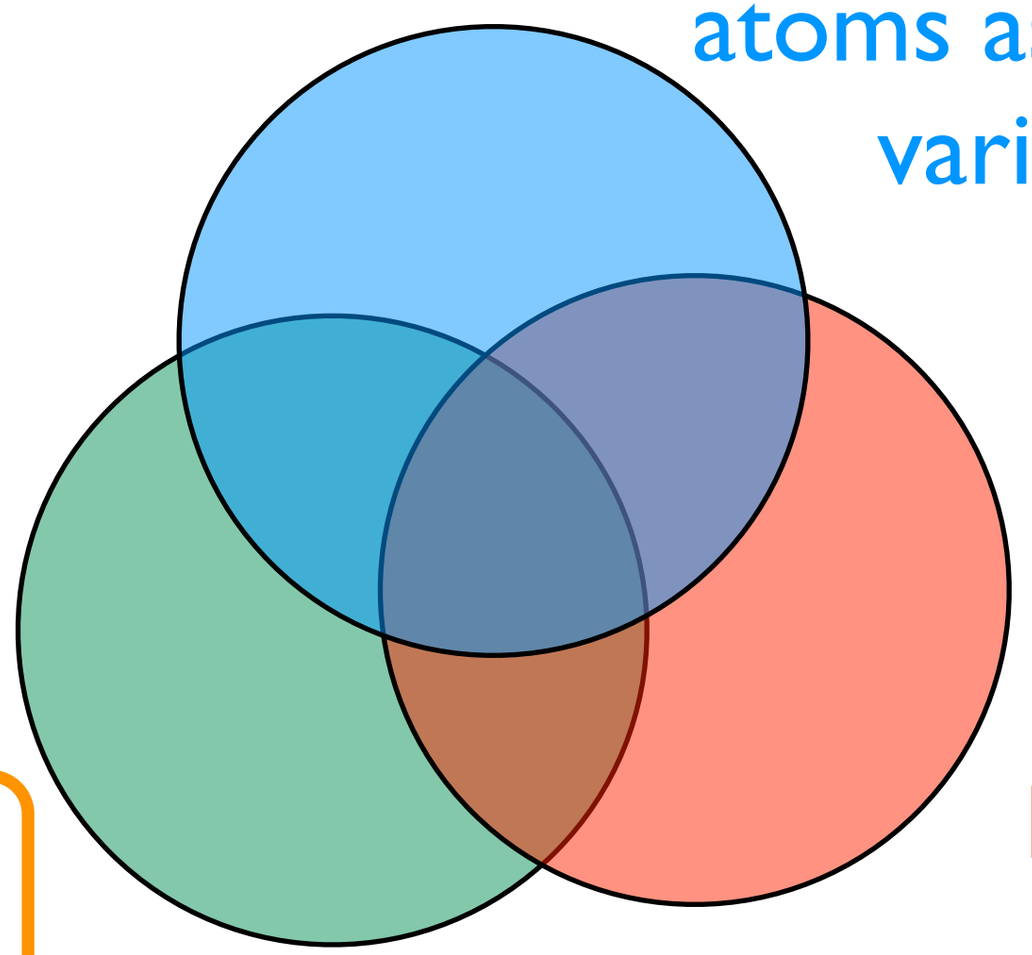

atoms as random variables



Prolog / logic programming

Learning

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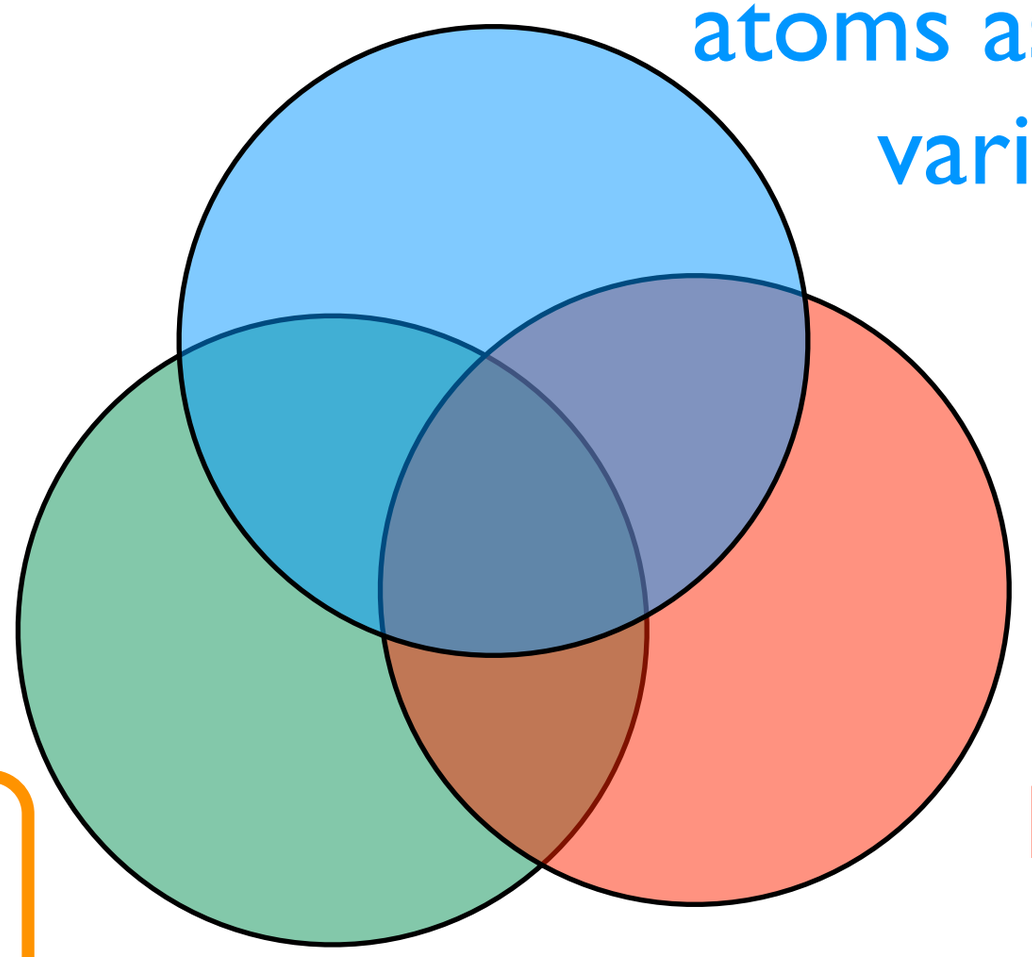

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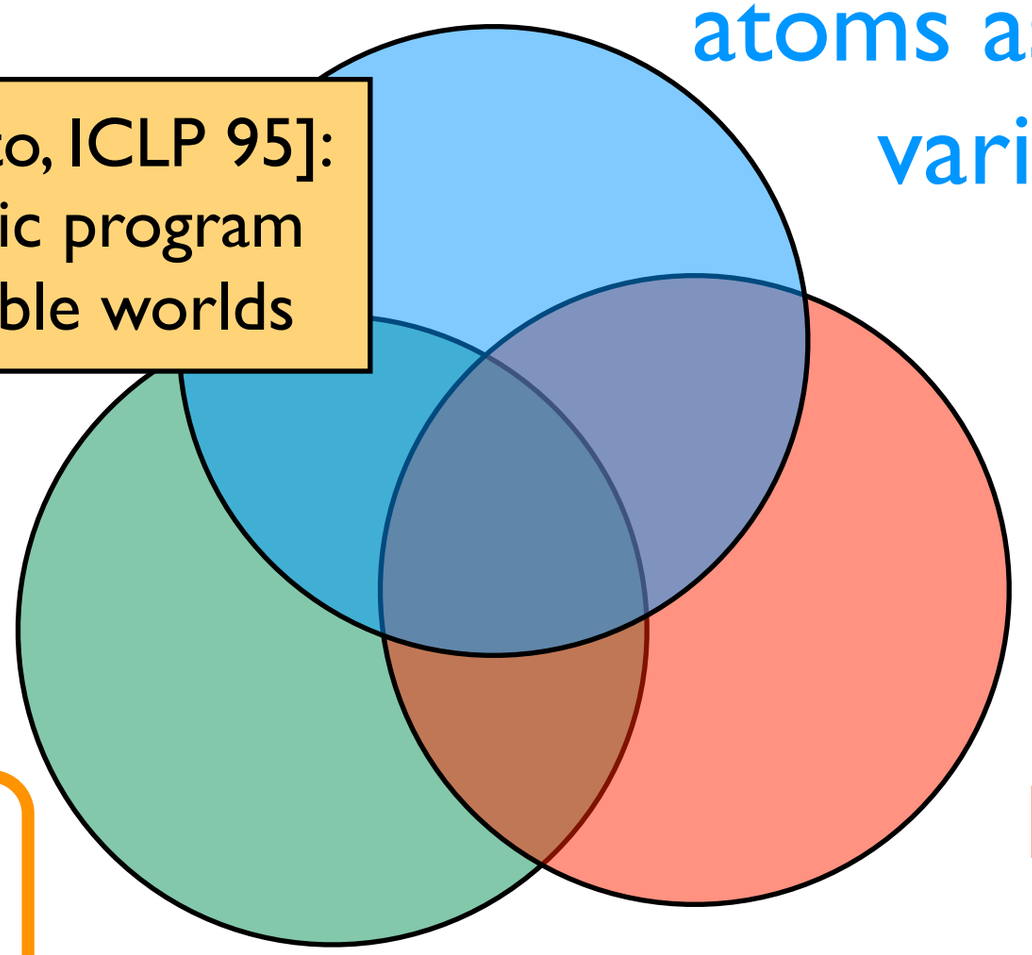
Distribution Semantics [Sato, ICLP 95]:
probabilistic choices + logic program
→ distribution over possible worlds



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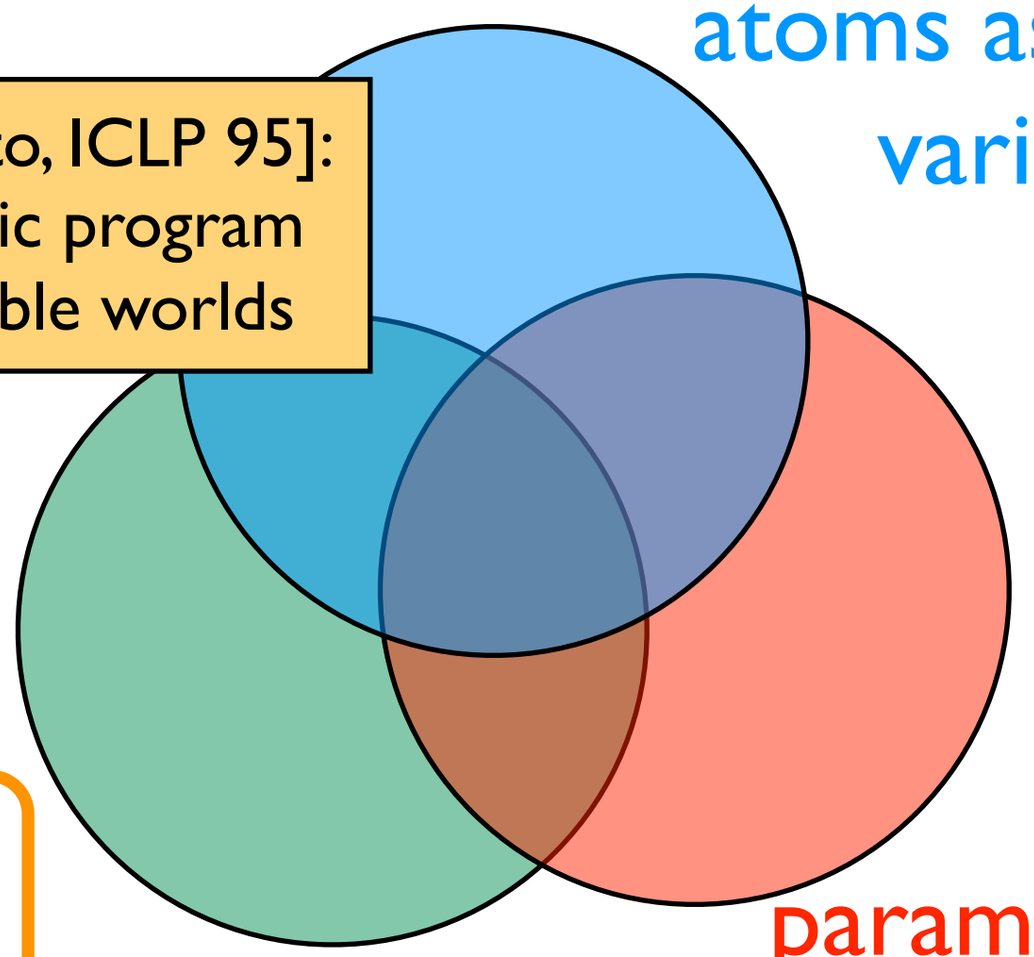

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parameter learning, adapted relational learning techniques

http://dtai.cs.kuleuven.be/problog/

Probabilistic Prologs: Two Views

- Distribution semantics:
  - probability distribution over interpretations
  - degree of belief
- Stochastic Logic Programs (SLPs):
  - probability distribution over query answers
  - like in probabilistic grammars

Graphs & Randomness

ProbLog, Phenetic, Prism, ICL, Probabilistic Databases, ...

- all based on a "random graph" model

Stochastic Logic Programs, ProPPR, PCFGs, ...

- based on a "random walk" model
- connected to PageRank

Probabilistic Logic Programming

Distribution Semantics [Sato, ICLP 95]:
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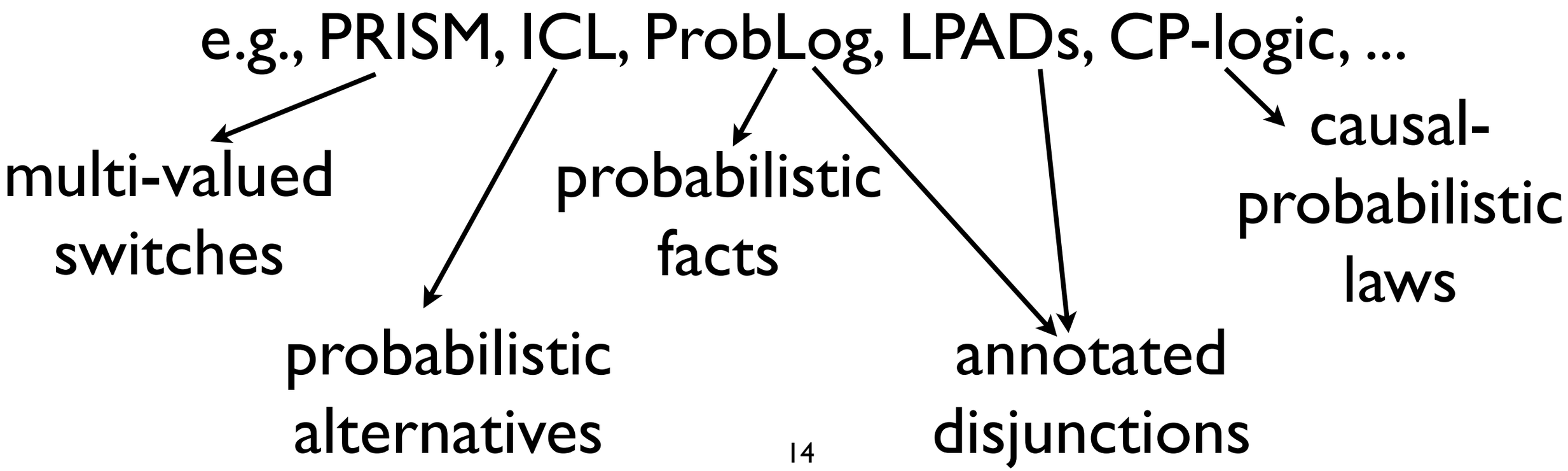
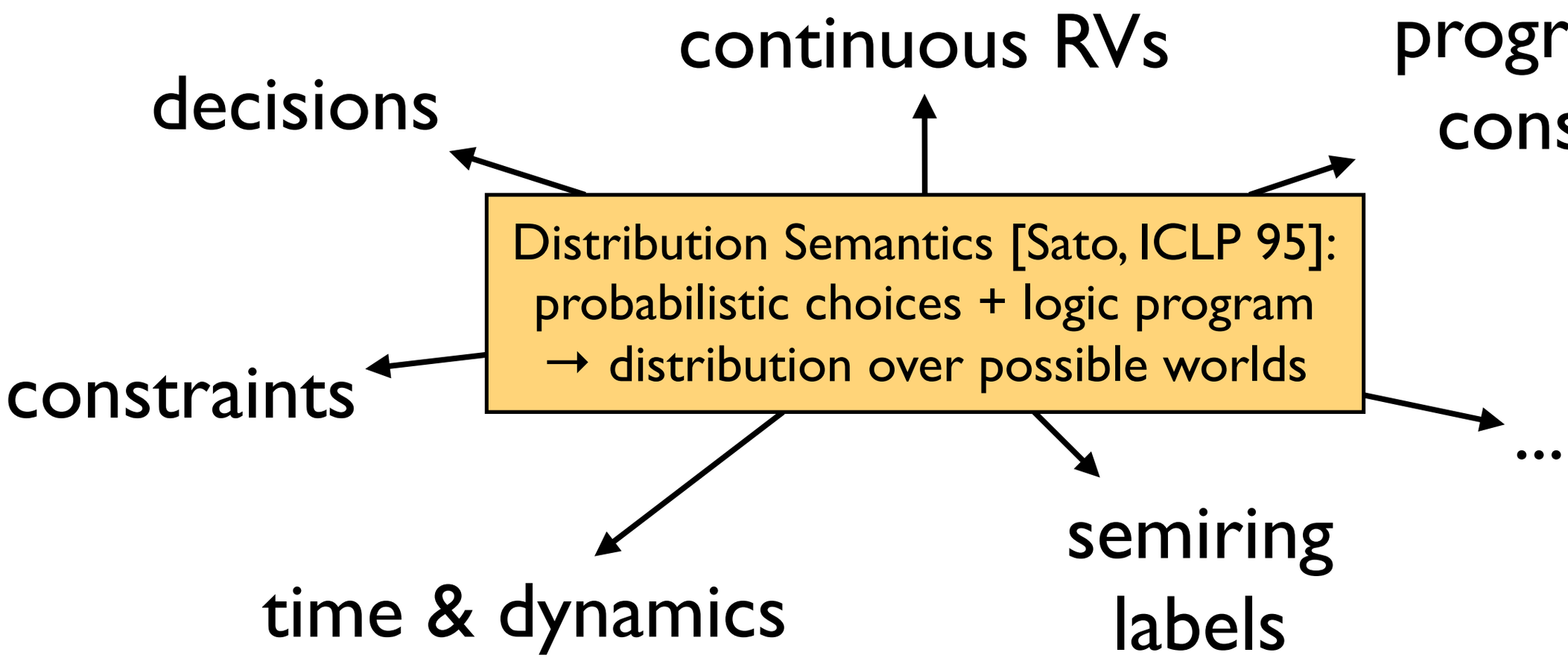


Extensions of basic PLP



Roadmap

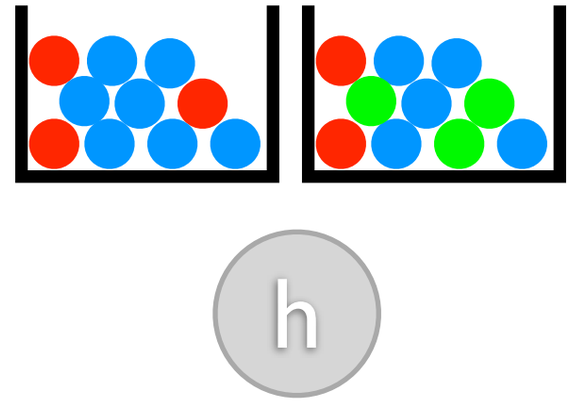
- Modeling
- Reasoning
- Language extensions
- Advanced topics

... with some detours on the way

Part 1 : Modeling

ProbLog by example:

A bit of gambling



- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

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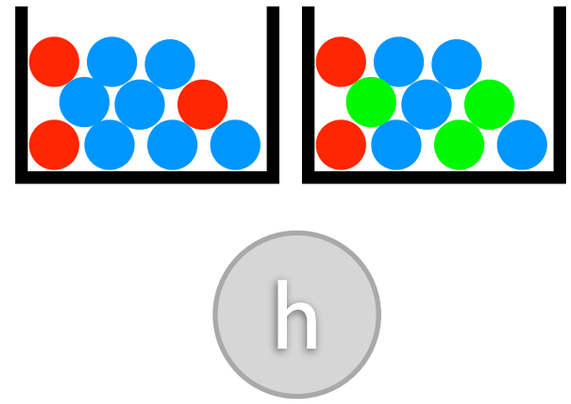

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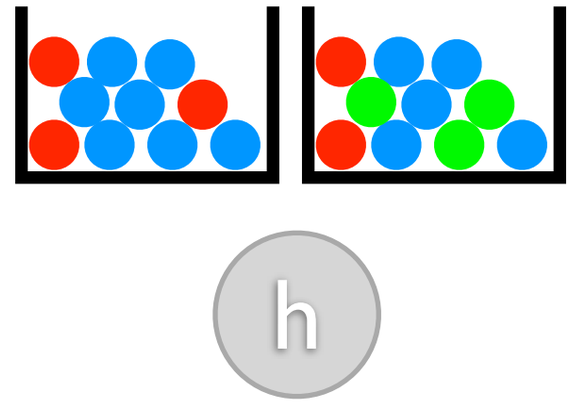
```
0.4 :: heads.
```

probabilistic fact: heads is true with probability 0.4 (and false with 0.6)

ProbLog by example:

A bit of gambling



- toss (biased) coin & draw ball from each urn
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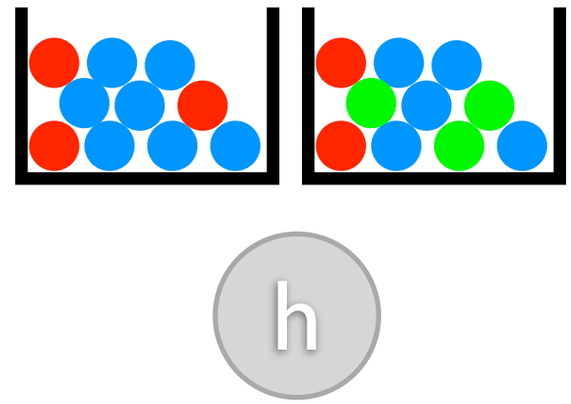
```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
```

annotated disjunction: first ball is red with probability 0.3 and blue with 0.7

ProbLog by example:

A bit of gambling



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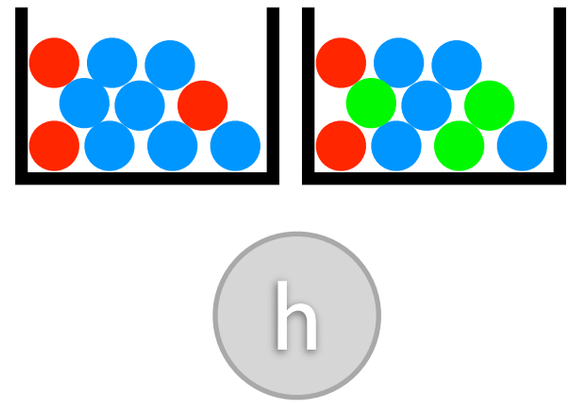
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green);
                   0.5 :: col(2,blue).
```

annotated disjunction: second ball is red with probability 0.2, green with 0.3, and blue with 0.5

ProbLog by example:

A bit of gambling



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win :- heads, col(_,red).
```
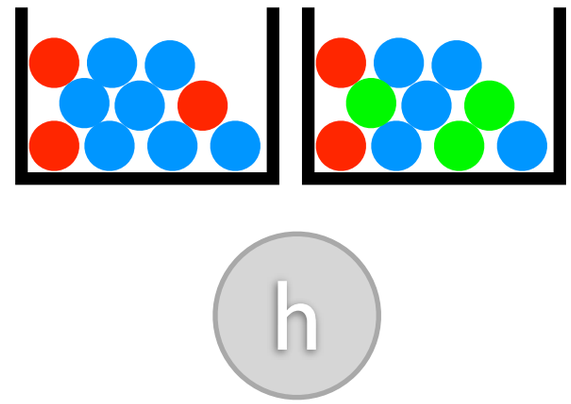

logical rule encoding background knowledge

ProbLog by example:

A bit of gambling



- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
0.2 :: col(2,red); 0.3 :: col(2,green);
                   0.5 :: col(2,blue).

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

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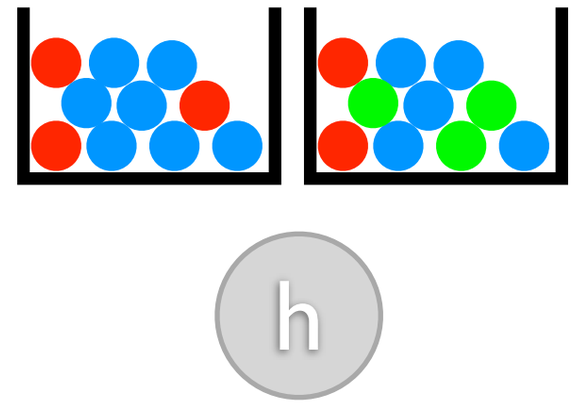

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```

probabilistic choices

```
win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

consequences

Questions

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
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win :- heads, col(_,red).
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```

- Probability of **`win`**?
- Probability of **`win`** given **`col(2,green)`**?
- Most probable world where **`win`** is true?

Questions

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0.4 :: heads.

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```

marginal probability

- Probability of `win`?

query

- Probability of `win` given `col(2,green)`?
- Most probable world where `win` is true?

Questions

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0.4 :: heads.

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```

marginal probability

- Probability of **`win`**?

conditional probability

- Probability of **`win`** given **`col(2,green)`**?

evidence

- Most probable world where **`win`** is true?

Questions

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
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marginal probability

- Probability of **`win`**?

conditional probability

- Probability of **`win`** given **`col(2,green)`**?

- Most probable world where **`win`** is true?

MPE inference

Possible Worlds

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue).
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0.4



Possible Worlds

```
0.4 :: heads.

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win :- heads, col(_,red).
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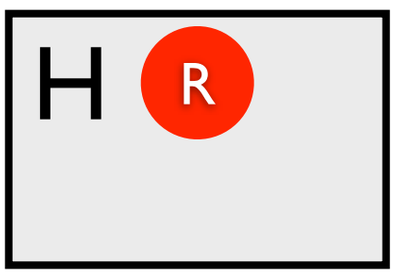
```

0.4 ×0.3



Possible Worlds

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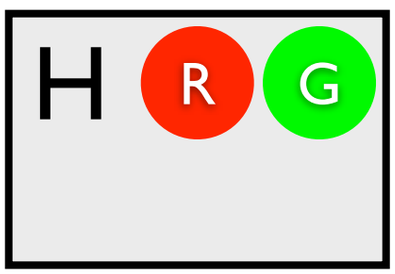

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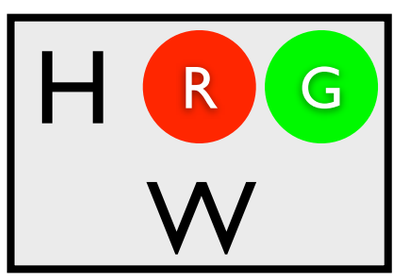

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```
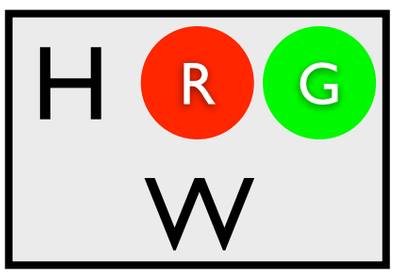

$0.4 \times 0.3 \times 0.3$



$(1-0.4)$



Possible Worlds

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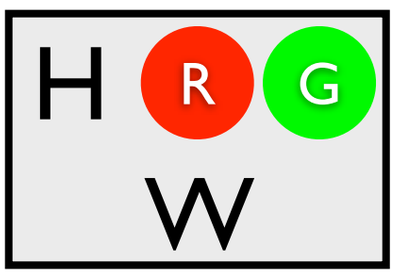
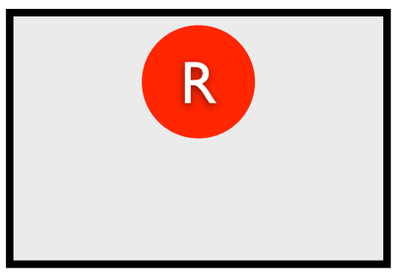
0.4 ×0.3 ×0.3



(1−0.4)×0.3



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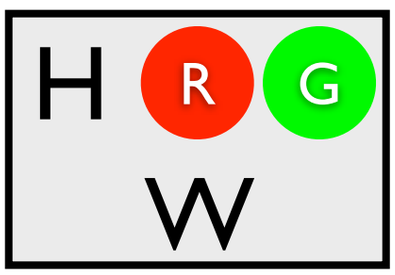
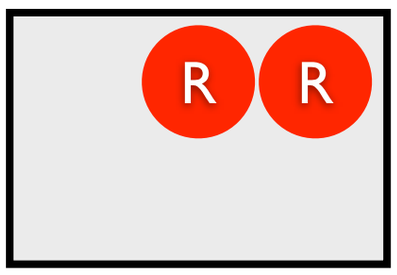
```

$0.4 \times 0.3 \times 0.3$



$(1-0.4) \times 0.3 \times 0.2$



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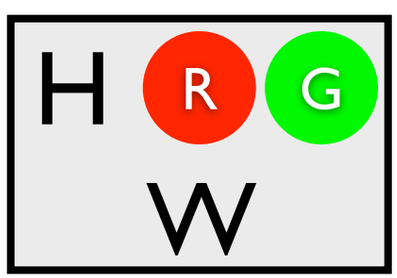
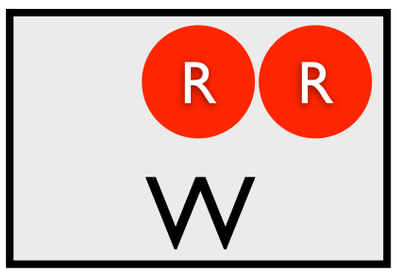


$(1-0.4) \times 0.3 \times 0.2$



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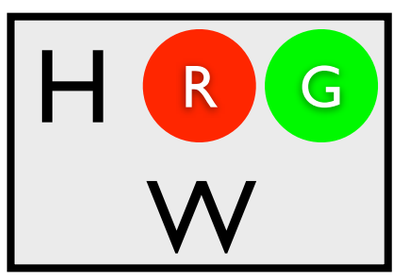
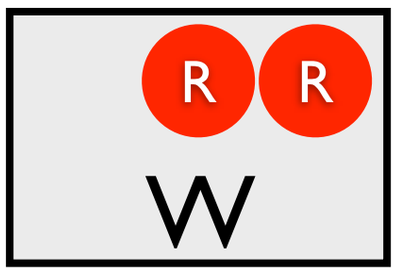


$(1-0.4) \times 0.3 \times 0.2$



$(1-0.4)$



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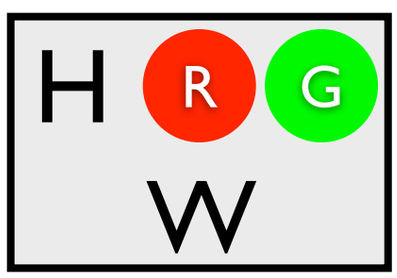
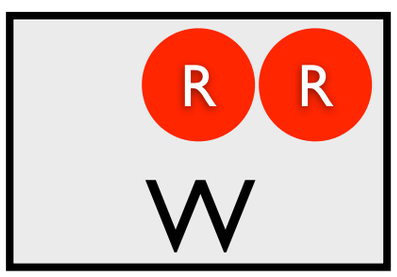


$(1-0.4) \times 0.3 \times 0.2$



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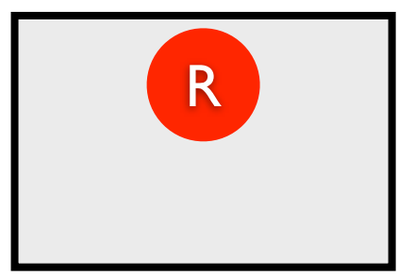

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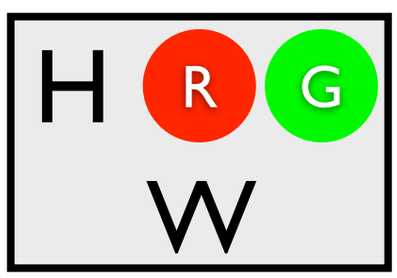
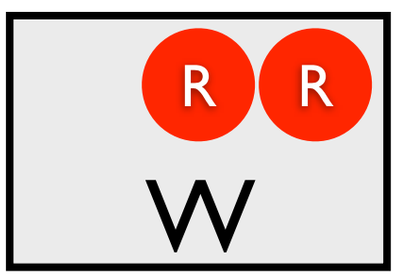


$(1-0.4) \times 0.3 \times 0.2$



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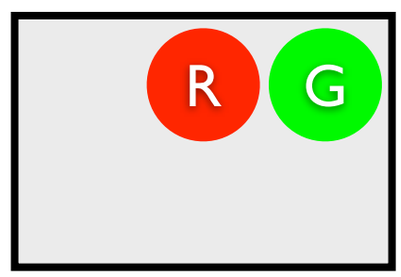

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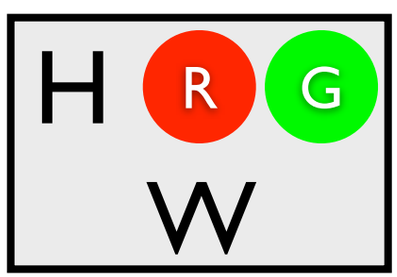
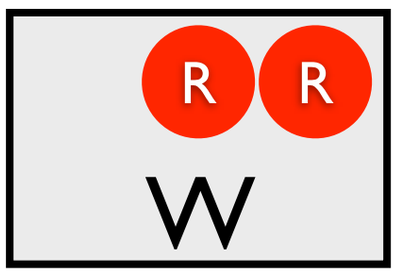
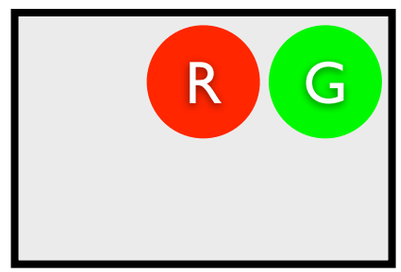


$(1-0.4) \times 0.3 \times 0.2$



$(1-0.4) \times 0.3 \times 0.3$



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```

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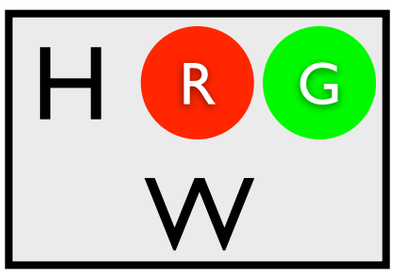
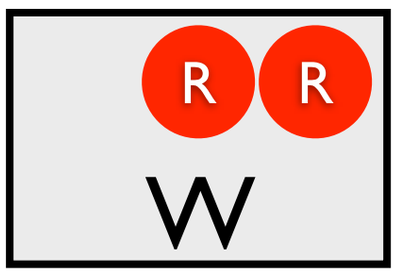
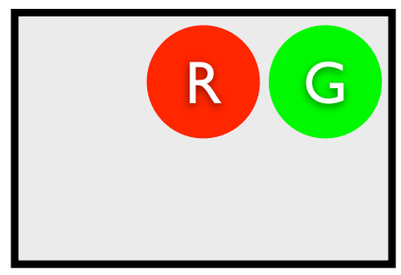


$(1-0.4) \times 0.3 \times 0.2$

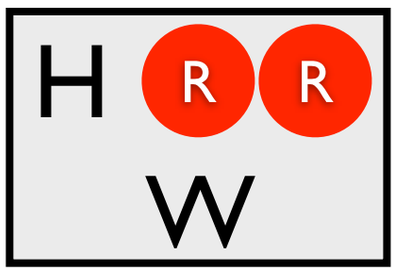


$(1-0.4) \times 0.3 \times 0.3$

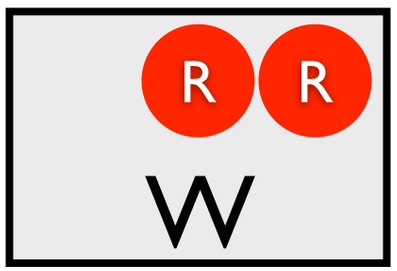


All Possible Worlds

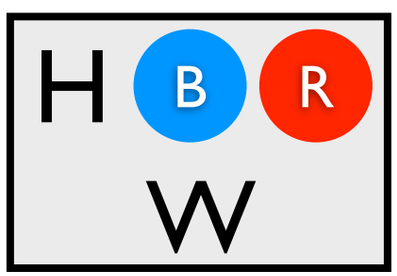
0.024



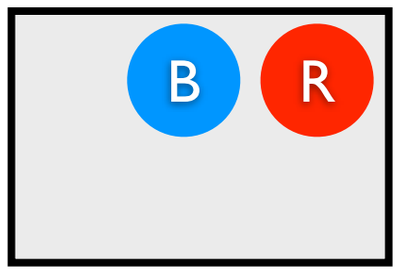
0.036



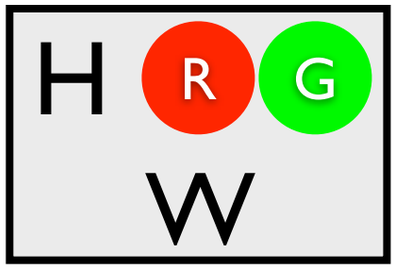
0.056



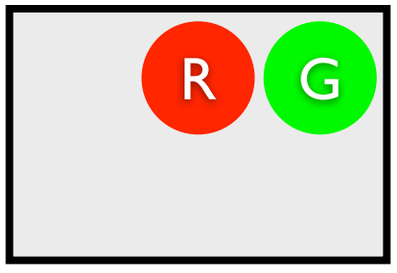
0.084



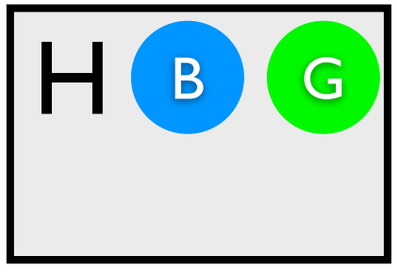
0.036



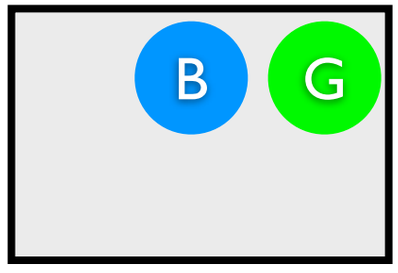
0.054



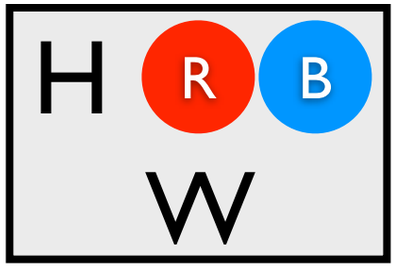
0.084



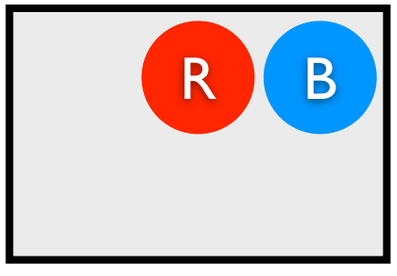
0.126



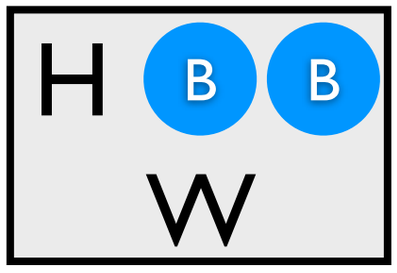
0.060



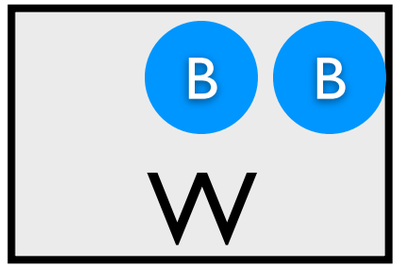
0.090



0.140

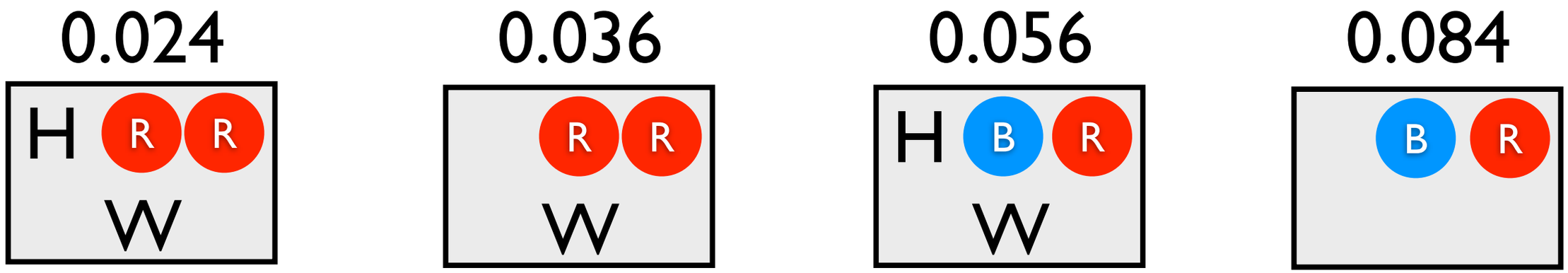


0.210



Most likely world where **win** is true?

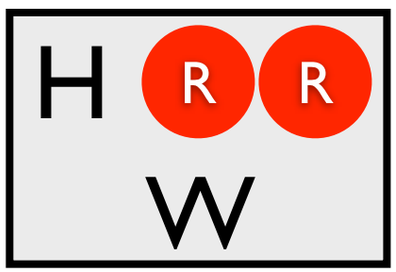
MPE Inference



Most likely world where **`win`** is true?

MPE Inference

0.024



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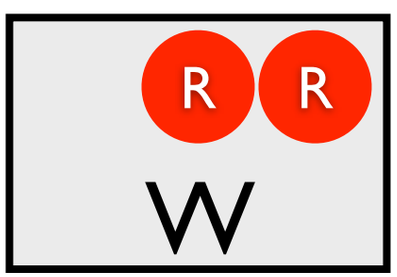

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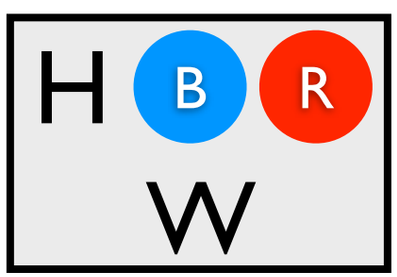

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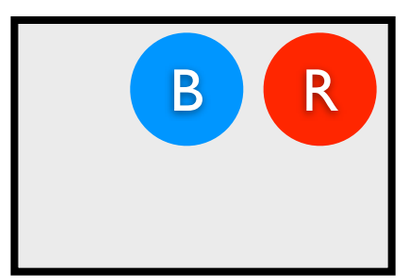

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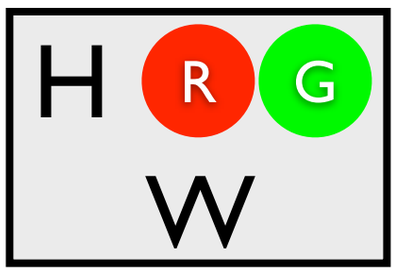

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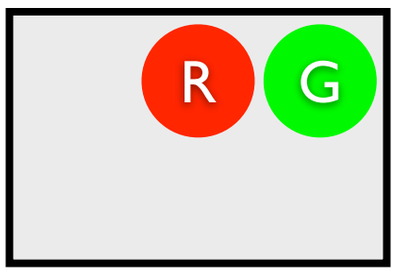

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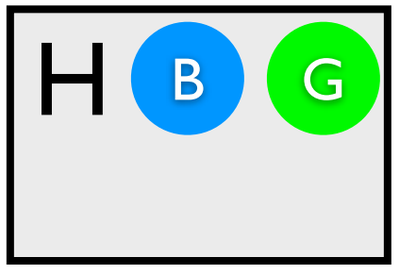

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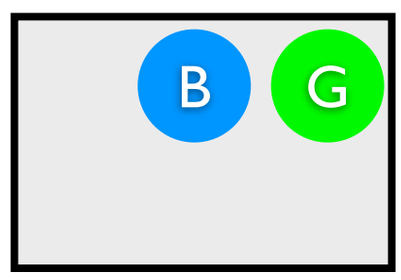

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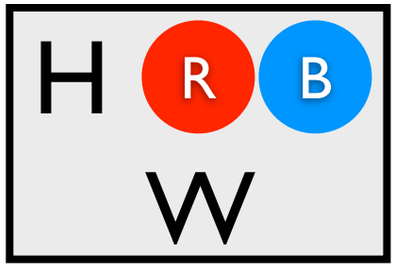

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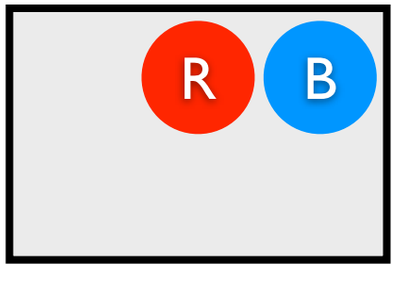

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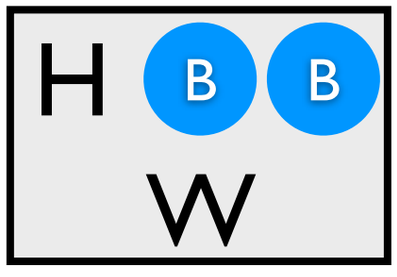

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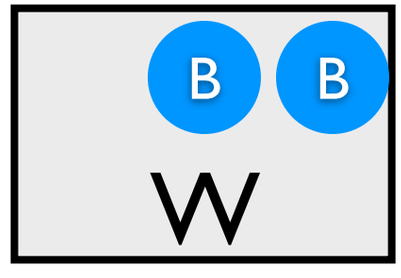
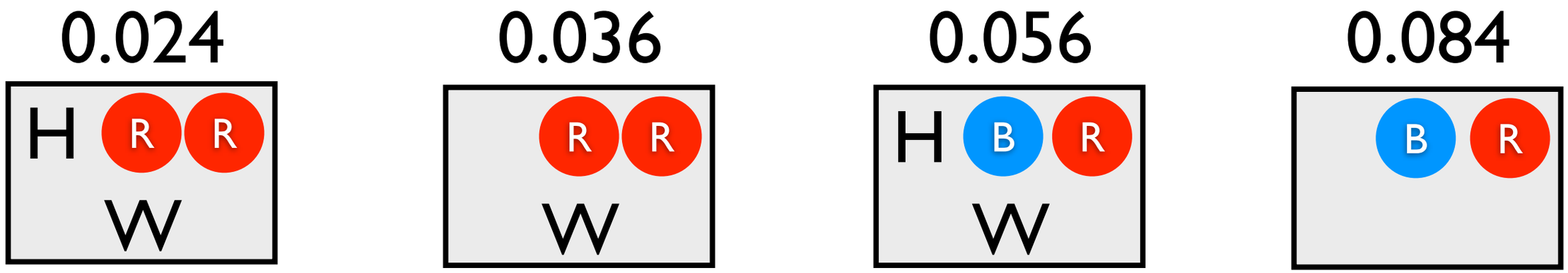


Most likely world where `col(2,blue)` is false?

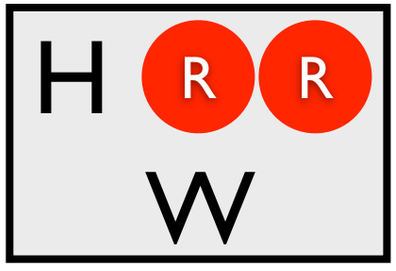
MPE Inference



Most likely world where `col(2,blue)` is false?

MPE Inference

0.024



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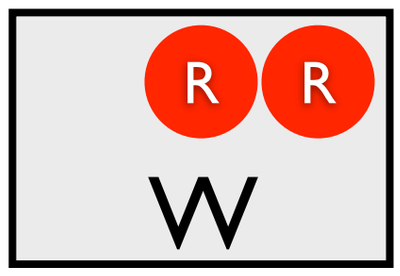

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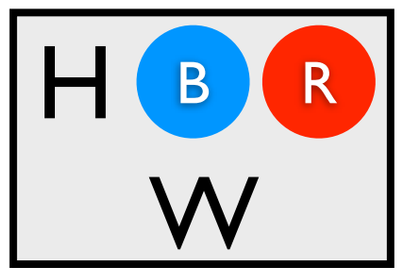

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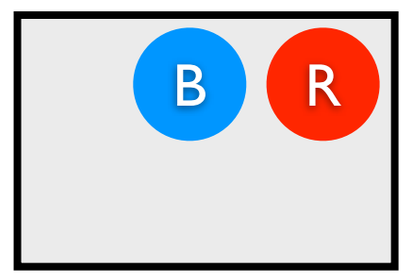

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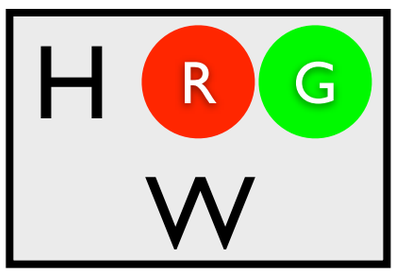

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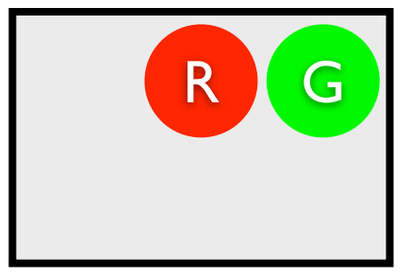

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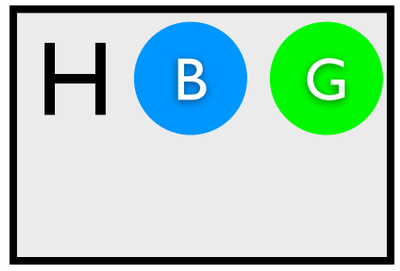

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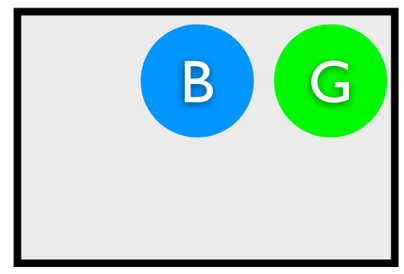

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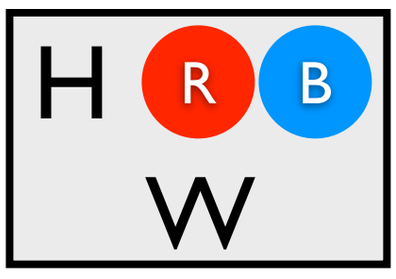

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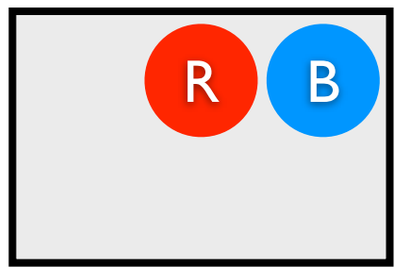

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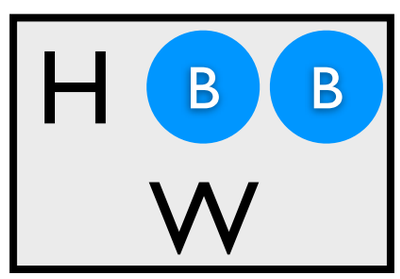

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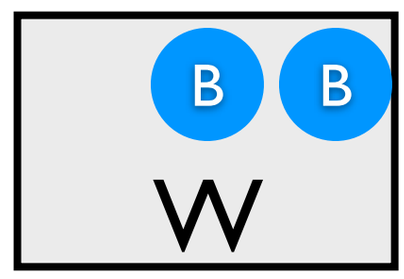
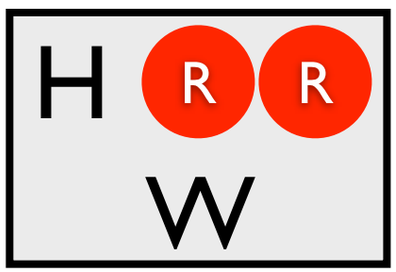


P(win)=?

Marginal Probability

0.024



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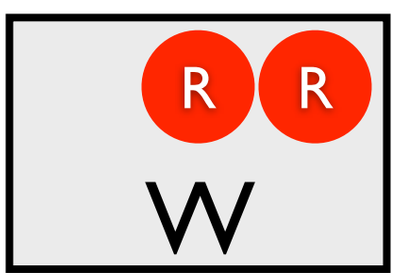

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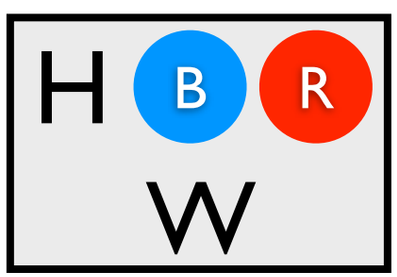

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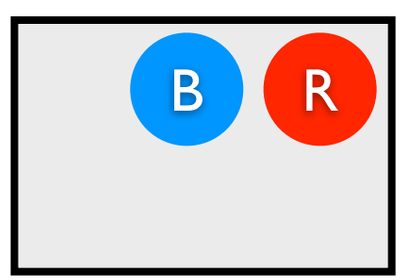

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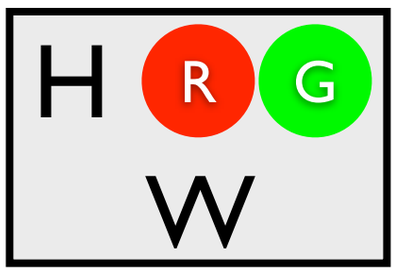

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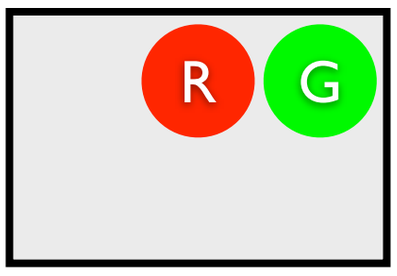

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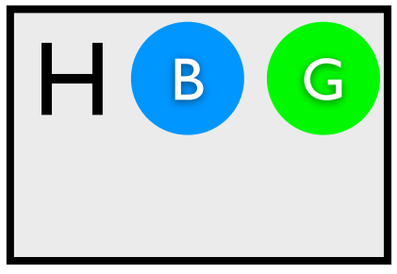

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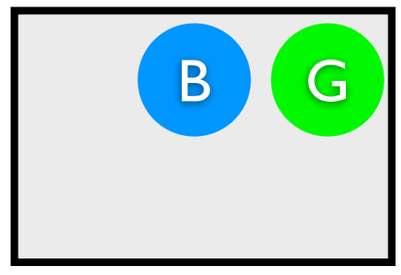

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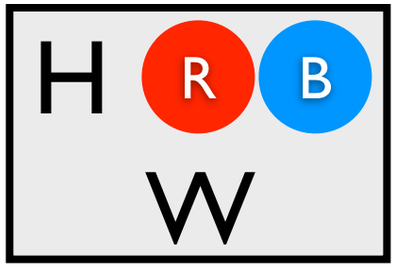

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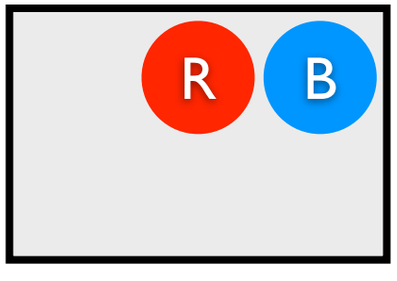

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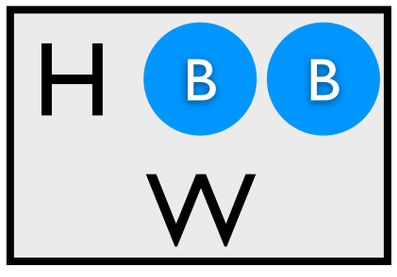

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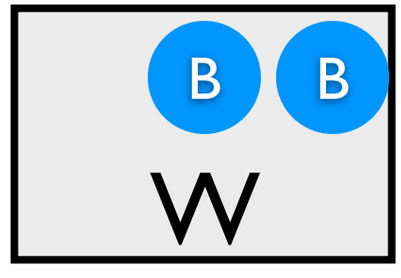
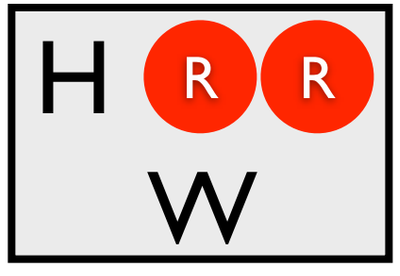


P(win)= Σ

Marginal Probability

0.024



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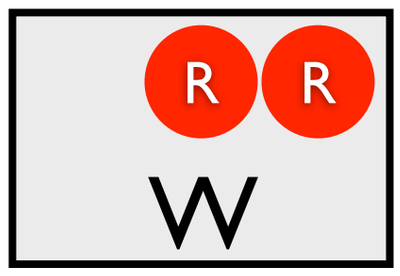

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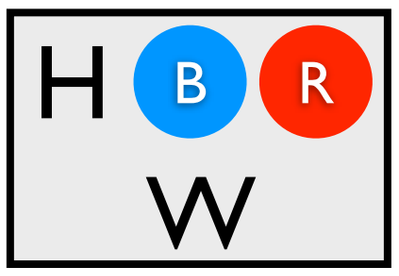

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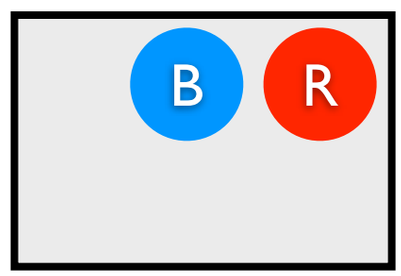

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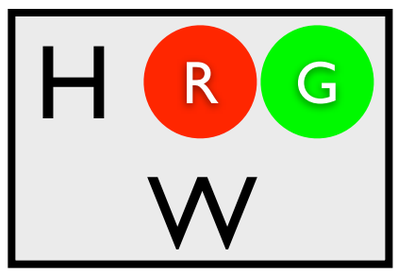

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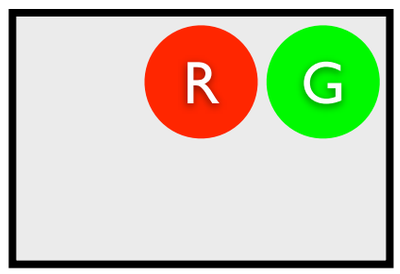

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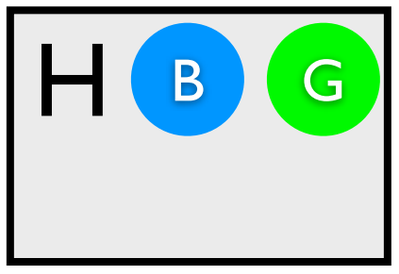

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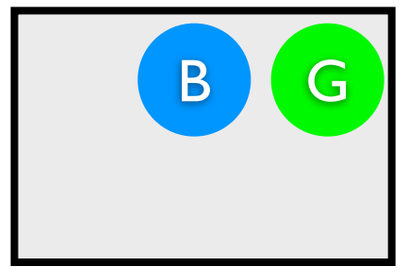

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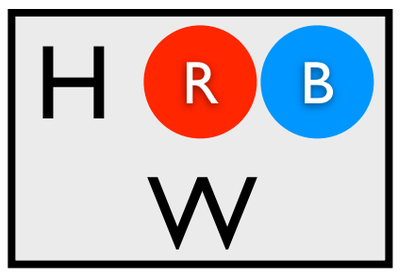

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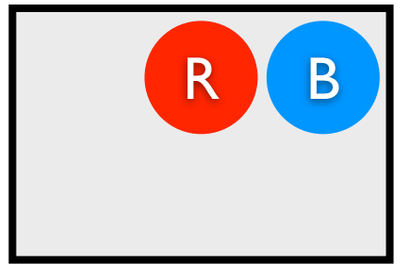

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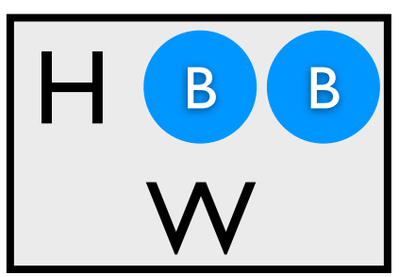

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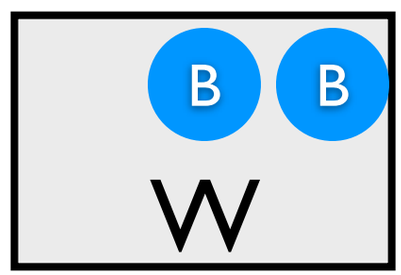
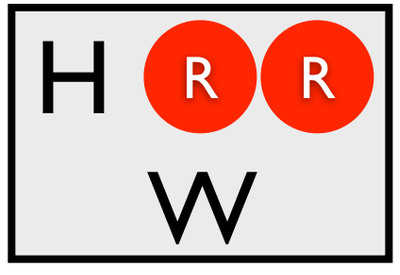


P(win)= Σ =0.562

Marginal Probability

0.024



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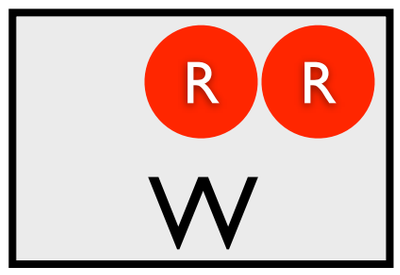

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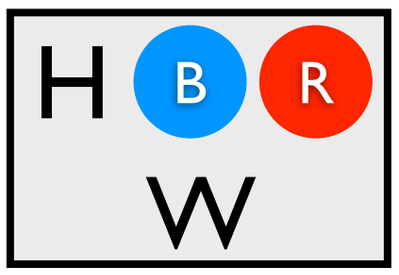

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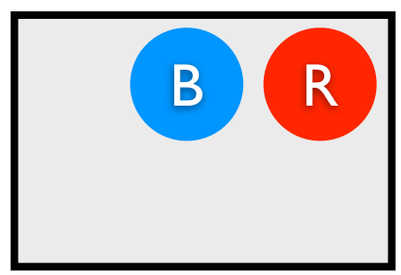

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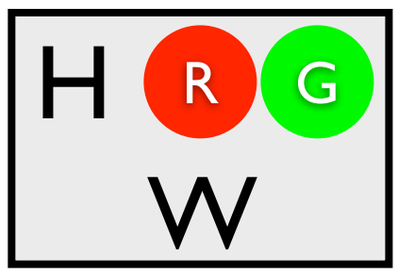

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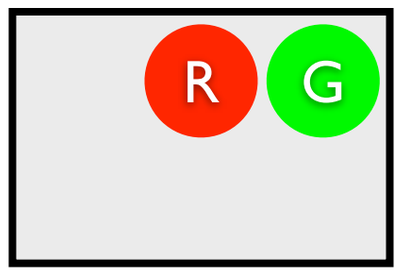

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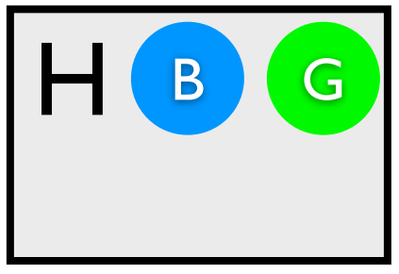

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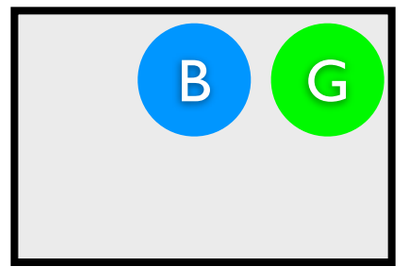

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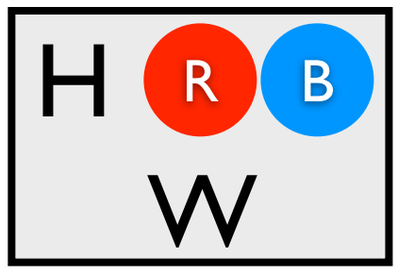

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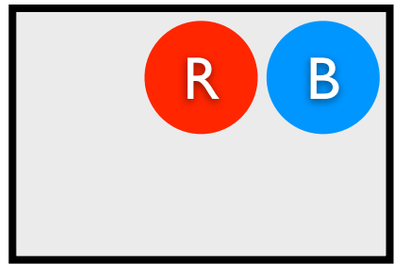

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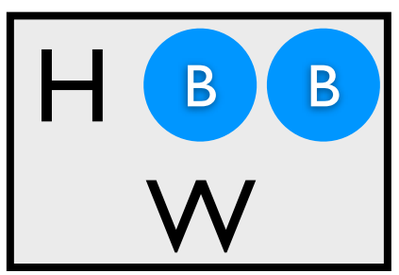
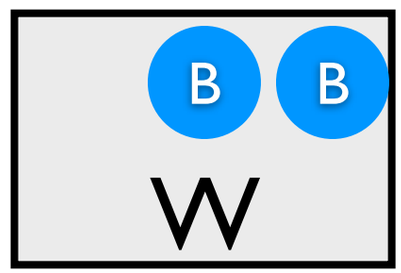


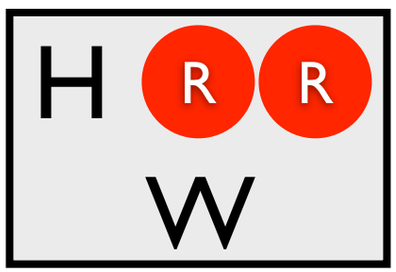
0.210



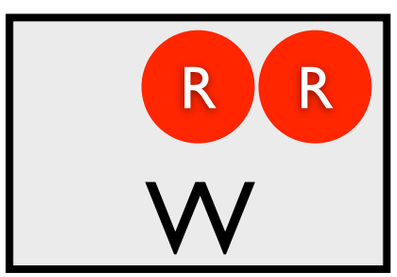
P(win|col(2,green))= ?

Conditional Probability

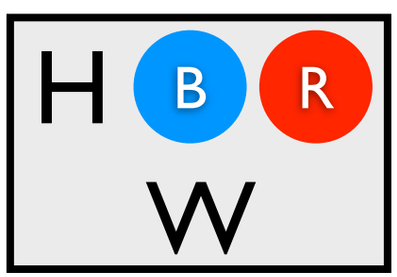
0.024



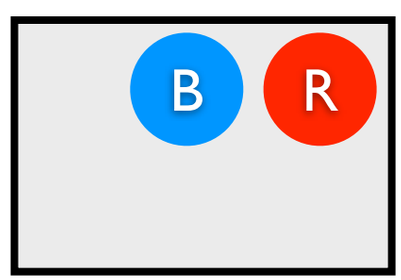
0.036



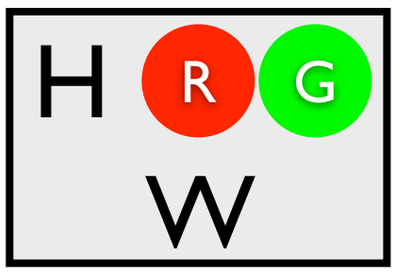
0.056



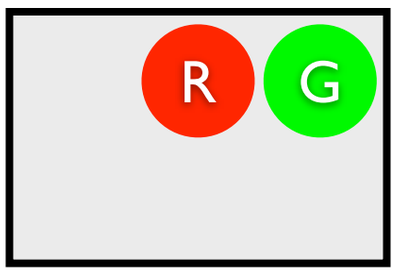
0.084



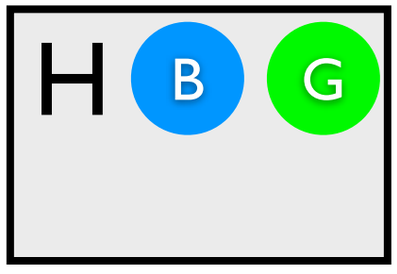
0.036



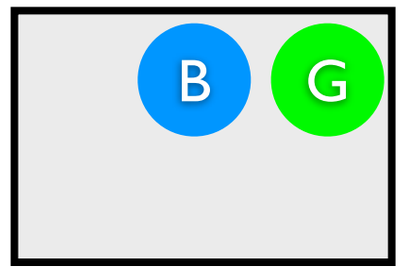
0.054



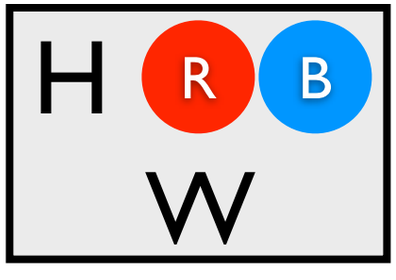
0.084



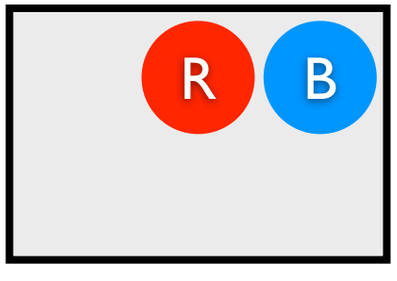
0.126



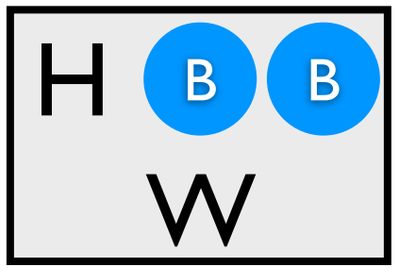
0.060



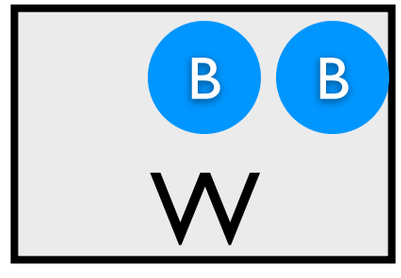
0.090



0.140



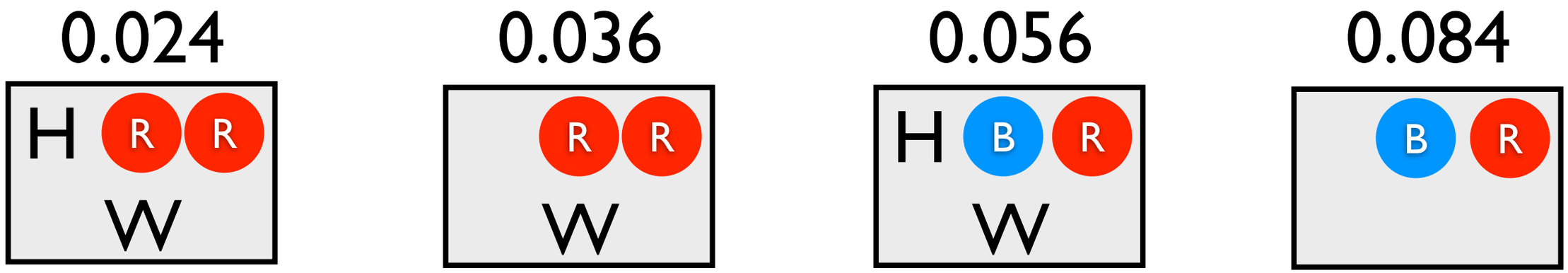
0.210



$$P(win|col(2,green)) = \Sigma/\Sigma$$

$$= P(win \wedge col(2,green))/P(col(2,green))$$

Conditional Probability



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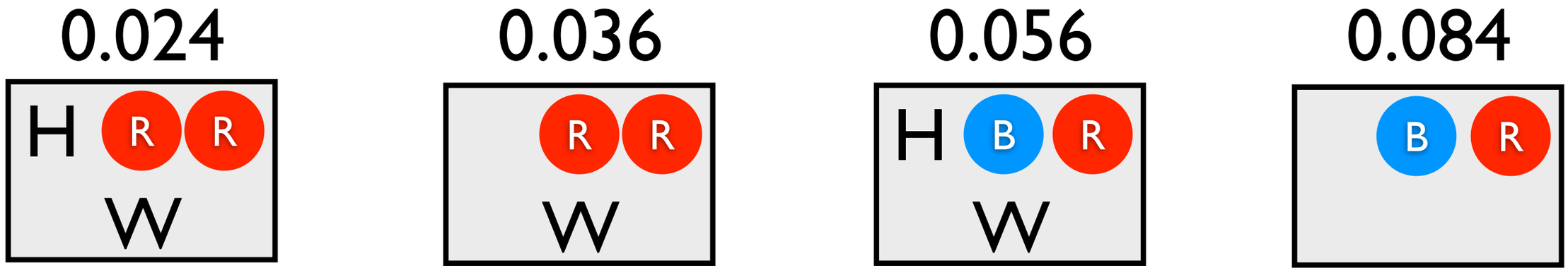
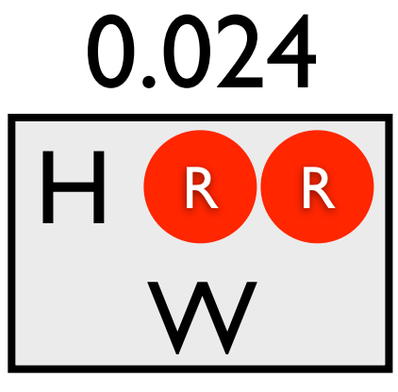
Conditional Probability

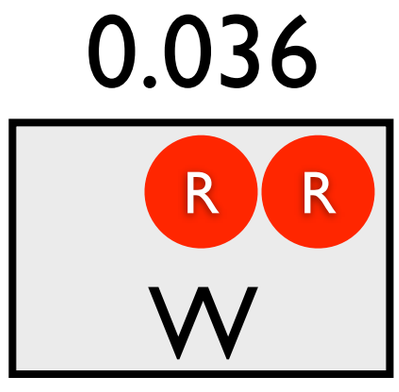


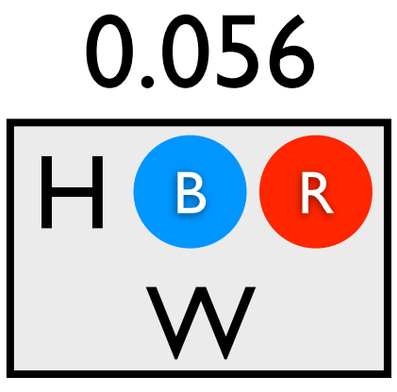
$$P(win|col(2,green)) = \Sigma/\Sigma$$

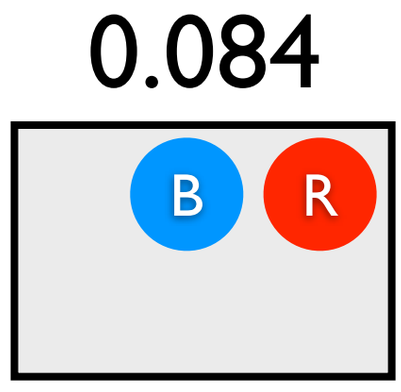
$$= 0.036/0.3 = 0.12$$

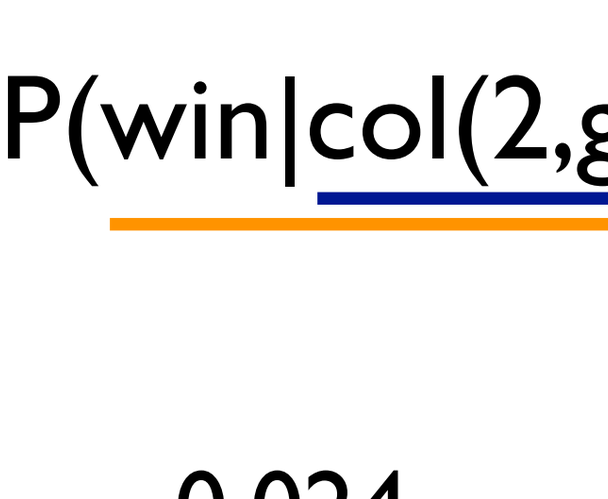
Conditional Probability

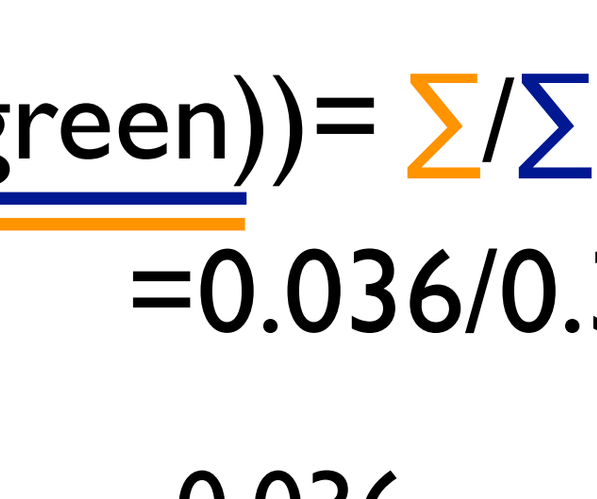


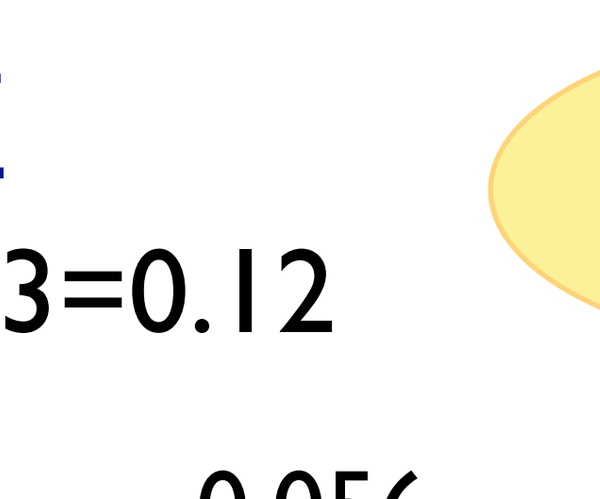


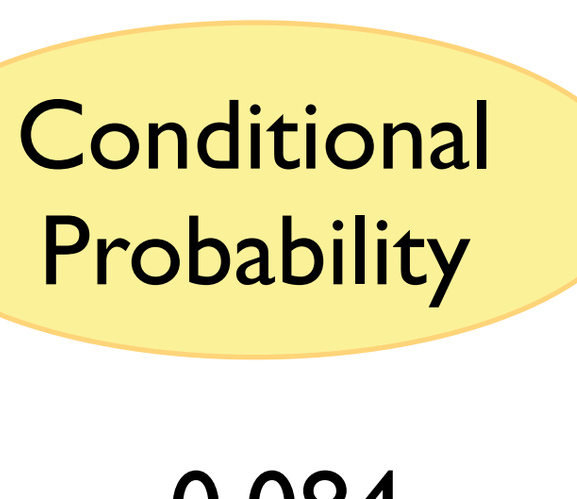


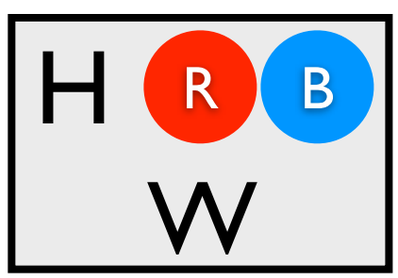


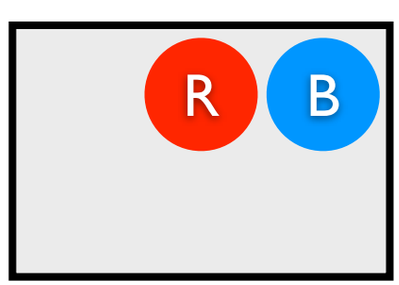


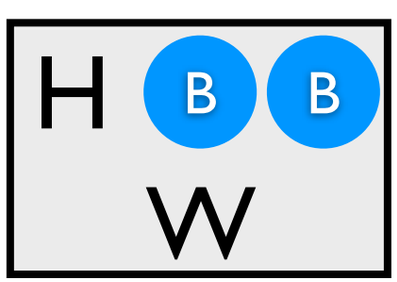


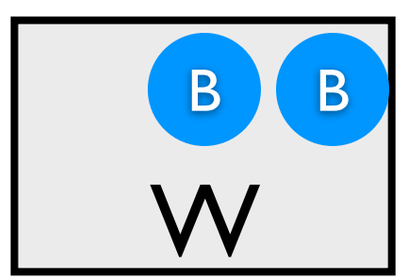












cProbLog: constraints on possible worlds

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :-
  weight(Item,Weight),
  P is 1.0/Weight.

excess(Limit) :- ...

not excess(10).
pack(helmet) v pack(boots).
```


[Fierens et al, PP 12; Shterionov et al]

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distribution over **all** possible worlds

[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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distribution over **all** possible worlds

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```

sbhg e(10)	sb g e(10)	sbh e(10)	sb
s hg e(10)	s g	s h	s
bhg	b g	bh	b
hg	g	h	

[Fierens et al, PP 12; Shterionov et al]

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constraints

as FOL formulas

treat as evidence

sbhg e(10)	sb g e(10)	sbh e(10)	sb
s hg e(10)	s g	s h	s
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[Fierens et al, PP 12; Shterionov et al]

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[Fierens et al, PP 12; Shterionov et al]

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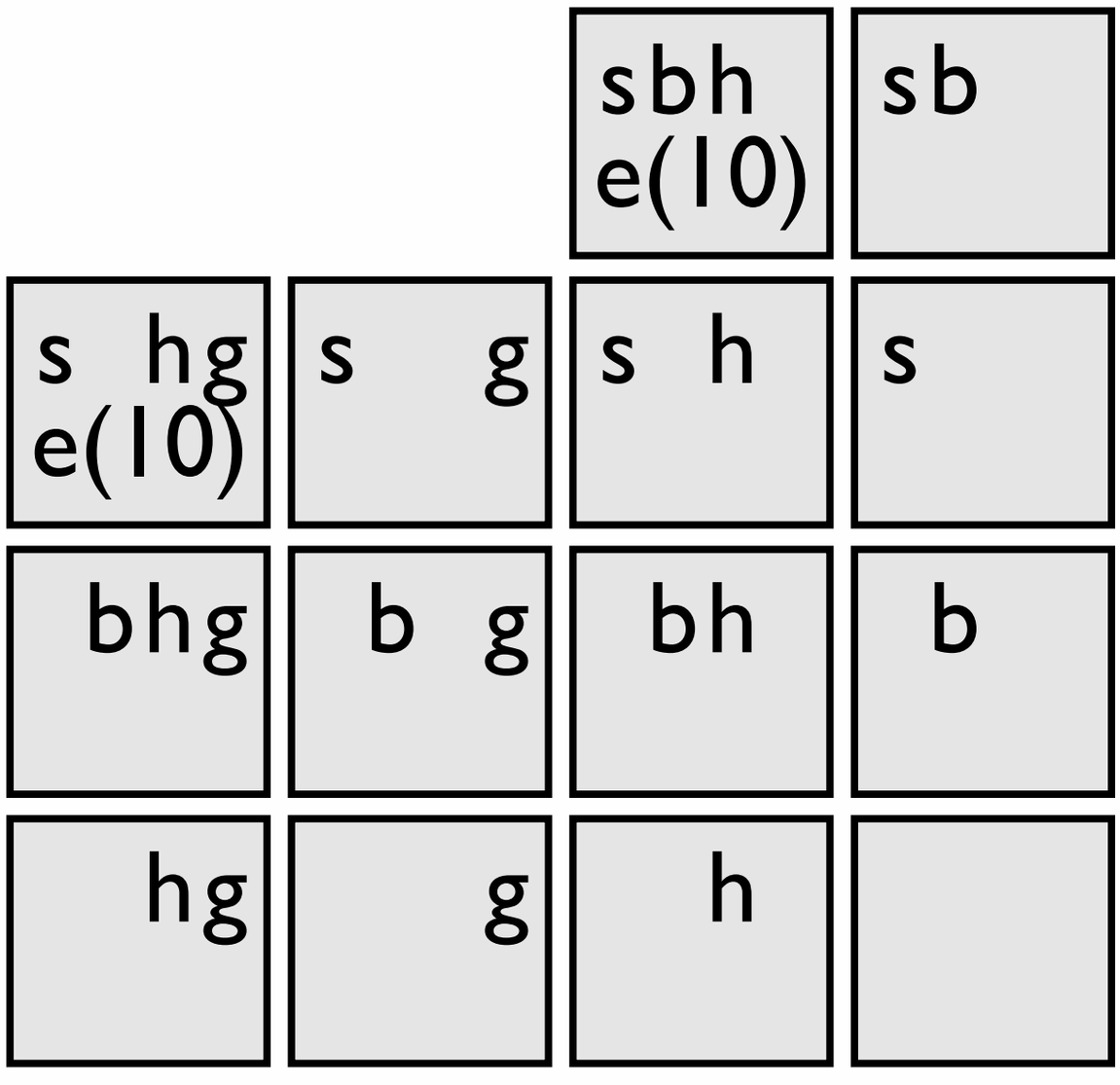

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sb			
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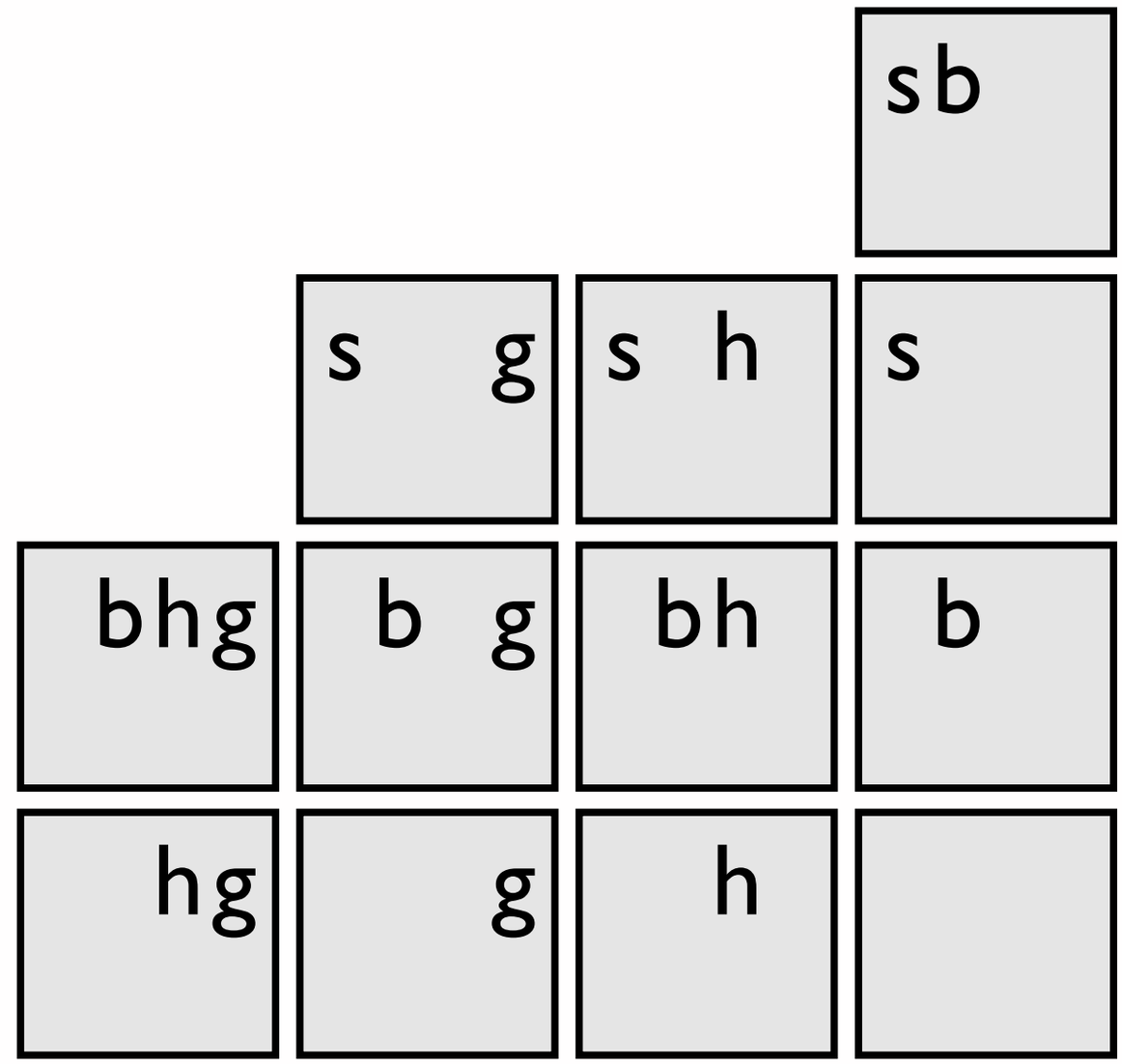

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[Fierens et al, PP 12; Shterionov et al]

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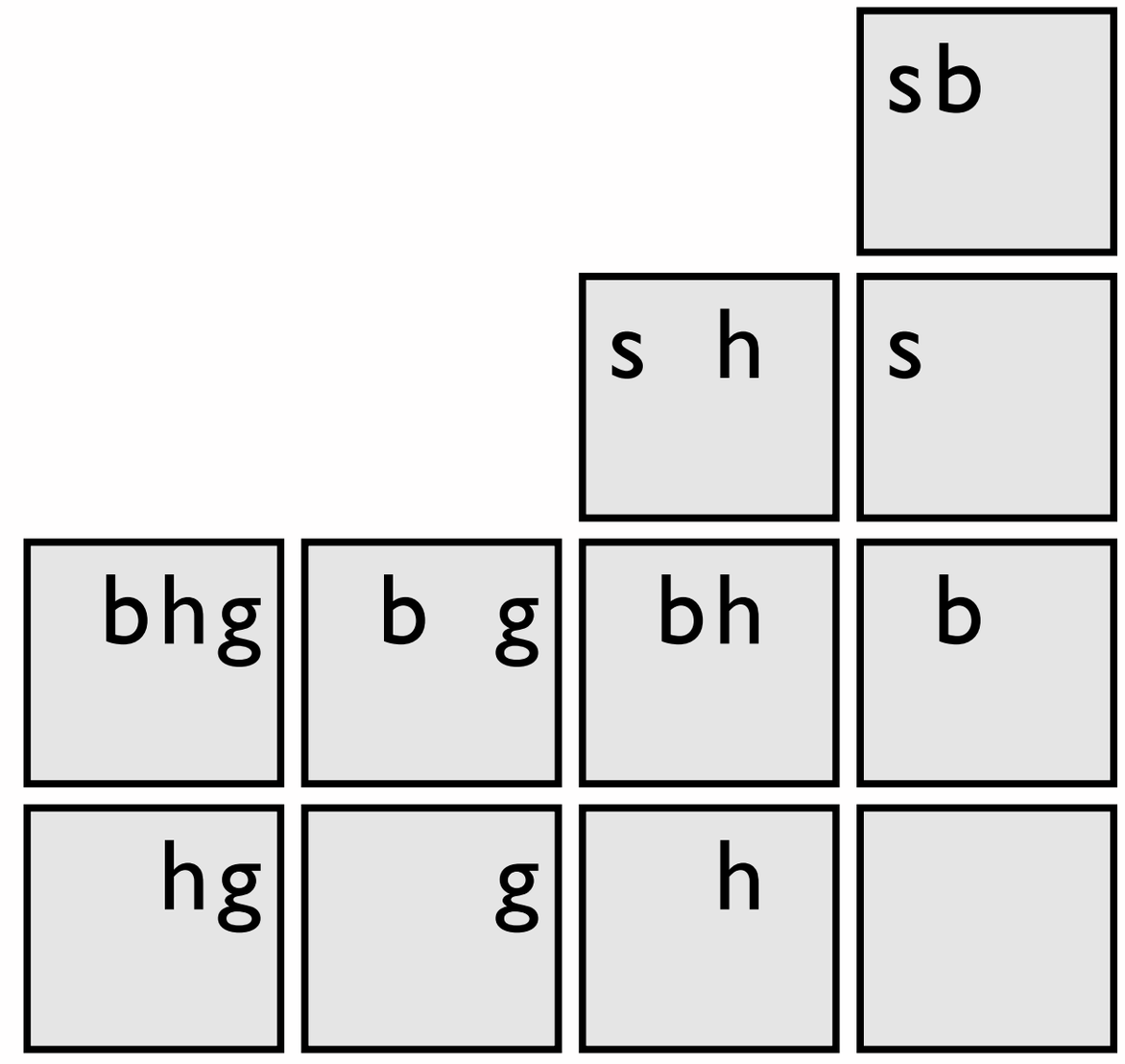

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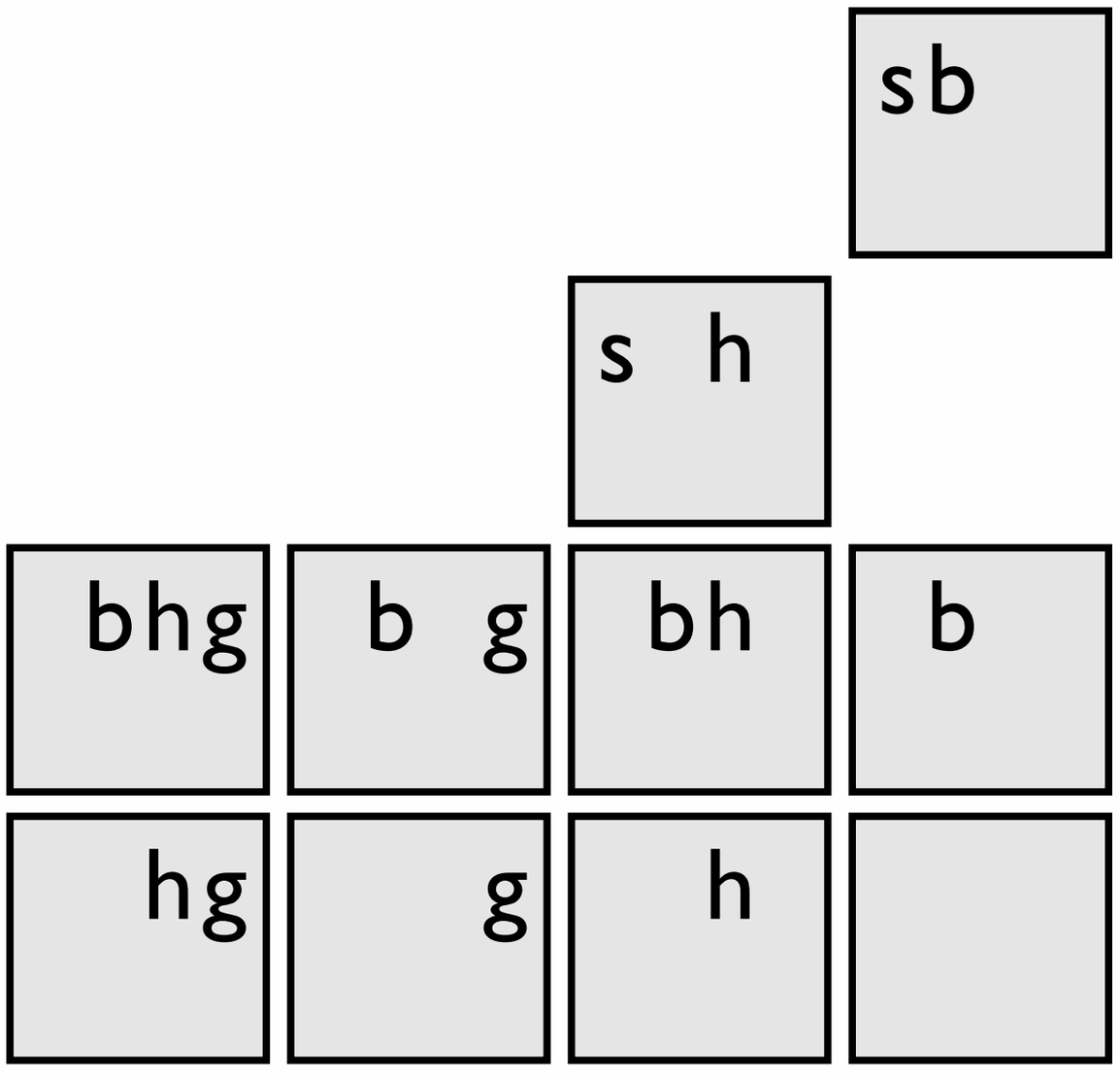

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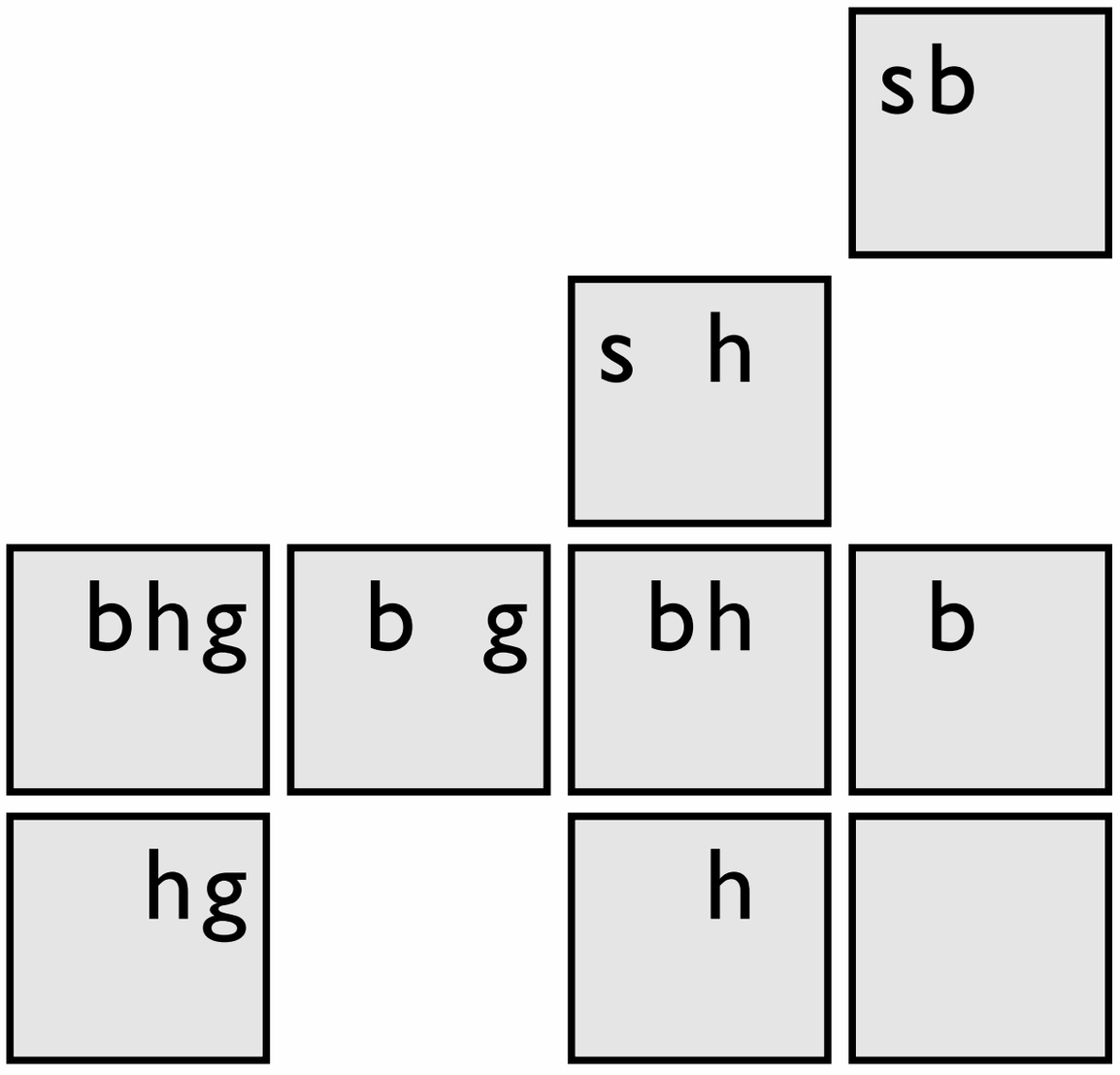

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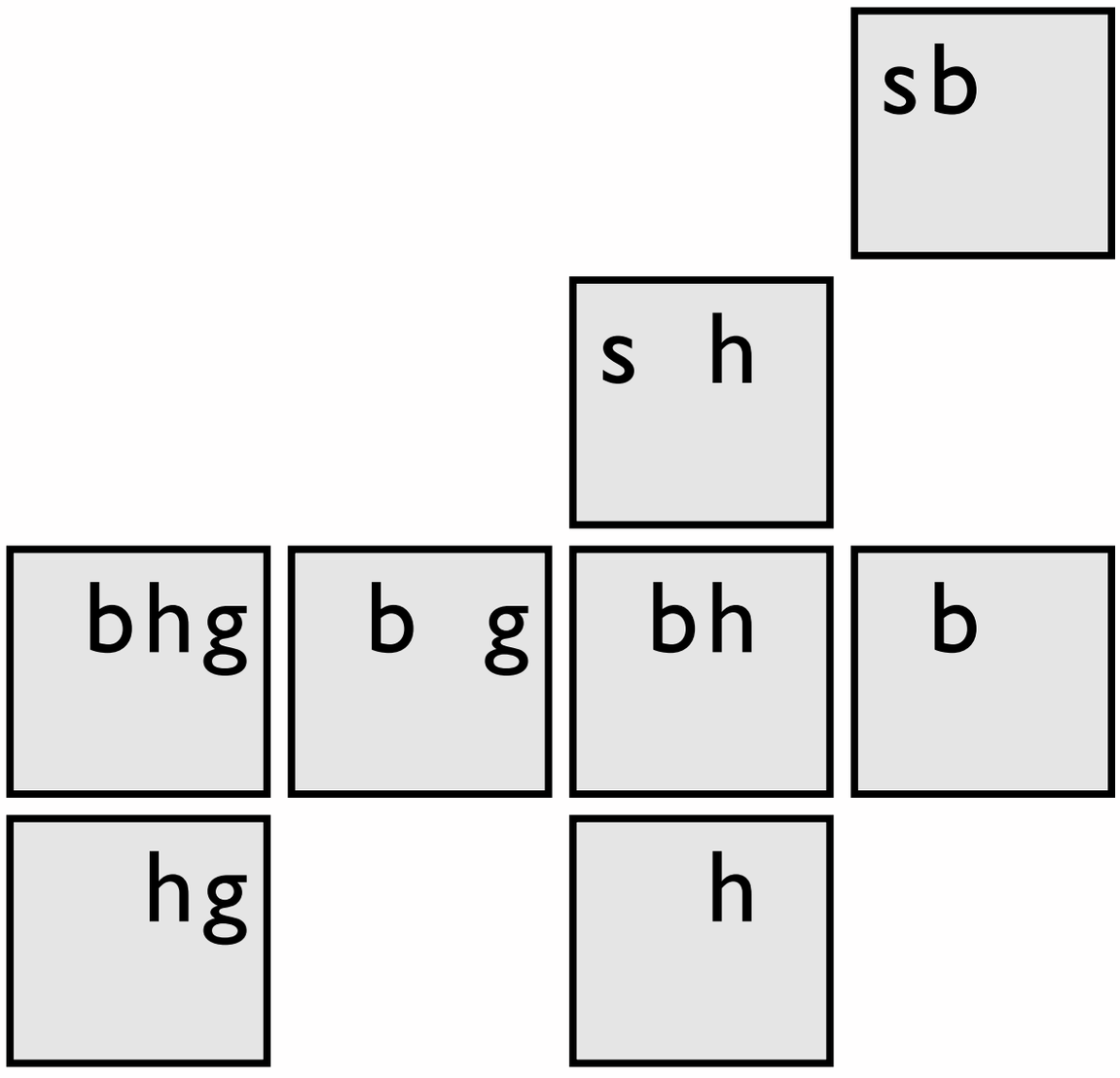

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```
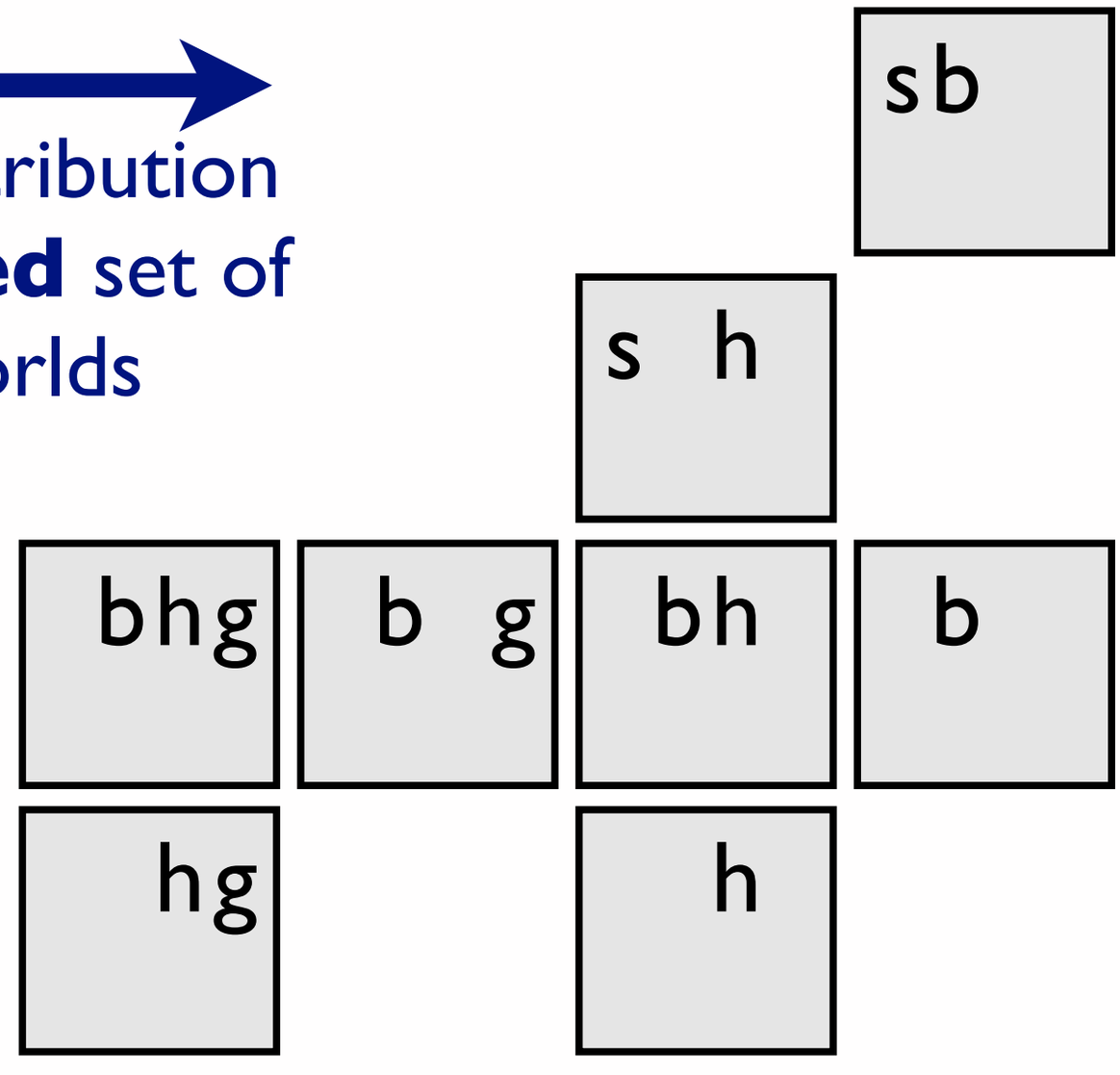

normalized distribution over **restricted** set of possible worlds



constraints

as FOL formulas

treat as evidence

[Fierens et al, PP 12; Shterionov et al]

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

Distribution Semantics

(with probabilistic facts)

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subset of probabilistic facts

Distribution Semantics

(with probabilistic facts)

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subset of probabilistic facts

Prolog rules

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

sum over possible worlds where Q is true

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Prolog rules

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$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Prolog rules

probability of possible world

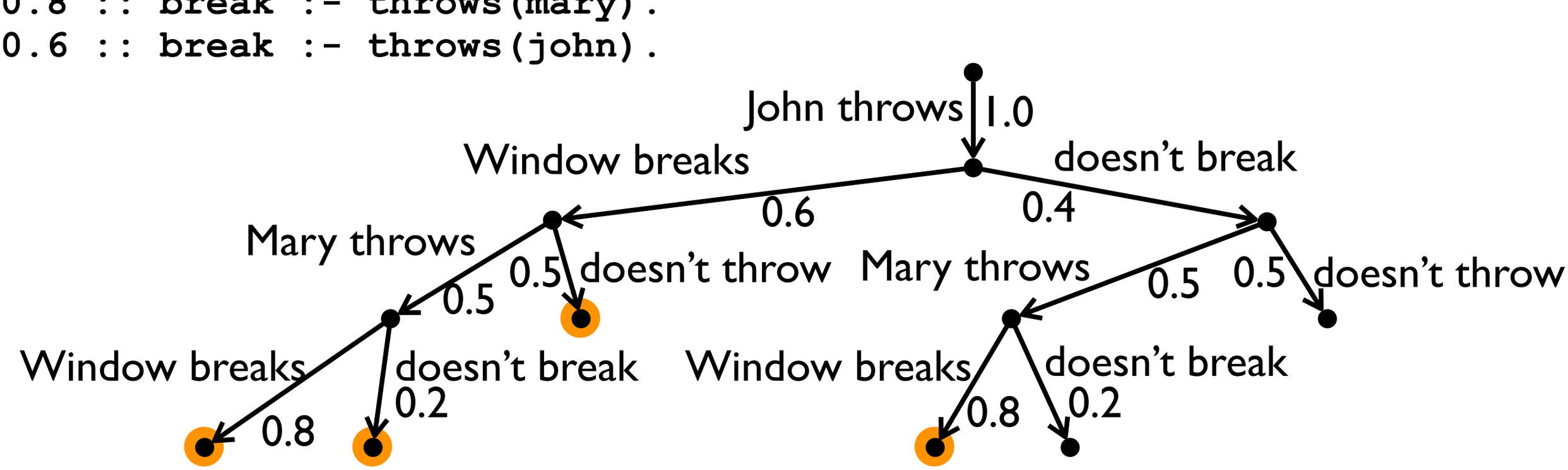
[Vennekens et al, ICLP 04]

Alternative view: CP-Logic

```
throws(john).
0.5::throws(mary).

0.8 :: break :- throws(mary).
0.6 :: break :- throws(john).
```

probabilistic causal laws



$$P(break)=0.6\times0.5\times0.8+0.6\times0.5\times0.2+0.6\times0.5+0.4\times0.5\times0.8$$

CP-logic [Vennekens et al.]

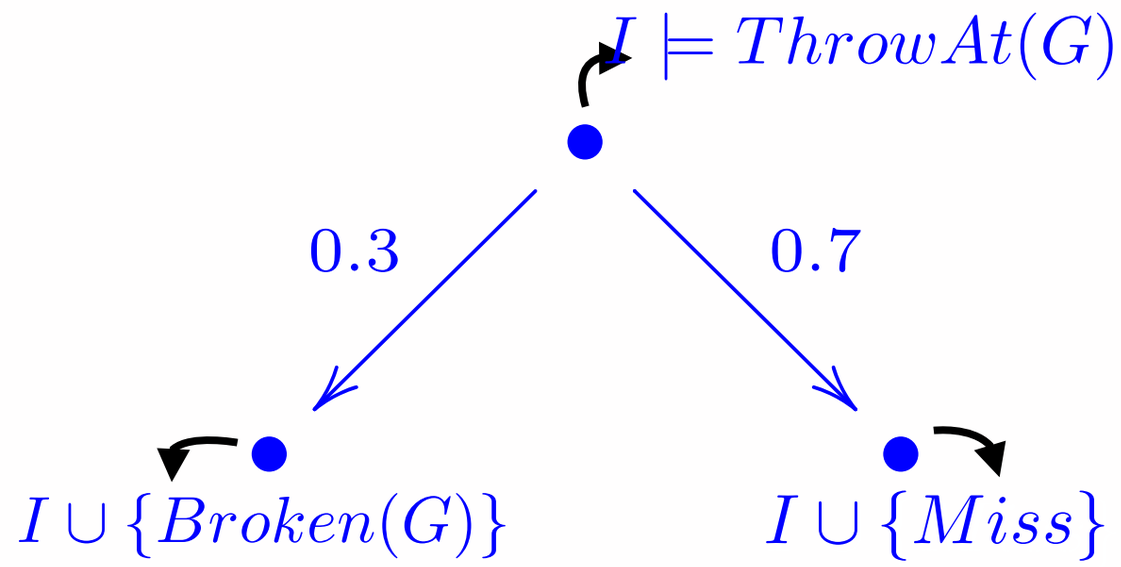
E.g., "throwing a rock at a glass breaks it with probability 0.3 and misses it with probability 0.7"

(Broken(G):0.3) ∨ (Miss 0.7) ← ThrowAt(G).

Note that the actual non-deterministic event ("rock flying at glass") is implicit

Slides CP-logic courtesy Joost Vennekens

Semantics



(Broken(G) 0.3) ∨ (Miss 0.7)
← ThrowAt(G)

Probability tree is an execution model of theory iff:

- Each tree-transition matches causal law
- The tree cannot be extended

Each execution model defines the same probability distribution over final states

Slides CP-logic courtesy Joost Vennekens

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables

random variable with Gaussian distribution

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
```



[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
stackable(OBot,OTop) :-
    ≃length(OBot) ≥ ≃length(OTop),
    ≃width(OBot) ≥ ≃width(OTop).
```

comparing values of random variables



[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
stackable(OBot,OTop) :-
      ≃length(OBot) ≥ ≃length(OTop),
      ≃width(OBot) ≥ ≃width(OTop).
ontype(Obj,plate) ~ finite([0 : glass, 0.0024 : cup,
                            0 : pitcher, 0.8676 : plate,
                            0.0284 : bowl, 0 : serving,
                            0.1016 : none])
                          :- obj(Obj), on(Obj,O2), type(O2,plate).
```



random variable with discrete distribution

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
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                            0 : pitcher, 0.8676 : plate,
                            0.0284 : bowl, 0 : serving,
                            0.1016 : none])
                    :- obj(Obj), on(Obj,O2), type(O2,plate).
```



[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

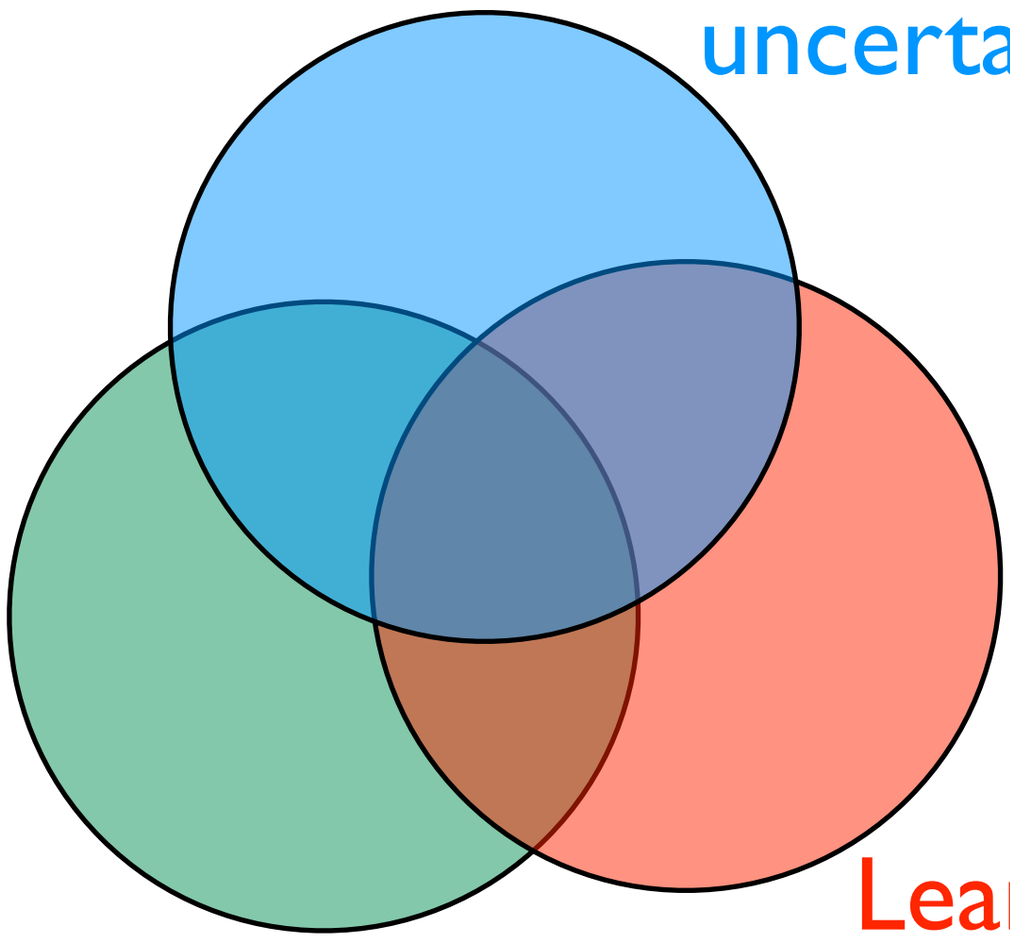
[Suciu et al 2011]

Probabilistic Databases

Dealing with uncertainty

Reasoning with relational data

Learning



[Suciu et al 2011]

Probabilistic Databases

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

bornIn

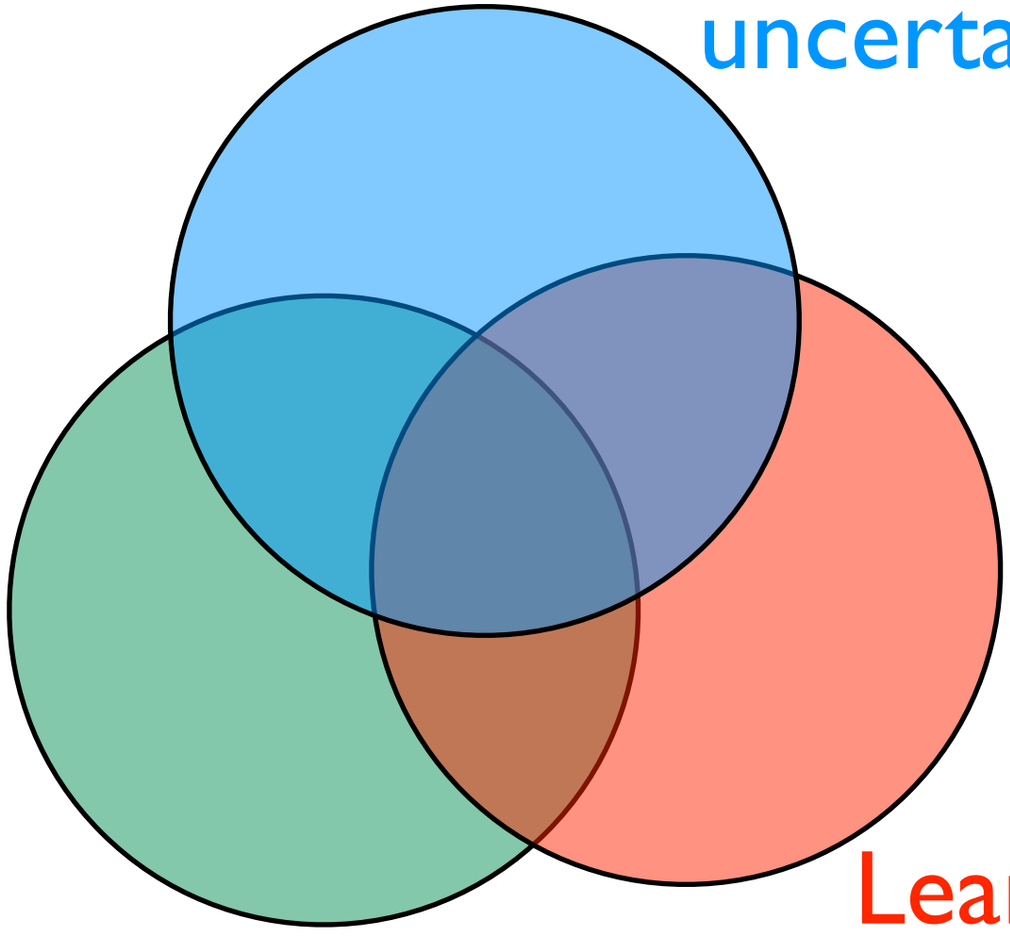
person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database

Dealing with uncertainty



Learning

[Suciu et al 2011]

Probabilistic Databases

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

one world

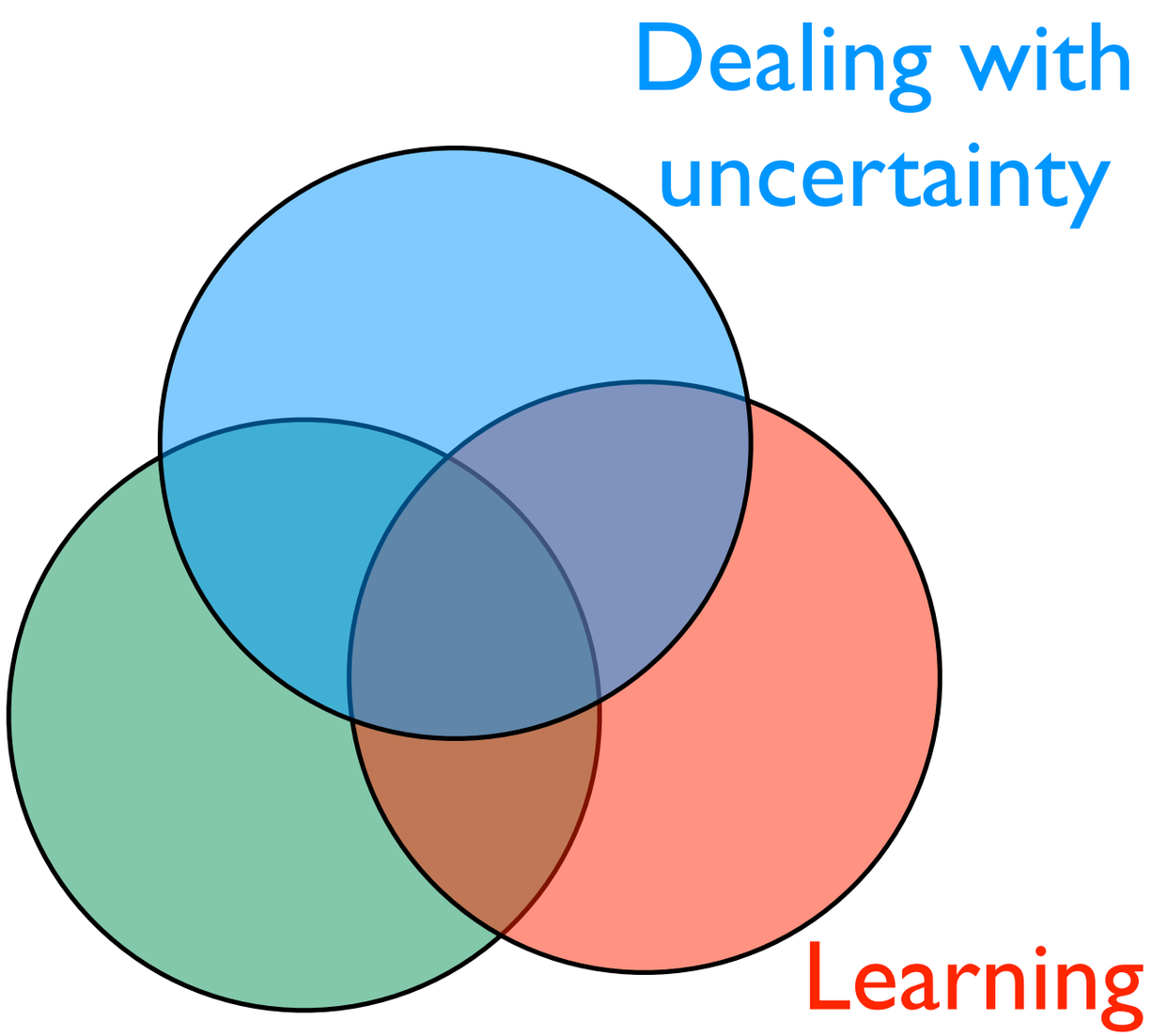
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



[Suciu et al 2011]

Probabilistic Databases

bornIn

person	city	P
ann	london	0.87
bob	york	0.95
eve	new york	0.90
tom	paris	0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random variables

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

one world

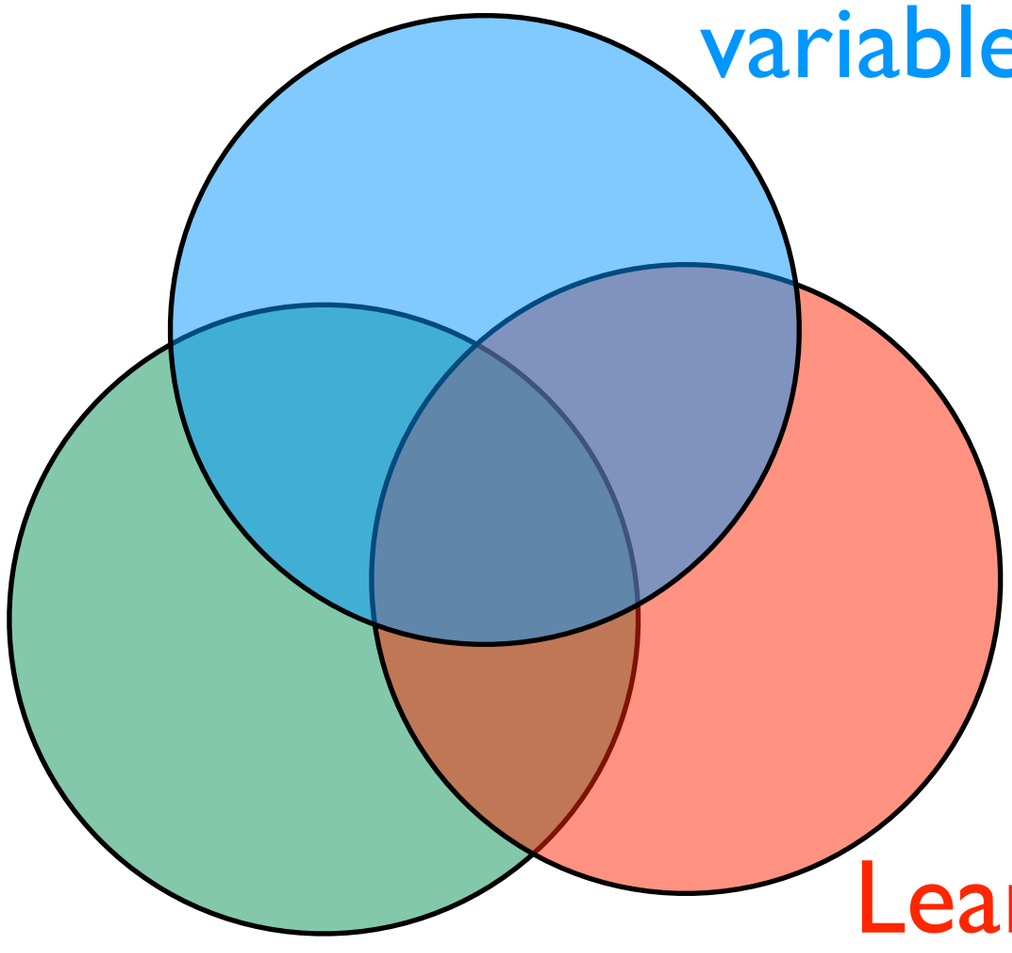
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



Learning

[Suciu et al 2011]

Probabilistic Databases

several possible worlds

bornIn

person	city	P
ann	london	0.87
bob	york	0.95
eve	new york	0.90
tom	paris	0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random variables

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select x.person, y.country
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one world

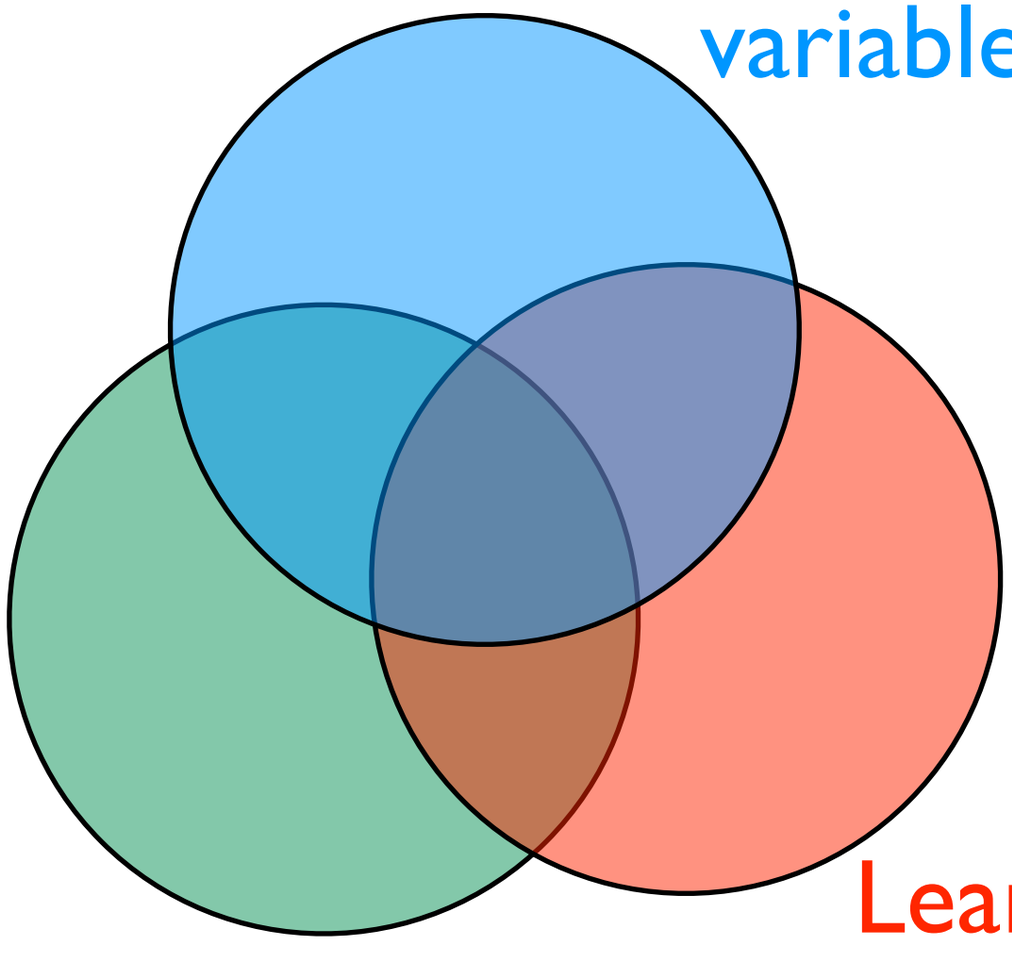
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



Learning

[Suciu et al 2011]

Probabilistic Databases

several possible worlds

bornIn

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bob	york	0.95
		0.90
		0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random

probabilistic tables + database queries
→ distribution over possible worlds

```
select
from bornIn x, cityIn y
where x.city=y.city
```

one world

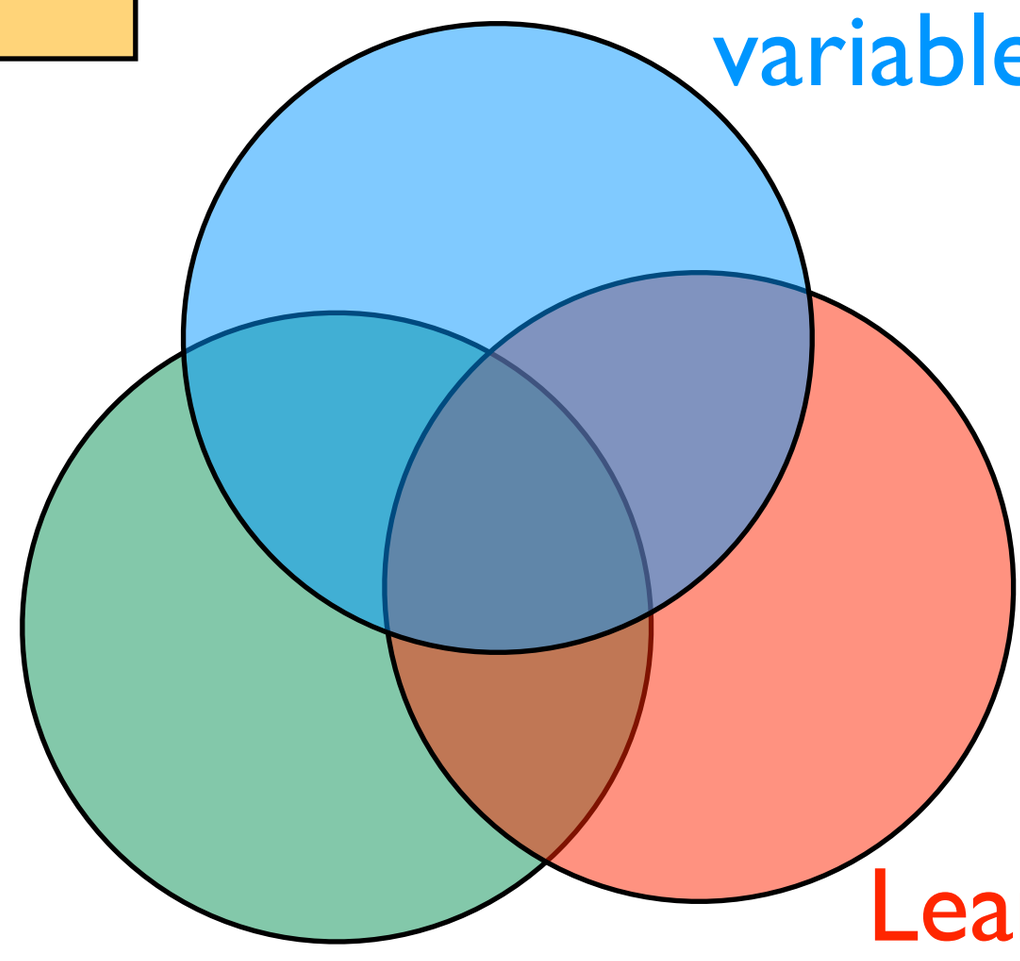
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



Example: Information Extraction

Recently-Learned Facts twitter Refresh

instance	iteration	date learned	confidence
kelly_andrews is a female	826	29-mar-2014	98.7
investment_next_year is an economic sector	829	10-apr-2014	95.3
shibenik is a geopolitical entity that is an organization	829	10-apr-2014	97.2
quality_web_design_work is a character trait	826	29-mar-2014	91.0
mercedes_benz_cls_by_carlsson is an automobile manufacturer	829	10-apr-2014	95.2
social_work is an academic program at the university rutgers_university	827	02-apr-2014	93.8
dante wrote the book the_divine_comedy	826	29-mar-2014	93.8
willie_aames was born in the city los_angeles	831	16-apr-2014	100.0
kitt_peak is a mountain in the state or province arizona	831	16-apr-2014	96.9
greenwich is a park in the city london	831	16-apr-2014	100.0

instances for many different relations

degree of certainty

NELL: http://rtw.ml.cmu.edu/rtw/

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

[Example from Suciu et al 2011]

Querying: probabilistic db

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Company	Product	P
sony	walkman	0.96
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adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
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honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

same query -

probabilities handled implicitly

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
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...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.96x0.99=0.95

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
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adobe	adobe_indesign	0.9
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...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.9x0.93=0.83

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
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ibm	san_jose	0.99
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honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

0.87x0.93=0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
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adobe	adobe_dreamweaver	0.87
...	...	...

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Company	City	P
microsoft	redmond	1.00
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honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

answer tuples ranked by probability

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

ProducesProduct

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ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

```
0.96::producesProduct(sony,walkman).
0.96::producesProduct(microsoft,mac_os_x).
0.96::producesProduct(ibm,personal_computer).
0.9::producesProduct(microsoft,mac_os).
0.9::producesProduct(adobe,adobe_indesign).
0.87::producesProduct(adobe,adobe_dreamweaver).
...
```


[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

```
result(Product,Company) :-
    producesProduct(Company,Product),
    headquarteredIn(Company,san_jose).
query(result(_,_)).
```

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

```
result(Product,Company) :-
    producesProduct(Company,Product),
    headquarteredIn(Company,san_jose).
query(result(_,_)).
```

PDB with attribute-level uncertainty in ProbLog?

color

item	color	P
mug	green blue	0.65 0.35
plate	pink red purple	0.23 0.14 0.63

PDB with attribute-level uncertainty in ProbLog?

color

item	color	P
mug	green blue	0.65 0.35
plate	pink red purple	0.23 0.14 0.63

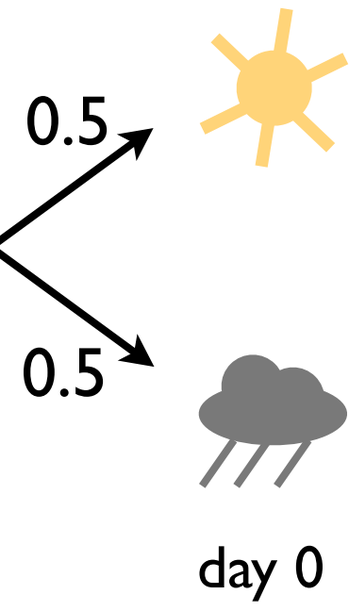
```
0.65::color(mug,green); 0.35::color(mug,blue).
0.23::color(plate,pink); 0.14::color(plate,red);
                                  0.63::color(plate,purple).
```

ProbLog by example:

Rain or sun?

ProbLog by example:

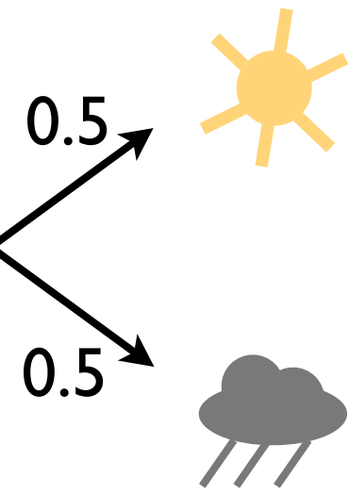
Rain or sun?



day 0

ProbLog by example:

Rain or sun?

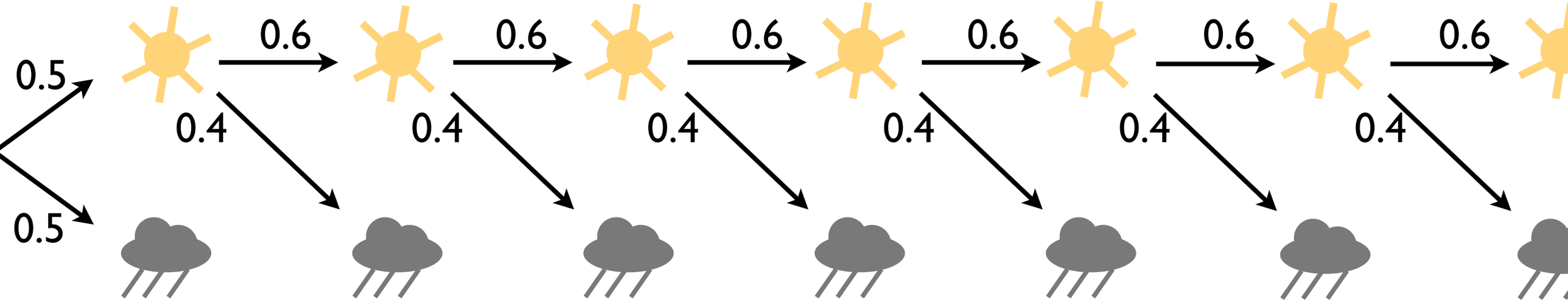


day 0

```
0.5::weather(sun,0) ; 0.5::weather(rain,0).
```

ProbLog by example:

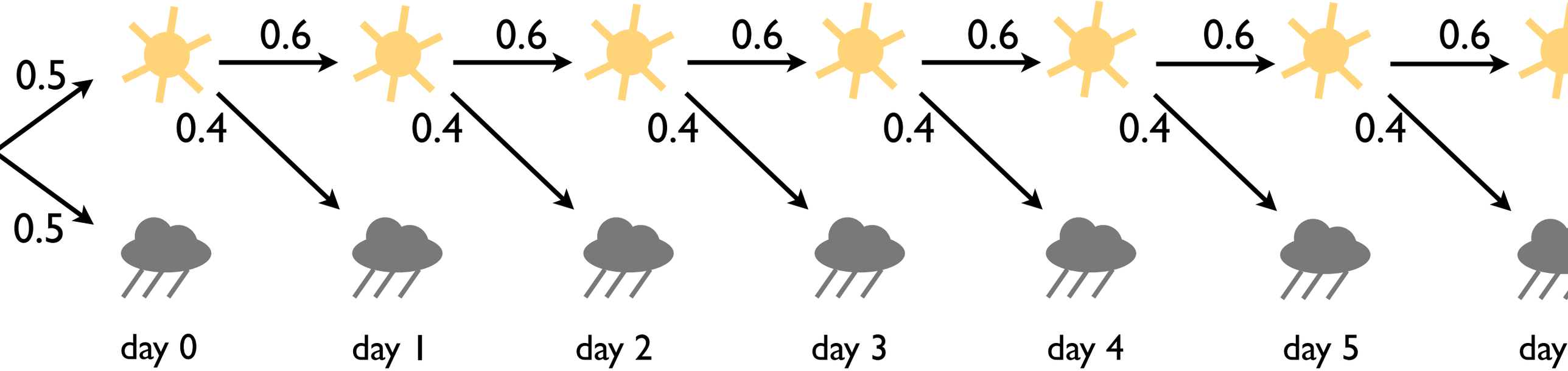
Rain or sun?




```
0.5::weather(sun,0) ; 0.5::weather(rain,0).
```

ProbLog by example:

Rain or sun?

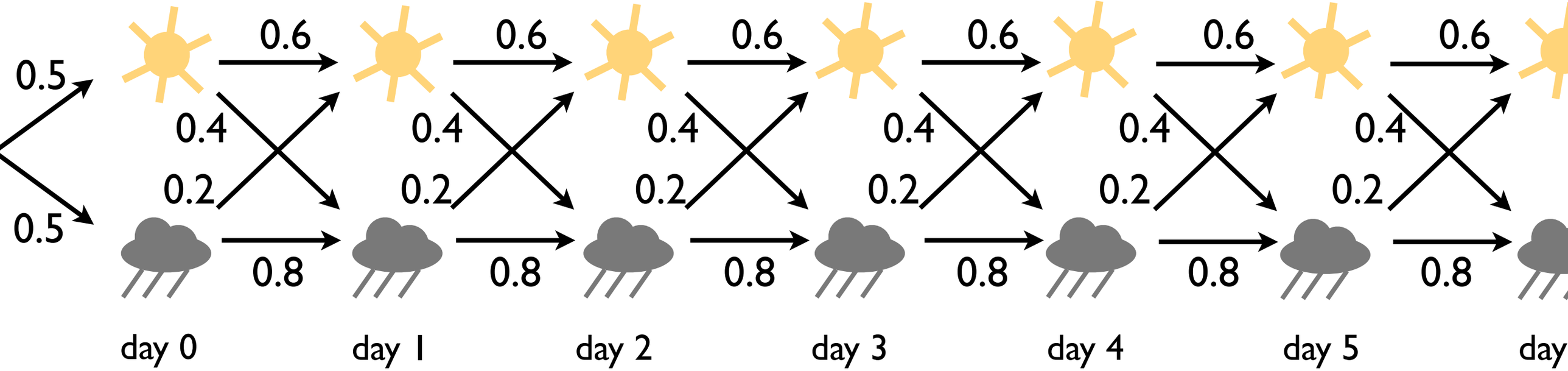



```
0.5::weather(sun,0) ; 0.5::weather(rain,0).

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                    :- T>0, Tprev is T-1, weather(sun,Tprev).
```

ProbLog by example:

Rain or sun?




```
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0.6::weather(sun,T) ; 0.4::weather(rain,T)
                    :- T>0, Tprev is T-1, weather(sun,Tprev).
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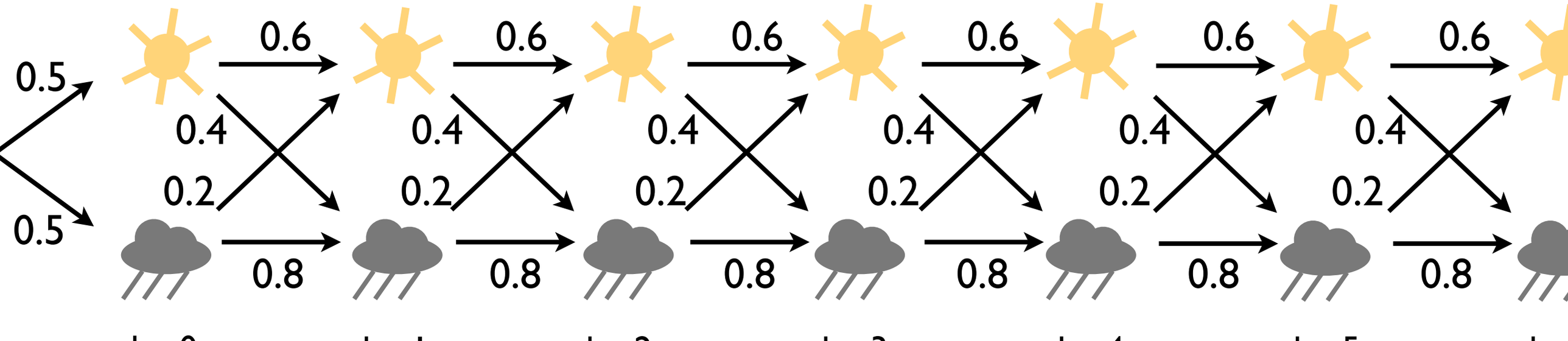
ProbLog by example:

Rain or sun?

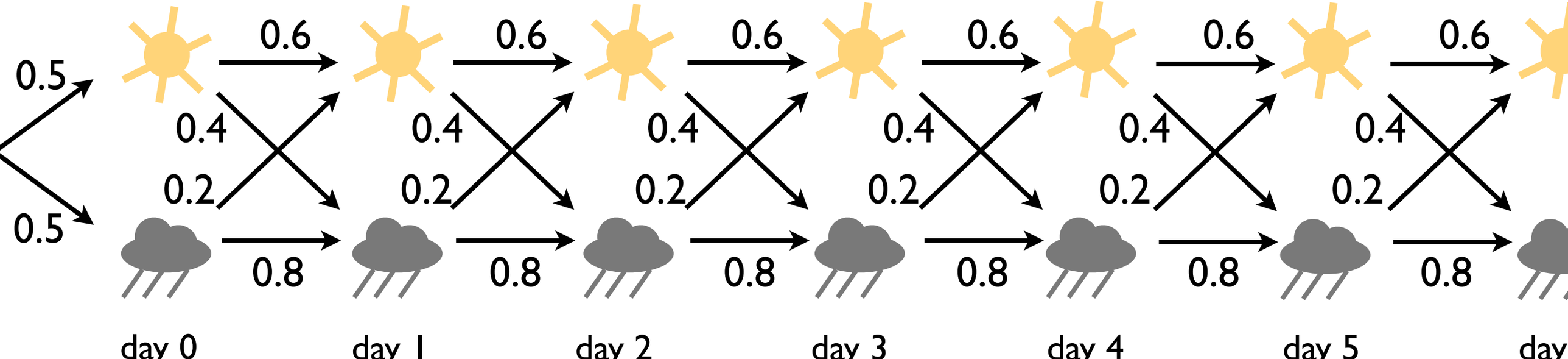



```
0.5::weather(sun,0) ; 0.5::weather(rain,0).

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                    :- T>0, Tprev is T-1, weather(sun,Tprev).
0.2::weather(sun,T) ; 0.8::weather(rain,T)
                    :- T>0, Tprev is T-1, weather(rain,Tprev).
```

ProbLog by example:

Rain or sun?

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0.2::weather(sun,T) ; 0.8::weather(rain,T)
                    :- T>0, Tprev is T-1, weather(rain,Tprev).
```
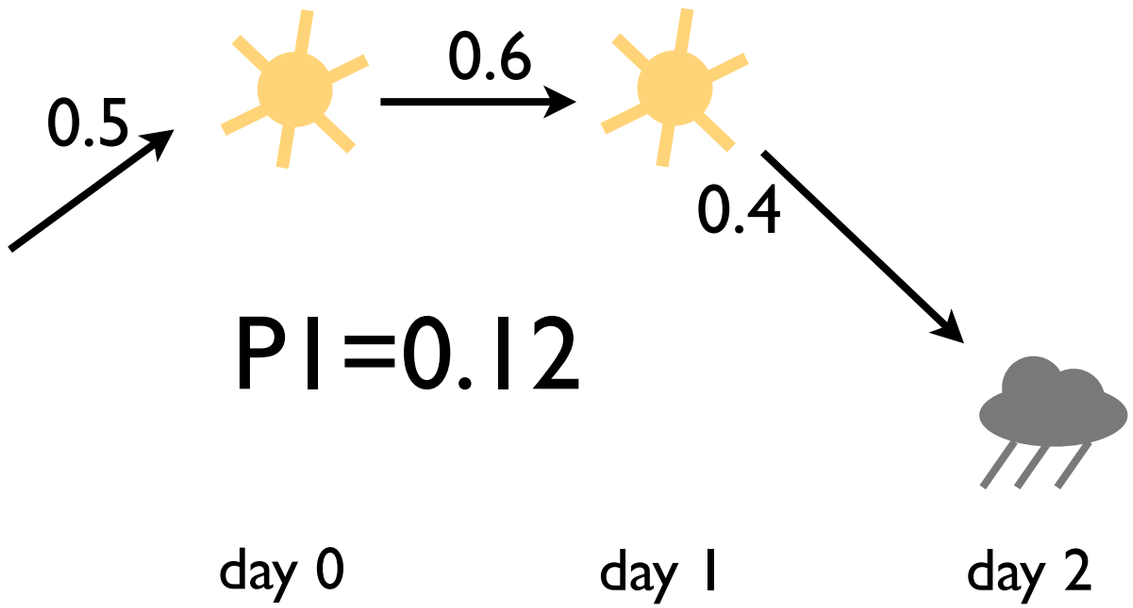
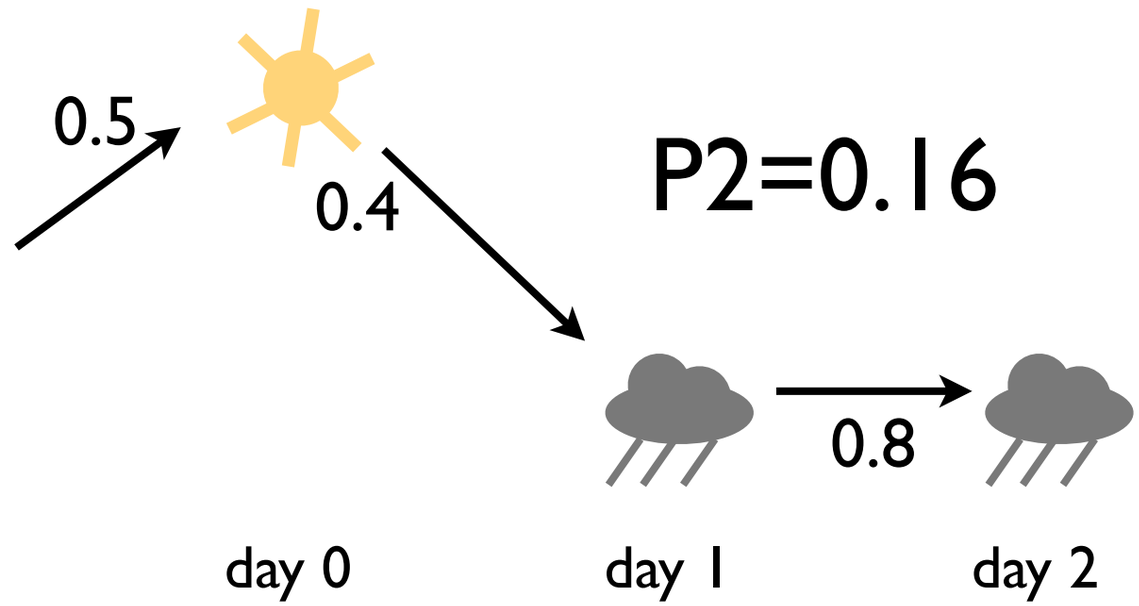

infinite possible worlds! BUT: finitely many partial worlds suffice to answer any given ground query

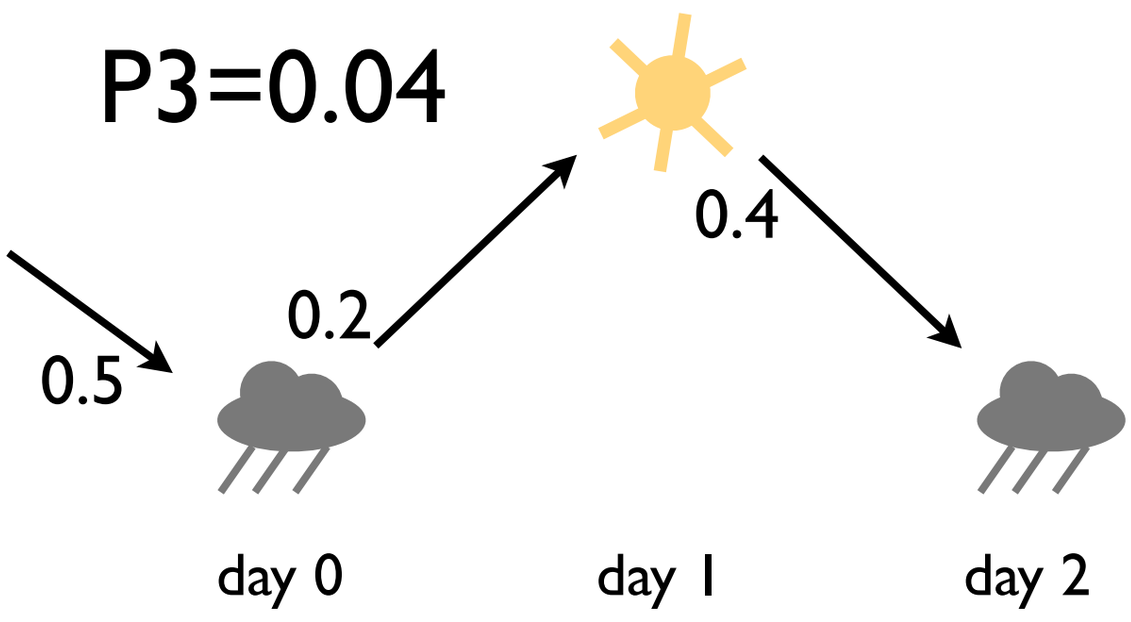
Possible worlds

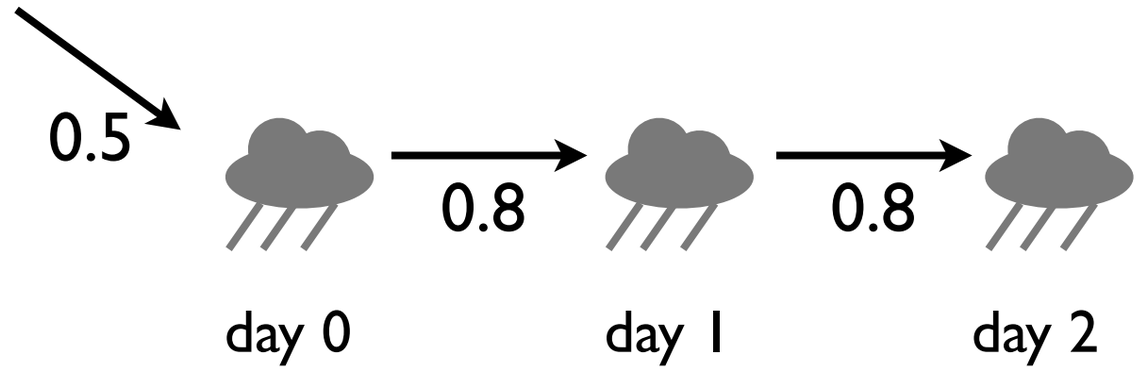
```
?- weather(rain,2).
```

P=P1+P2+P3+P4









ProbLog

- **probabilistic choices** + their **consequences**
- probability distribution over **possible worlds**
- how to efficiently answer **questions**?
  - most probable world (MPE inference)
  - probability of query (computing marginals)
  - probability of query given evidence

Summary: ProbLog Syntax

- input database: ground facts

```
person(bob).
```

- probabilistic facts

```
0.5::stress(bob).
```

- annotated disjunctions

```
0.5::stress(X) :- person(X).
0.4::a(X); 0.3::b(X); 0.2::c(X); 0.1::d(X) :- q(X).
0.5::weather(sun,0) ; 0.5::weather(rain,0).
```

- flexible probabilities

```
P::pack(Item) :- weight(Item,W), P is 1.0/W.
```

- Prolog clauses

```
smokes(X) :- influences(Y,X), smokes(Y).
excess([I|R],Limit) :- \+pack(I), excess(R,Limit).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

```
0.5::weather(sun,0) ; 0.5::weather(rain,0).

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                :- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                :- T>0, Tprev is T-1, weather(rain,Tprev).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

first rule for day 0, others for later days

```
0.5::weather(sun,0) ; 0.5::weather(rain,0).

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                :- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                :- T>0, Tprev is T-1, weather(rain,Tprev).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

first rule for day 0, others for later days

day 0: either sun or rain

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```

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no two rules apply simultaneously

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                    :- T>0, Tprev is T-1, weather(rain,Tprev).
```

rules for T>0 cover mutually exclusive cases on previous day

[Sato and Kameya, JAIR 01]

PRISM

- Another probabilistic Prolog based on the distribution semantics
- Mutual exclusiveness assumption
  - allows for efficient inference by dynamic programming, cf. probabilistic grammars
  - but excludes certain models, e.g., smokers
- http://sato-www.cs.titech.ac.jp/prism/

PRISM

- "multi-valued random switches" = annotated disjunctions with body *true*
- switch gives fresh result on each call
- Prolog rules
- limited support for negation (compiling away)

stochastic memoization

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain]).

:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

```
weather(W,Time) :-
  Time >= 0,
  msw(init,W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

```
weather(W,Time) :-
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   w(Next,T,WNext,WT).
```

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

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:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

probability distributions

```
weather(W,Time) :-
  Time >= 0,
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w(T,T,W,W).
w(Now,T,WNow,WT) :-
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   msw(tr(WNow),WNext),
   Next is Now+1,
   w(Next,T,WNext,WT).
```

Weather in PRISM

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values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

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```

probability distributions

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weather(W,Time) :-
   Time >= 0,
   msw(init,W0),
   w(0,Time,W0,W).

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w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

set **W0** to random value of **init**

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
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```

probability distributions

```
weather(W,Time) :-
   Time >= 0,
   msw(init,W0),
   w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

set **`W0`** to random value of **`init`**

set **`WNext`** to random value of **`tr(WNow)`**, using **fresh** value on every call

Weather in PRISM / ProbLog

```
values(init,[sun,rain]).
values(tr(_),[sun,rain]).

:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

```
weather(W,Time) :-
  Time >= 0,
  msw(init,W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

```
0.5::init(sun) ;
  0.5::init(rain).
0.6::tr(T,sun,sun) ;
  0.4::tr(T,sun,rain).
0.2::tr(T,rain,sun) ;
  0.8::tr(T,rain,rain).
```

```
weather(W,Time) :-
  Time >= 0,
  init(W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    tr(Now,WNow,WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

ProbLog needs to explicitly use different facts at each call

Probabilistic Prologs: Two Views

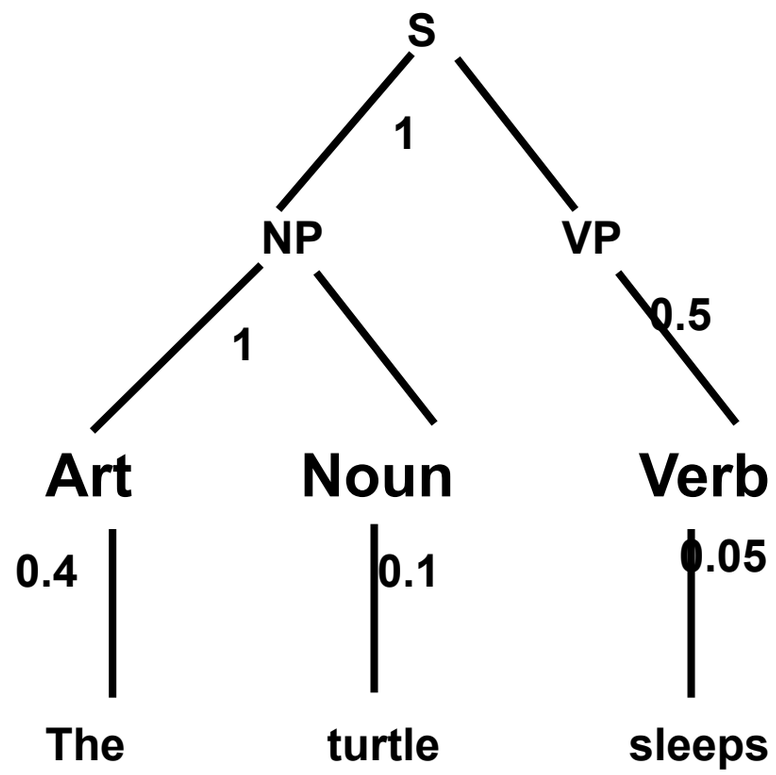
- Distribution semantics:
  - probability distribution over interpretations
  - degree of belief
- Stochastic Logic Programs (SLPs):
  - probability distribution over query answers
  - like in probabilistic grammars

Probabilistic Prologs: Two Views

- Distribution semantics:
  - probability distribution over interpretations
  - degree of belief
- Stochastic Logic Programs (SLPs):
  - probability distribution over query answers
  - like in probabilistic grammars

Probabilistic Context Free Grammars

1.0 : S -> NP, VP
1.0 : NP -> Art, Noun
0.6 : Art -> a
0.4 : Art -> the
0.1 : Noun -> turtle
0.1 : Noun -> turtles
…
0.5 : VP -> Verb
0.5 : VP -> Verb, NP
0.05 : Verb -> sleep
0.05 : Verb -> sleeps
….



P(parse tree) = 1x1x.5x.1x.4x.05

PCFGs

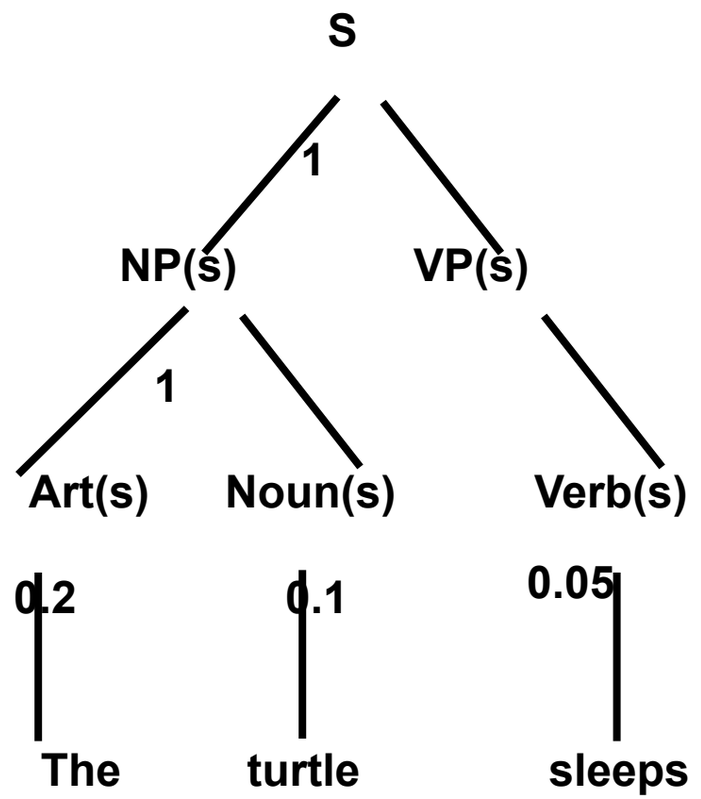
$P(parse\ tree) = \prod_i p_i^{c_i}$
where p_i is the probability of rule i
and c_i the number of times
it is used in the parse tree

$P(sentence) = \sum_{p:parsetree} P(p)$

Observe that derivations always succeed, that is
$S \to T, Q$ and $T \to R, U$
always yields
$S \to R, U, Q$

Probabilistic Definite Clause Grammar

1.0 S -> NP(Num), VP(Num)
1.0 NP(Num) -> Art(Num), Noun(Num)
0.6 Art(sing) -> a
0.2 Art(sing) -> the
0.2 Art(plur) -> the
0.1 Noun(sing) -> turtle
0.1 Noun(plur) -> turtles
…
0.5 VP(Num) -> Verb(Num)
0.5 VP(Num) -> Verb(Num), NP(Num)
0.05 Verb(sing) -> sleeps
0.05 Verb(plur) -> sleep
….



P(derivation tree) = 1x1x.5x.1x .2 x.05

Stochastic Logic Programs

[49][Muggleton, Cussens]

In SLP notation

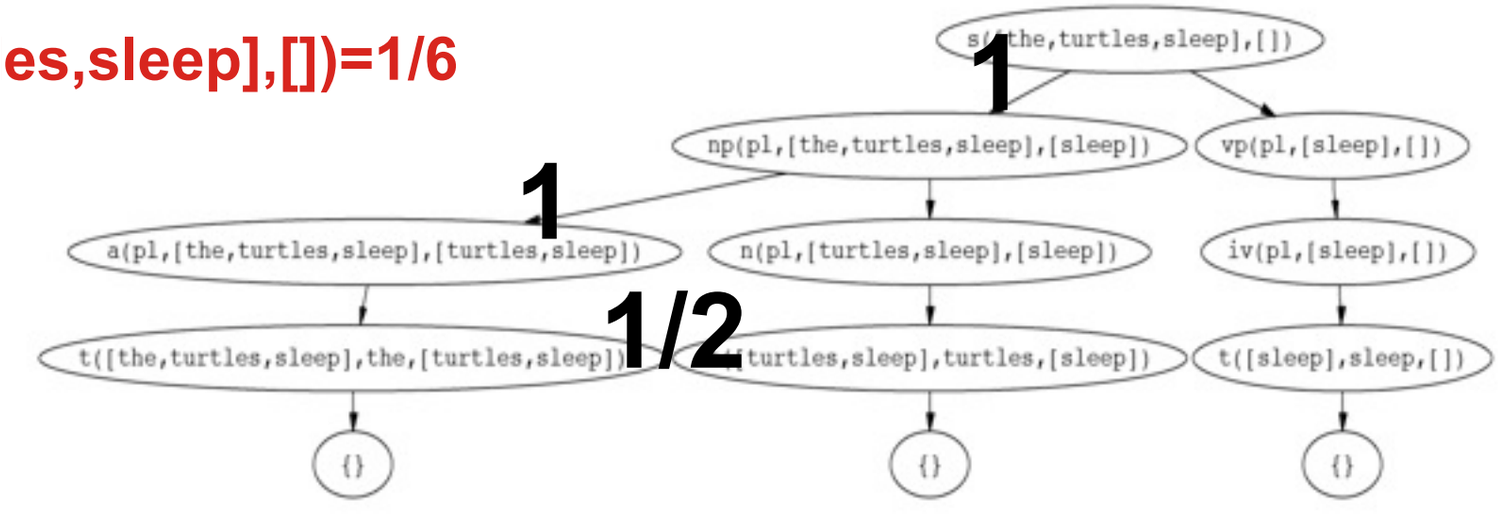
```
sentence(A, B) :- noun_phrase(C, A, D), verb_phrase(C, D, B).
noun_phrase(A, B, C) :- article(A, B, D), noun(A, D, C).
verb_phrase(A, B, C) :- intransitive_verb(A, B, C).
article(singular, A, B) :- terminal(A, a, B).
article(singular, A, B) :- terminal(A, the, B).
article(plural, A, B)   :- terminal(A, the, B).
noun(singular, A, B) :- terminal(A, turtle, B).
noun(plural, A, B)   :- terminal(A, turtles, B).
intransitive_verb(singular, A, B) :- terminal(A, sleeps, B).
intransitive_verb(plural, A, B) :- terminal(A, sleep, B).
terminal([A|B],A,B).
```

1

1/3

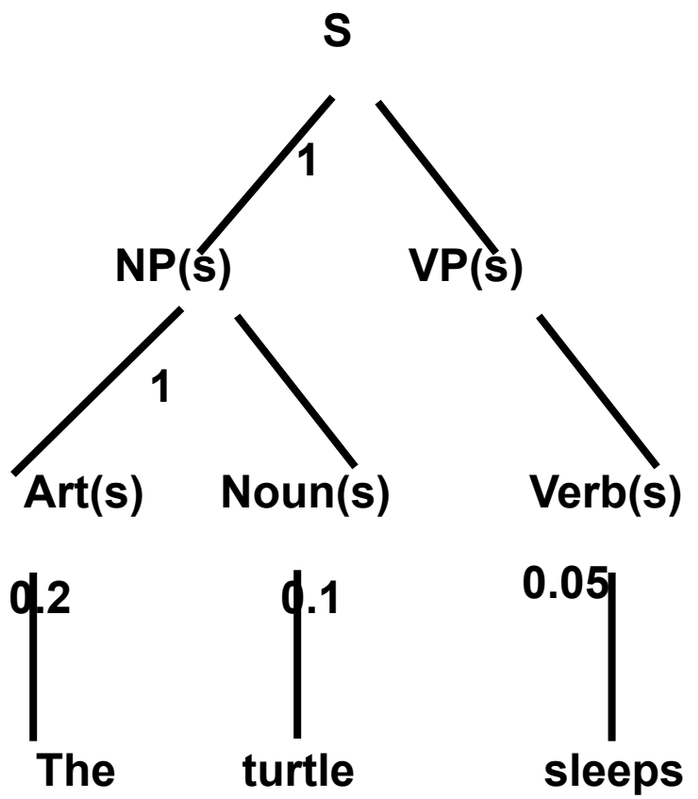
1/2

P(s([the,turtles,sleep],[])=1/6



Probabilistic DCG

1.0 S -> NP(Num), VP(Num)
1.0 NP(Num) -> Art(Num), Noun(Num)
0.6 Art(sing) -> a
0.2 Art(sing) -> the
0.2 Art(plur) -> the
0.1 Noun(sing) -> turtle
0.1 Noun(plur) -> turtles
…
0.5 VP(Num) -> Verb(Num)
0.5 VP(Num) -> Verb(Num), NP(Num)
0.05 Verb(sing) -> sleeps
0.05 Verb(plur) -> sleep
….



P(derivation tree) = 1x1x.5x.1x .2 x.05

Probabilistic DCG

1.0 S -> NP(Num), VP(Num)
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0.05 Verb(sing) -> sleeps
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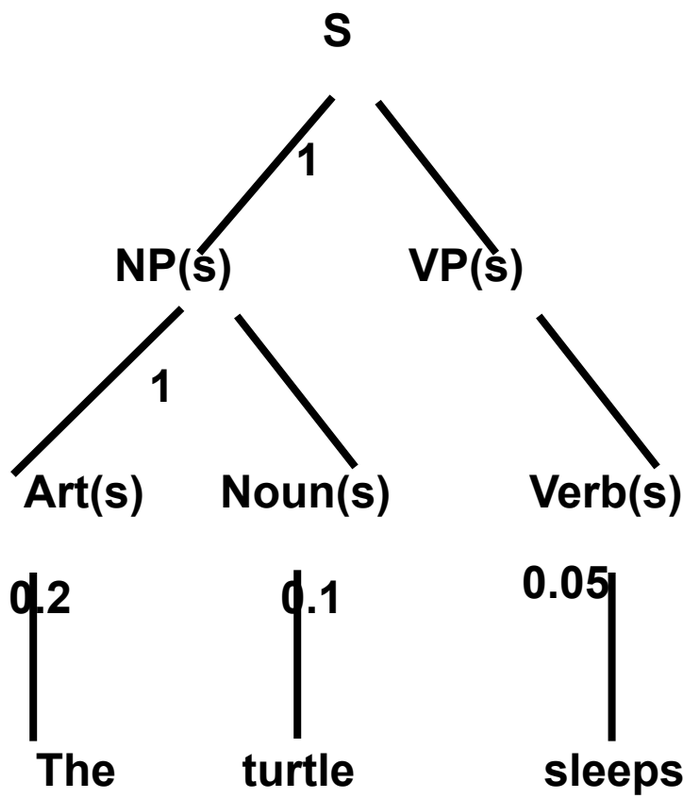
….



P(derivation tree) = 1x1x.5x.1x .2 x.05

What about "A turtles sleeps" ?

SLPs

$P_d(derivation) = \prod_i p_i^{c_i}$
where p_i is the probability of rule i
and c_i the number of times
it is used in the parse tree

Observe that some derivations now fail due to unification,
$np(Num) \rightarrow art(Num), noun(Num)$ and $art(sing) \rightarrow a$
$noun(plural) \rightarrow turtles$

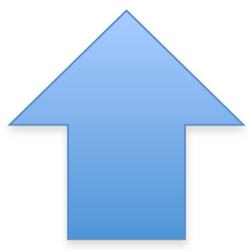
Normalization necessary
$P_s(proof) = \frac{P_d(proof)}{\sum_i P_d(proof_i)}$

ProPPR

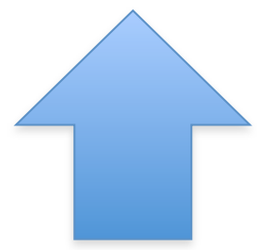
- A variation on SLPs
- Integrating concepts from Personalized Page Rank
- Fast inference and rule learning abilities
- Used by CMU group for NELL (Never Ending Learning)
- See [Wang et al., CIKM 13, arXiv:1404.3301, Van Daele et al., ILP 14]

Sample ProPPR program….

```
about(X,Z) :- handLabeled(X,Z)          # base.
about(X,Z) :- sim(X,Y),about(Y,Z)       # prop.
sim(X,Y) :- links(X,Y)                  # sim,link.
sim(X,Y) :-
    hasWord(X,W),hasWord(Y,W),
    linkedBy(X,Y,W)                     # sim,word.
linkedBy(X,Y,W) :- true                 # by(W).
```

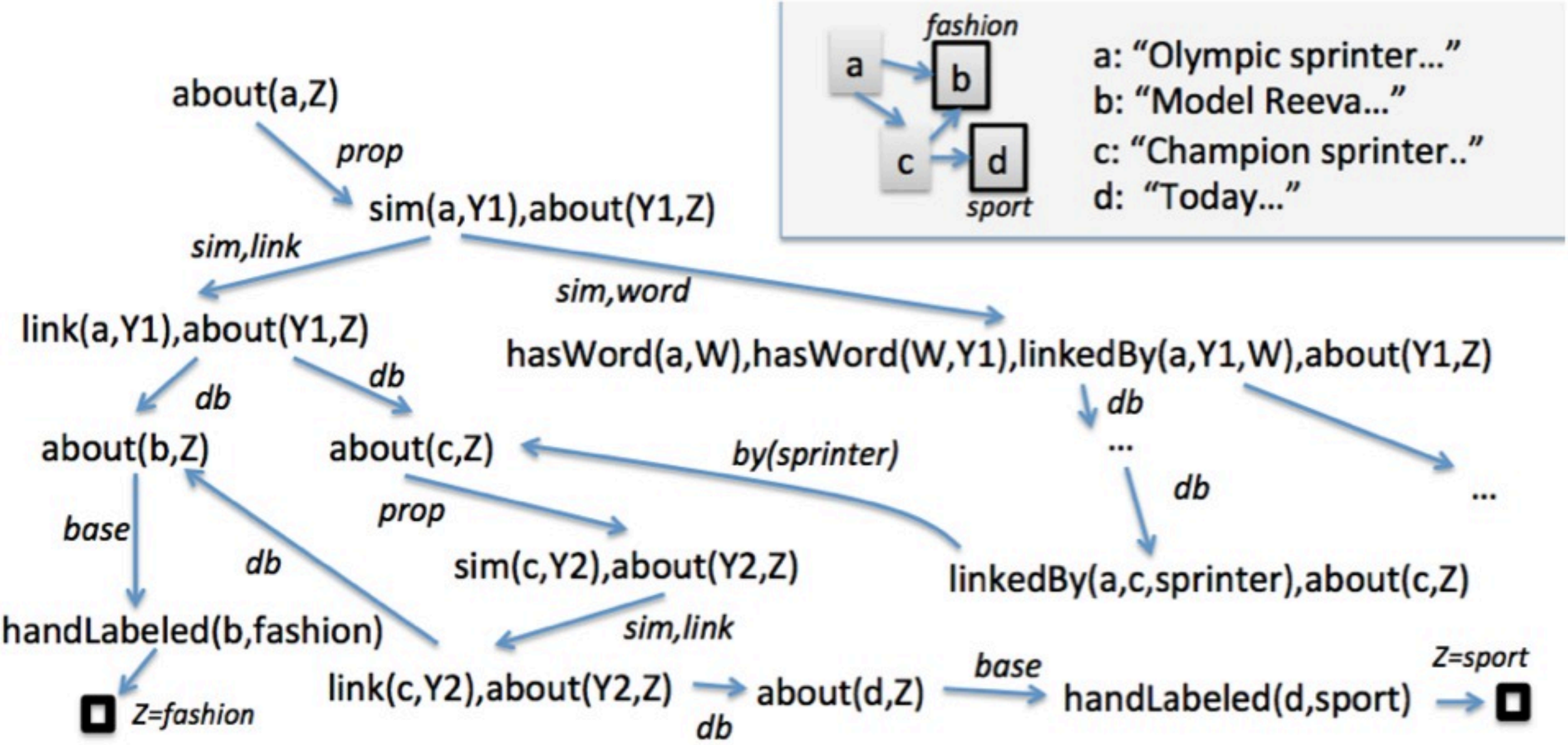


Horn rules



features of rules
(vars from head ok)

[Slide by William Cohen]



.. and search space…

[Slide by William Cohen]

```
about(X,Z) :- handLabeled(X,Z)          # base.
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    hasWord(X,W),hasWord(Y,W),
    linkedBy(X,Y,W)                     # sim,word.
linkedBy(X,Y,W) :- true                 # by(W).
```
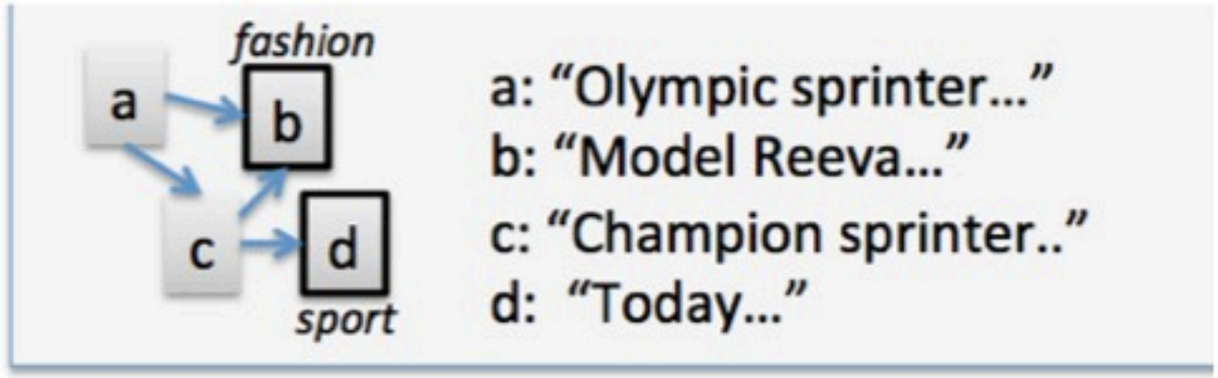

D'oh! This is a graph!



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```
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```
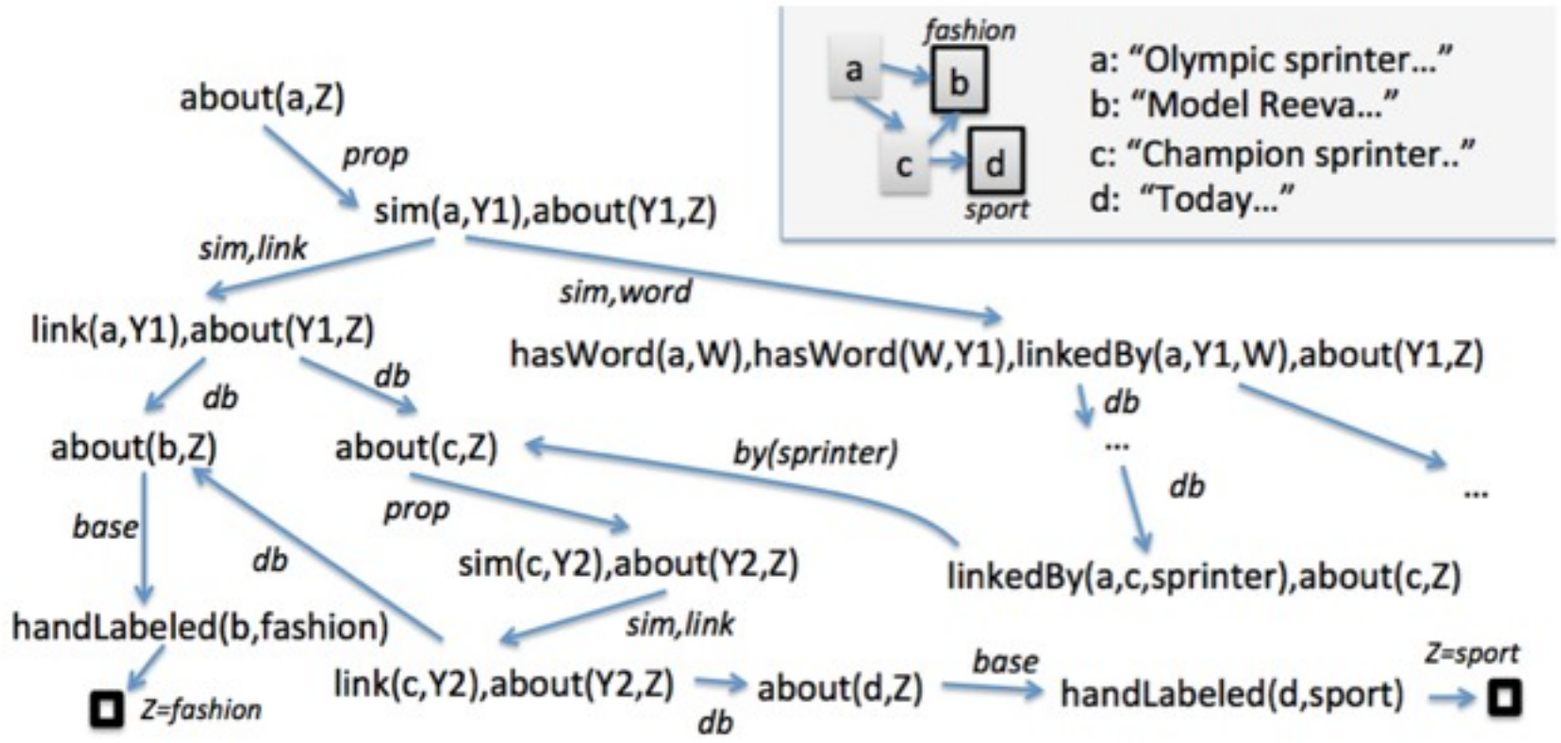

- Score for a query soln (e.g., "Z=sport" for "about(a,Z)") depends on probability of reaching a □ node*
  - learn transition probabilities based on features of the rules
  - implicit "reset" transitions with (p≥α) back to query node
- Looking for answers supported by many short proofs



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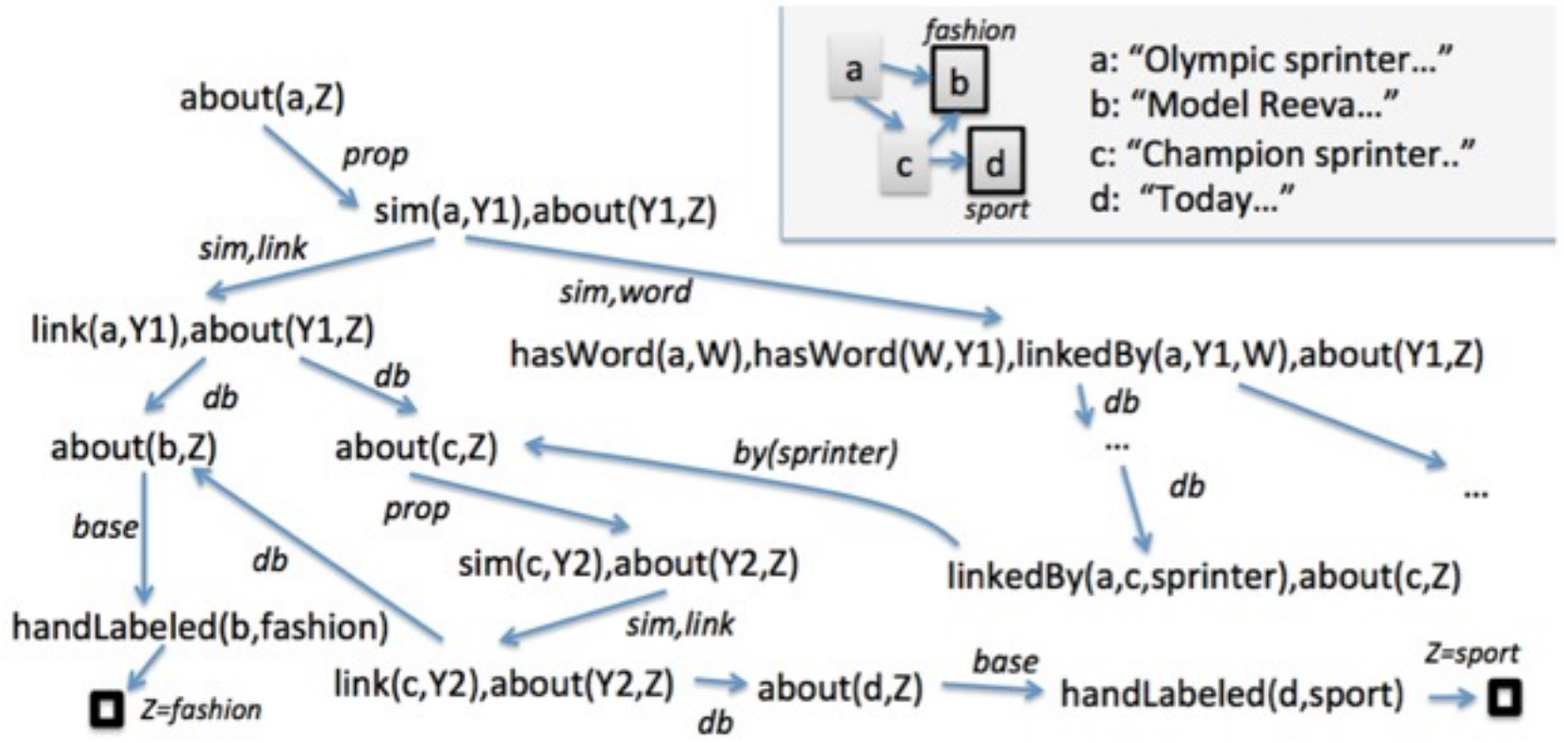
*Exactly as in Stochastic Logic Programs [Cussens, 2001]



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"Grounding" size is O(1/αε) … ie independent of DB size ➔ fast approx incremental inference (Reid,Lang,Chung, 08)

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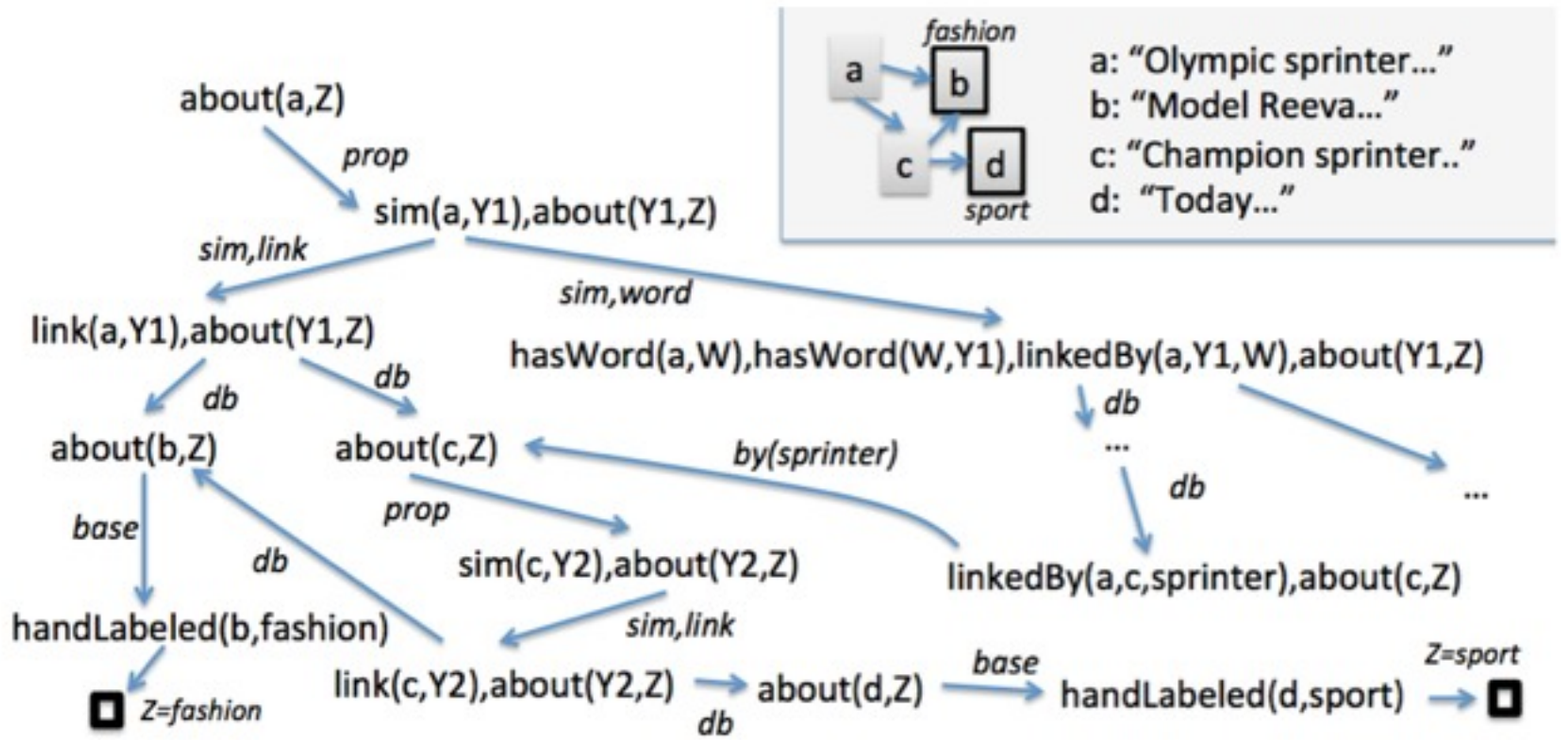

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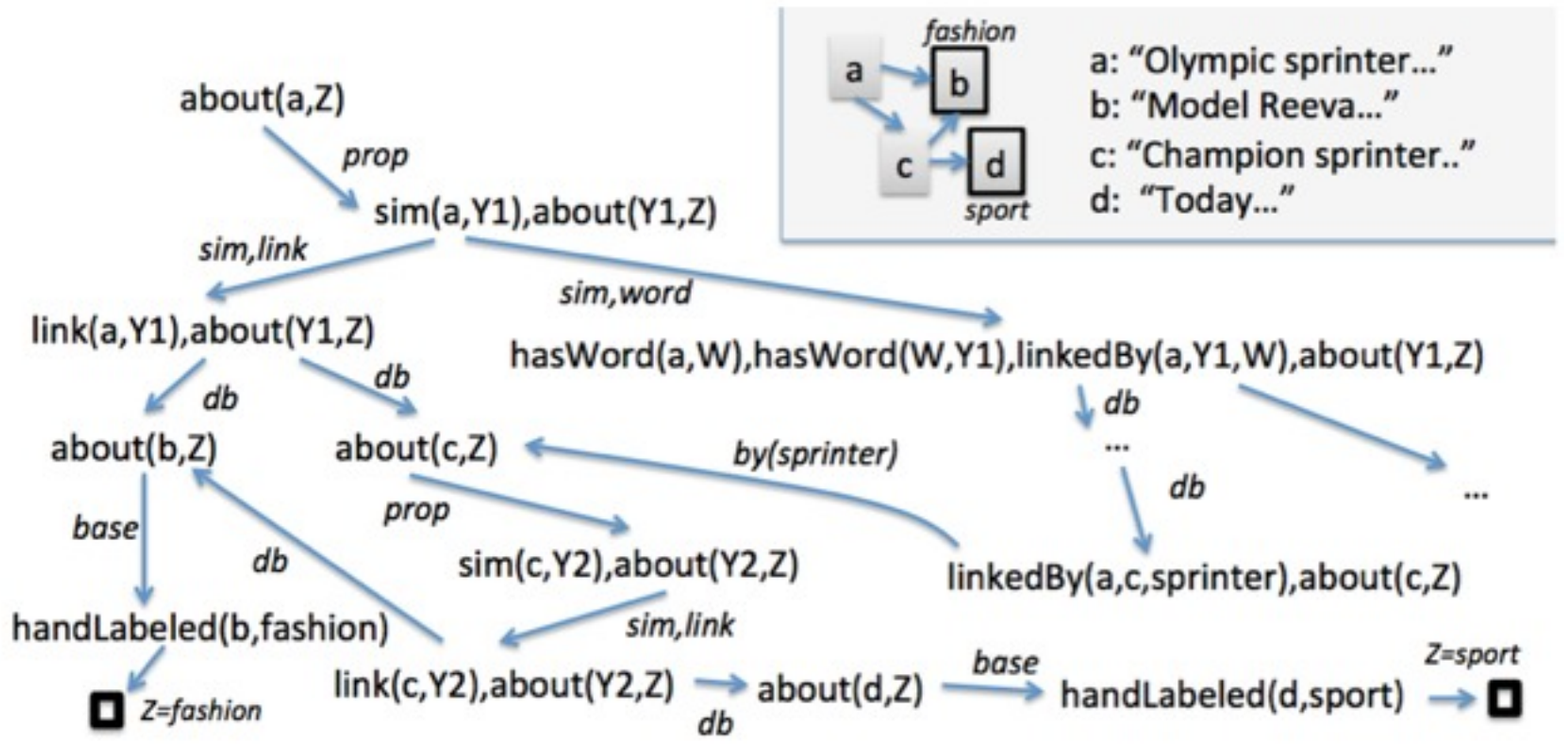
Learning: supervised variant of personalized PageRank (Backstrom & Leskovic, 2011)



[Slide by William Cohen]

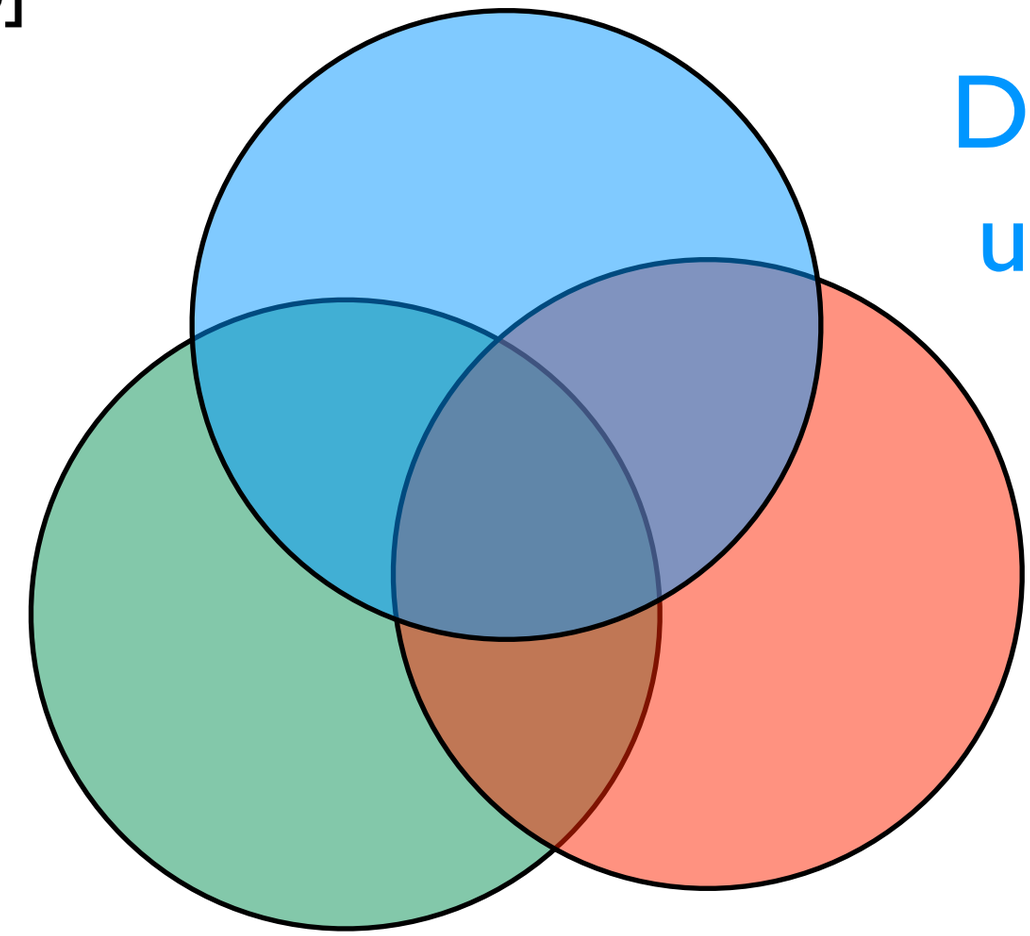
Probabilistic Programming Languages outside LP

- IBAL [Pfeffer 01]
- Figaro [Pfeffer 09]
- Church [Goodman et al 08]
- BLOG [Milch et al 05]
- and many more appearing recently

Church

probabilistic functional programming

[Goodman et al, UAI 08]



Dealing with uncertainty

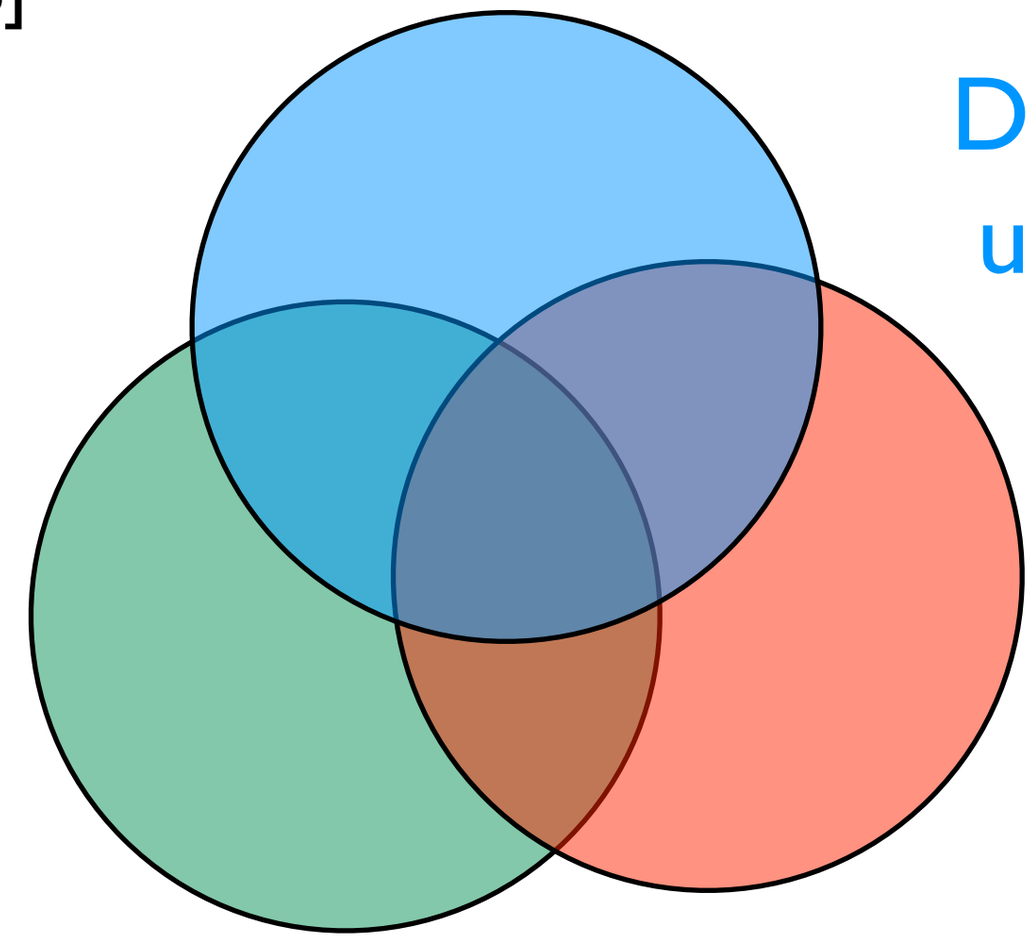
Reasoning with relational data

Learning

Church

probabilistic functional programming

[Goodman et al, UAI 08]



Dealing with uncertainty

functional programming

Learning

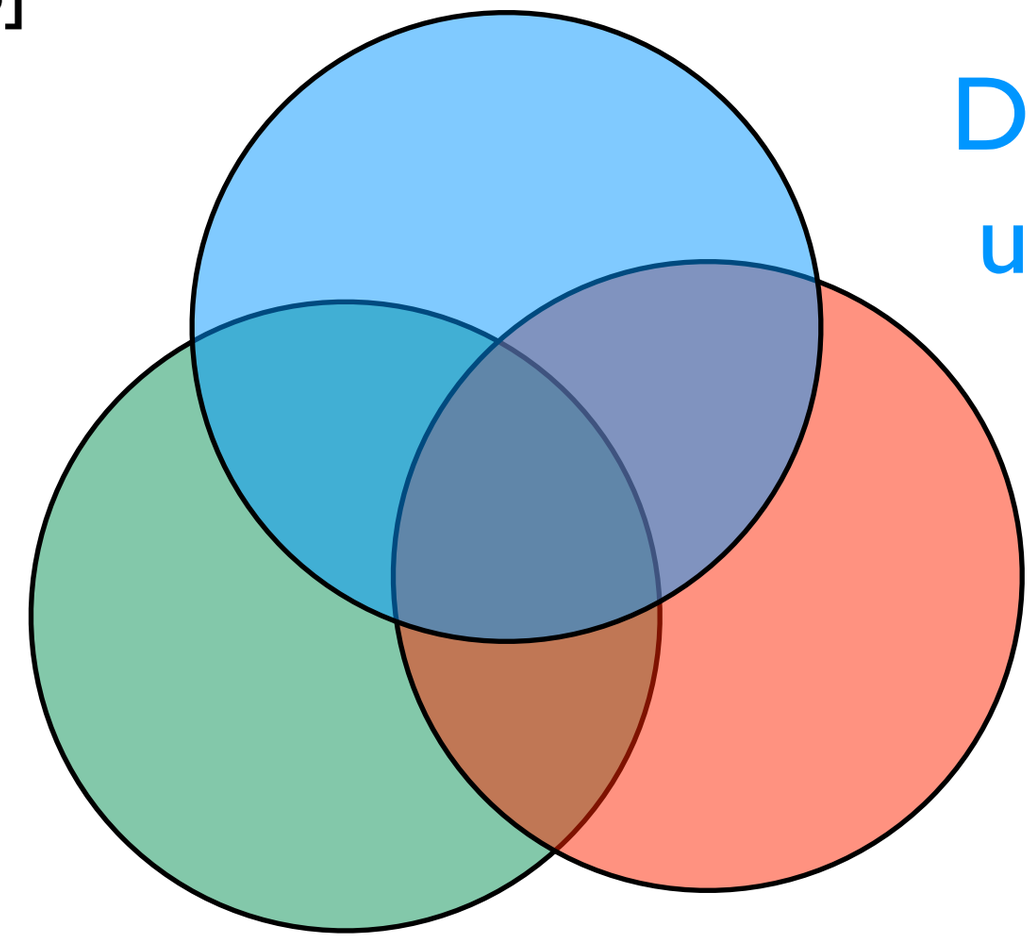
```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

Church

probabilistic functional programming

[Goodman et al, UAI 08]



Dealing with uncertainty

functional programming

Learning

one execution

```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

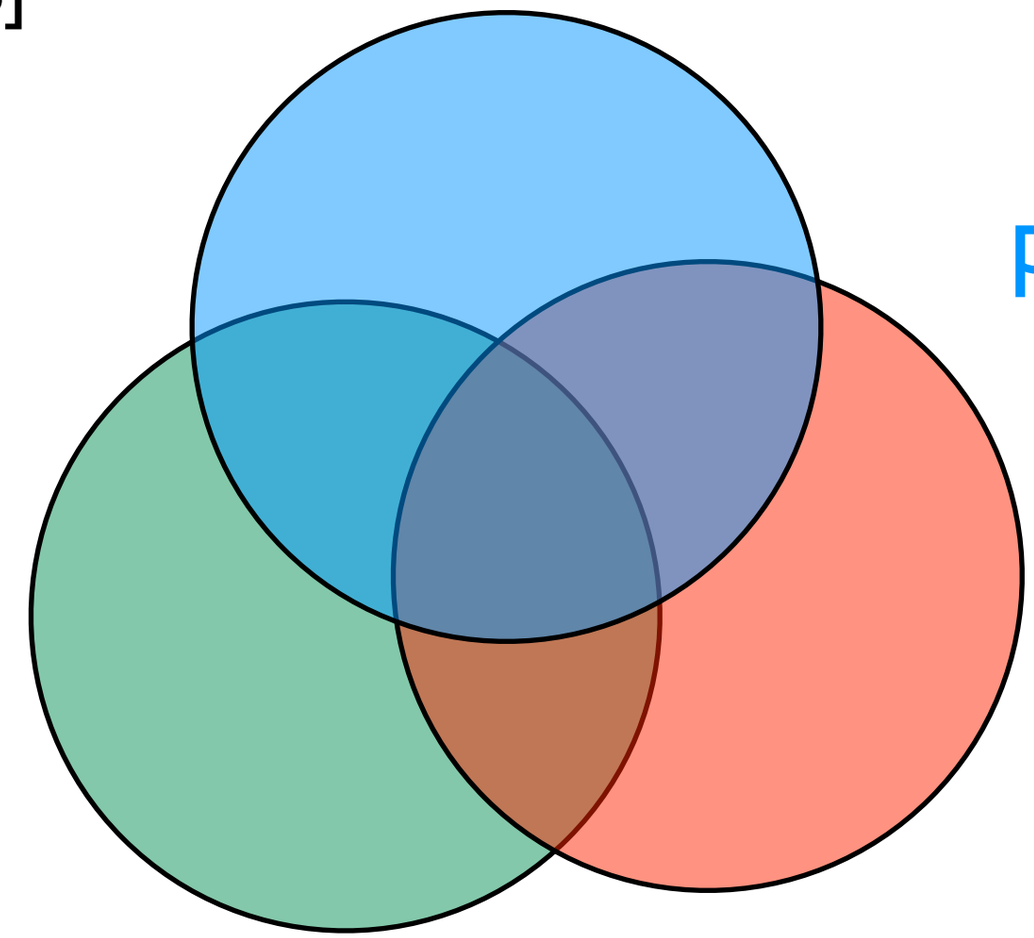
Church

probabilistic functional programming

[Goodman et al, UAI 08]

```
(define randplus5
 (lambda (x) (if (flip 0.6)
                 (+ x 5)
                 x)))

(map randplus5 '(1 2 3))
```



random primitives

functional programming

Learning

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(define plus5 (lambda (x) (+ x 5)))

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Church

probabilistic functional programming

[Goodman et al, UAI 08]

several possible executions

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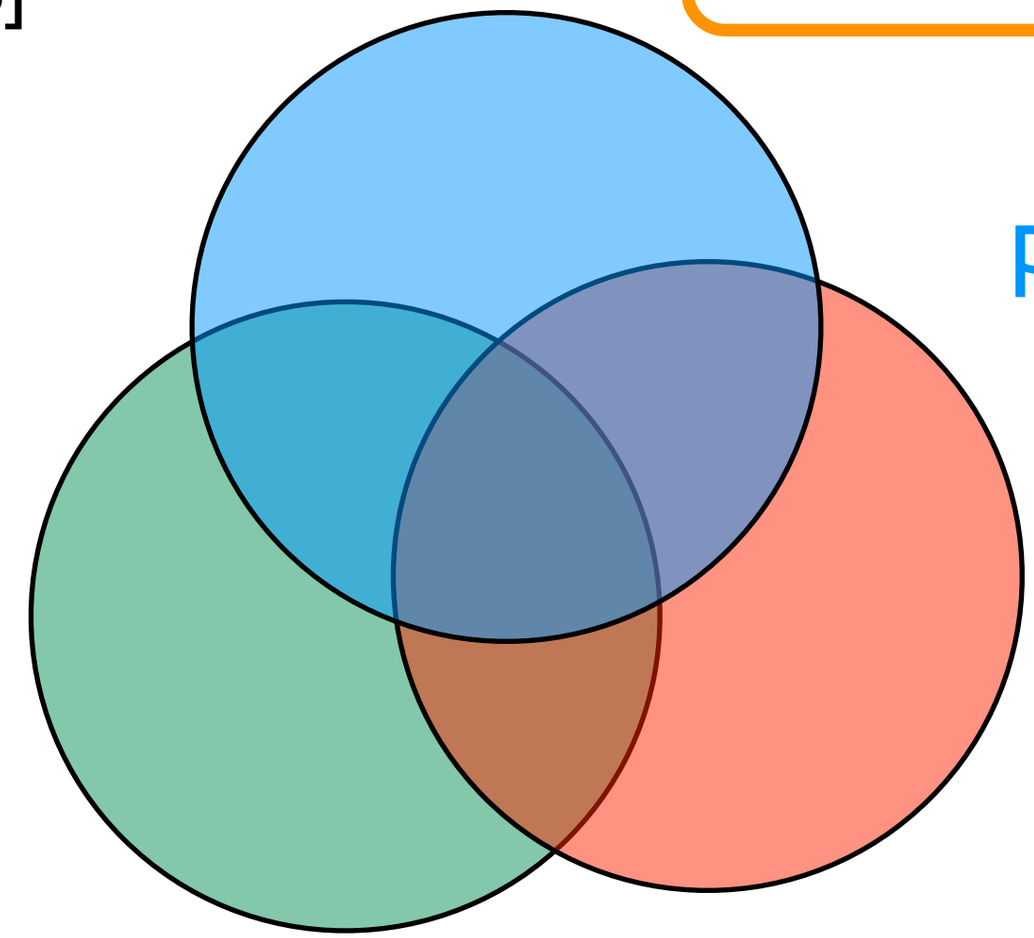

random primitives

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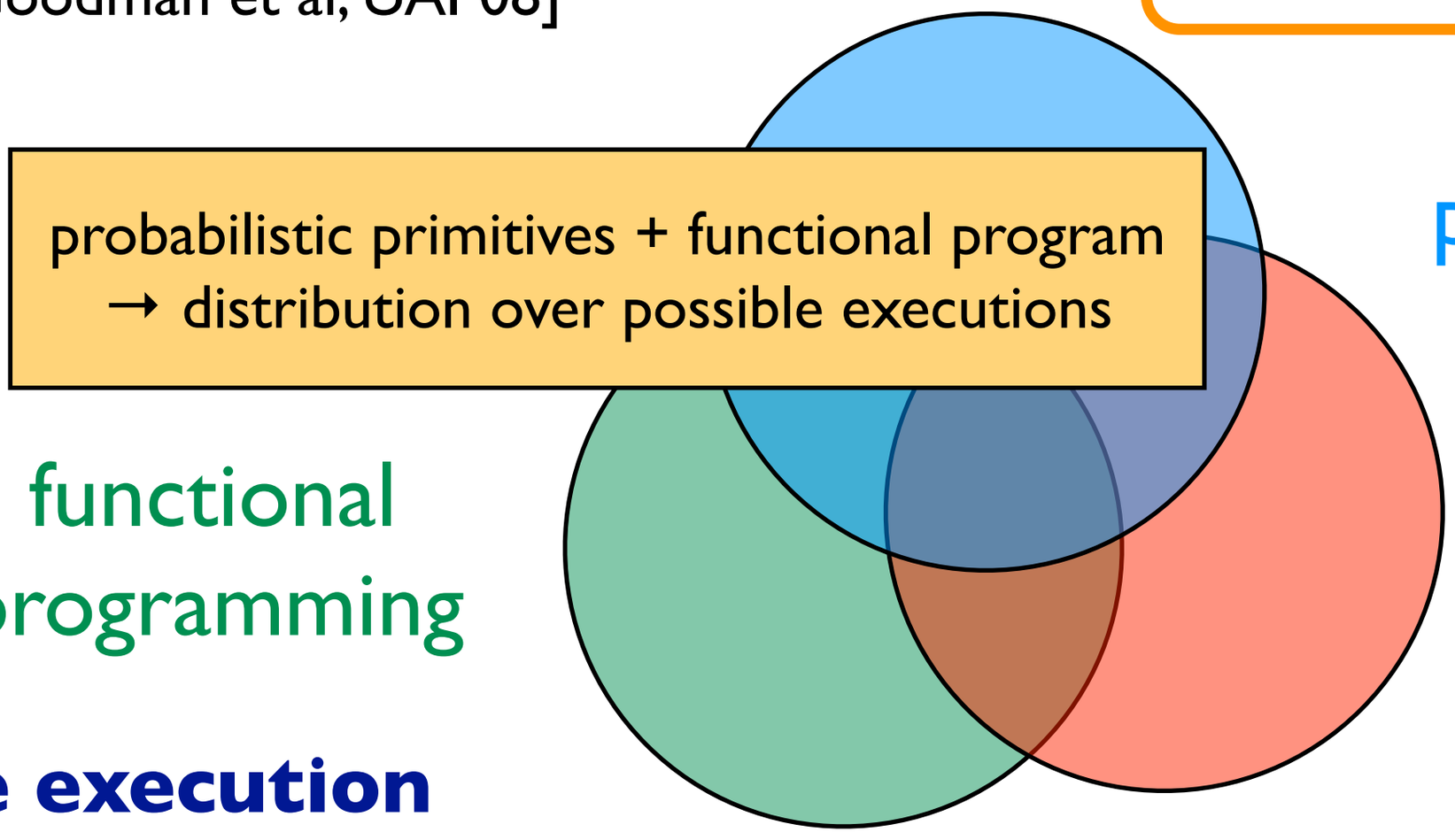

one execution

```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

Church vs ProbLog

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 2))
```

Church vs ProbLog

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 2))
```

Church result: (1 2) with 0.4×0.4
(1 7) with 0.4×0.6
(6 2) with 0.6×0.4
(6 7) with 0.6×0.6

Church vs ProbLog

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 2))
```

Church result: (1 2) with 0.4×0.4
(1 7) with 0.4×0.6
(6 2) with 0.6×0.4
(6 7) with 0.6×0.6

```
0.4::p5(N,N);0.6::p5(N,M) :- M is N+5.

lp5([],[]).
lp5([N|L],[M|K]) :-
    p5(N,M),
    lp5(L,K).

query(lp5([1,2],_)).
```

Church vs ProbLog

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 2))
```

Church result: (1 2) with 0.4×0.4
(1 7) with 0.4×0.6
(6 2) with 0.6×0.4
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```
0.4::p5(N,N);0.6::p5(N,M) :- M is N+5.

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query(lp5([1,2],_)).
```

ProbLog result: (1 2) with 0.4×0.4
(1 7) with 0.4×0.6
(6 2) with 0.6×0.4
(6 7) with 0.6×0.6

results for [1,1]?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 1))
```

```
0.4::p5(N,N);0.6::p5(N,M) :- M is N+5.
lp5([],[]).
lp5([N|L],[M|K]) :-
    p5(N,M),
    lp5(L,K).

query(lp5([1,1],_)).
```

results for [1,1]?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 1))
```

Church result: (1 1) with 0.4×0.4
(1 6) with 0.4×0.6
(6 1) with 0.6×0.4
(6 6) with 0.6×0.6

```
0.4::p5(N,N);0.6::p5(N,M) :- M is N+5.
lp5([],[]).
lp5([N|L],[M|K]) :-
    p5(N,M),
    lp5(L,K).

query(lp5([1,1],_)).
```

results for [1,1]?

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```

ProbLog result: (1 1) with 0.4
(1 6) with 0.0
(6 1) with 0.0
(6 6) with 0.6

results for [1,1]?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

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ProbLog result: (1 1) with 0.4
(1 6) with 0.0
(6 1) with 0.0
(6 6) with 0.6

stochastic memoization

Solution

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 1))
```

```
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Solution

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

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```
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identifier distinguishes calls

Stochastic Memoization

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(define randplus5 (mem (lambda (x) (if (flip 0.6) (+ x 5) x))))

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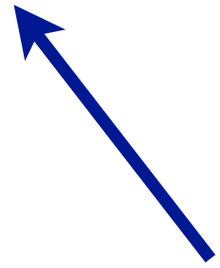

remember first value &
reuse for all later calls

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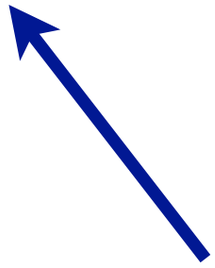

remember first value &
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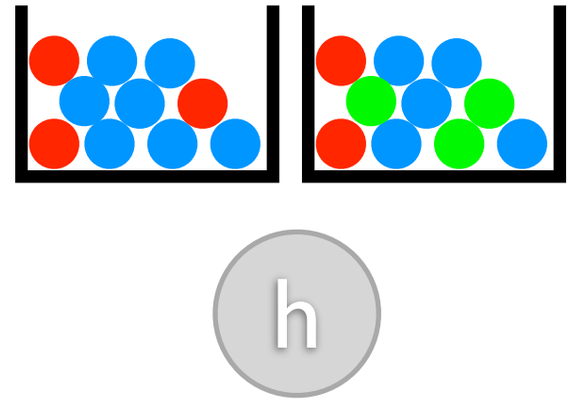
ProbLog always memoizes
PRISM never memoizes
Church allows fine-grained choice

Church by example:

A bit of gambling



- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

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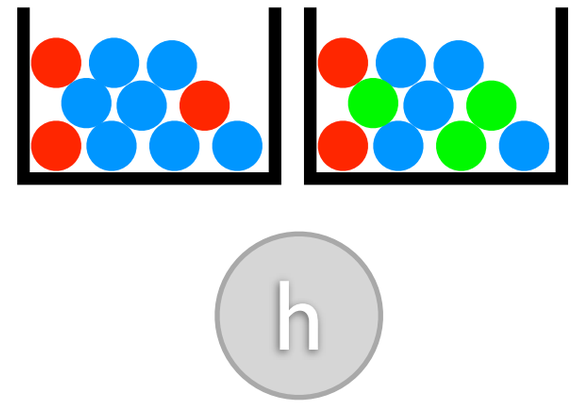

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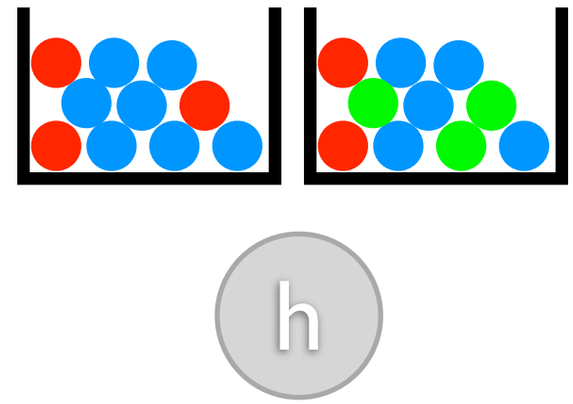

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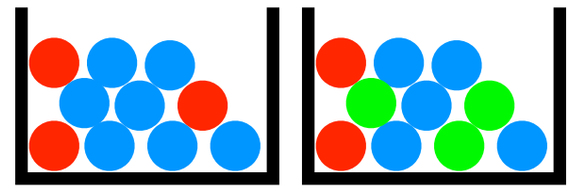



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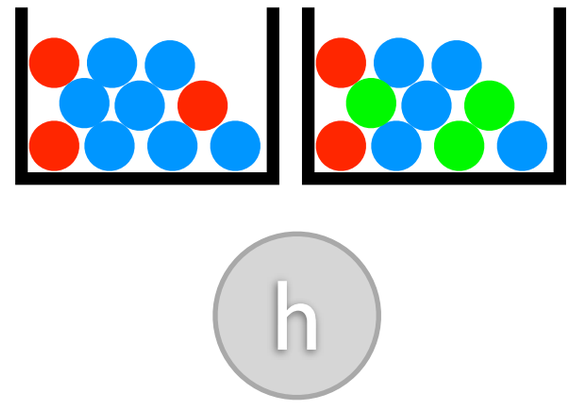

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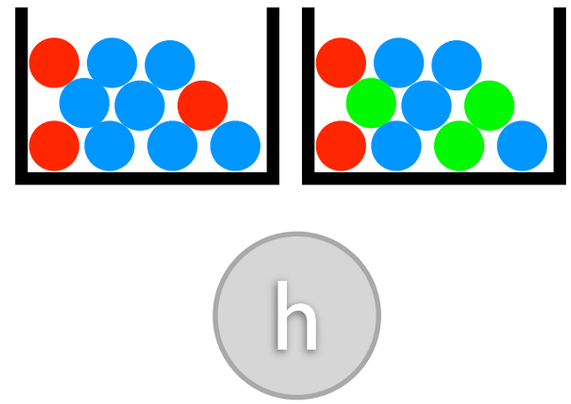

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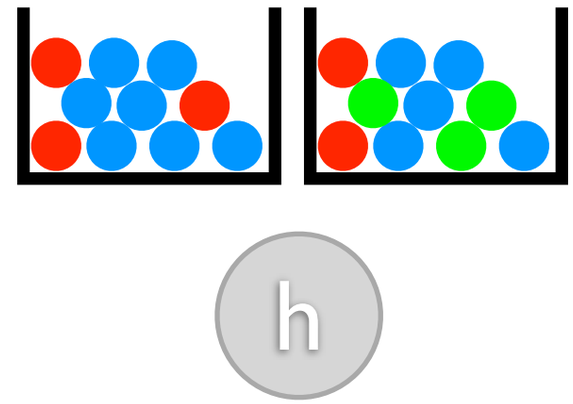

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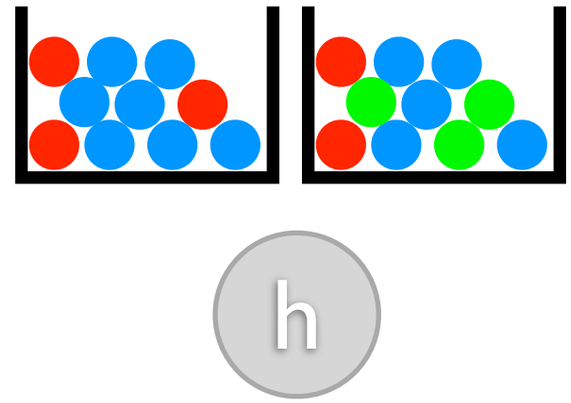

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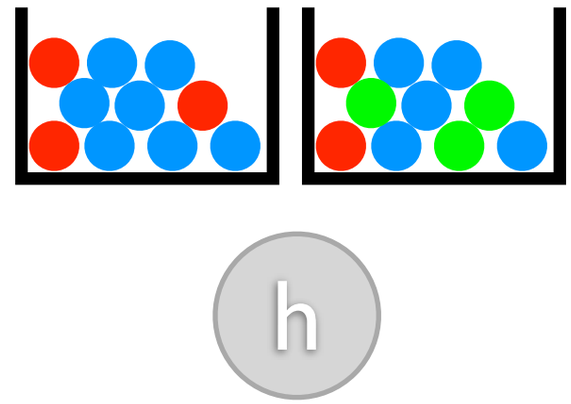

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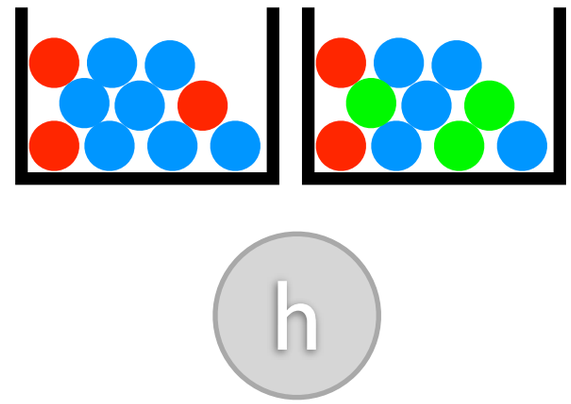

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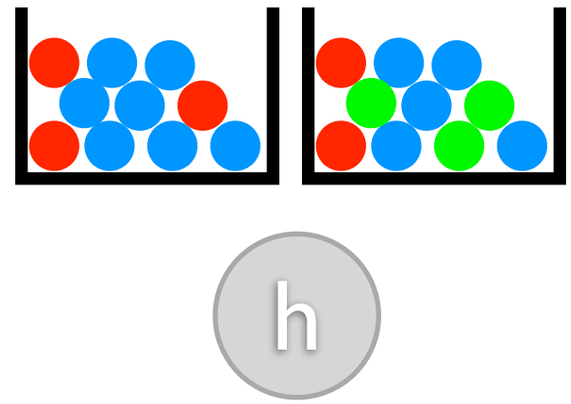

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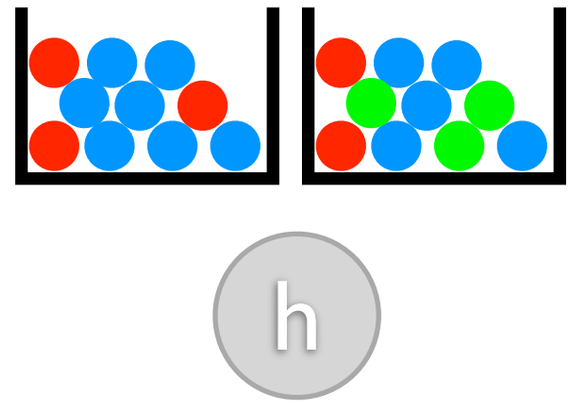

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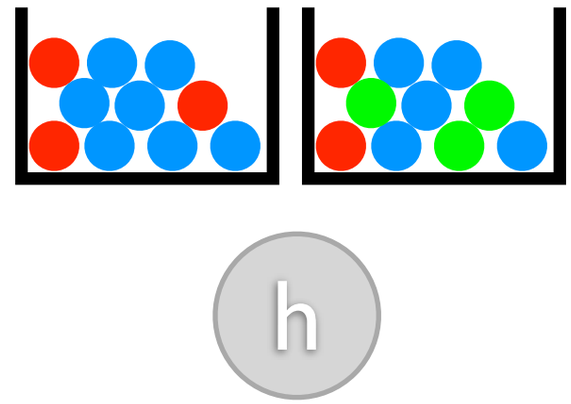

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```

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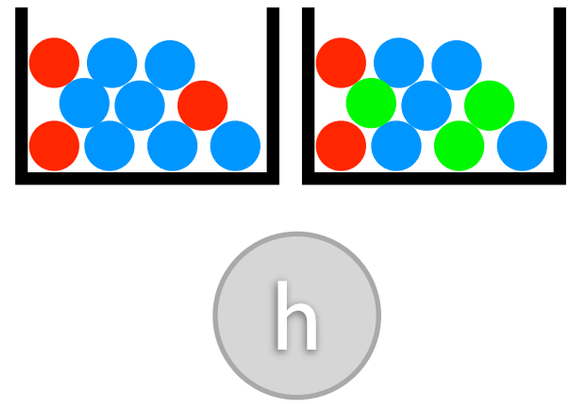

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```

Sampling execution

```
(define heads (mem (lambda () (flip 0.4))))

(define color1 (mem (lambda () (if (flip 0.3) 'red 'blue))))

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                      (multinomial '(red green blue) '(0.2 0.3 0.5)))))

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(define win1 (and (heads) redball))

(define win2 (equal? (color1) (color2)))

(define win (or win1 win2))

win
```



query

Marginals via enumeration

```
(enumeration-query
  (define heads (mem (lambda () (flip 0.4))))
  (define color1 (mem (lambda () (if (flip 0.3) 'red 'blue))))
  (define color2 (mem (lambda ()
                        (multinomial '(red green blue) '(0.2 0.3 0.5)))))
(define redball (or (equal? (color1) 'red) (equal? (color2) 'red)))
(define win1 (and (heads) redball))
(define win2 (equal? (color1) (color2)))
(define win (or win1 win2))

win ← query

true )
  ← evidence
```

Histogram via sampling

```
(repeat 1000 (lambda ()
               (rejection-query

  (define heads (mem (lambda () (flip 0.4))))

  (define color1 (mem (lambda () (if (flip 0.3) 'red 'blue))))

  (define color2 (mem (lambda ()
                        (multinomial '(red green blue) '(0.2 0.3 0.5)))))

(define redball (or (equal? (color1) 'red) (equal? (color2) 'red)))

(define win1 (and (heads) redball))

(define win2 (equal? (color1) (color2)))

(define win (or win1 win2))

win

true ))))
```

query

evidence

Probabilistic Programming Summary

- Church: functional programming + random primitives
- probabilistic generative model
- stochastic memoization
- sampling
- increasing number of probabilistic programming languages using various underlying paradigms

ProbLog	**PRISM**	**Church**
probabilistic facts & choices	probabilistic choices	random primitives
all RVs memoized	no RVs memoized	user-defined per RV
Prolog	Prolog with mutually exclusive derivations	λ-calculus functions
distribution over worlds	distribution over derivations / answers	distribution over computations / answers

Roadmap

- Modeling
- Inference
- Learning & Dynamics
- Advanced Inference and KBMC

... with some detours on the way

PART II : Inference

Reasoning

- Exact inference with knowledge compilation
  - using proofs
  - using models
  - in PRISM
- Approximate inference

Answering Questions

Given:

program

queries

evidence

?

Find:

marginal probabilities

conditional probabilities

MPE state

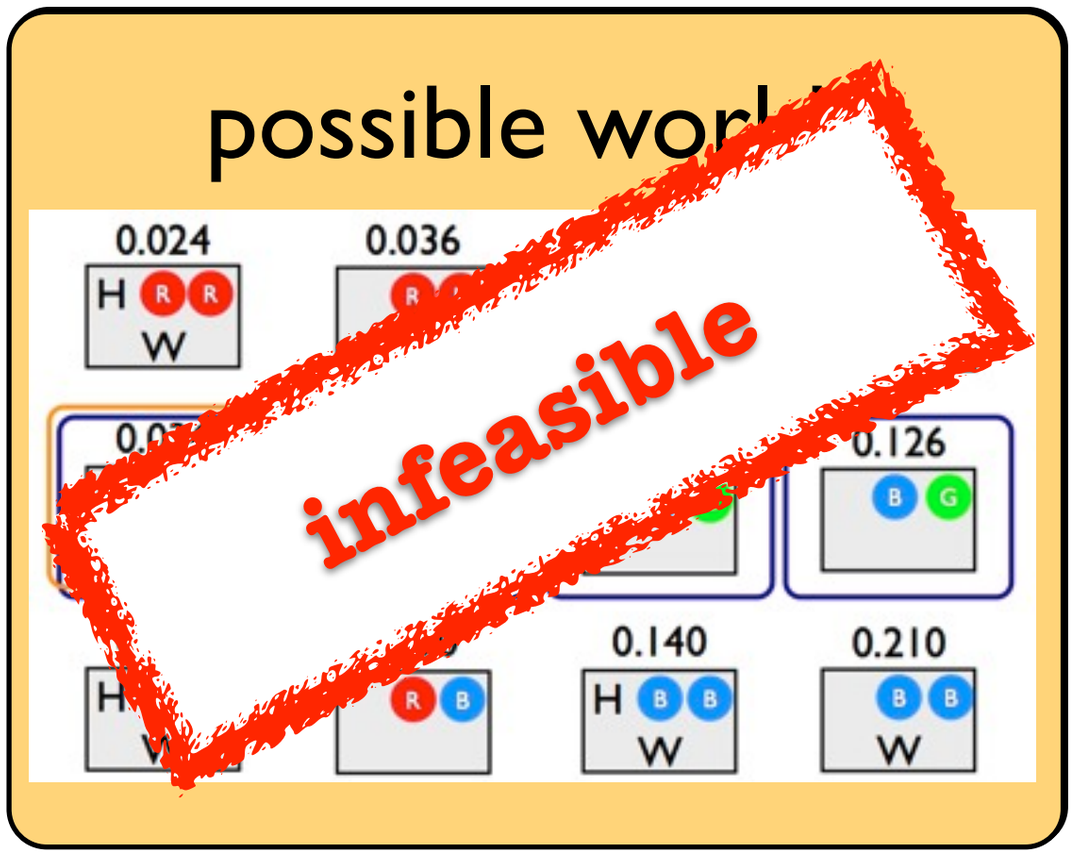
Answering Questions

Given:

program

queries

evidence



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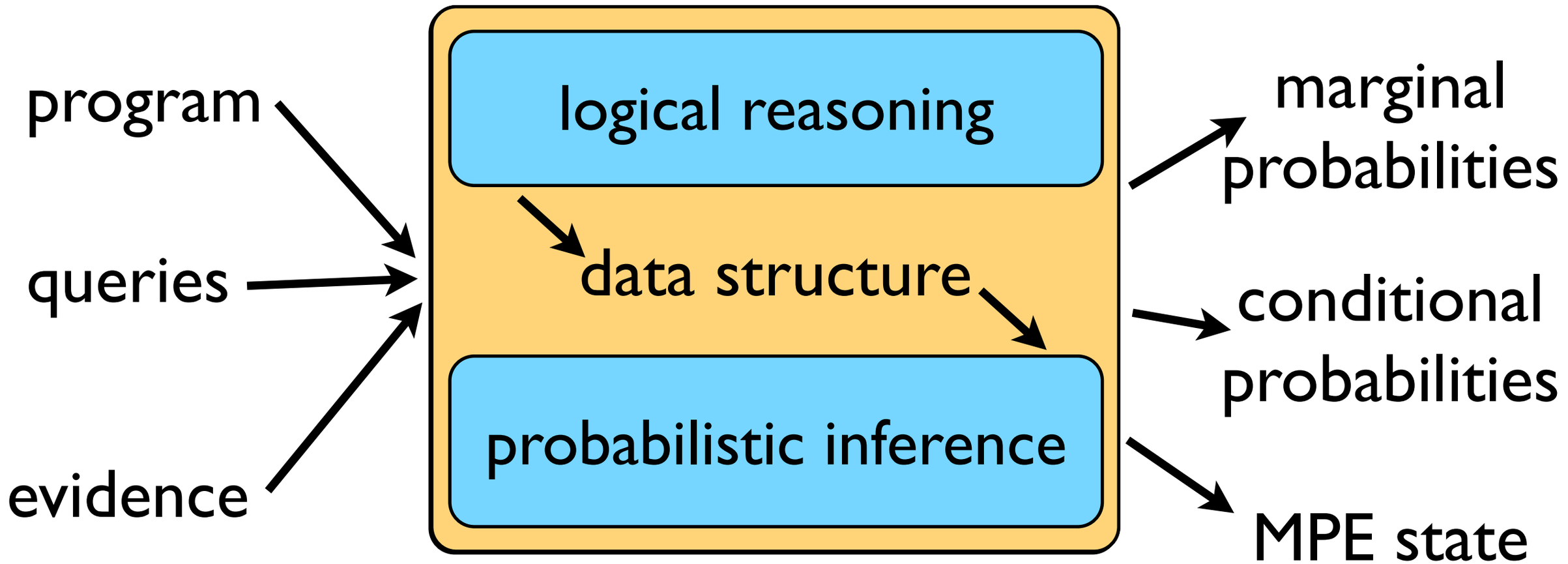

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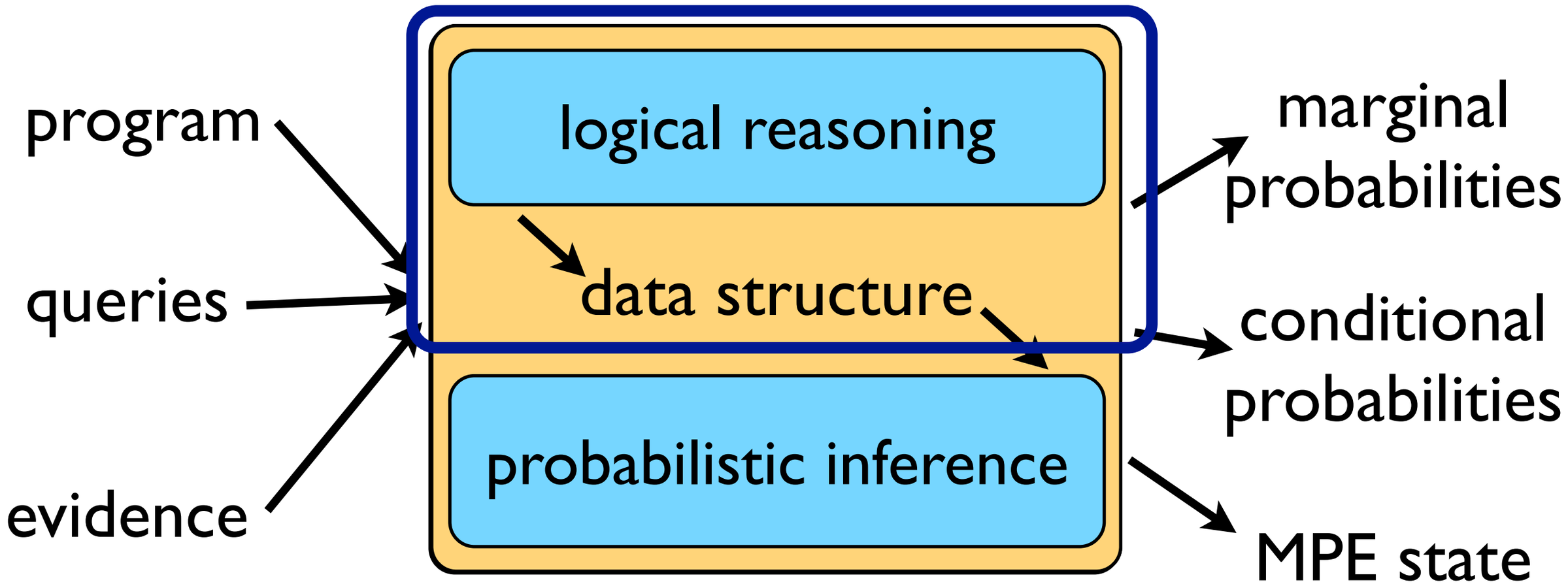

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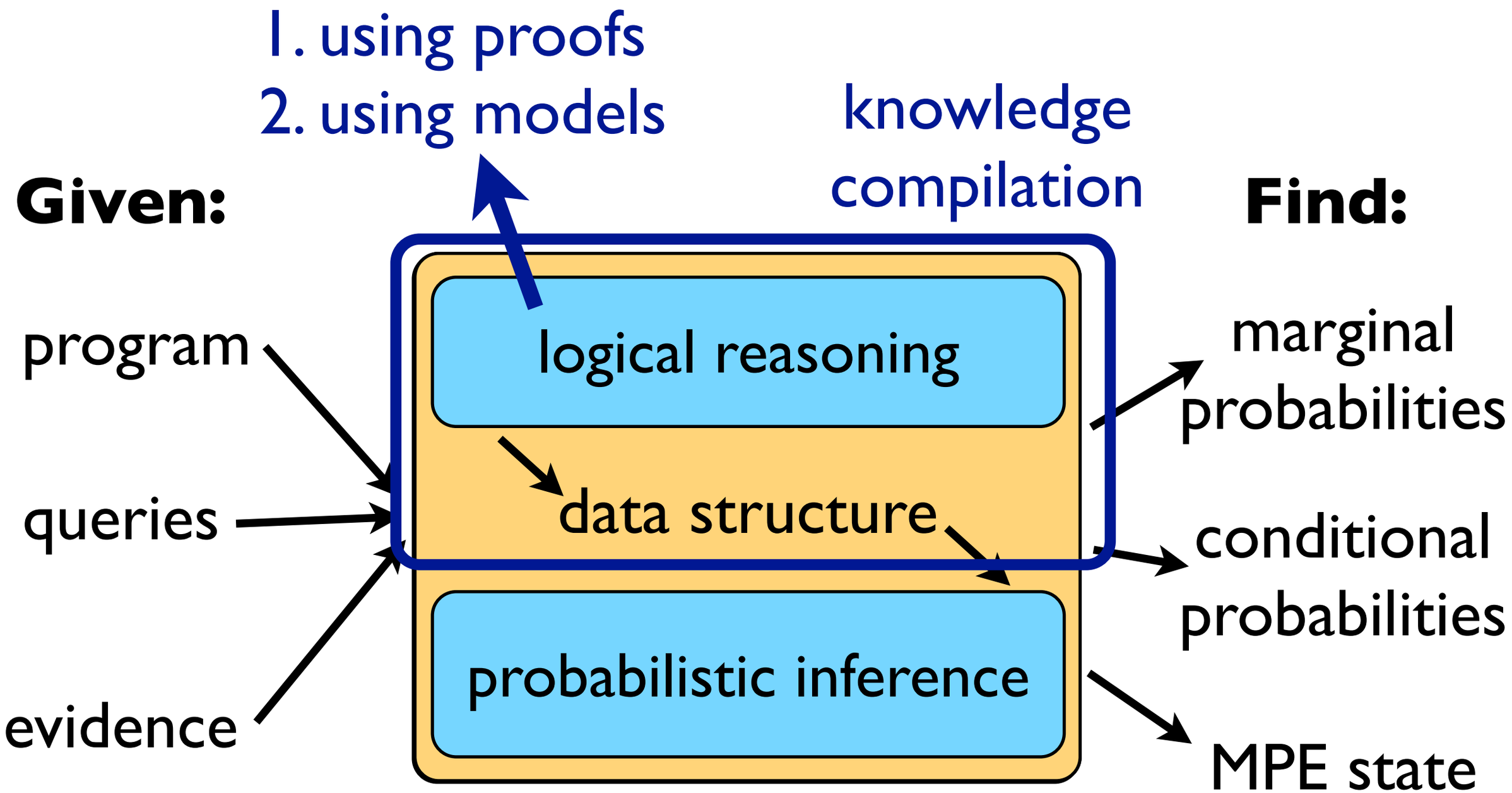

Logical Reasoning: Proofs in Prolog

```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
      influences(Y,X),
      smokes(Y).
```

Logical Reasoning: Proofs in Prolog

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?- smokes(carl).
```

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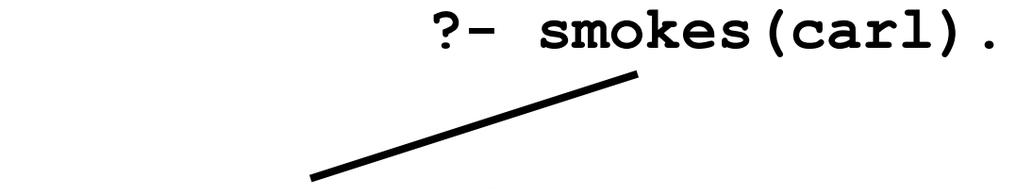

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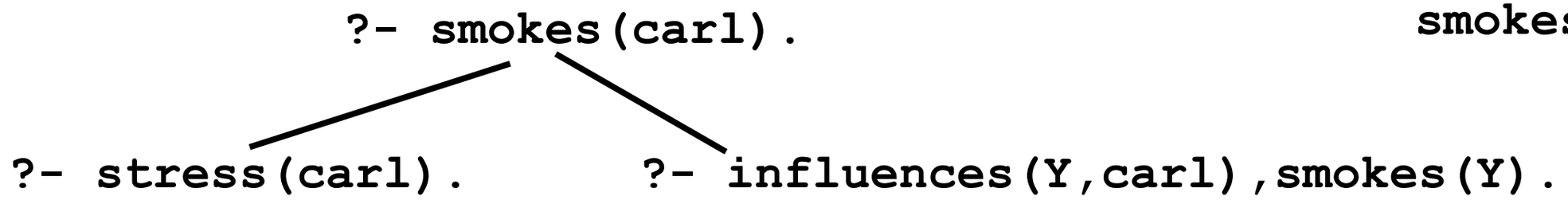

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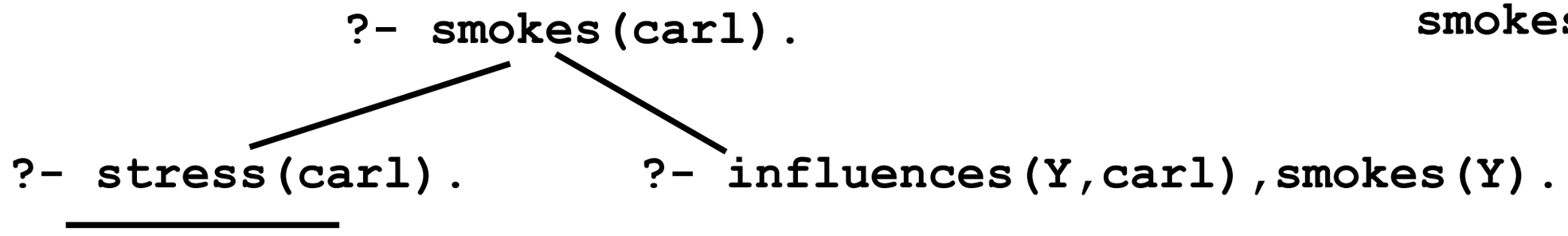

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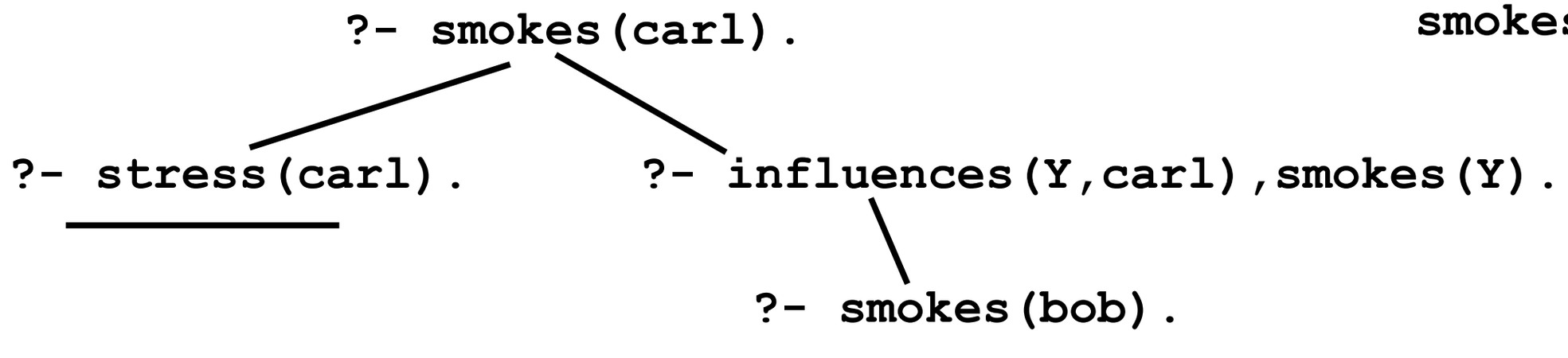

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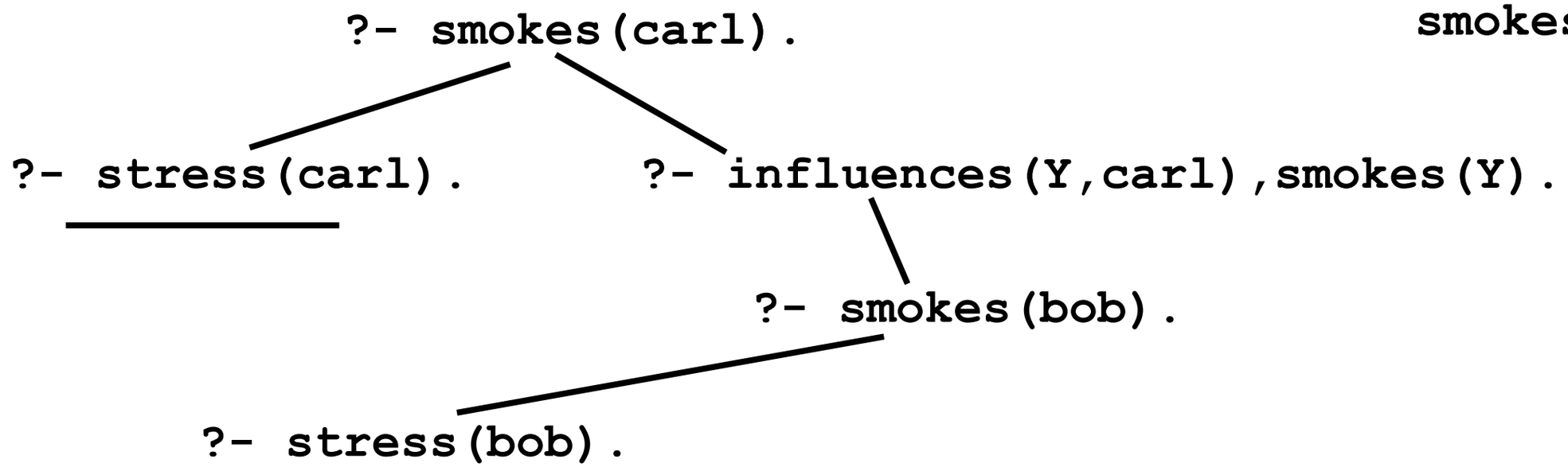

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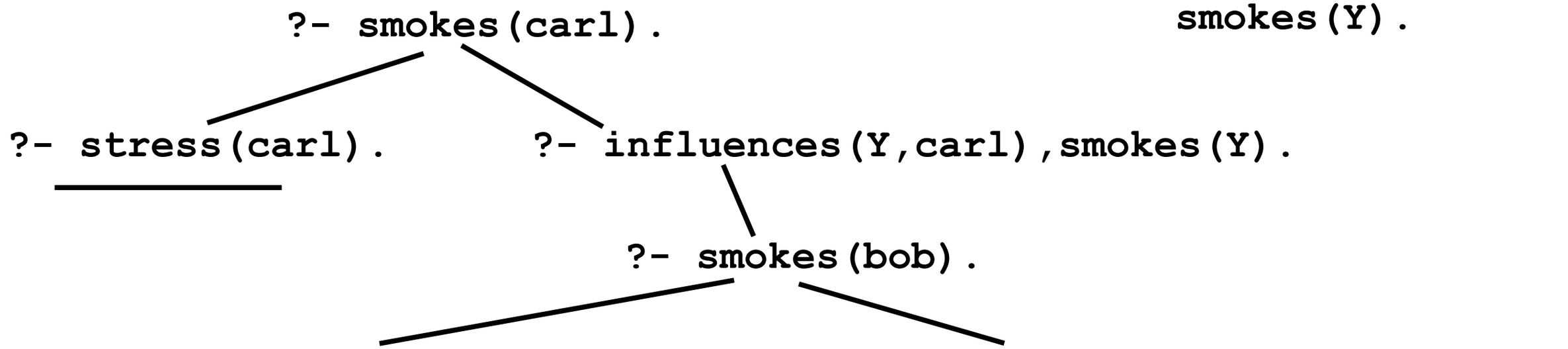

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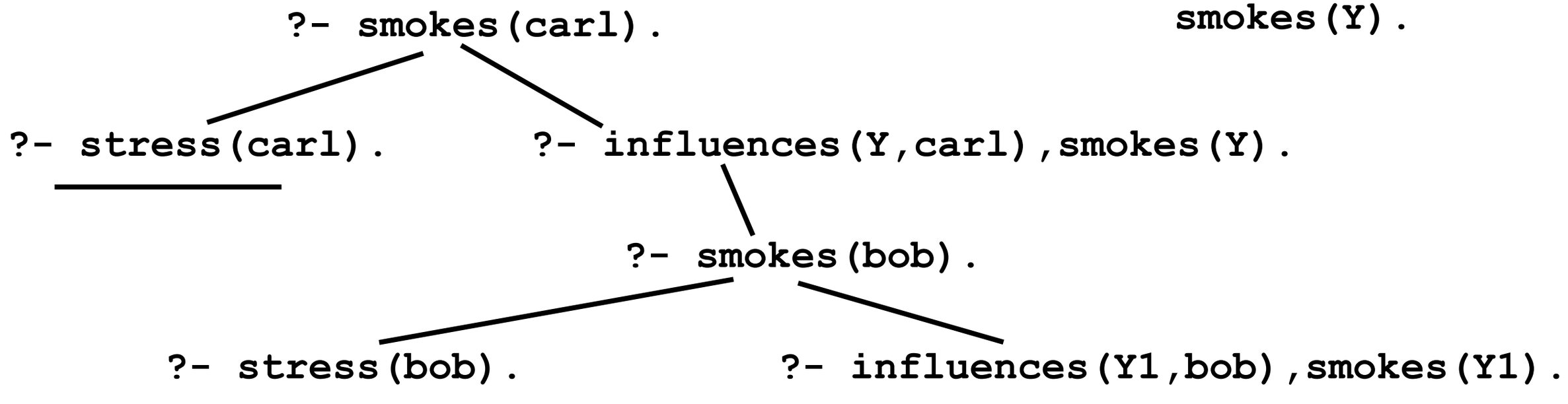
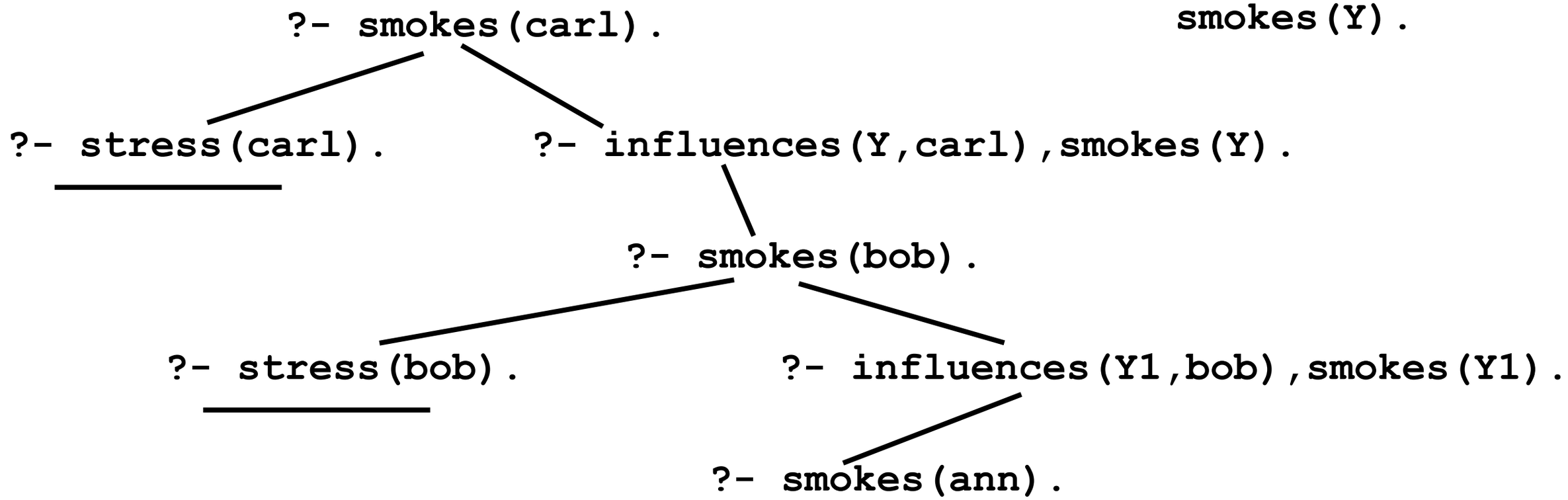


Logical Reasoning: Proofs in Prolog

```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X)  :- stress(X).
smokes(X)  :-
      influences(Y,X),
      smokes(Y).
```



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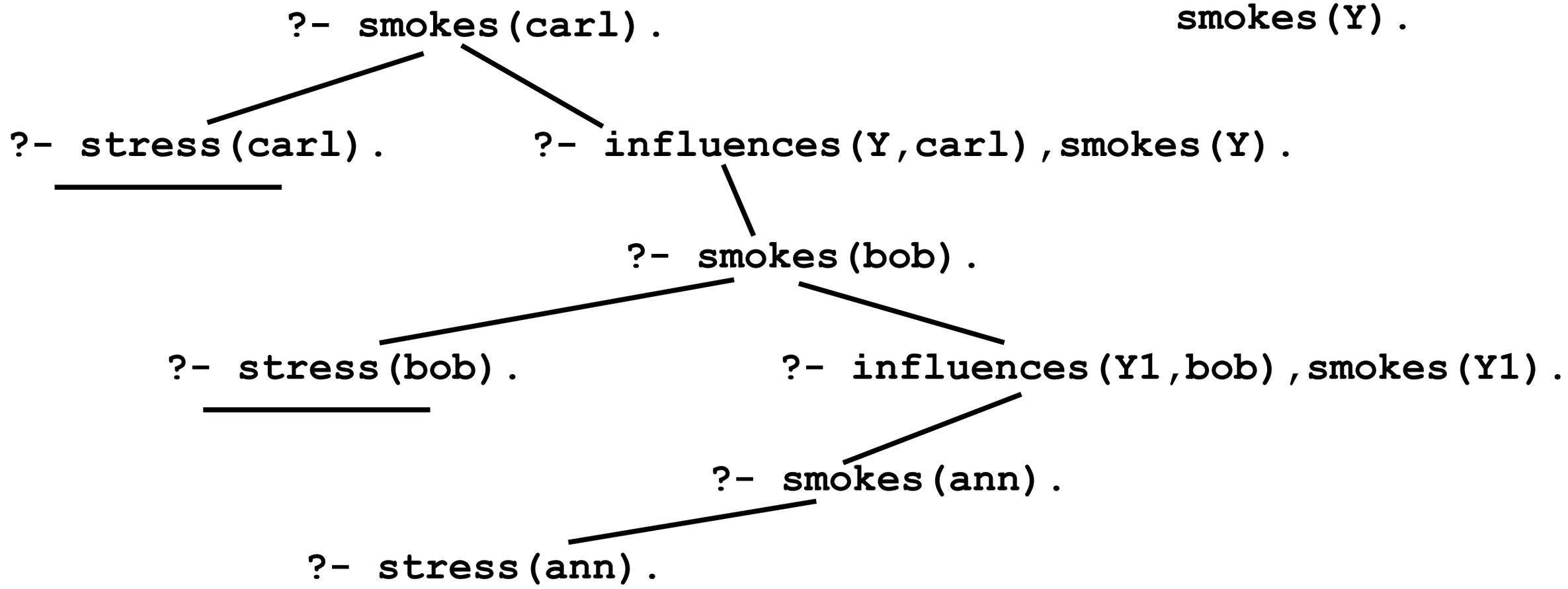

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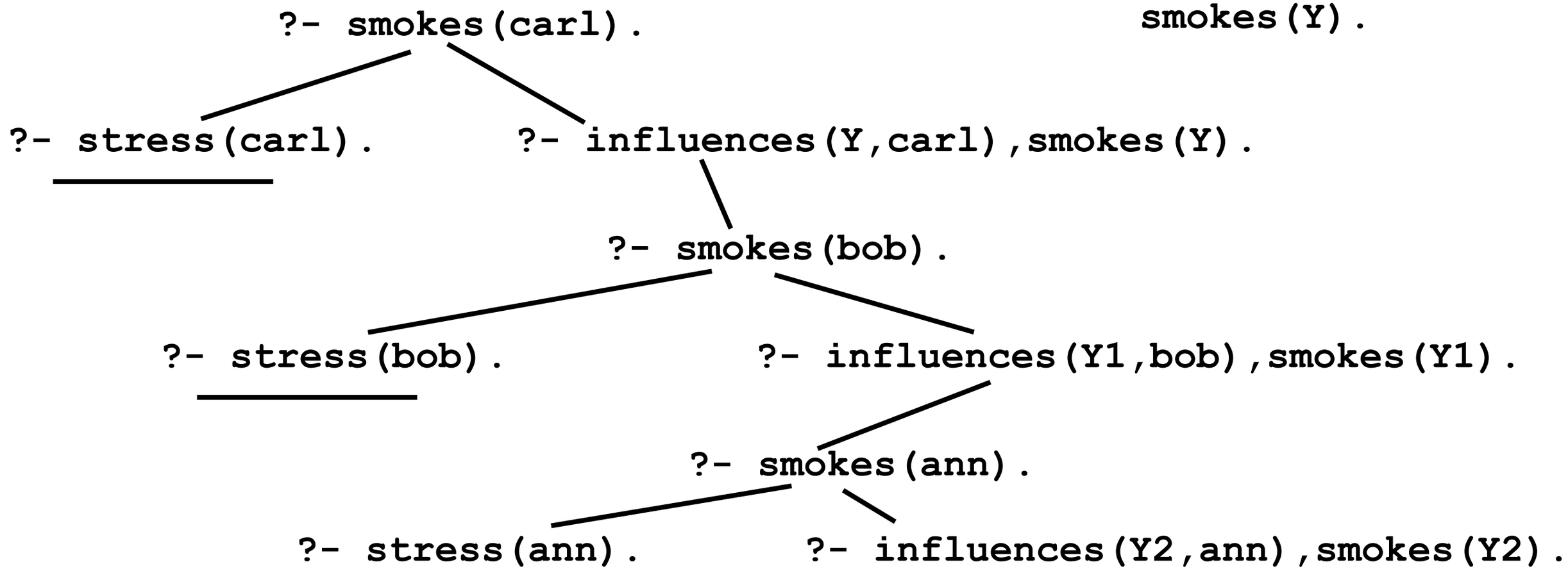

Logical Reasoning: Proofs in Prolog

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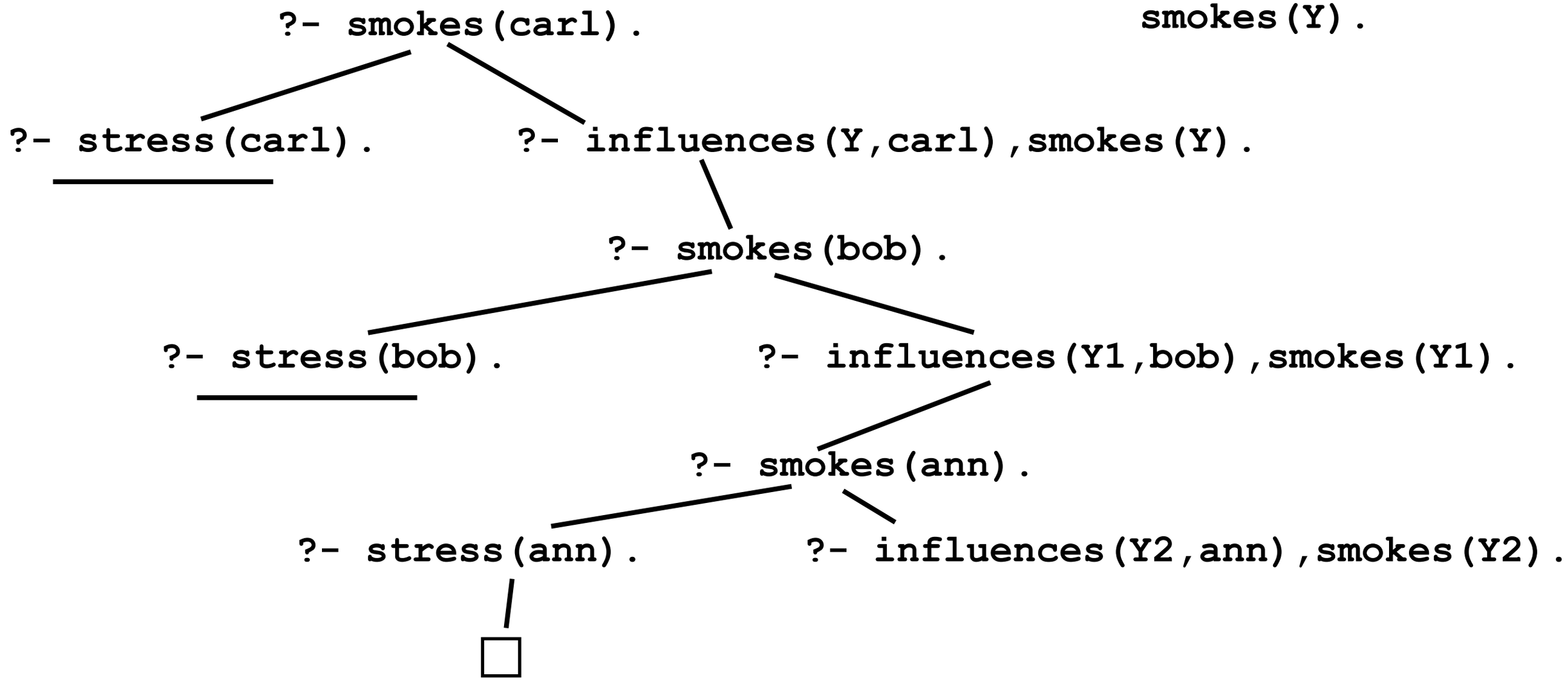

Logical Reasoning: Proofs in Prolog

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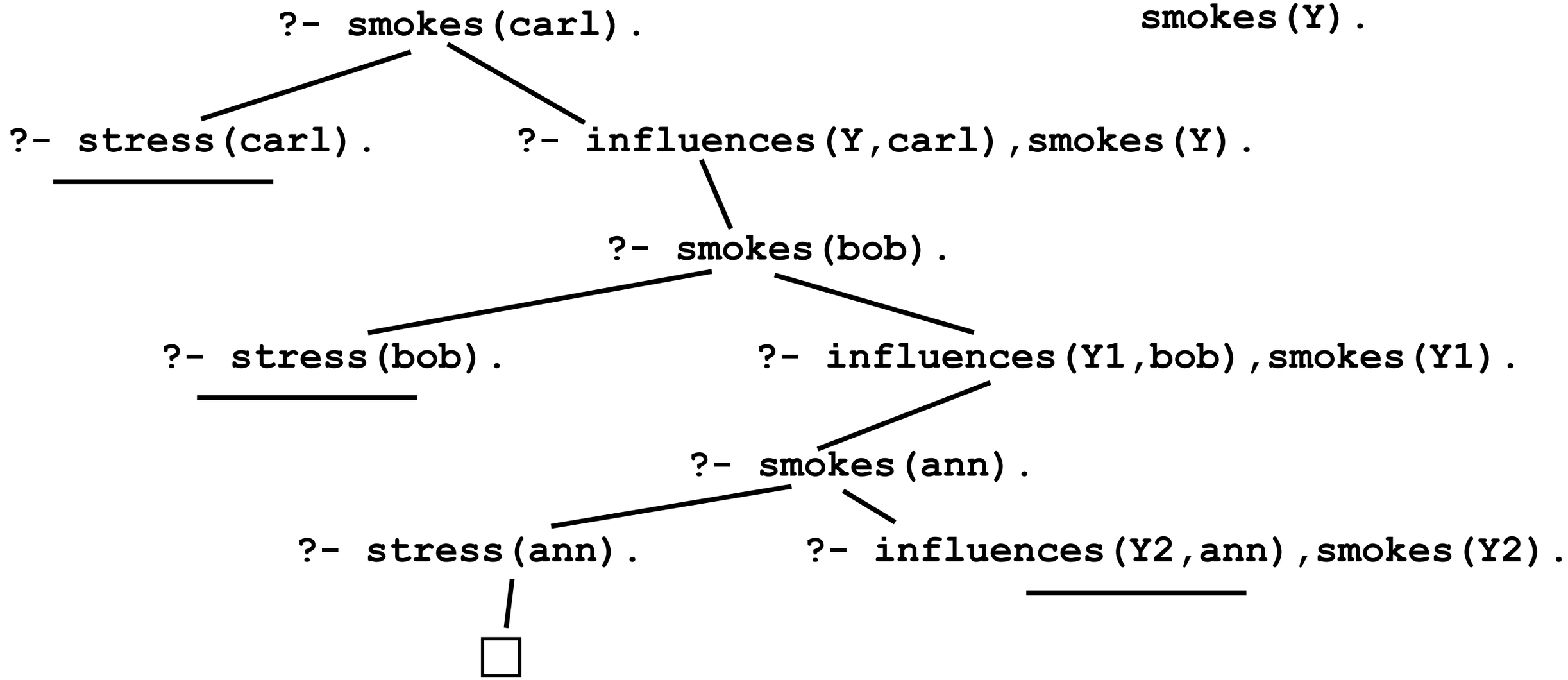

Logical Reasoning: Proofs in Prolog

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stress(ann).
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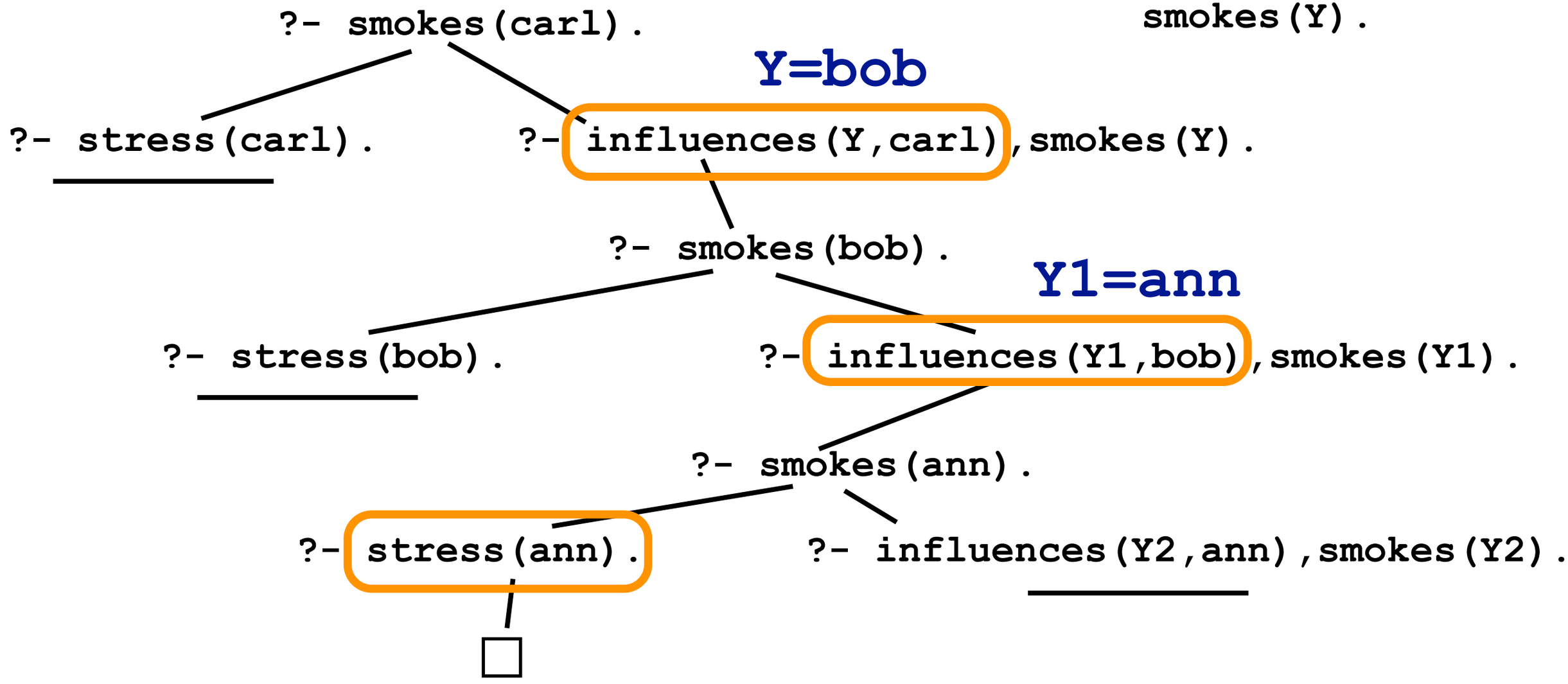


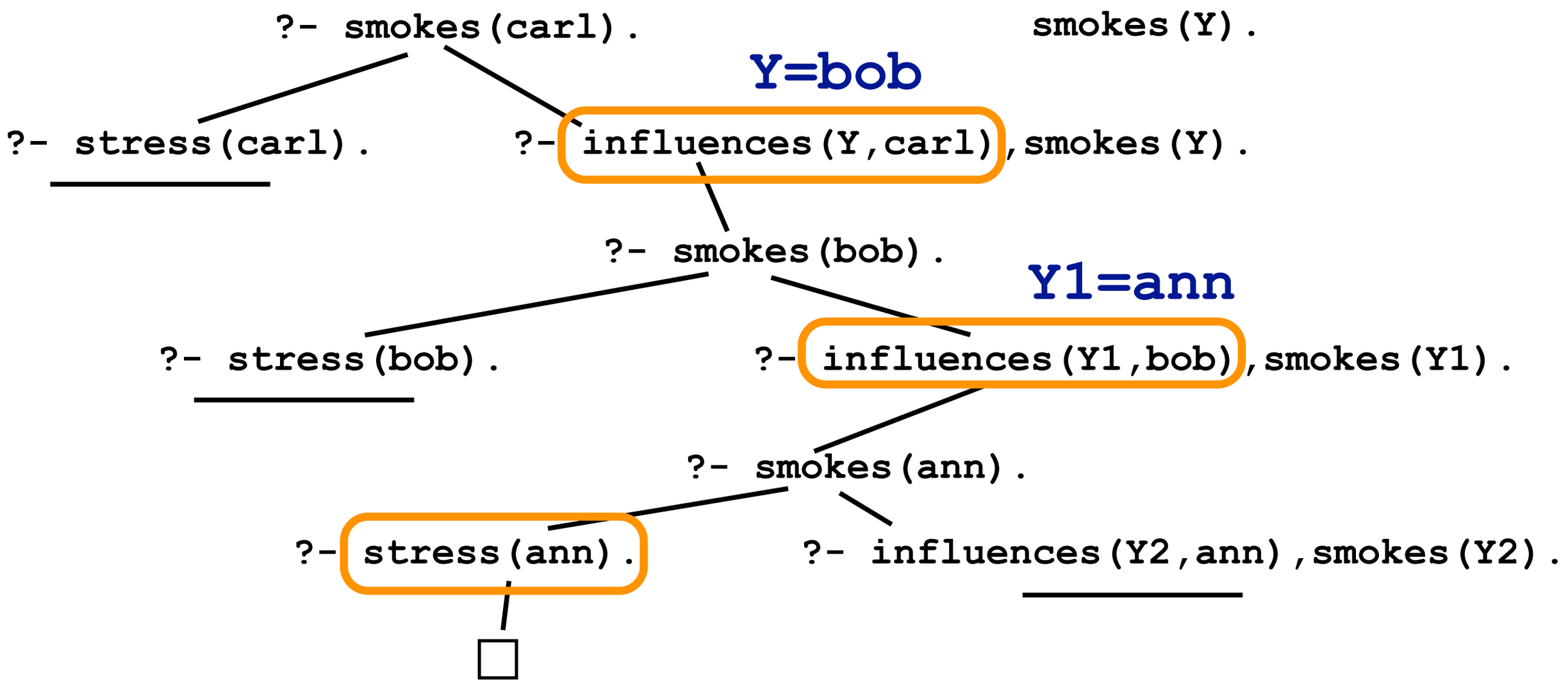
proof = facts used in successful derivation:

`influences(bob,carl) &influences(ann,bob) &stress(ann)`

Proofs in ProbLog

```
0.8::stress(ann).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X),
     smokes(Y).
```



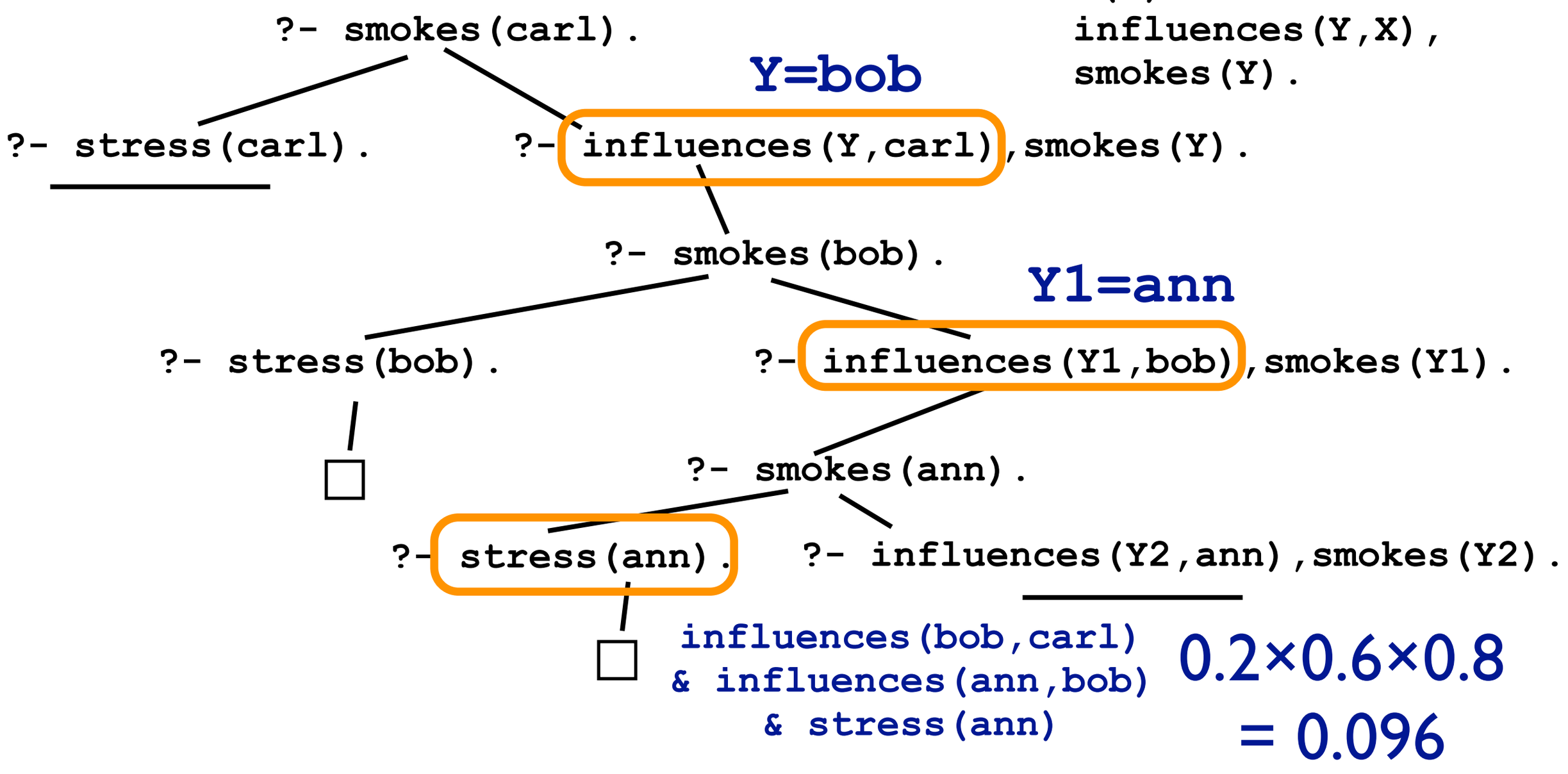
`influences(bob,carl)&influences(ann,bob)&stress(ann)`

probability of proof = 0.2 × 0.6 × 0.8 = 0.096

Proofs in ProbLog

```
0.8::stress(ann).
0.4::stress(bob).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X),
     smokes(Y).
```



Proofs in ProbLog

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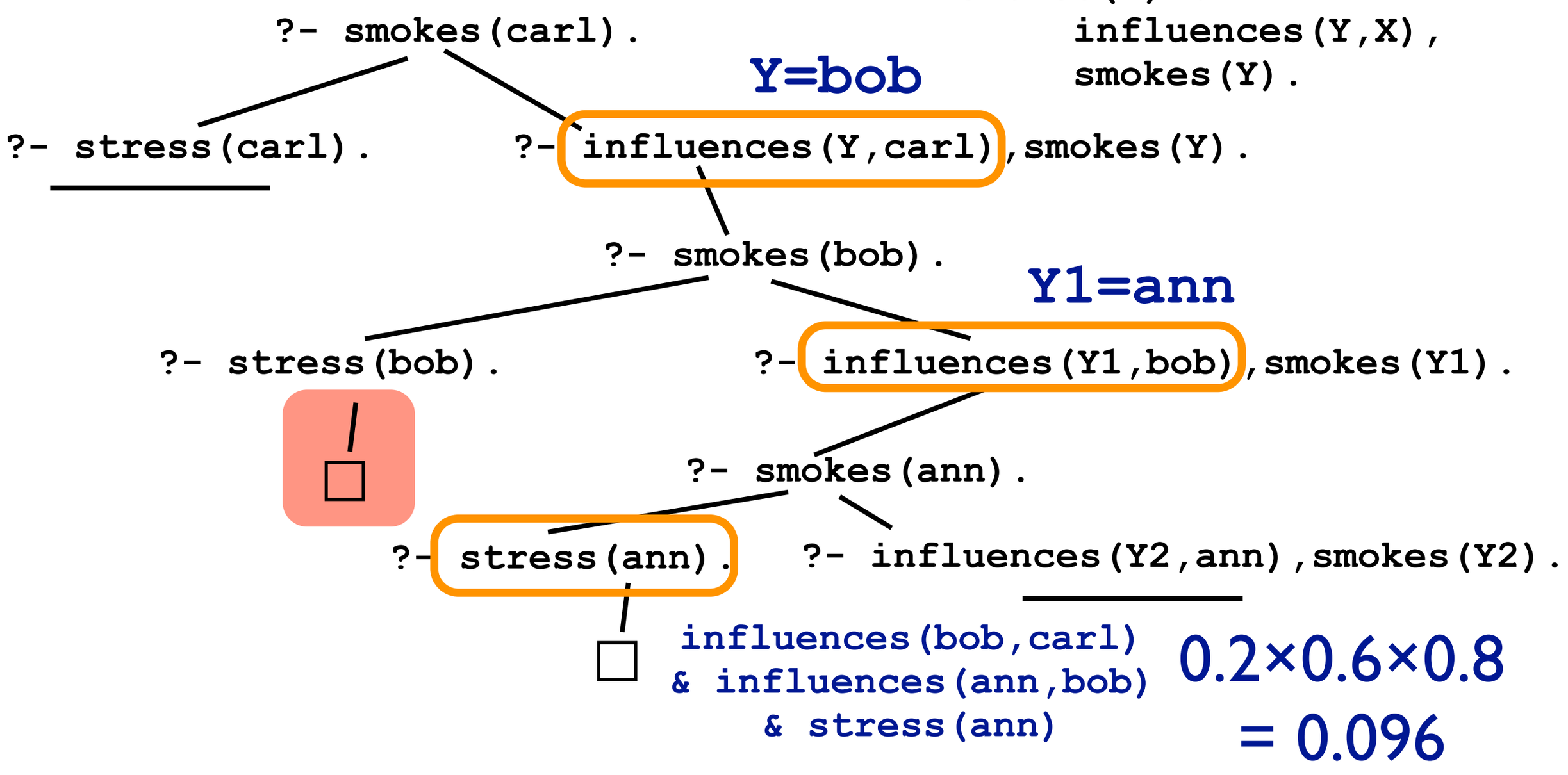
```



Proofs in ProbLog

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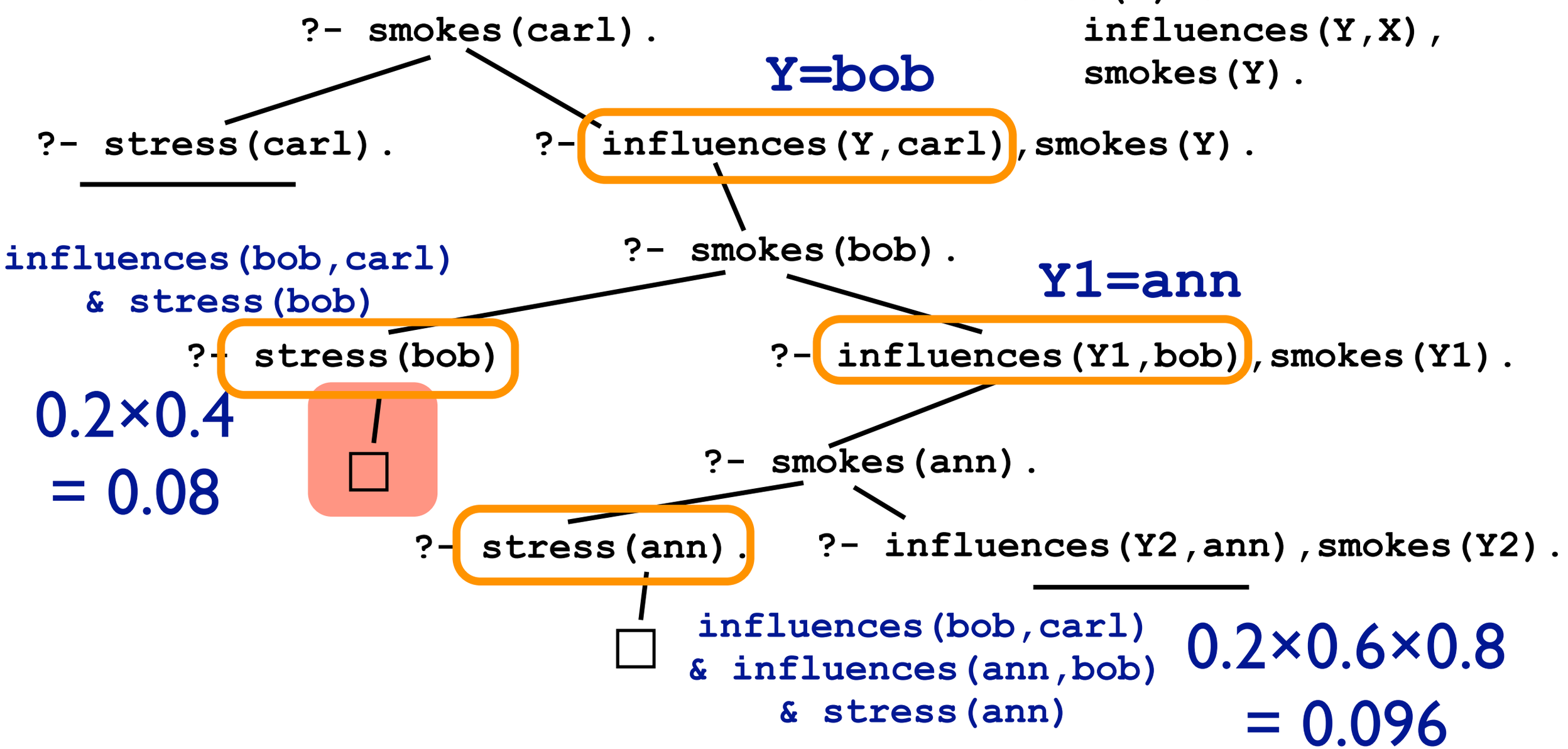

Proofs in ProbLog

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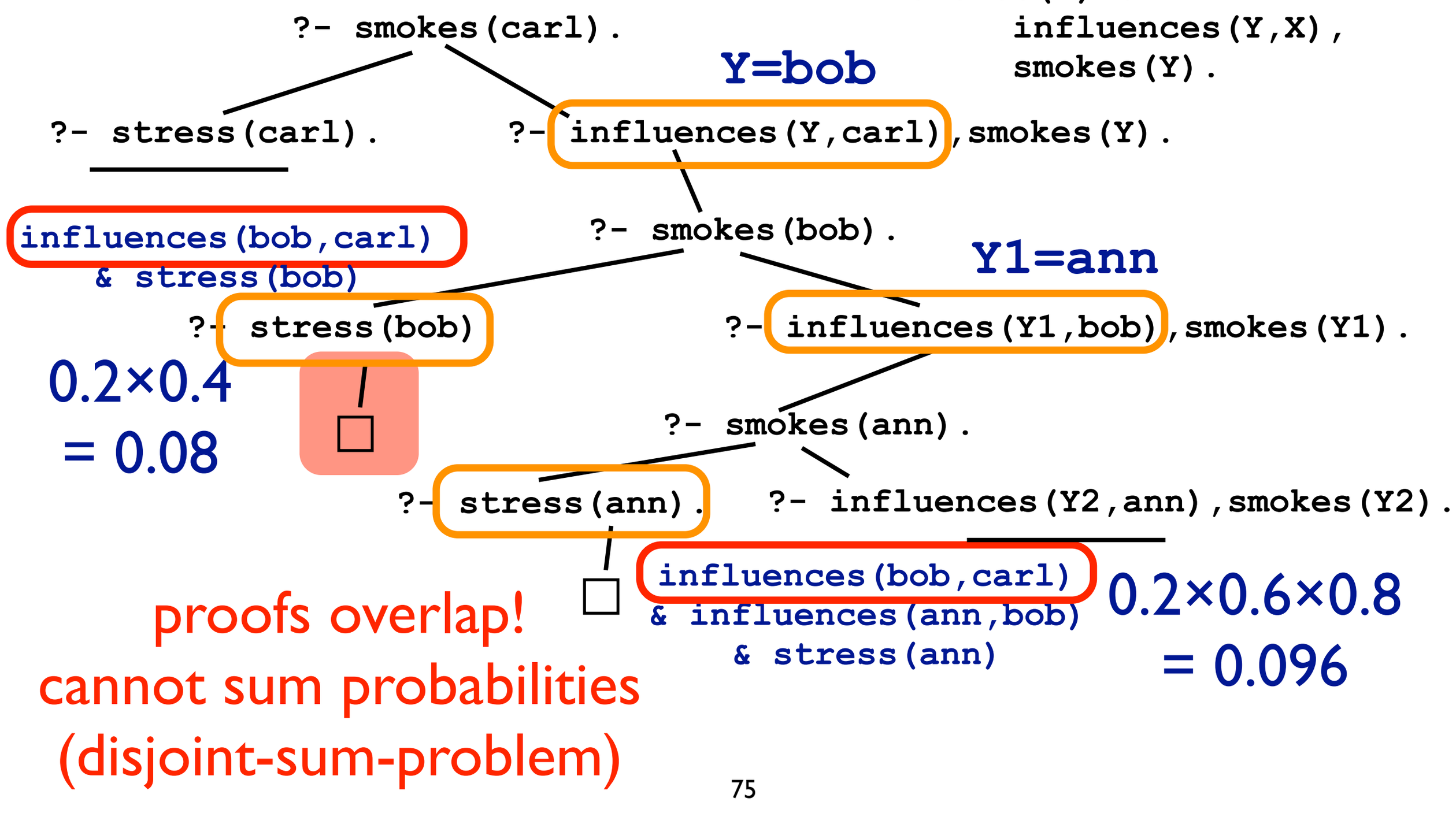

proofs overlap!
cannot sum probabilities
(disjoint-sum-problem)

Disjoint-Sum-Problem

possible worlds

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)

infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)

infl(bob,carl) & \+infl(ann,bob) &   st(ann) &   st(bob)

infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)

infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)

...
```

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)

infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)

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infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)

infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)

...
```

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
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infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)
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infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)
```

... **influences(bob,carl) & stress(bob)**

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

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infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)
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infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)
```

... **influences(bob,carl) & stress(bob)**

sum of proof probabilities: 0.096+0.08 = 0.1760

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

`infl(bob,carl) & infl(ann,bob) & st(ann) & \+st(bob)`	0.0576
`infl(bob,carl) & infl(ann,bob) & st(ann) & st(bob)`	0.0384
`infl(bob,carl) & \+infl(ann,bob) & st(ann) & st(bob)`	0.0256
`infl(bob,carl) & infl(ann,bob) & \+st(ann) & st(bob)`	0.0096
`infl(bob,carl) & \+infl(ann,bob) & \+st(ann) & st(bob)`	0.0064
... **influences(bob,carl) & stress(bob)**	∑ = 0.1376

sum of proof probabilities: 0.096+0.08 = 0.1760

Disjoint-Sum-Problem

possible worlds

solution: knowledge compilation

`infl(bob,carl) & infl(ann,bob) & st(ann) & \+st(bob)`	0.0576
`infl(bob,carl) & infl(ann,bob) & st(ann) & st(bob)`	0.0384
`infl(bob,carl) & \+infl(ann,bob) & st(ann) & st(bob)`	0.0256
`infl(bob,carl) & infl(ann,bob) & \+st(ann) & st(bob)`	0.0096
`infl(bob,carl) & \+infl(ann,bob) & \+st(ann) & st(bob)`	0.0064
`...` `influences(bob,carl) & stress(bob)`	$\sum = 0.1376$

sum of proof probabilities: 0.096+0.08 = 0.1760

Binary Decision Diagrams [Bryant 86]

- compact graphical representation of Boolean formula
- popular in many branches of CS
- automatically disjoins proofs
  → efficient probability computation

Binary Decision Diagrams
[Bryant 86]

$$X \vee Y \vee Z$$

Binary Decision Diagrams
[Bryant 86]

$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

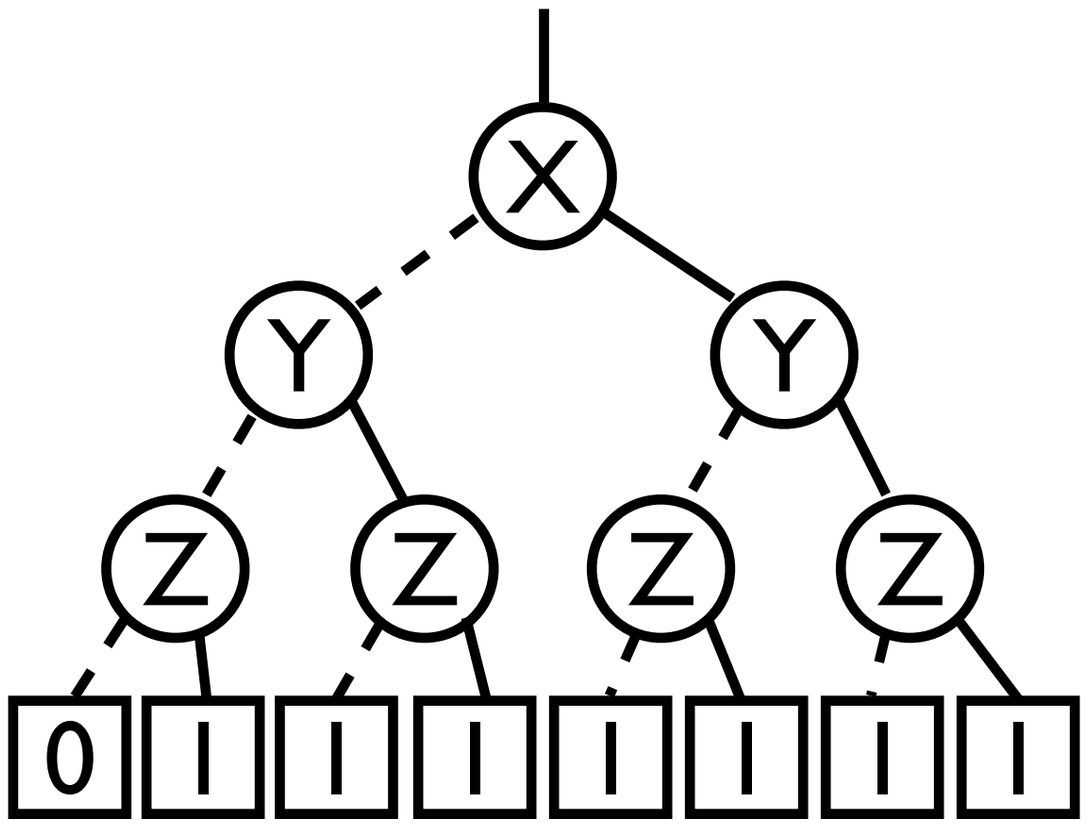
Binary Decision Diagrams
[Bryant 86]

$X \vee Y \vee Z$

X	0	0	0	0	I	I	I	I
Y	0	0	I	I	0	0	I	I
Z	0	I	0	I	0	I	0	I

Binary Decision Diagrams

[Bryant 86]

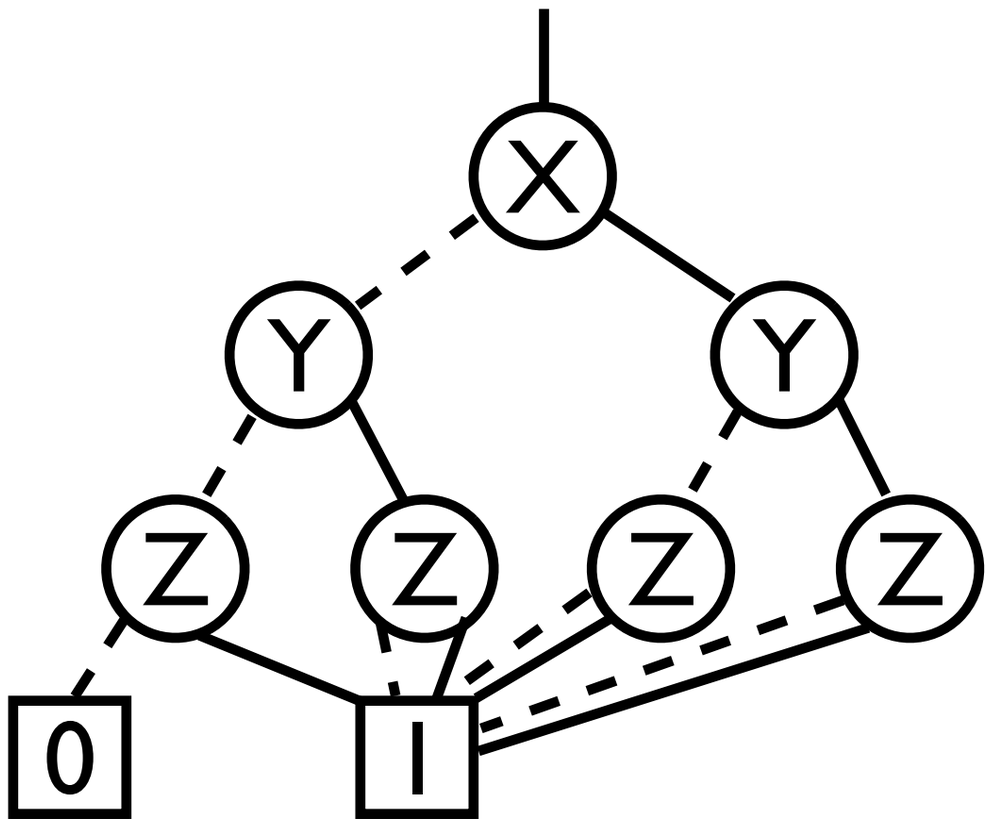


$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

[Bryant 86]



$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
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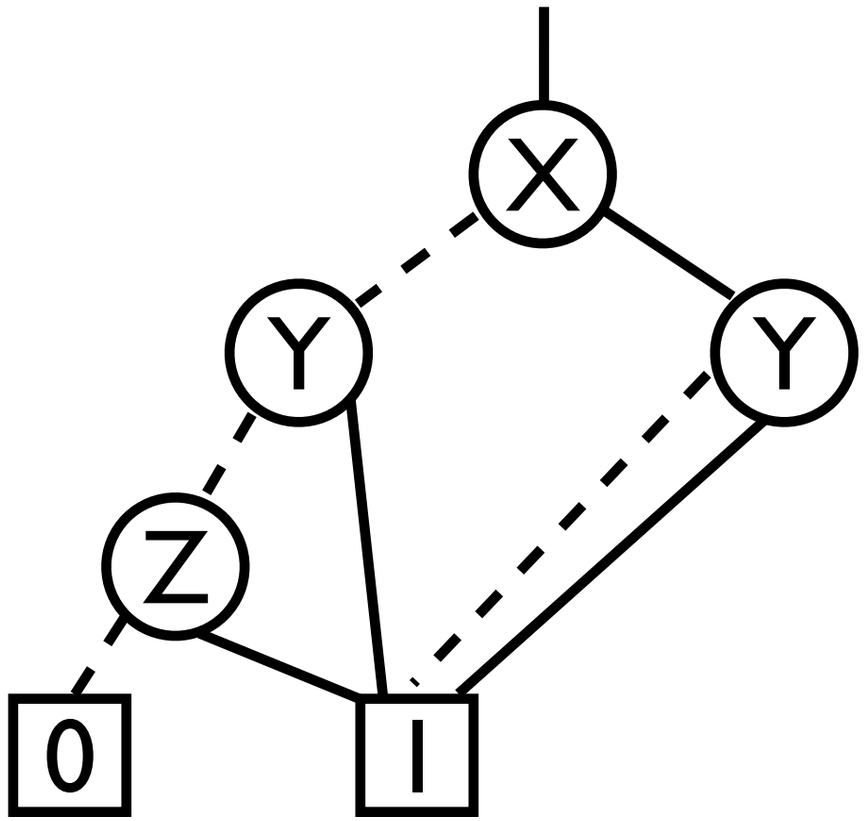
Binary Decision Diagrams

[Bryant 86]



$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
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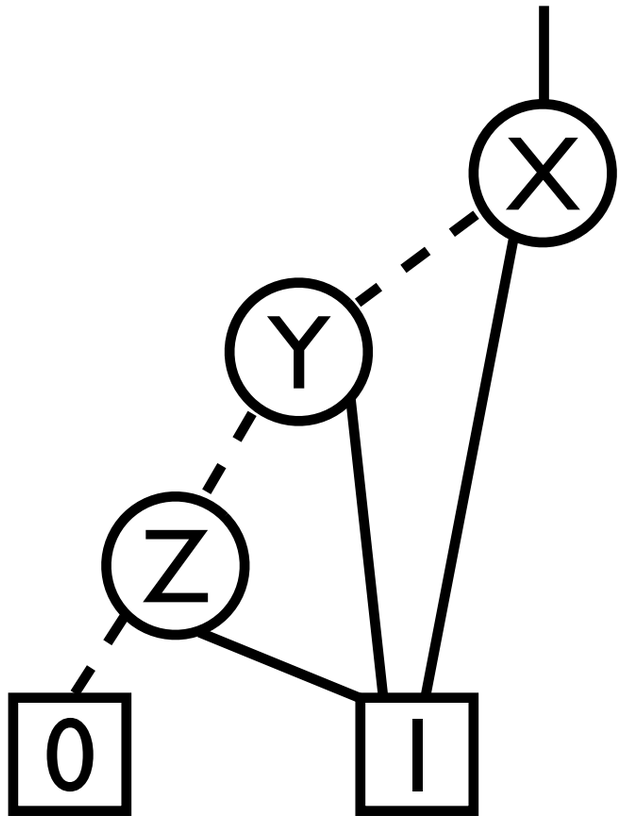
Binary Decision Diagrams

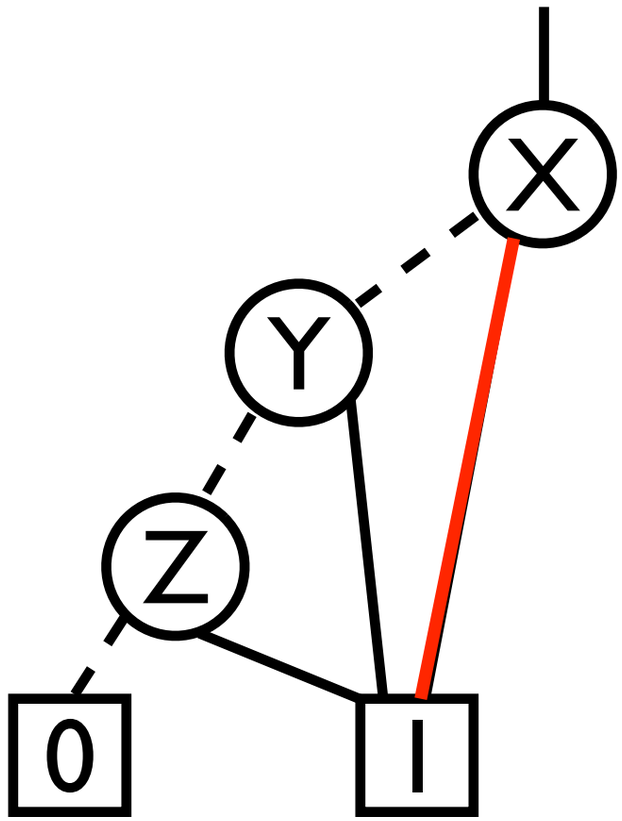
[Bryant 86]



$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]

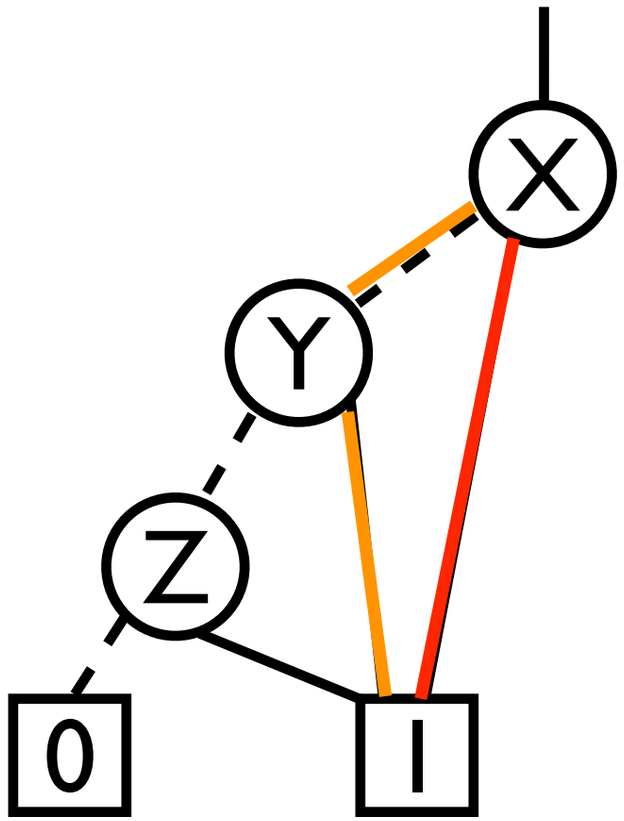


$X \vee Y \vee Z$

$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]

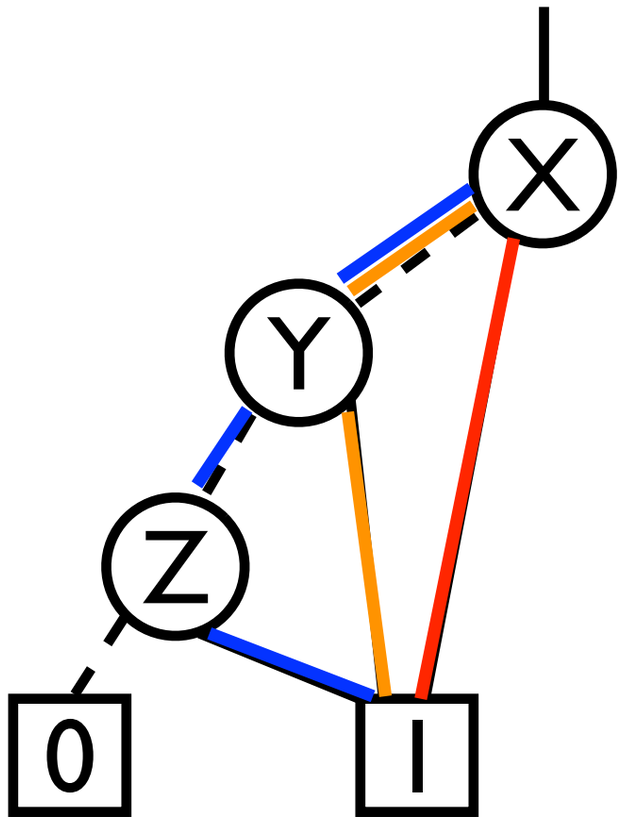


$X \vee Y \vee Z$

$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]

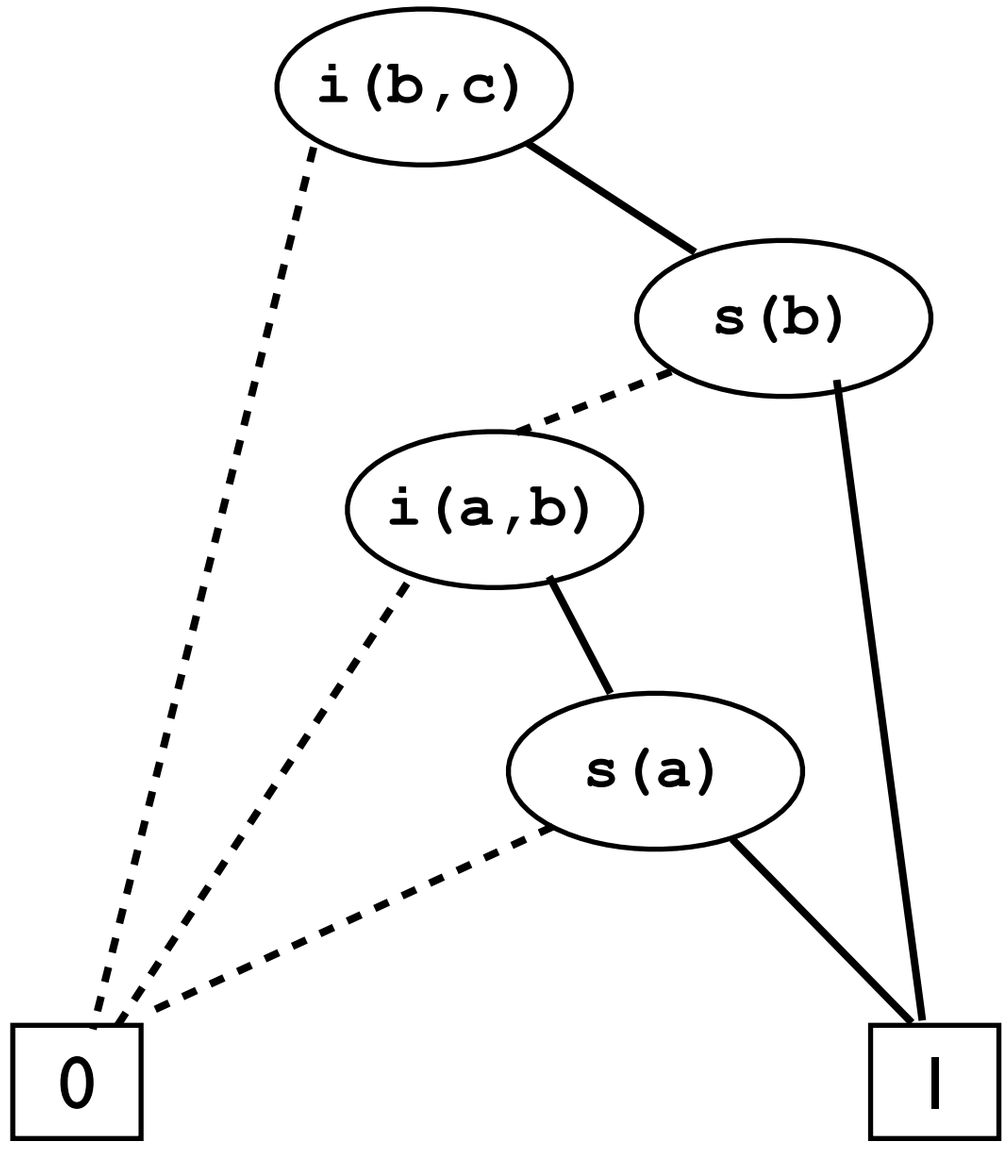


$X \vee Y \vee Z$

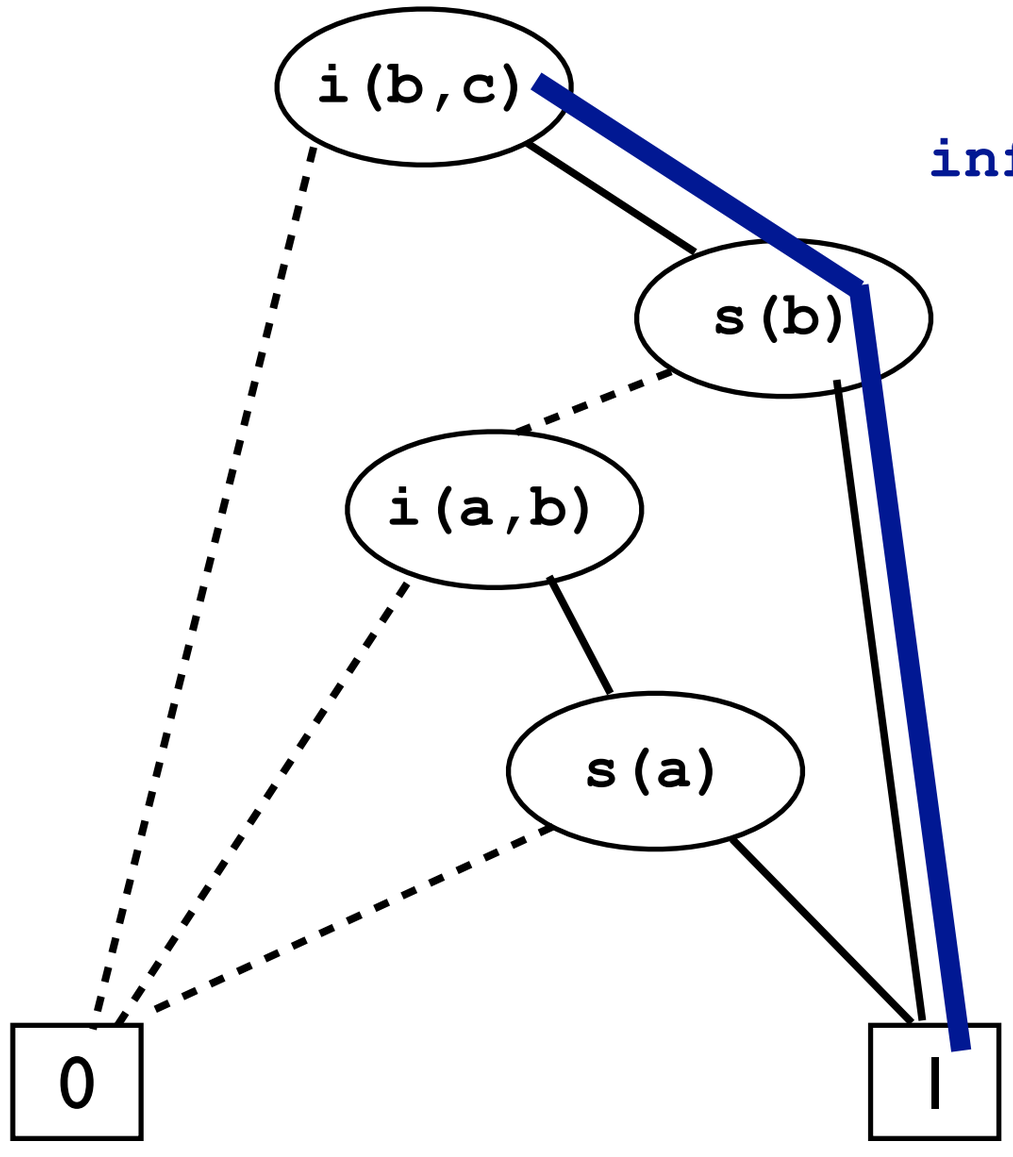
$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]



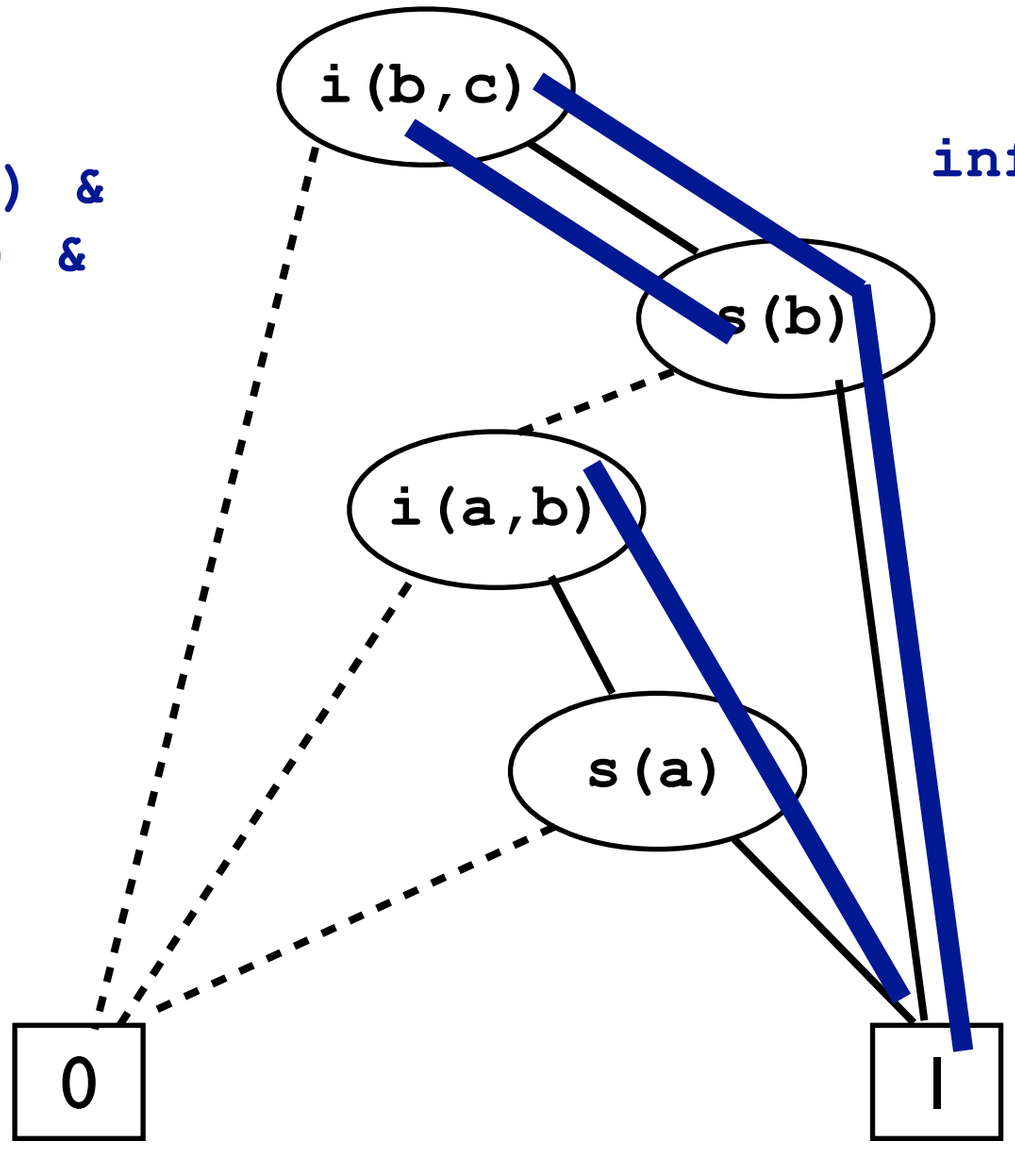
Binary Decision Diagrams
[Bryant 86]



influences(bob,carl) & stress(bob)

Binary Decision Diagrams
[Bryant 86]

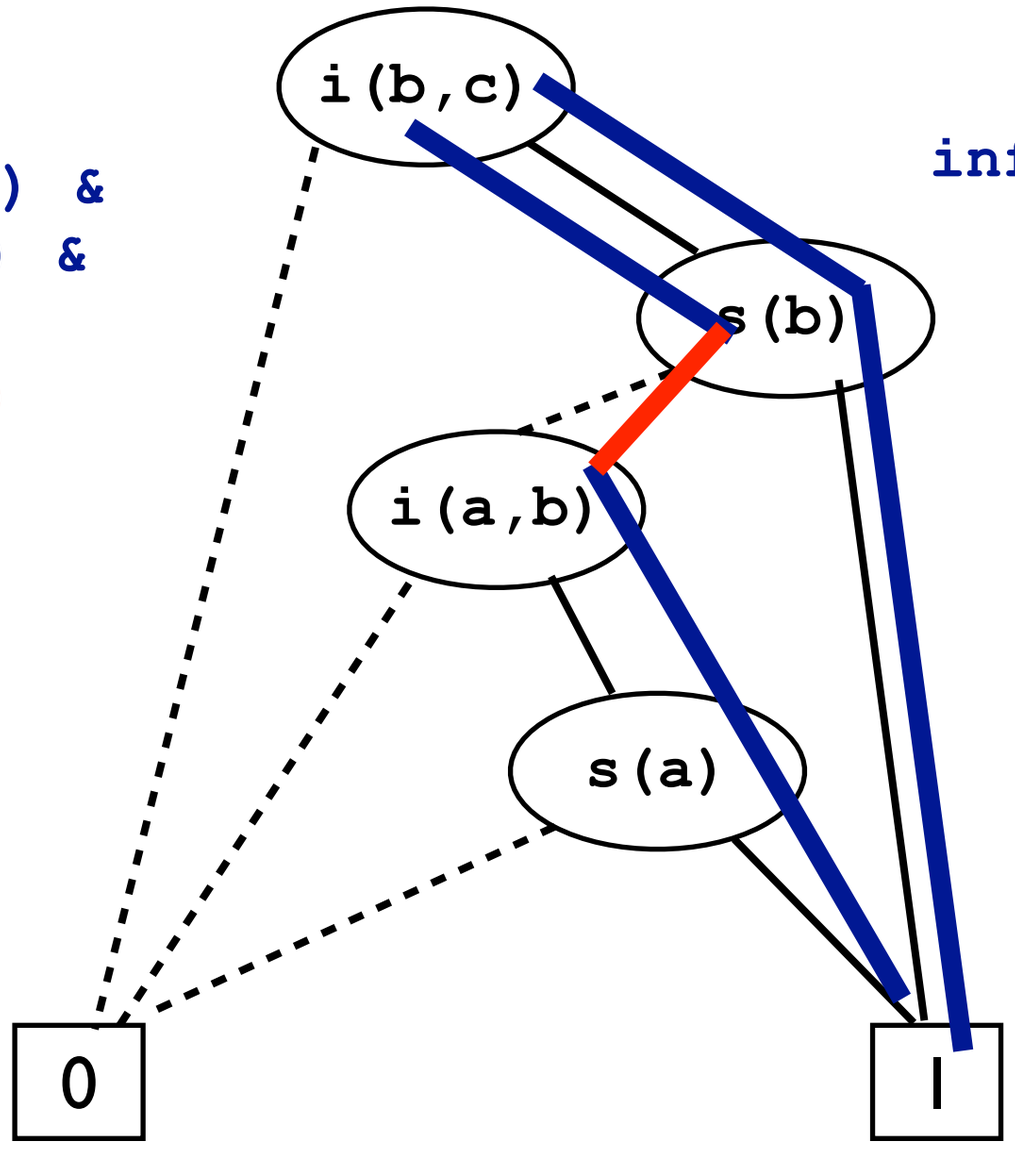
influences(bob,carl) & influences(ann,bob) & stress(ann)



influences(bob,carl) & stress(bob)

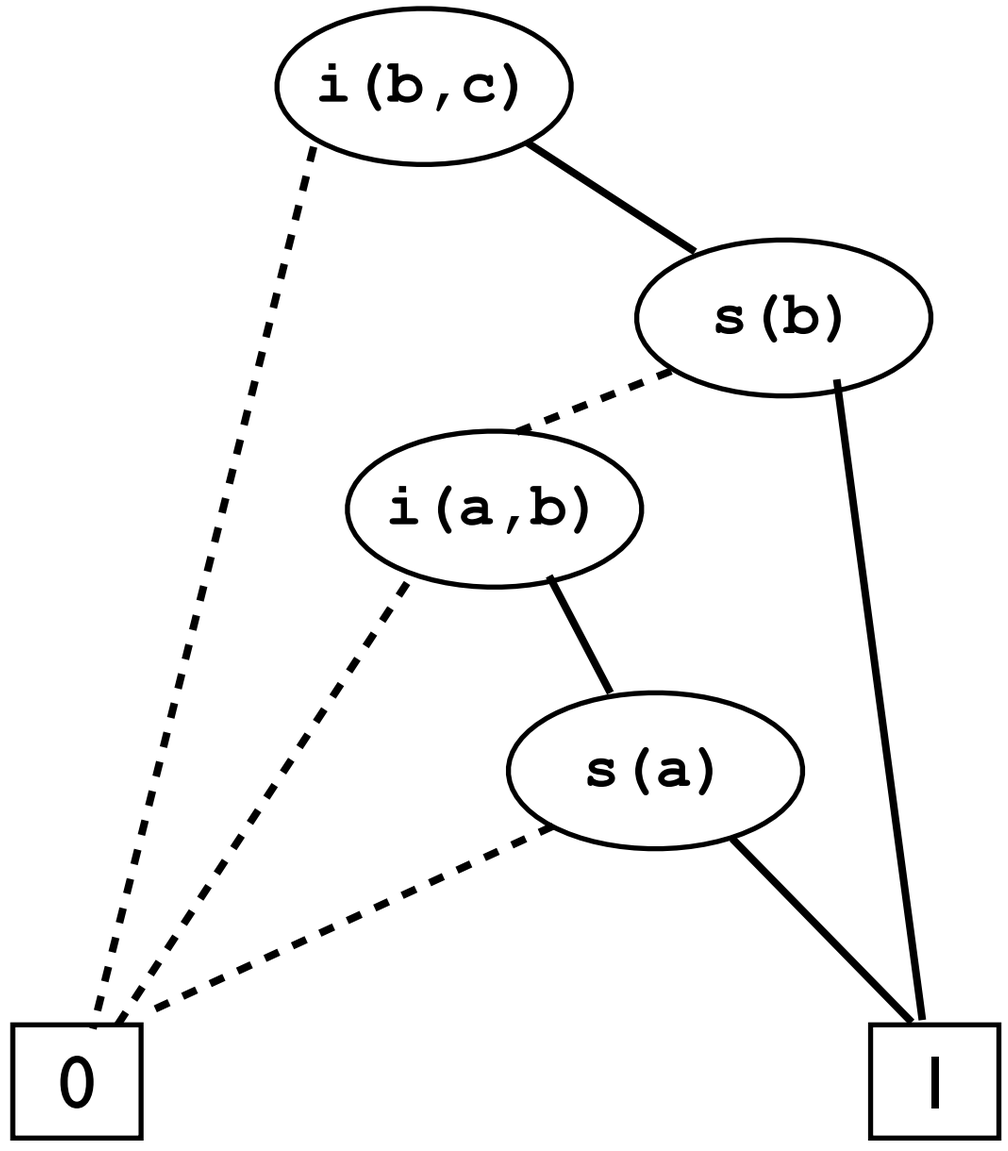
Binary Decision Diagrams
[Bryant 86]

influences(bob,carl) &
influences(ann,bob) &
stress(ann)
& not stress(bob)

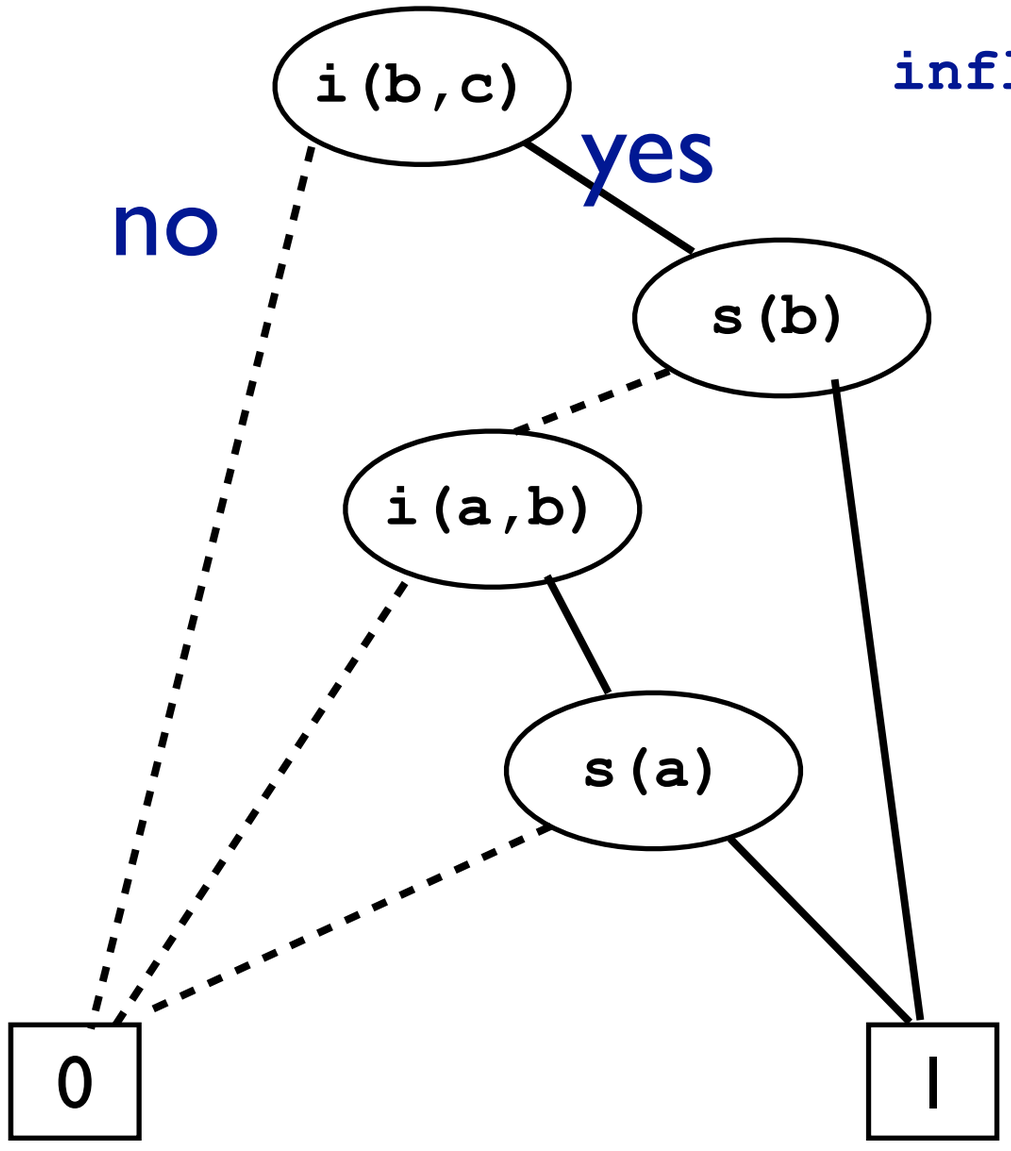


influences(bob,carl) &
stress(bob)

Binary Decision Diagrams

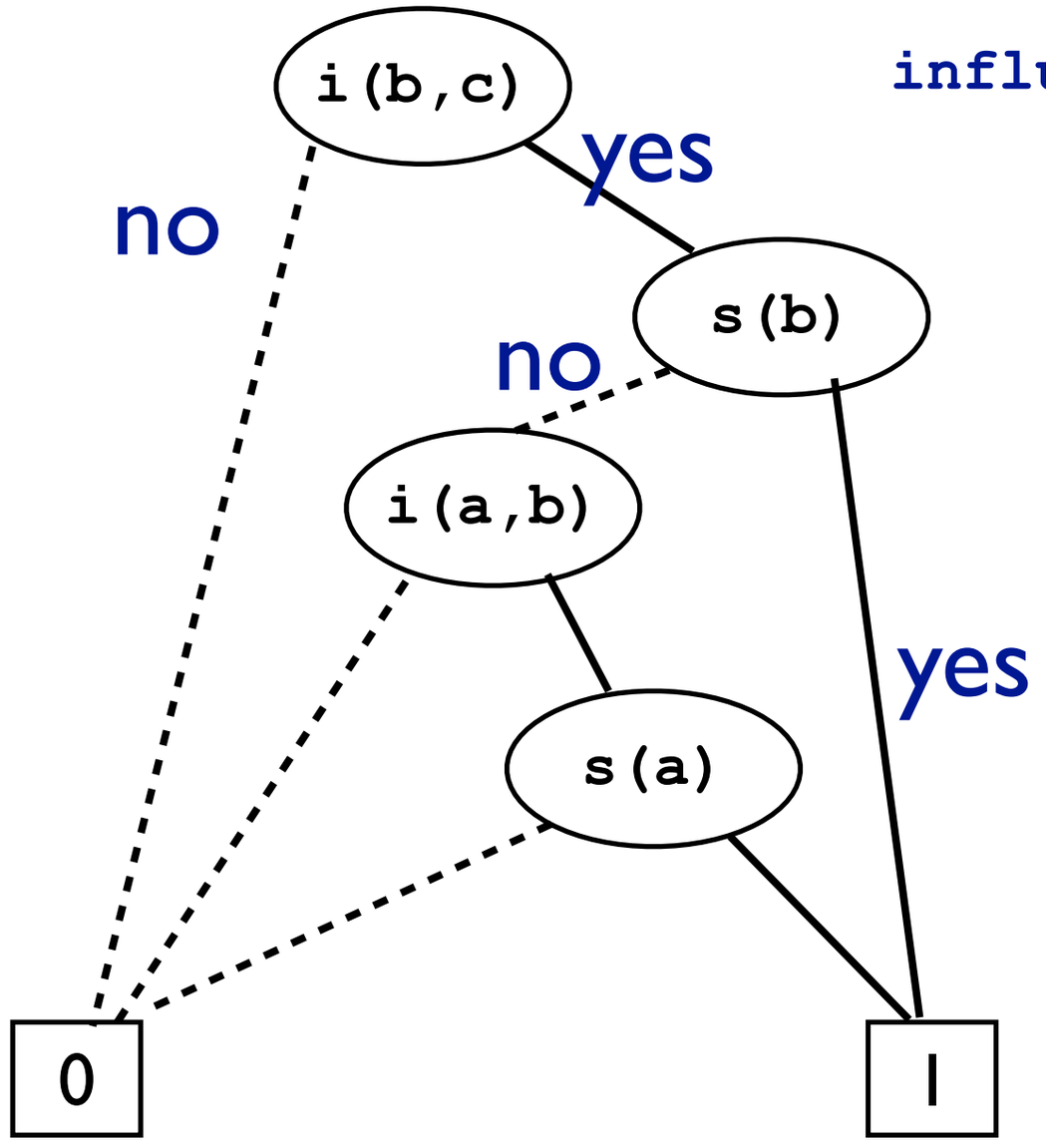


Binary Decision Diagrams



influences(bob,carl)?

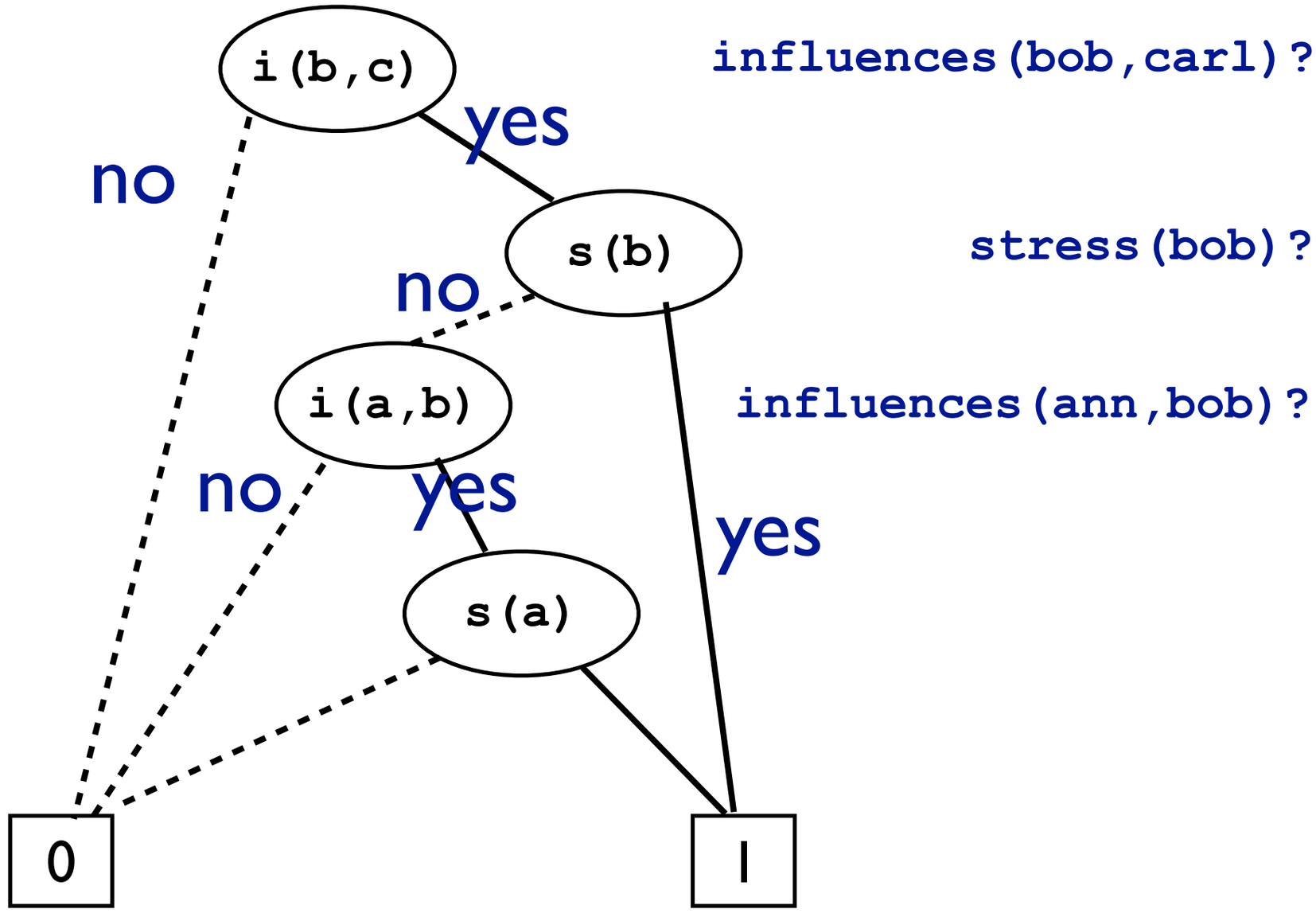
Binary Decision Diagrams



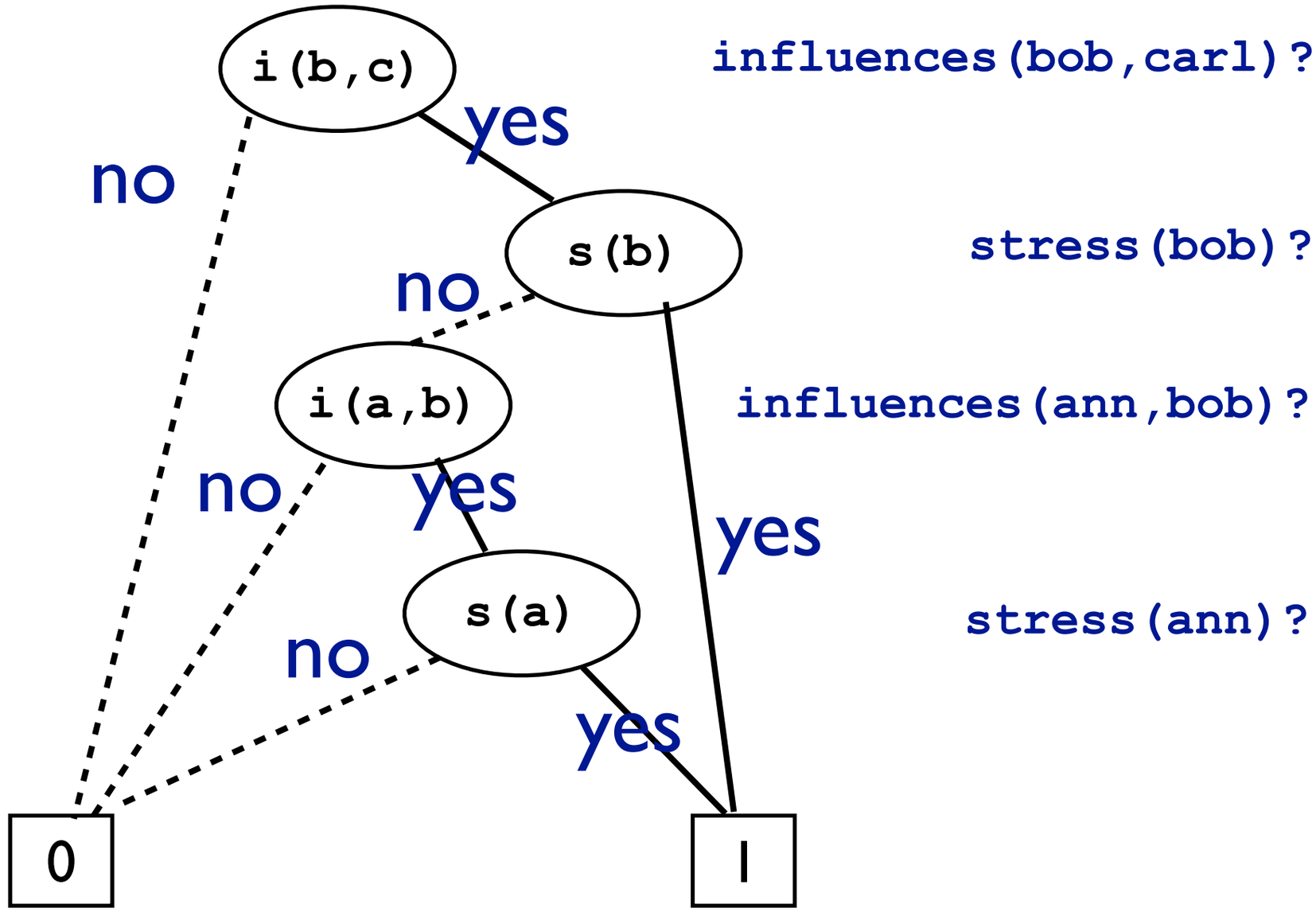
influences(bob,carl)?

stress(bob)?

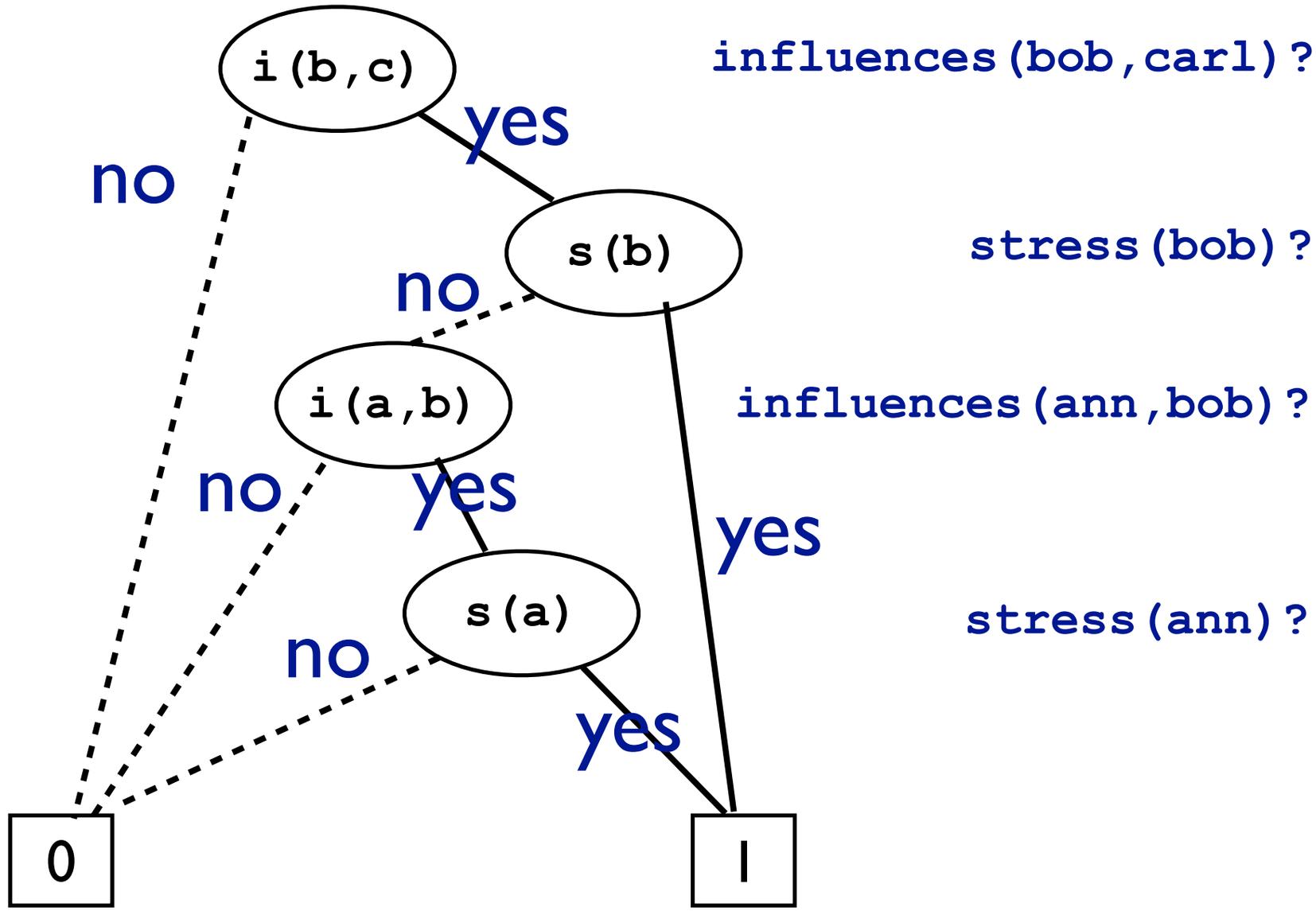
Binary Decision Diagrams



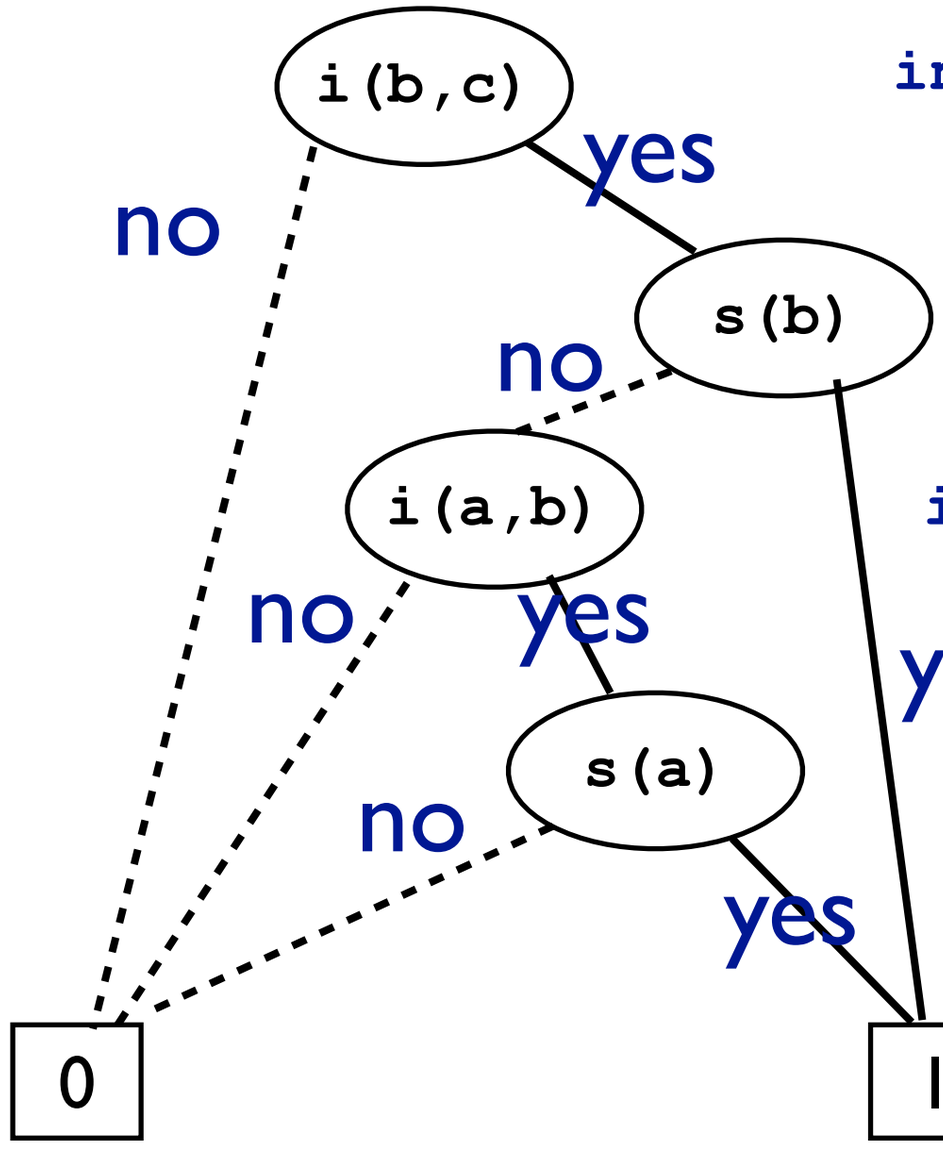
Binary Decision Diagrams



Binary Decision Diagrams



Binary Decision Diagrams



influences(bob,carl)?

stress(bob)?

influences(ann,bob)?

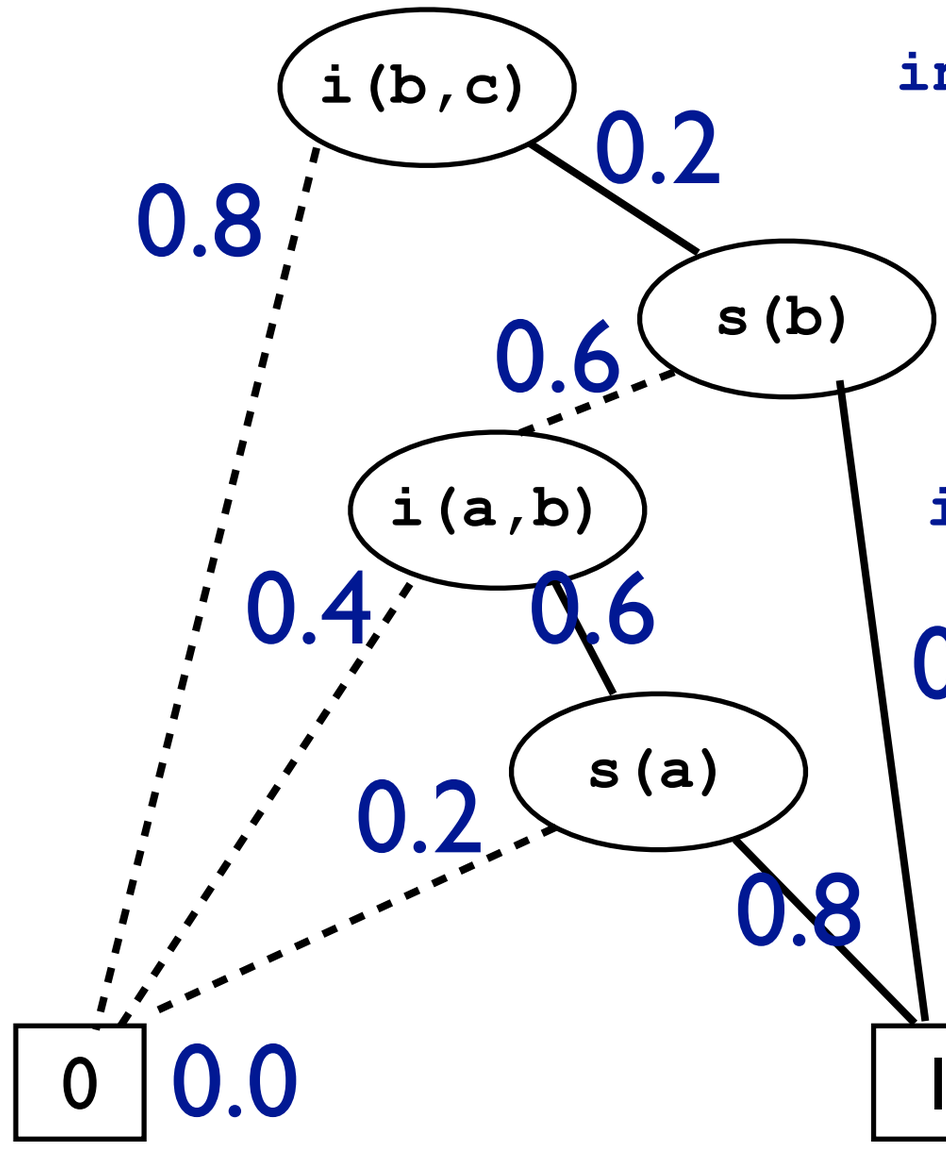
stress(ann)?

probability of

smokes(c)?

smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



`influences(bob,carl)?`

`stress(bob)?`

`influences(ann,bob)?`

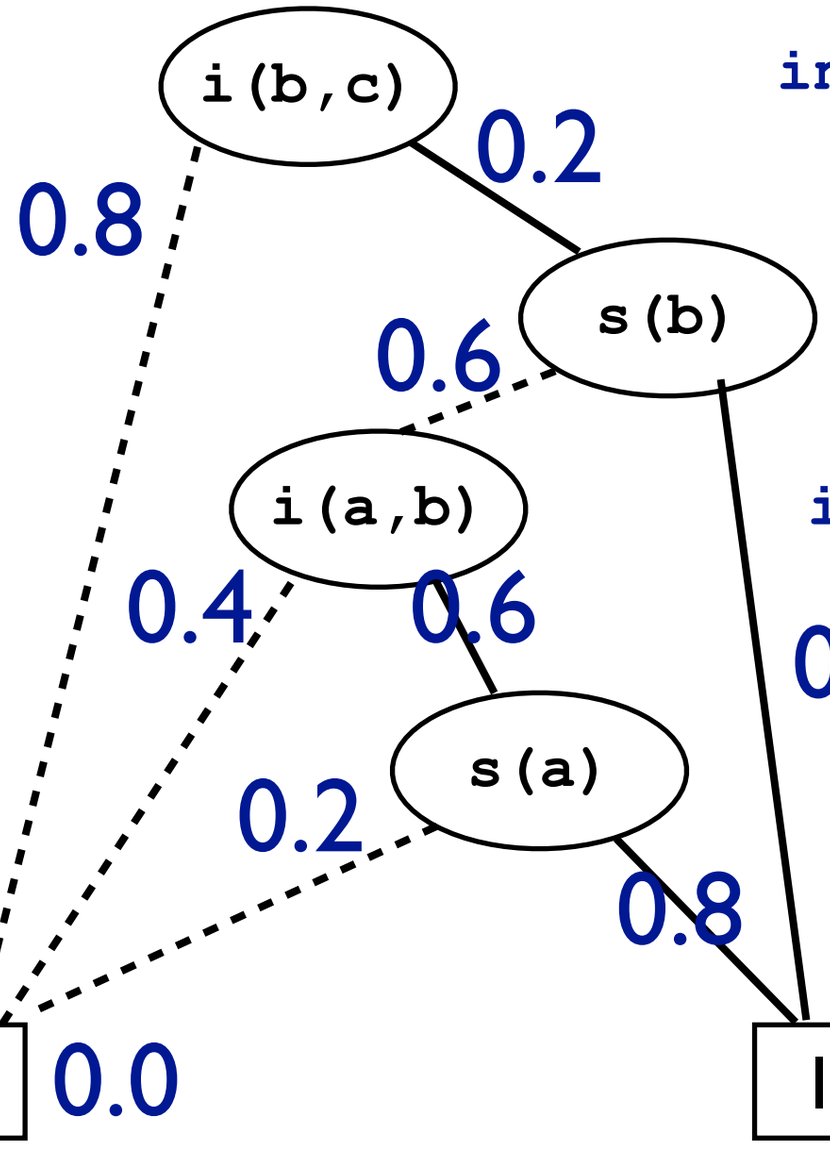
`stress(ann)?`

probability of

`smokes(c)?`

`smokes(c) = i(b,c)&s(b) v i(b,c)&i(a,b)&s(a)`

Binary Decision Diagrams



`influences(bob,carl)?`

`stress(bob)?`

`influences(ann,bob)?`

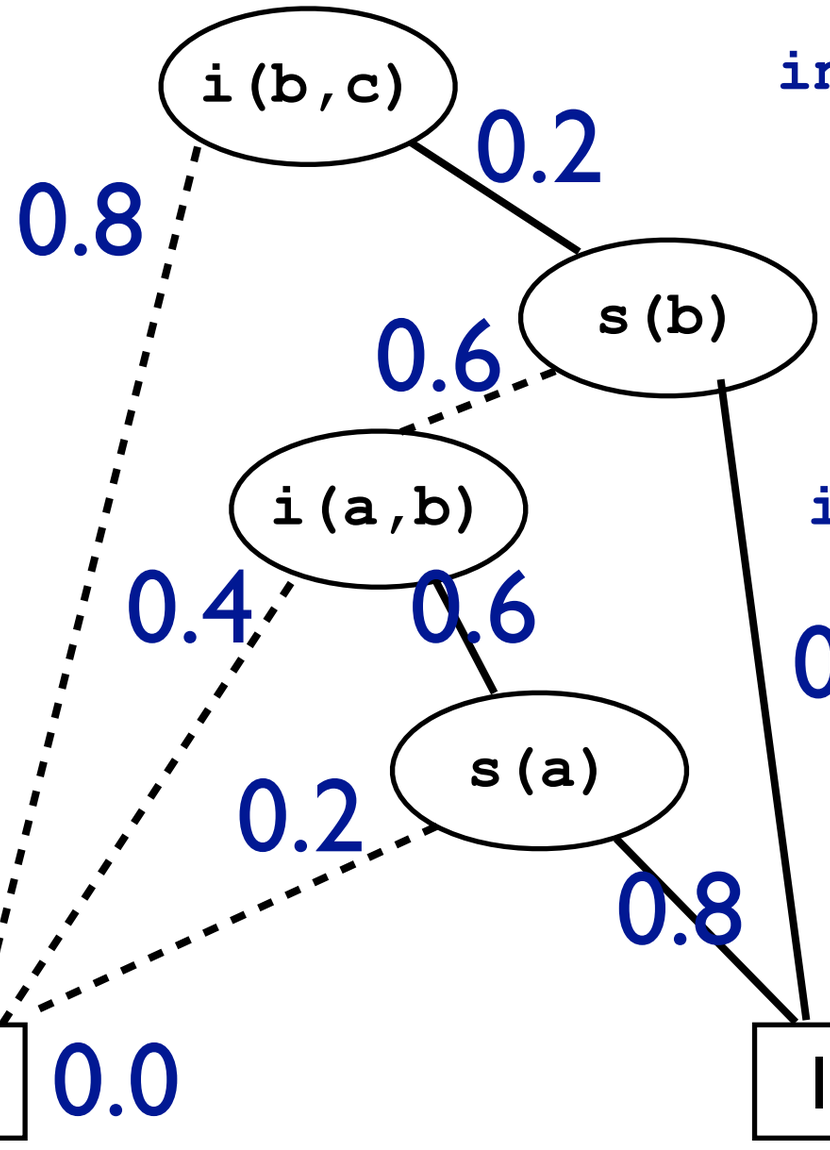
`stress(ann)?`

0.2×0.0+0.8×1.0=0.8

probability of `smokes(c)`?

`smokes(c) = i(b,c)&s(b) v i(b,c)&i(a,b)&s(a)`

Binary Decision Diagrams



`influences(bob,carl)?`

`stress(bob)?`

`influences(ann,bob)?`

`stress(ann)?`

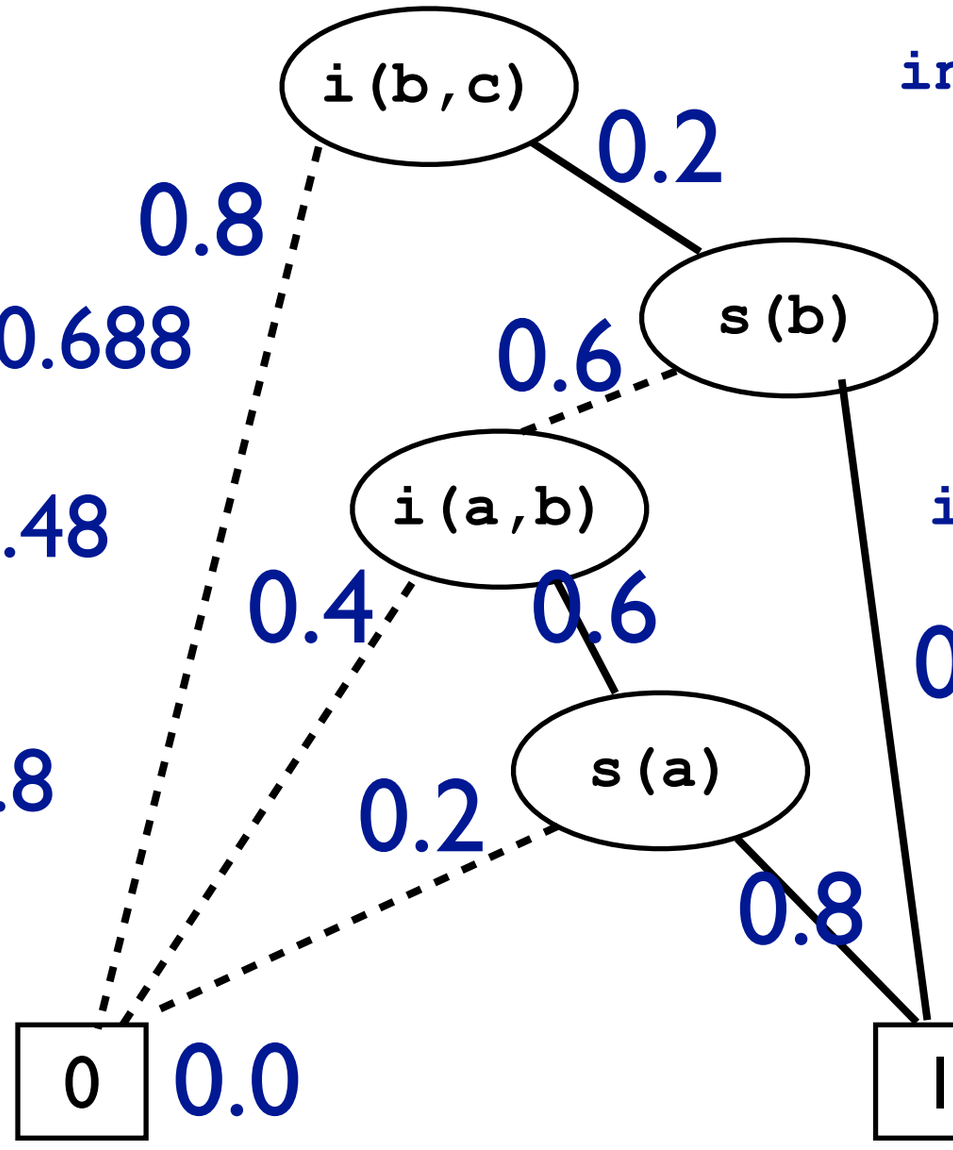
0.4×0.0+0.6×0.8=0.48

0.2×0.0+0.8×1.0=0.8

probability of `smokes(c)`?

`smokes(c) = i(b,c)&s(b) v i(b,c)&i(a,b)&s(a)`

Binary Decision Diagrams



influences(bob,carl)?

stress(bob)?

influences(ann,bob)?

stress(ann)?

0.6×0.48+0.4×1.0=0.688

0.4×0.0+0.6×0.8=0.48

0.2×0.0+0.8×1.0=0.8

probability of

smokes(c)?

smokes(c) = i(b,c)&s(b) v
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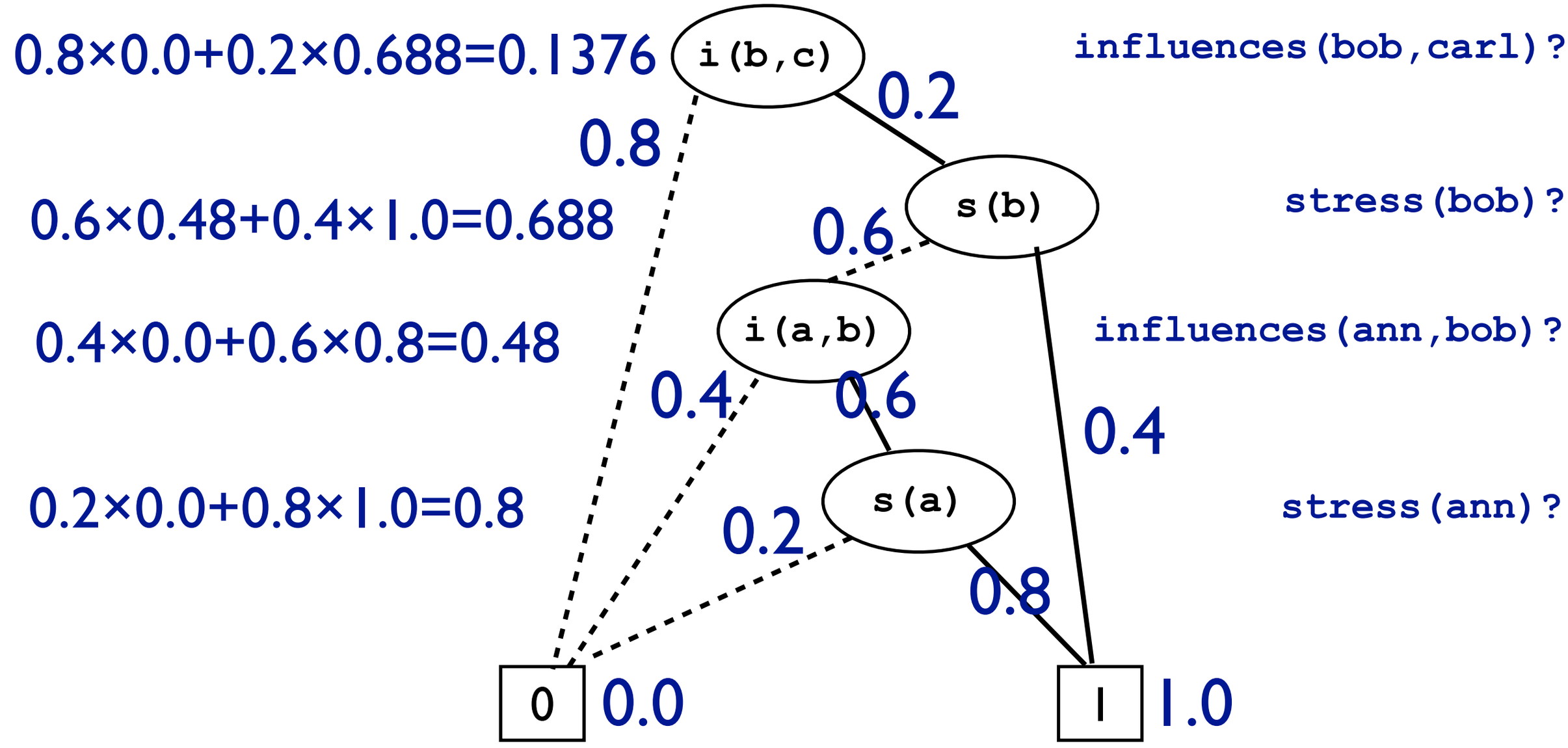
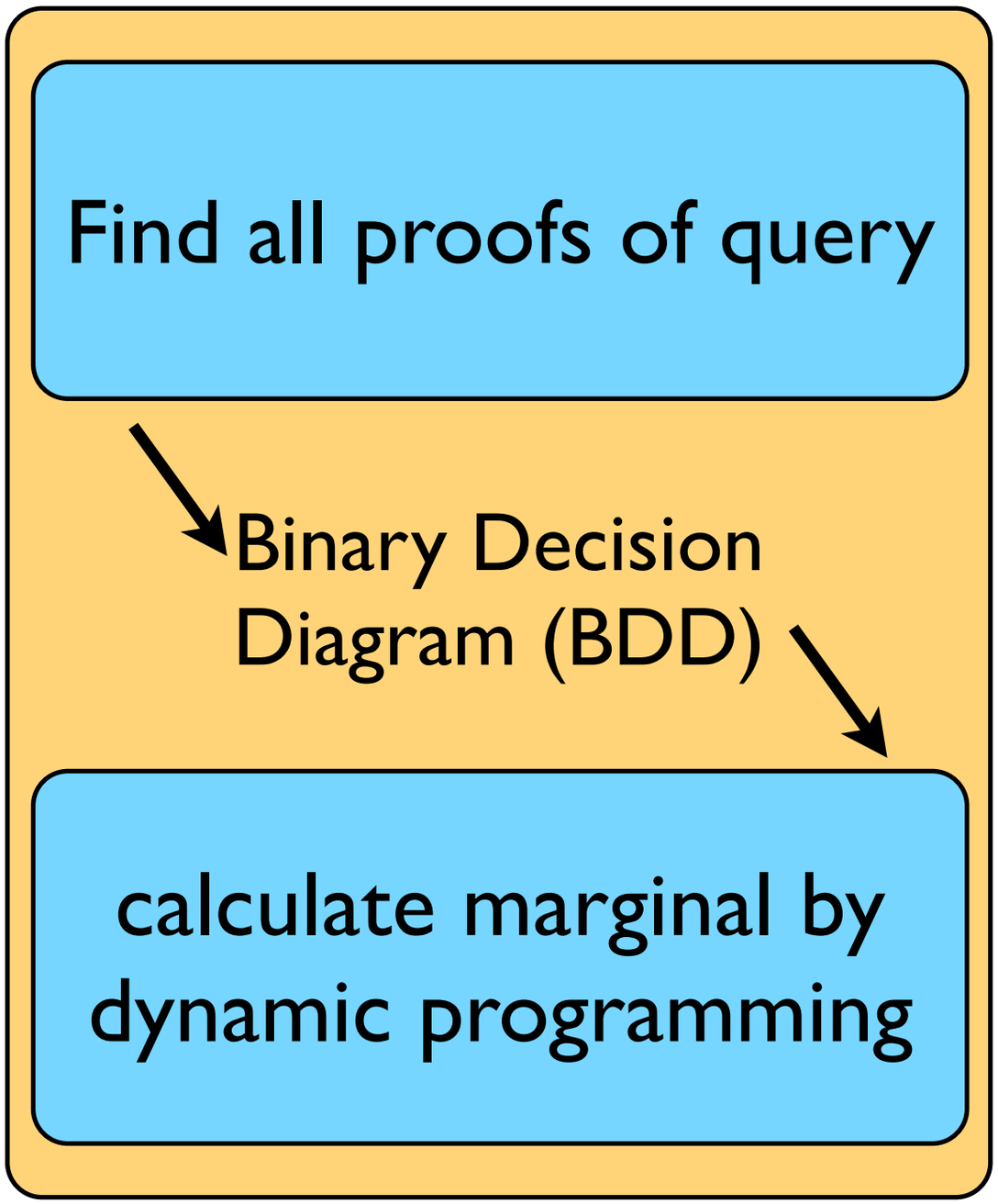


Initial Approach

(ProbLog1 & others)



[De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

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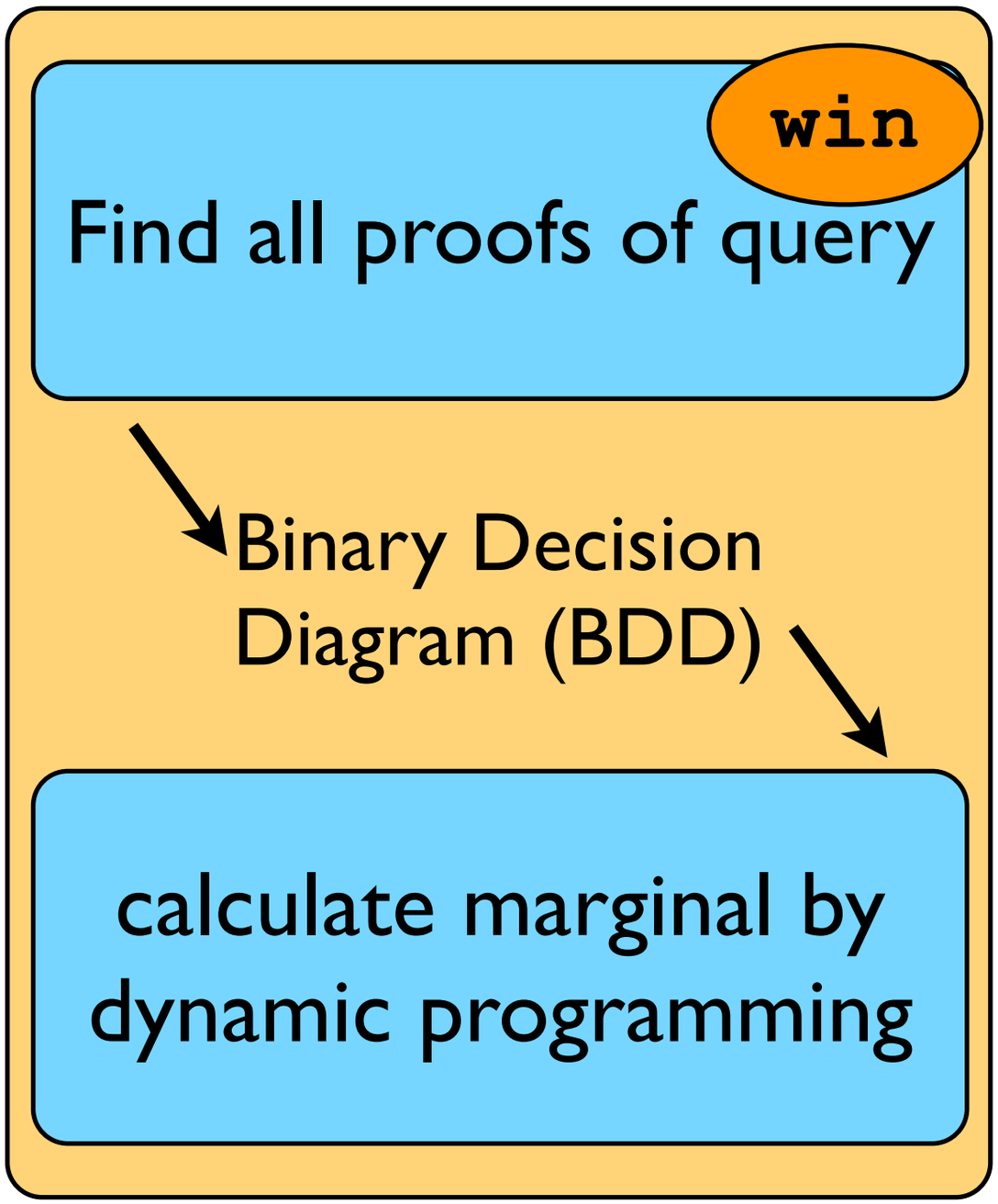

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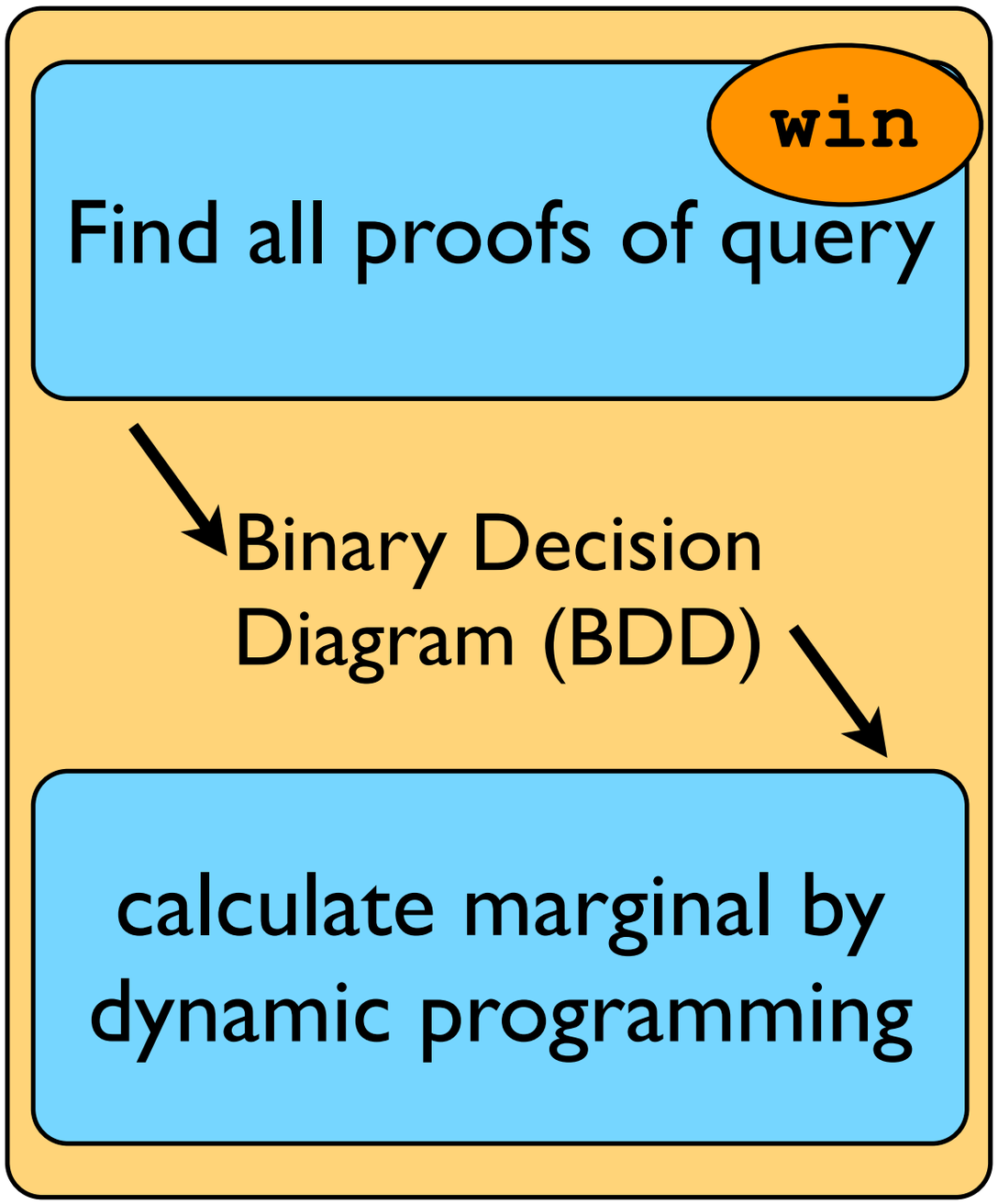


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heads(1)
heads(2)  & heads(3)
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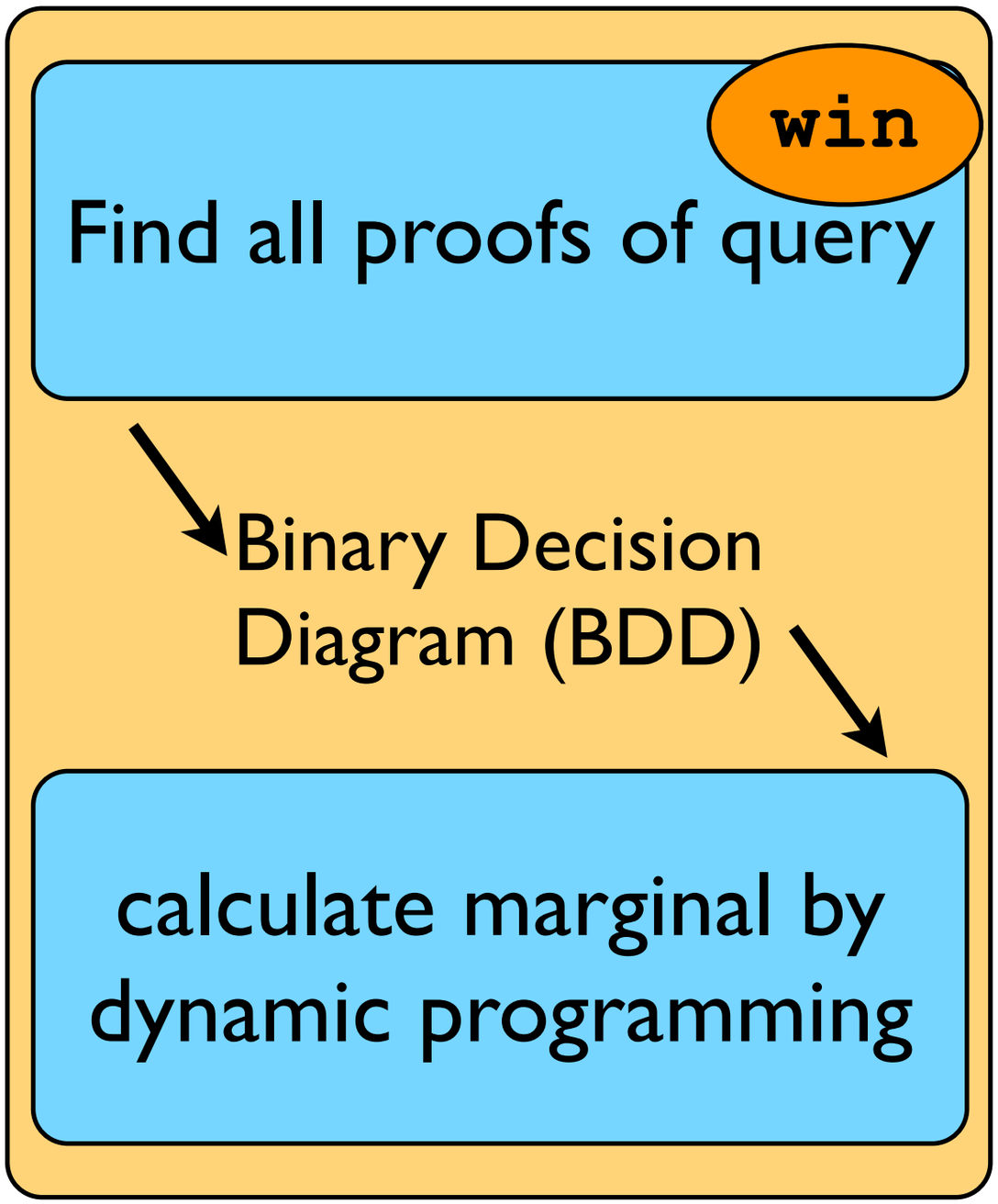


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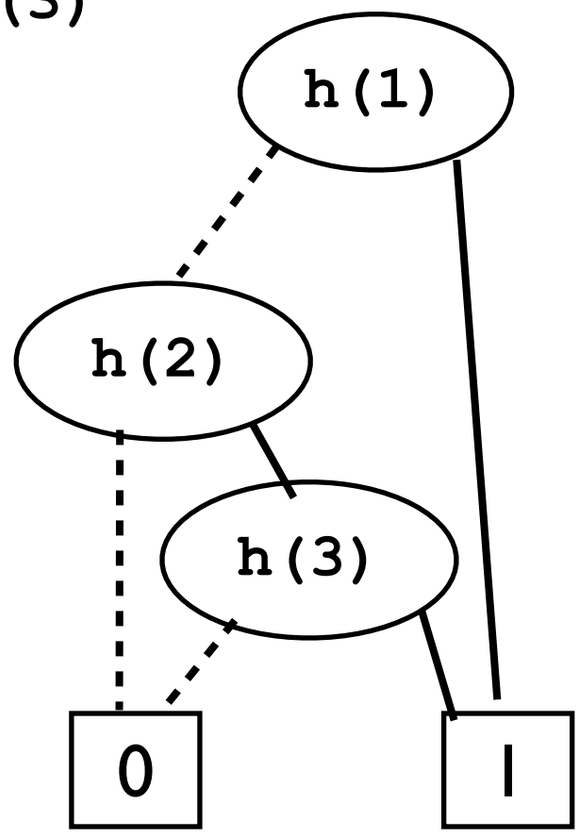

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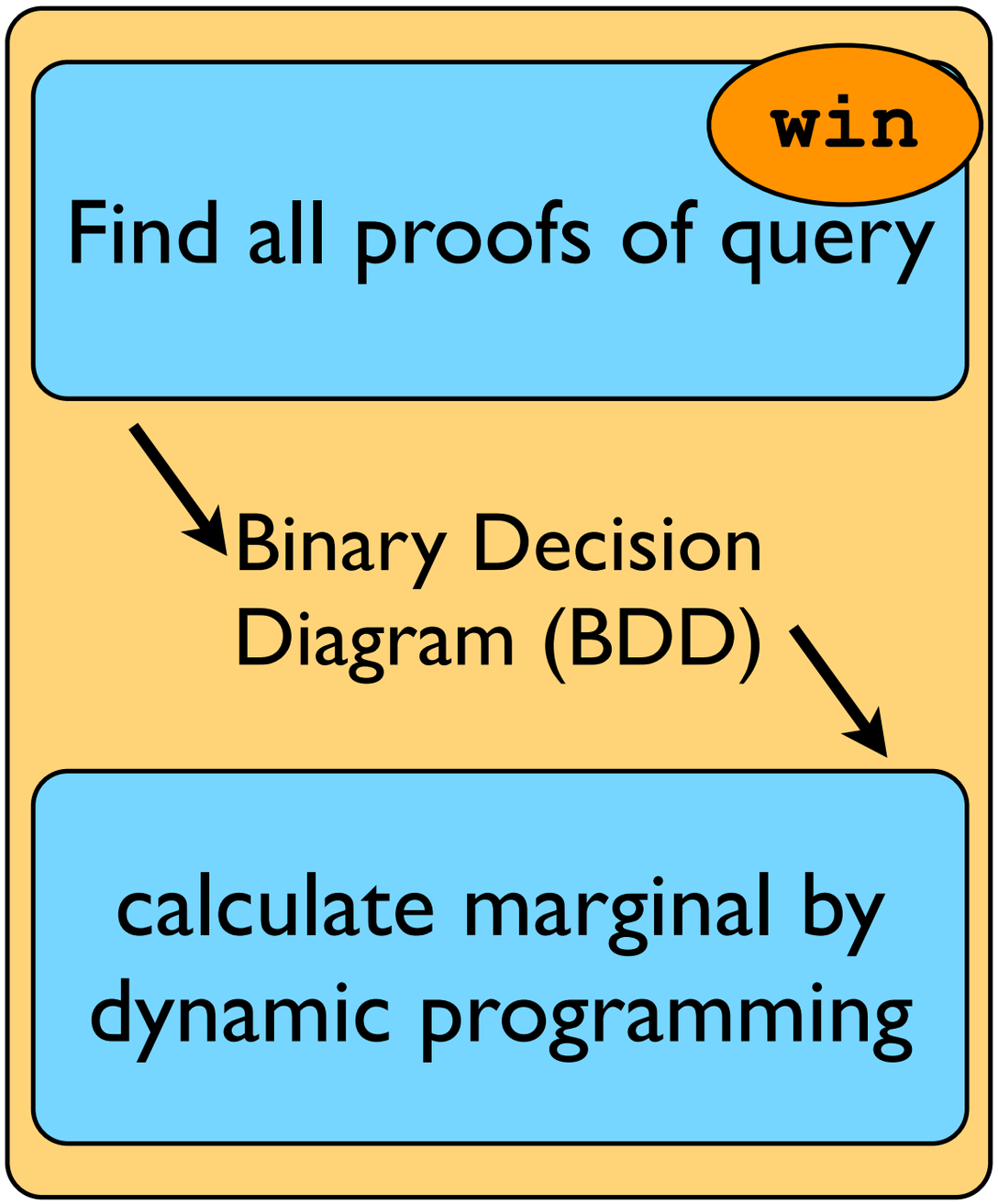


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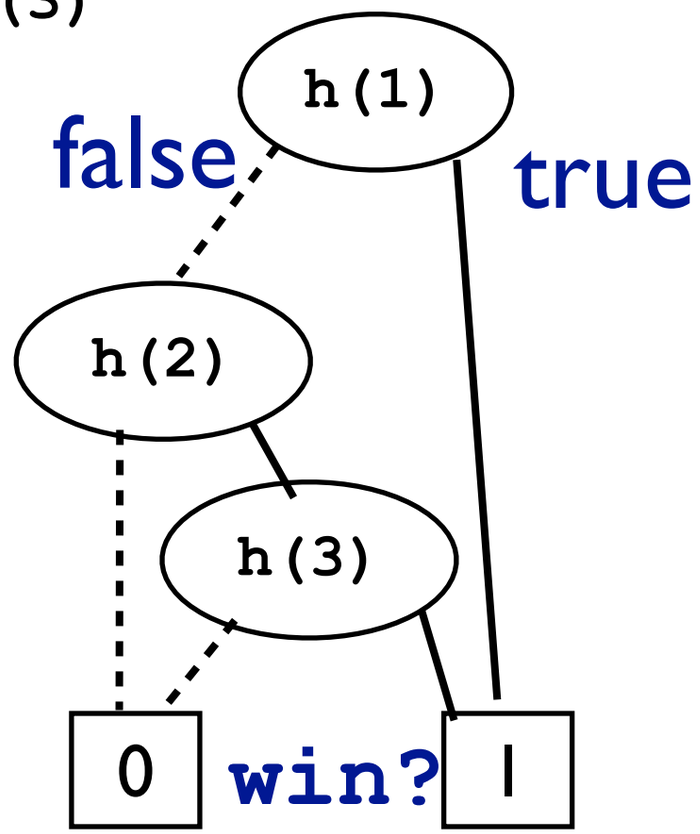

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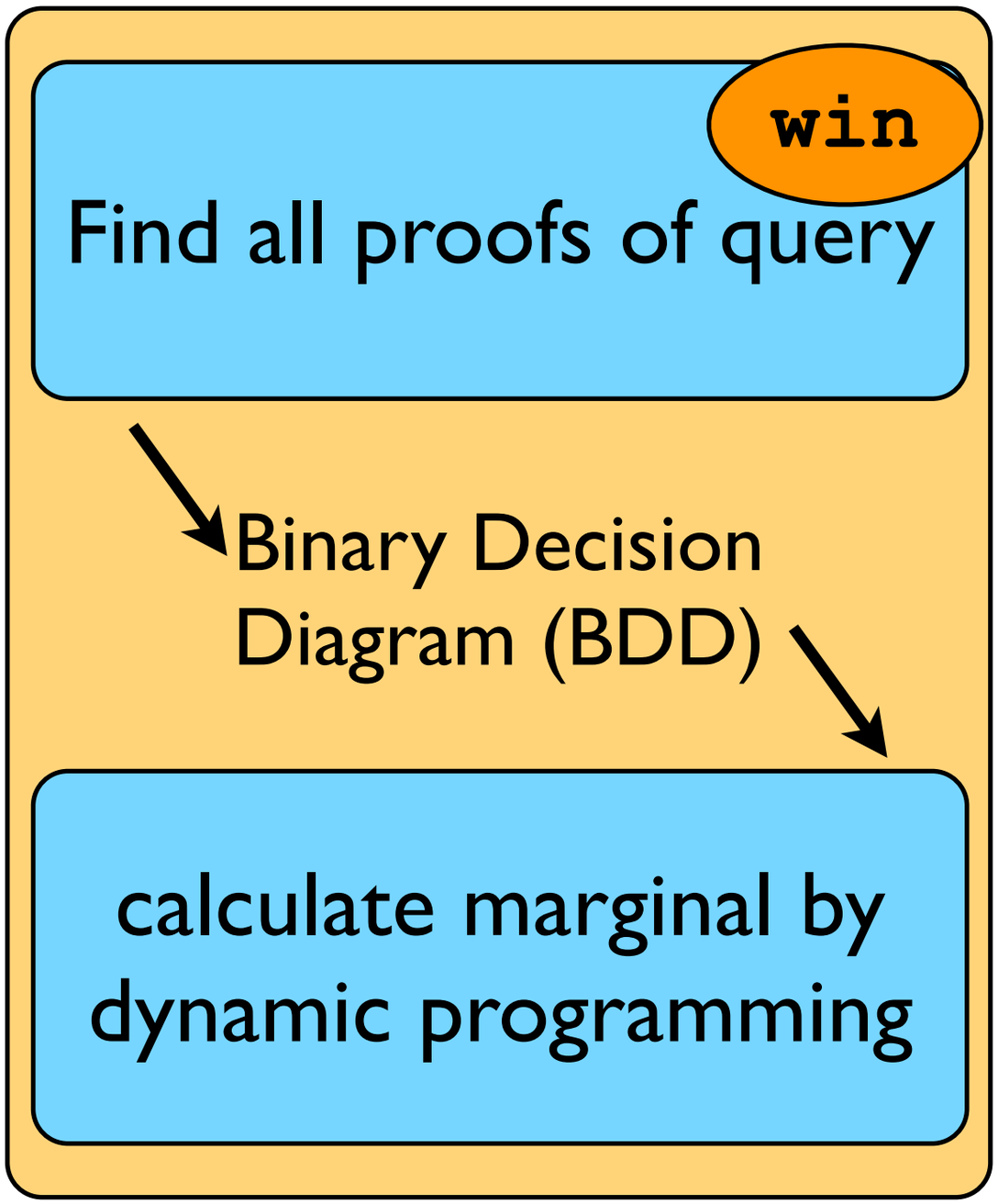


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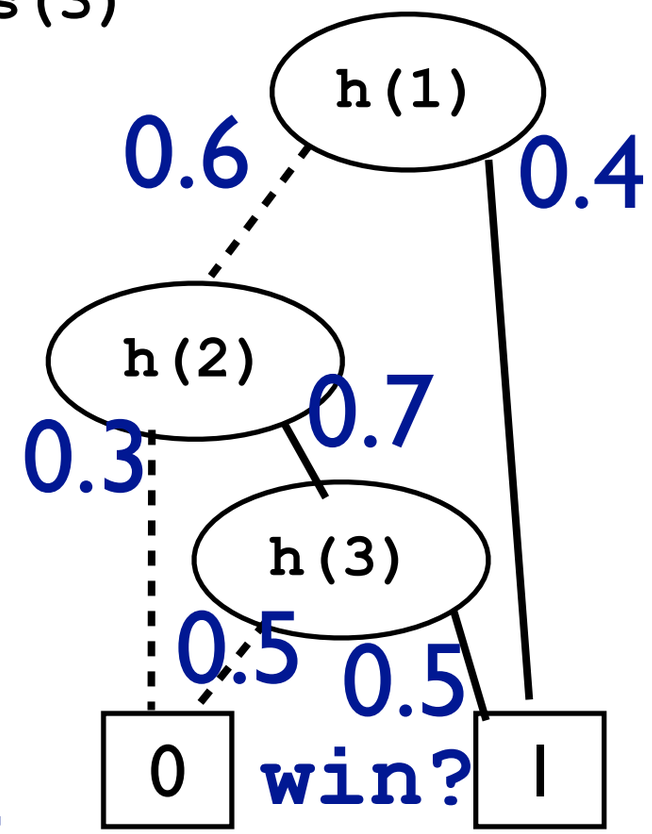


P(**win**) = probability of reaching 1-leaf

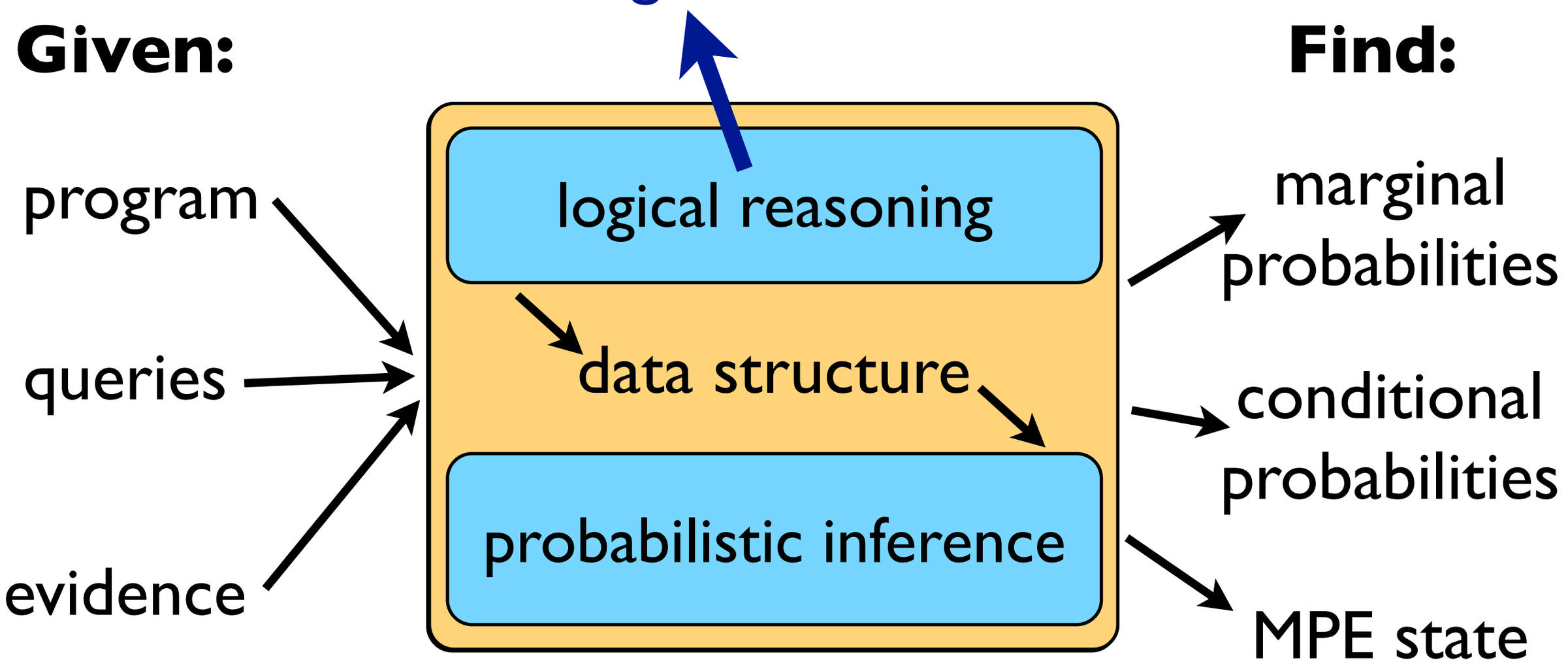
 [De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Answering Questions

1. using proofs
2. using models

Given:

Find:



Logical Reasoning: Models in Prolog

```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
      influences(Y,X),
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?- smokes(carl).
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- Forward reasoning to construct unique model:

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- Forward reasoning to construct unique model:
  - Start with database facts
  - Use rules to add more facts
- Query true iff in model

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- **ProbLog**: each possible world is a model, probability of query is sum over models where query is true

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→ weighted model counting

Weighted Model Counting

$$WMC(\phi) = \sum_{I_V \models \phi} \prod_{l \in I_V} w(l)$$

Weighted Model Counting

propositional formula in conjunctive normal form (CNF)

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interpretations (truth value assignments) of propositional variables

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propositional formula in conjunctive normal form (CNF)

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interpretations (truth value assignments) of propositional variables

weight of literal

Weighted Model Counting

propositional formula in conjunctive normal form (CNF)

given by ProbLog program & query

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interpretations (truth value assignments) of propositional variables

possible worlds

weight of literal

for p::f,
w(f) = p
w(not f) = 1−p

Weighte

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

propositional formula in conjunctive normal form (CNF)

given by ProbLog program & query

$$WMC(\phi) = \sum_{I_V \models \phi} \prod_{l \in I_V} w(l)$$

interpretations (truth value assignments) of propositional variables

possible worlds

weight of literal

for p::f,
w(f) = p
w(not f) = 1−p

ProbLog → CNF

```
?- smokes(carl).
```

```
0.8::stress(ann).
0.4::stress(bob).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
      influences(Y,X),
      smokes(Y).
```

ProbLog → CNF

```
0.8::stress(ann).
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- Find relevant ground rules by backward reasoning

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```
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```

- Find relevant ground rules by backward reasoning

```
smokes(carl) :- influences(bob,carl),smokes(bob).
smokes(bob) :- stress(bob).
smokes(bob) :- influences(ann,bob),smokes(ann).
smokes(ann) :- stress(ann).
```

ProbLog → CNF

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0.8::stress(ann).
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smokes(ann) :- stress(ann).
```

- Convert to propositional logic formula

ProbLog → CNF

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smokes(bob)  :- influences(ann,bob),smokes(ann).
smokes(ann)  :- stress(ann).
```

- Convert to propositional logic formula

may require loop-breaking

$$sm(c) \leftrightarrow (i(b,c) \wedge sm(b)) \wedge sm(b) \leftrightarrow (st(b) \vee (i(a,b) \wedge sm(a))) \wedge sm(a) \leftrightarrow st(a)$$

ProbLog → CNF

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- Convert to propositional logic formula

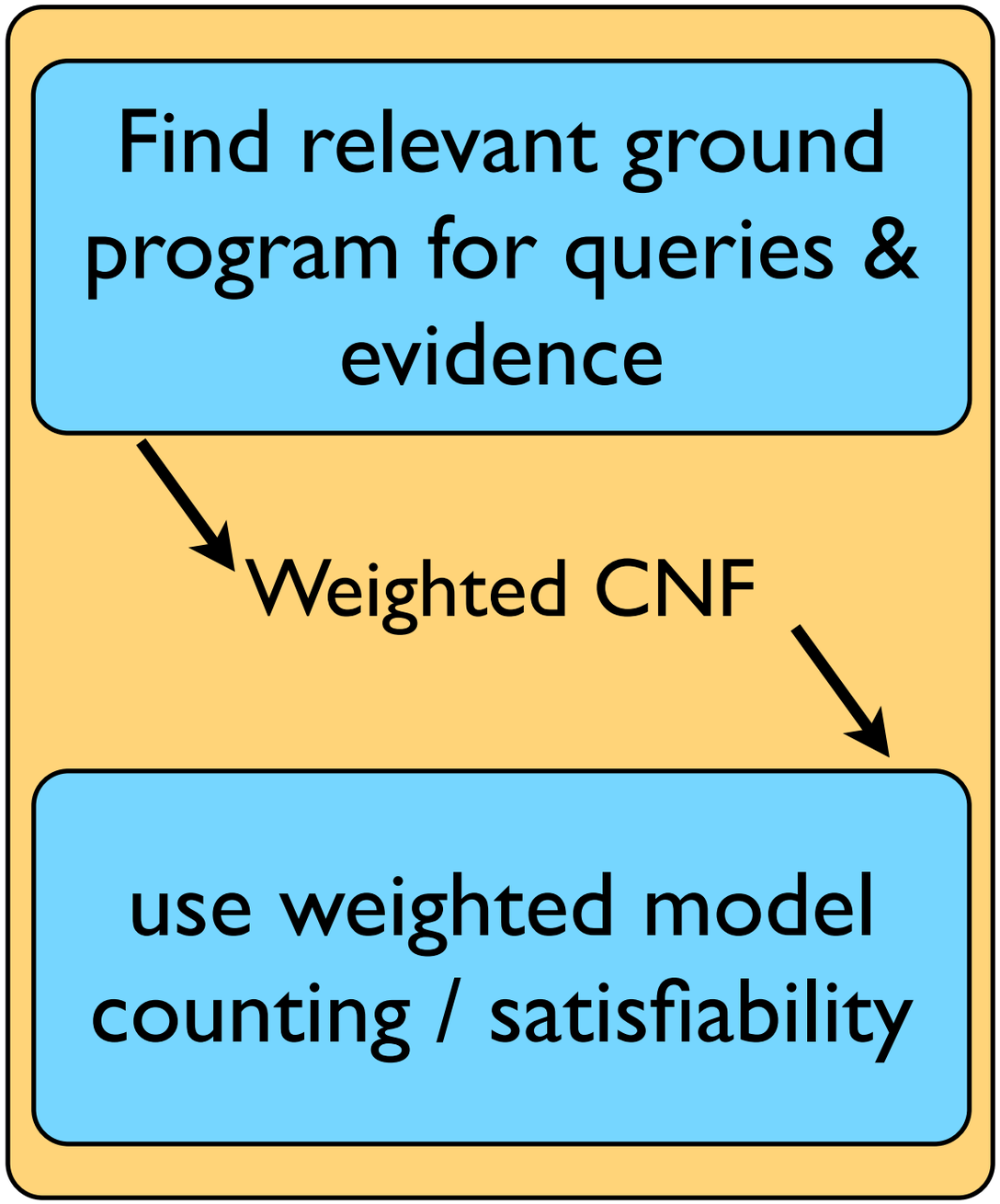
may require loop-breaking

$$sm(c) \leftrightarrow (i(b,c) \wedge sm(b)) \wedge sm(b) \leftrightarrow (st(b) \vee (i(a,b) \wedge sm(a))) \wedge sm(a) \leftrightarrow st(a)$$

- Rewrite in CNF (as usual)

Current Approach

(ProbLog2)



[Fierens et al, TPLP 14]

Current Approach
(ProbLog2)

```
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win :- heads(1).
win :- heads(2),
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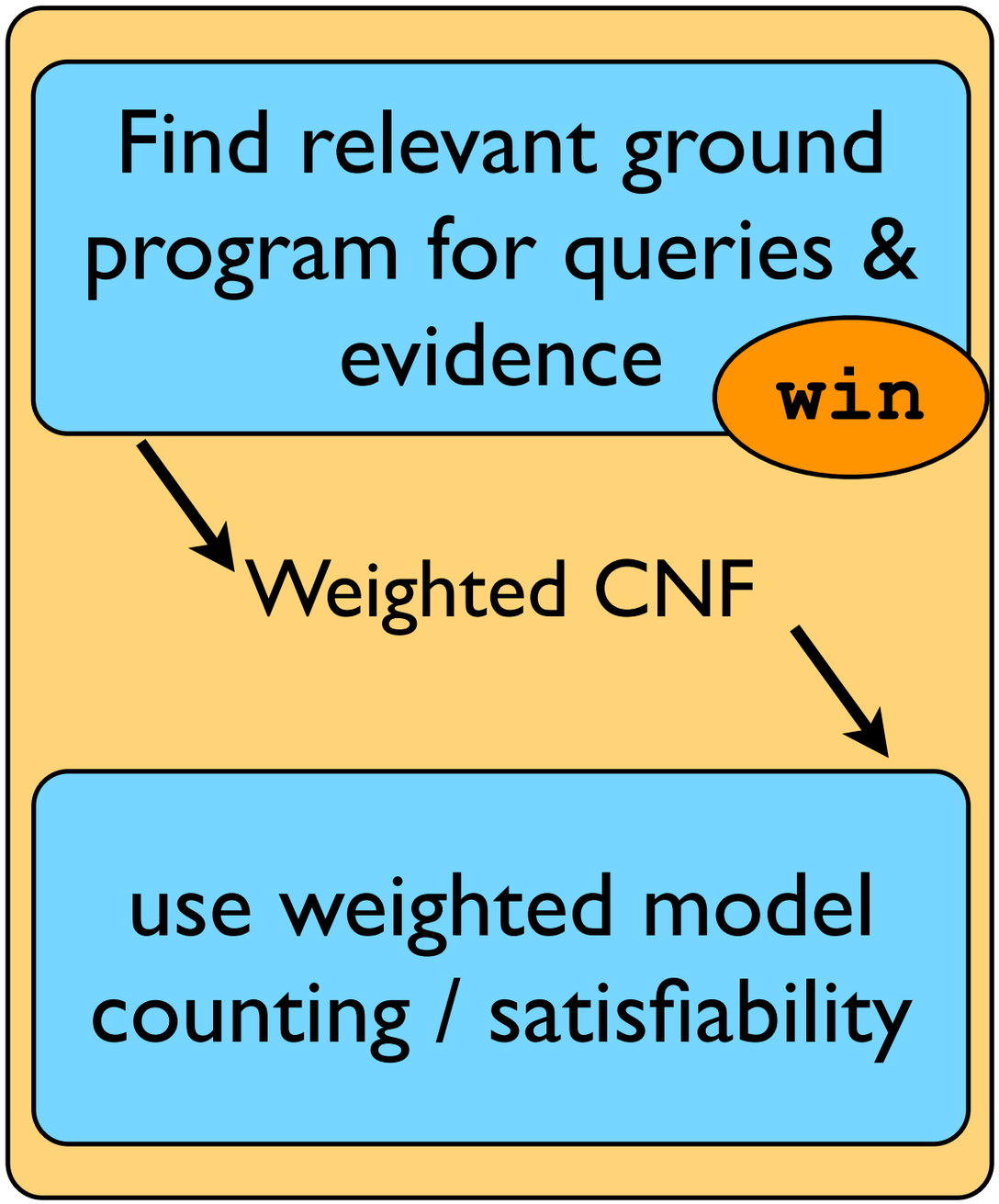

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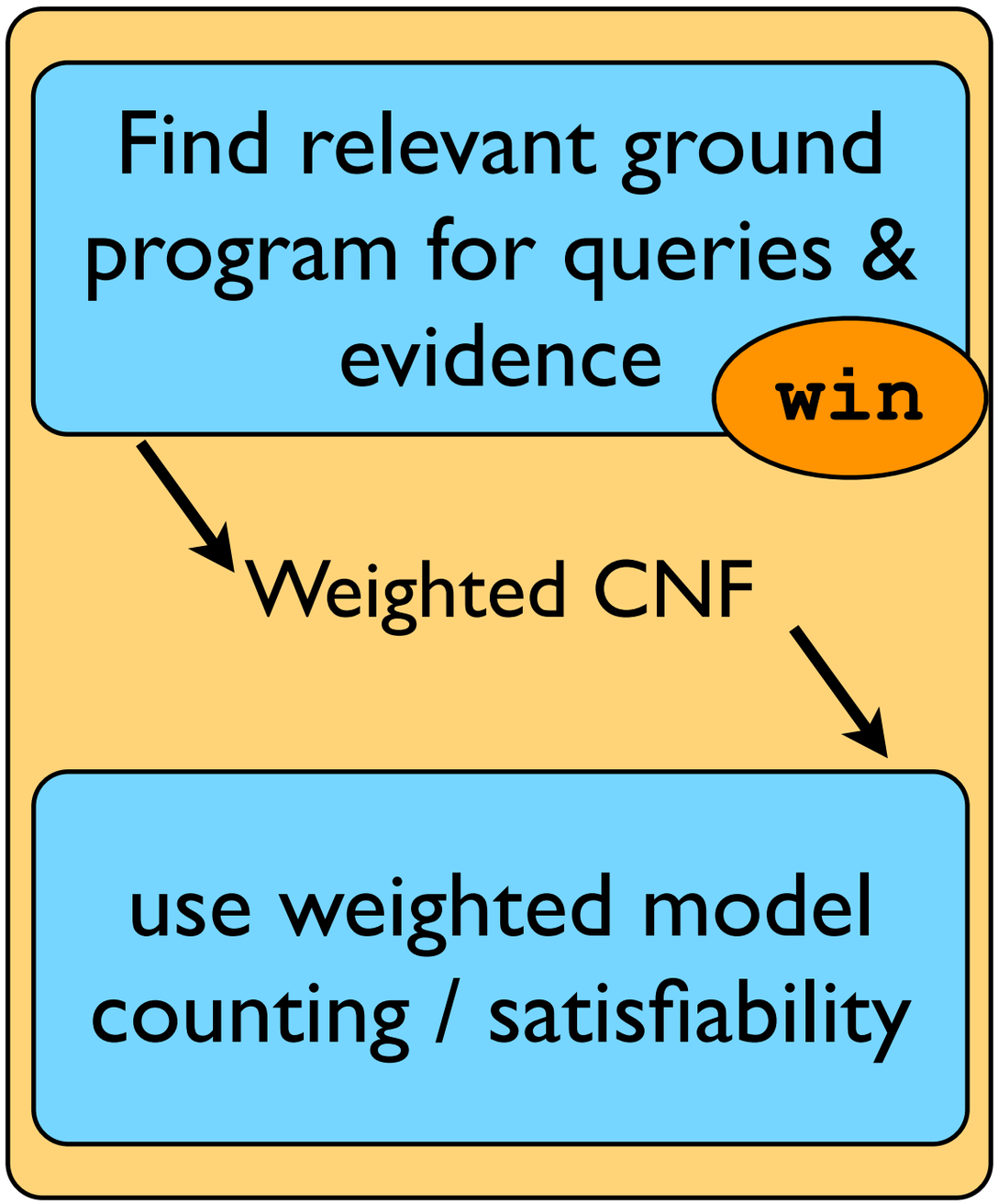


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[Fierens et al, TPLP 14]

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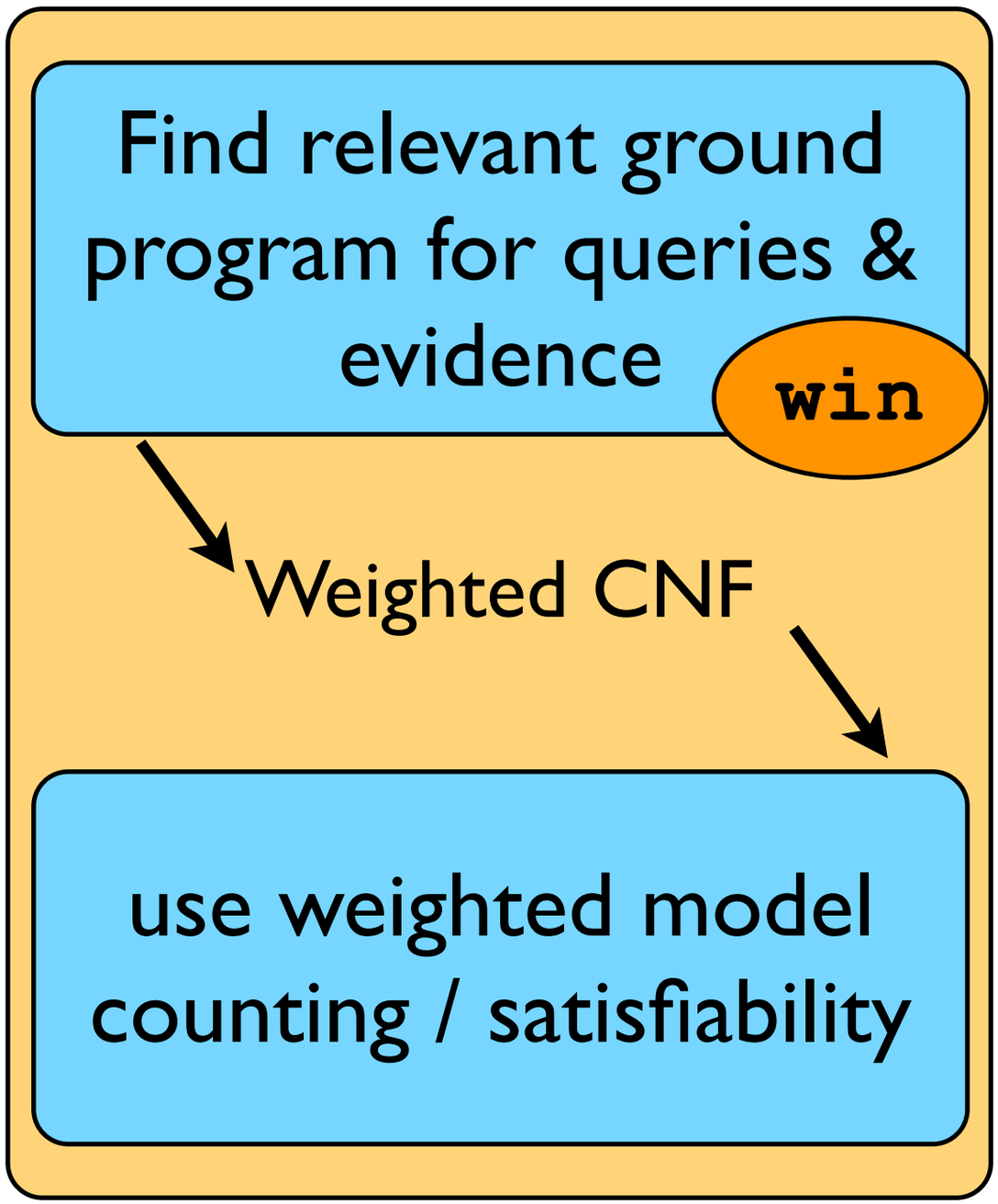


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win :- heads(1).
win :- heads(2), heads(3).
```

$$\text{win} \leftrightarrow h(1) \vee (h(2) \wedge h(3))$$

[Fierens et al, TPLP 14]

Current Approach
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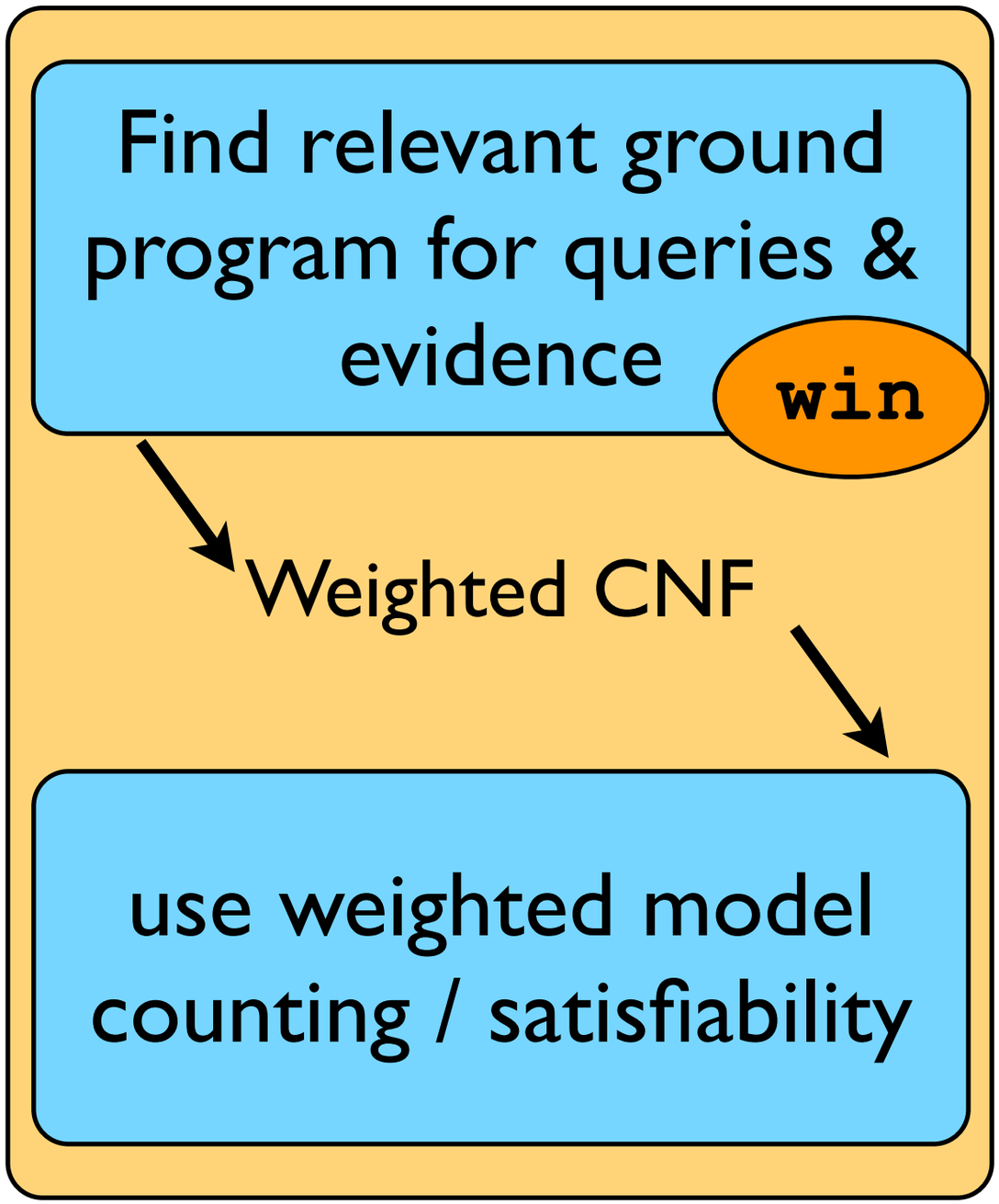


```
win :- heads(1).
win :- heads(2), heads(3).
```

$$\text{win} \leftrightarrow h(1) \vee (h(2) \wedge h(3))$$

$$(\neg\text{win} \vee h(1) \vee h(2)) \wedge (\neg\text{win} \vee h(1) \vee h(3)) \wedge (\text{win} \vee \neg h(1)) \wedge (\text{win} \vee \neg h(2) \vee \neg h(3))$$

[Fierens et al, TPLP 14]

Current Approach
(ProbLog2)

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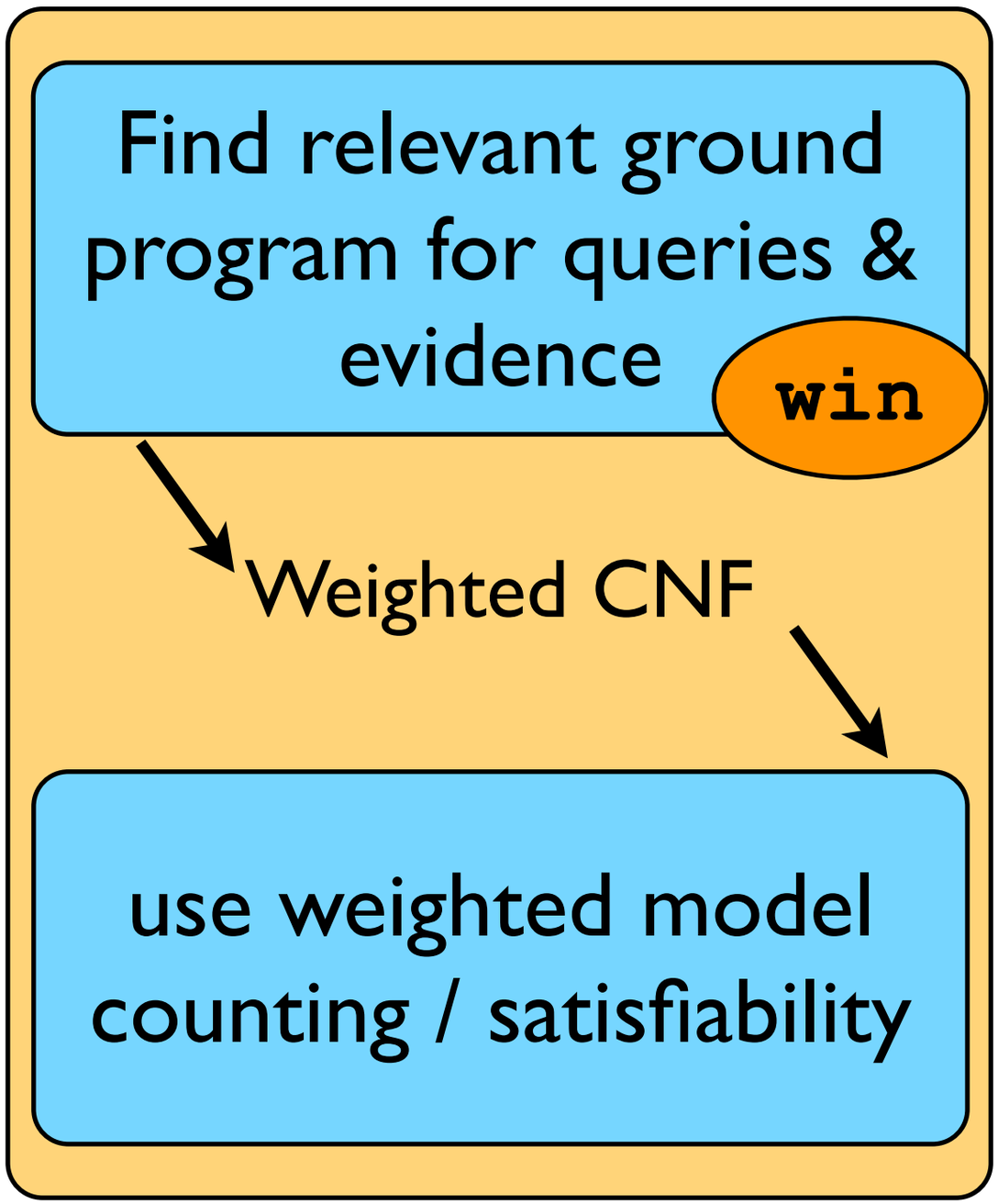


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win :- heads(1).
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$h(1) \rightarrow 0.4$	$h(2) \rightarrow 0.7$	$h(3) \rightarrow 0.5$
$\neg h(1) \rightarrow 0.6$	$\neg h(2) \rightarrow 0.3$	$\neg h(3) \rightarrow 0.5$

[Fierens et al, TPLP 14]

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Find relevant ground program for queries & evidence

win

Weighted CNF

use weighted model counting / satisfiability

```
win :- heads(1).
win :- heads(2), heads(3).
```

$$\text{win} \leftrightarrow h(1) \vee (h(2) \wedge h(3))$$

$$(\neg\text{win} \vee h(1) \vee h(2)) \wedge (\neg\text{win} \vee h(1) \vee h(3)) \wedge (\text{win} \vee \neg h(1)) \wedge (\text{win} \vee \neg h(2) \vee \neg h(3))$$

use standard tool

$h(1) \rightarrow 0.4$ $\quad h(2) \rightarrow 0.7$ $\quad h(3) \rightarrow 0.5$

$\neg h(1) \rightarrow 0.6$ $\quad \neg h(2) \rightarrow 0.3$ $\quad \neg h(3) \rightarrow 0.5$

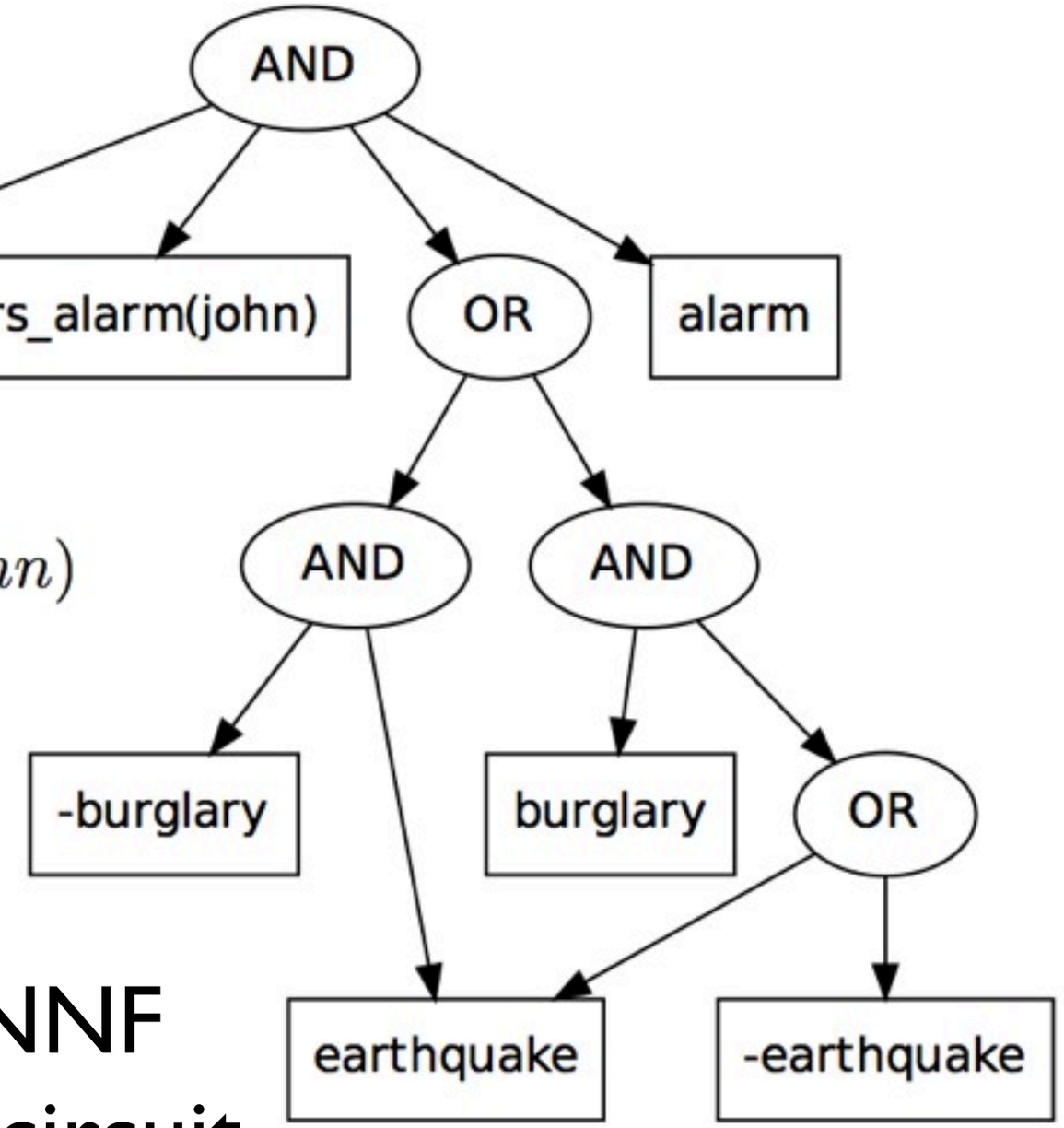
[Fierens et al, TPLP 14]

WMC using d-DNNFs

$$alarm \leftrightarrow burglary \vee earthquake$$

$$calls(john) \leftrightarrow alarm, hears_alarm(john)$$

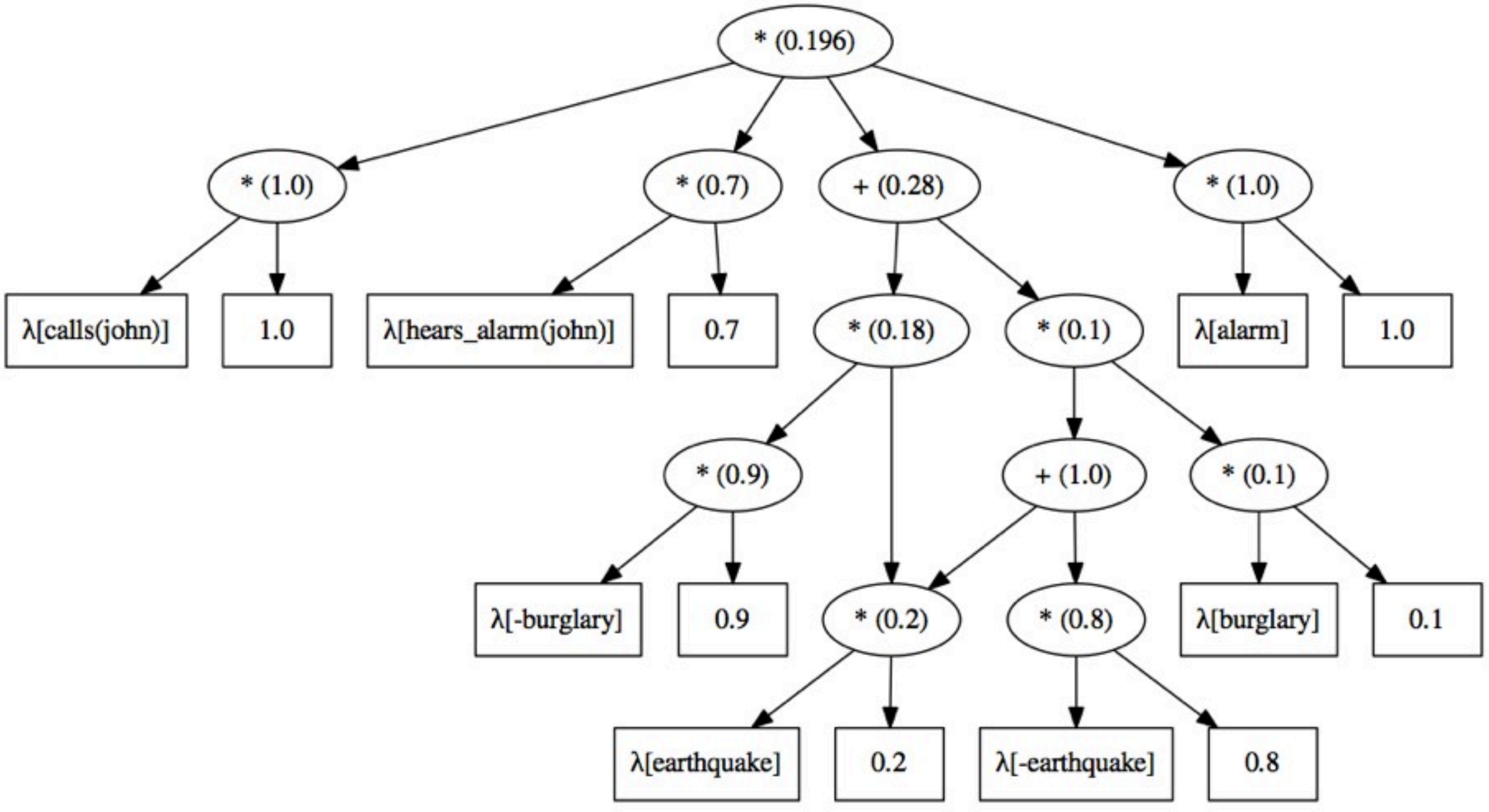
$$calls(john)$$



1. represent formula as d-DNNF
2. transform into arithmetic circuit
3. evaluate bottom-up

[Figure: Fierens et al, TPLP 14]

WMC using d-DNNFs



3. evaluate bottom-up

[Figure: Fierens et al, TPLP 14]

ProbLog Inference

- reduction to propositional formula
- addresses disjoint-sum-problem
- **but**: not all probabilistic logic programs face this problem! e.g., weather
- more generally: mutually exclusive proofs as assumed in PRISM

PRISM

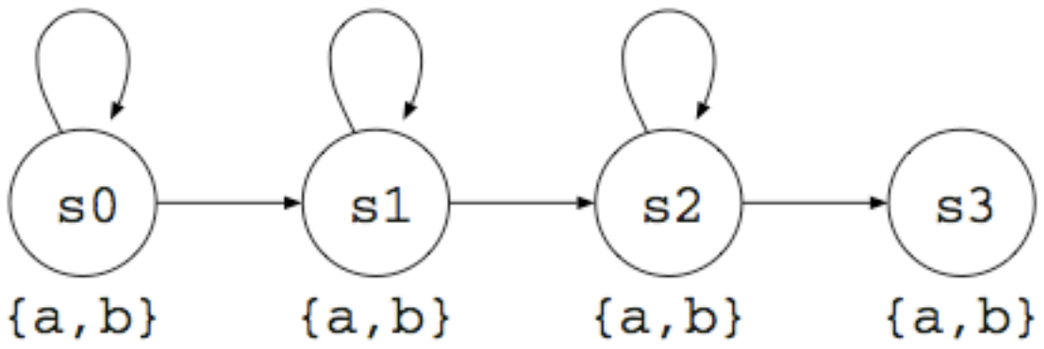


Fig. 1. Example of a left-to-right HMM with four states

```
target(hmm/1).
values(tr(s0),[s0,s1]).
values(tr(s1),[s1,s2]).
values(tr(s2),[s2,s3]).
values(out(_),[a,b]).

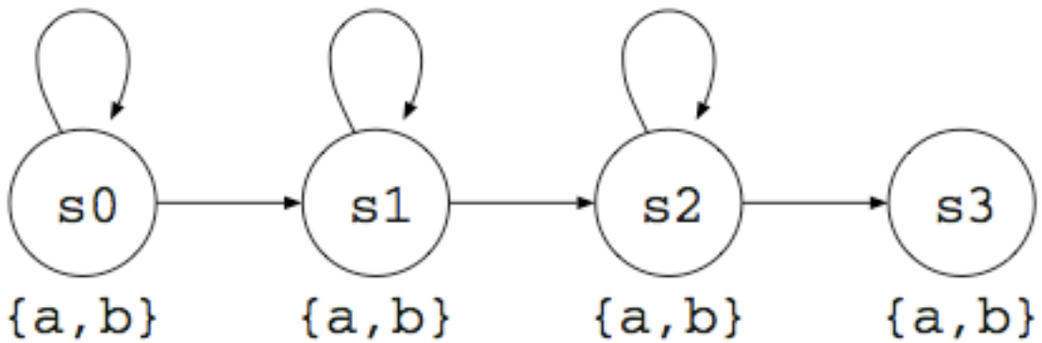
hmm(Cs):- hmm(0,s0,Cs).

hmm(T,s3,[C]):- msw(out(s3),  C). % If at the final state:
                                  %   output a symbol and then terminate.
hmm(T,S,[C|Cs]):- S\==s3,         % If not at the final state:
  msw(out(S),  C),                %   choose a symbol to be output,
  msw(tr(S),  Next),              %   choose the next state,
  T1 is T+1,                      %   Put the clock ahead,
  hmm(T1,Next,Cs).                %   and enter the next loop.
```

Fig. 2. PRISM program for the left-to-right HMM

[Figures: Sato and Kameya 08]

PRISM



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define switches

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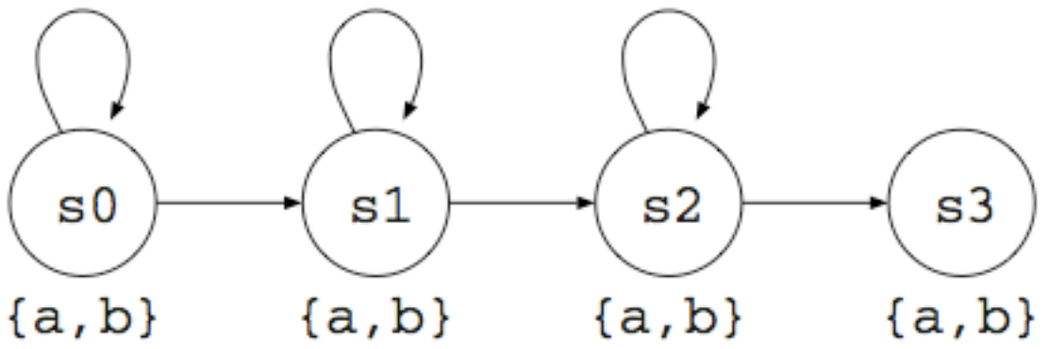
PRISM



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```

use switches

Fig. 2. PRISM program for the left-to-right HMM

[Figures: Sato and Kameya 08]

PRISM: compute probability by dynamic programming

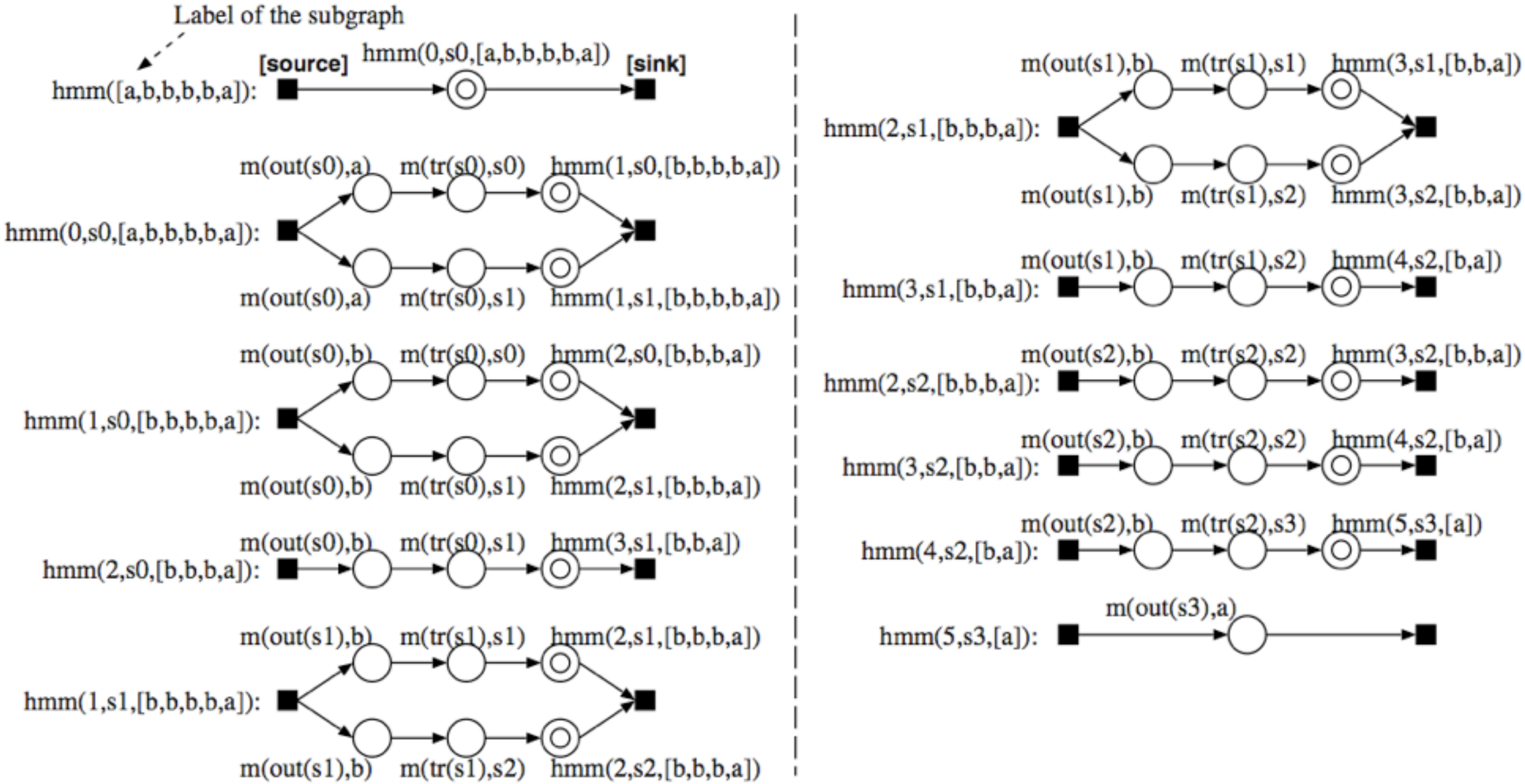

Fig. 5. Explanation graph

[Figures: Sato and Kameya 08]

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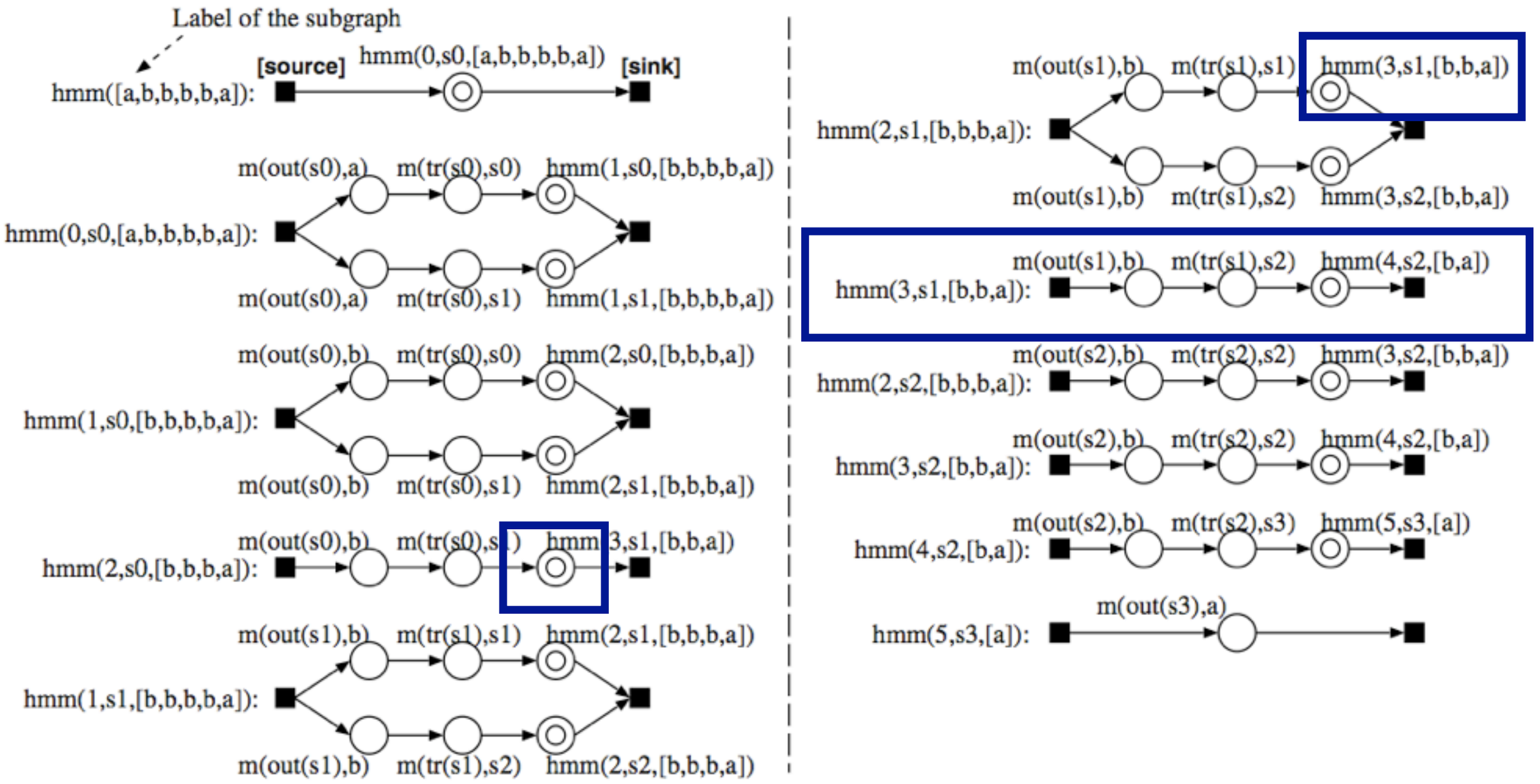


Fig. 5. Explanation graph

[Figures: Sato and Kameya 08]

Dyna (Eisner et al.)

word(john,0,1), word(loves,1,2), word(Mary,2,3)
0.003:: np -> Mary as rewrite(np,Mary)=0.003
0.5::vp -> verb, np as rewrite(vp,verb,np)=0.5

- CKY Algorithm in Dyna

```
constit(X,I,J) += rewrite(X,W) * word(W,I,J).
constit(X,I,K) += rewrite(X,Y,Z) * constit(Y,I,J) * constit(Z,J,K).
goal += constit(S,0,N) * end(N).
```

- $\sum_{Y,Z,J}$ $\sum_{W}$

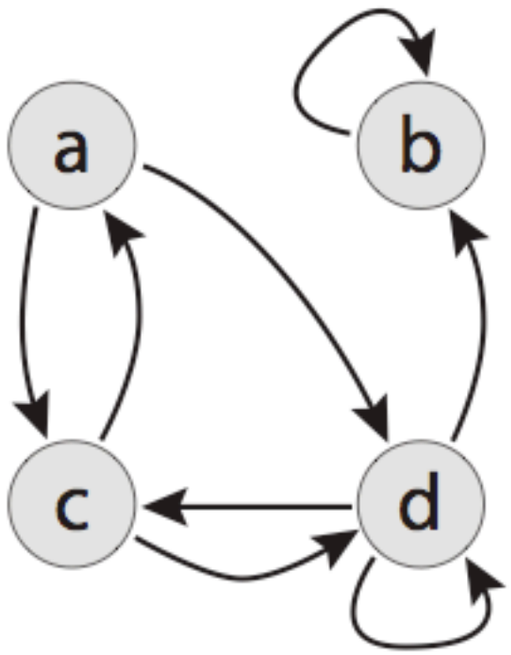
Origins in Natural Language Processing Context
Based on Dynamic Programming
Useful for SRL/NLP/Programming

Dyna

- Weighted Logic Programs (but not Prolog)
- Inspired by NLP
- Arbitrary semiring weights
- Forward reasoning

```
reachable(Q) :- initial(Q).
reachable(Q) :- reachable(P), edge(P, Q).
```

Fig. 1. A simple bottom-up logic program for graph reachability



$$\begin{aligned}
&\texttt{initial(a)} = \mathrm{T} \quad \texttt{edge(c, d)} = \mathrm{T} \\
&\texttt{edge(a, c)} = \mathrm{T} \quad \texttt{edge(d, b)} = \mathrm{T} \\
&\texttt{edge(a, d)} = \mathrm{T} \quad \texttt{edge(d, c)} = \mathrm{T} \\
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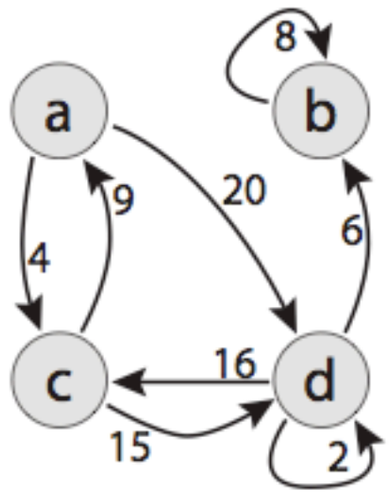
\end{aligned}$$

Fig. 2. A directed graph and the corresponding initial database

[Figures: Cohen et al, ICLP 08]

$$\texttt{reachable(Q)} \oplus\!= \texttt{initial(Q)}.$$

$$\texttt{reachable(Q)} \oplus\!= \texttt{reachable(P)} \otimes \texttt{edge(P, Q)}.$$



$$\begin{aligned}
\texttt{initial(a)} &= 0 & \texttt{edge(c, d)} &= 15\\
\texttt{edge(a, c)} &= 4 & \texttt{edge(d, b)} &= 6\\
\texttt{edge(a, d)} &= 20 & \texttt{edge(d, c)} &= 16\\
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\texttt{edge(c, a)} &= 9
\end{aligned}$$

Fig. 3. A cost graph and the corresponding initial database

[Figures: Cohen et al, ICLP 08]

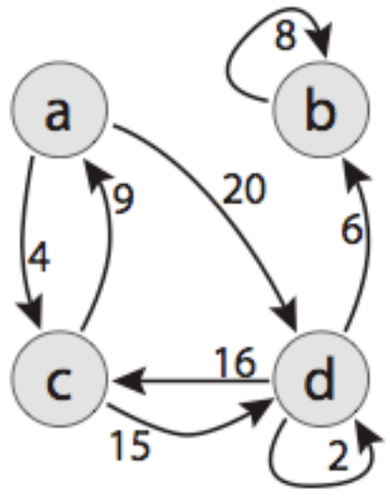
$$\texttt{reachable}(\texttt{Q}) \oplus\!= \texttt{initial}(\texttt{Q}).$$

$$\texttt{reachable}(\texttt{Q}) \oplus\!= \texttt{reachable}(\texttt{P}) \otimes \texttt{edge}(\texttt{P}, \texttt{Q}).$$

$\oplus \quad \rightarrow \quad \min$

$\otimes \quad \rightarrow \quad +$

shortest path



$$\texttt{initial}(\texttt{a}) = 0 \qquad \texttt{edge}(\texttt{c}, \texttt{d}) = 15$$
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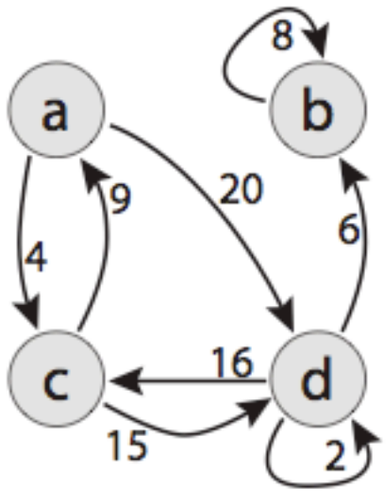

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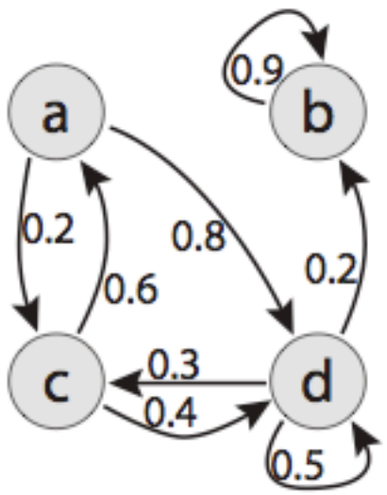

$$\texttt{initial}(\texttt{a}) = 1 \quad \texttt{edge}(\texttt{c},\texttt{d}) = 0.4$$
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Fig. 4. A probabilistic graph and the corresponding initial database. With stopping probabilities made explicit, this would encode a Markov model.

[Figures: Cohen et al, ICLP 08]

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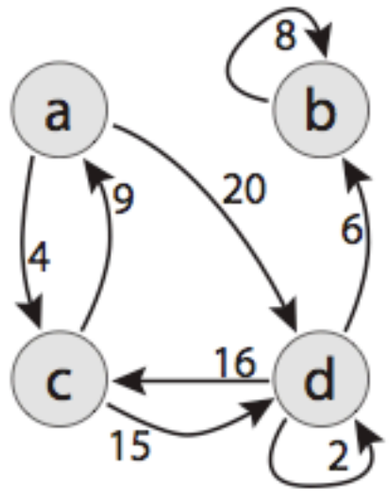

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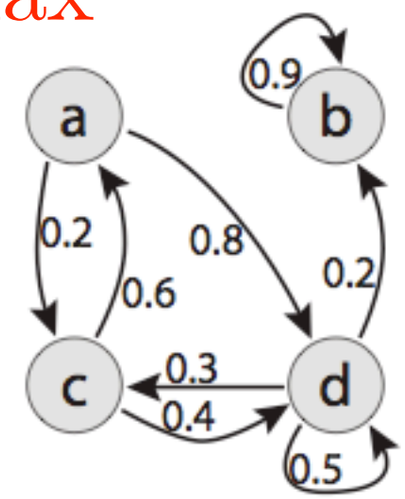
$\oplus \rightarrow \max$

$\otimes \rightarrow \cdot$

most likely path



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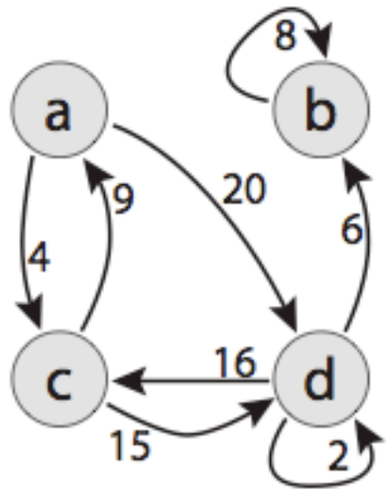

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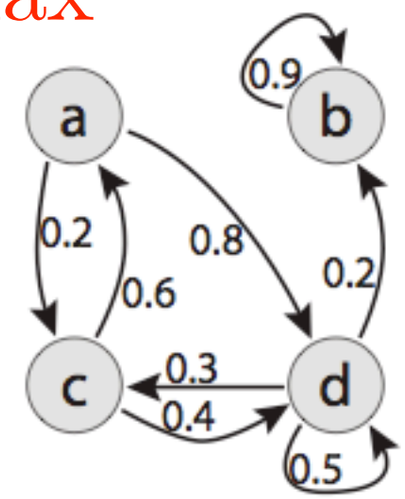

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PRISM

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aProbLog: algebraic Prolog

[Kimmig et al, AAAI 11]

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- Prolog + labeled facts

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	neutral sum	NSP
disjoint sum	**SAT**	**MPE**
DSP	**PROB**	**#SAT**

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one world - several proofs

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Algebraic Prolog (aProbLog)

- commutative semiring $(A,\oplus,\otimes,e^{\oplus},e^{\otimes})$
- algebraic ground literals
  $L(F) = \{f_1,...,f_n\} \cup \{\neg f_1,...,\neg f_n\}$
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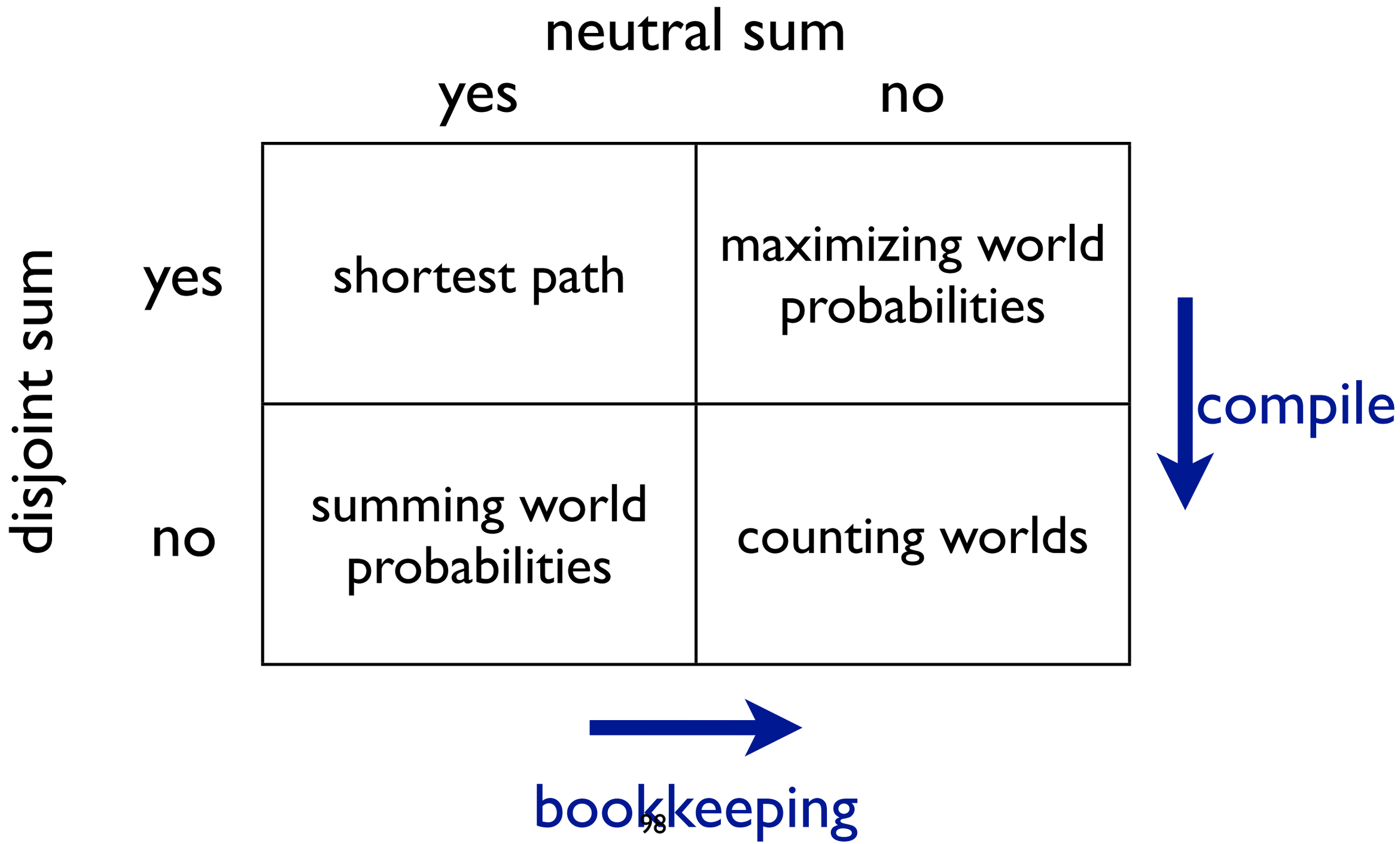
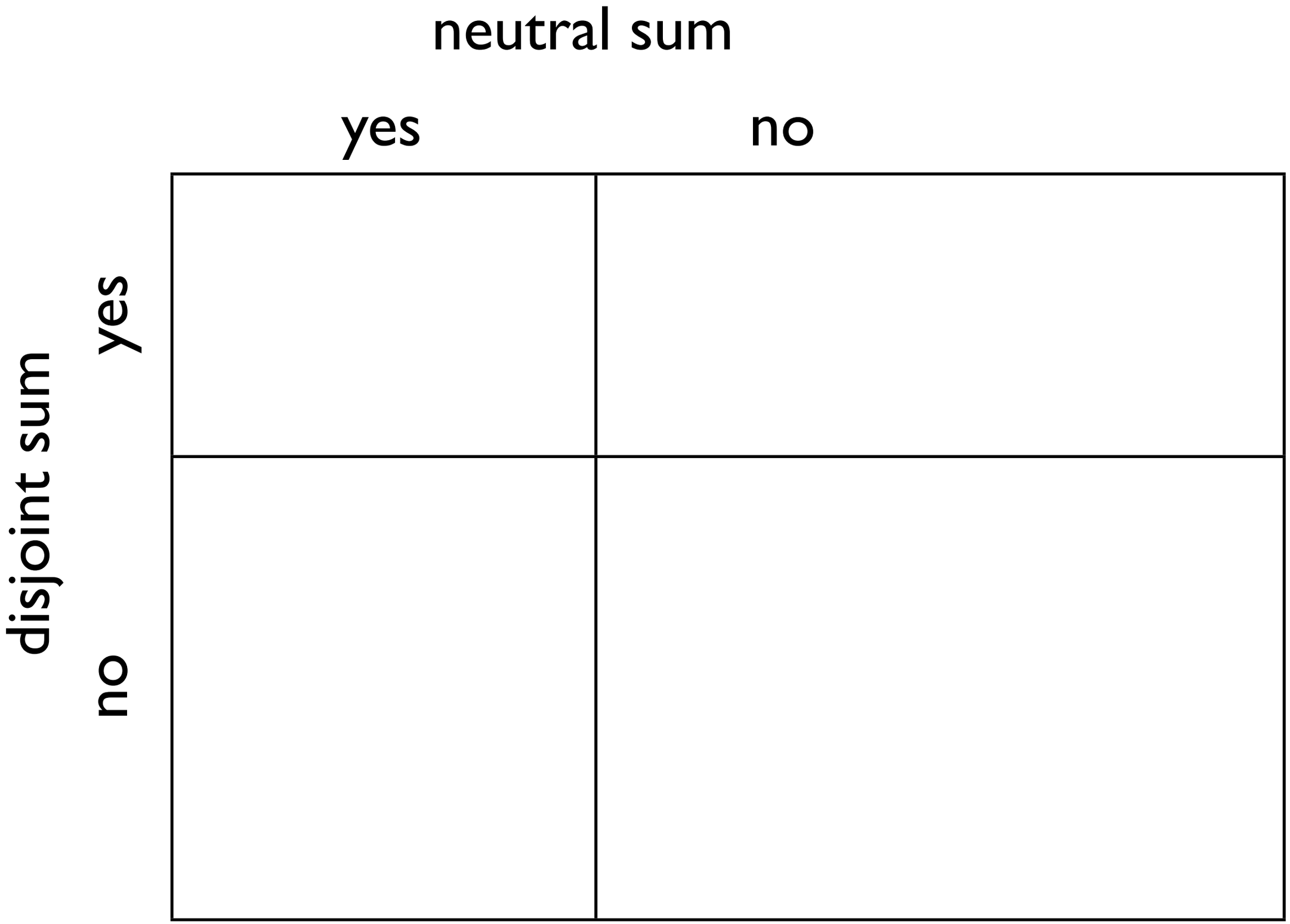
- labeling function $\alpha: L(F) \rightarrow A$

$$L(\text{query}) = \bigoplus_{\text{worlds}} \bigotimes_{\text{literals}} \alpha(l)$$

Inference settings




neutral sum
yes
no
disjoint sum
yes
no

neutral sum

min-sum

disjoint sum	yes	no
yes	$(a \wedge b) \vee (a \wedge c)$	
no		

neutral sum

min-sum | yes | no | *most likely*

disjoint sum

	yes	no
yes	$(a \wedge b) \vee (a \wedge c)$	$(a \wedge b \wedge (c \vee \neg c)) \vee (a \wedge c \wedge (b \vee \neg b))$
no		

expand

neutral sum

min-sum yes no *most likely*

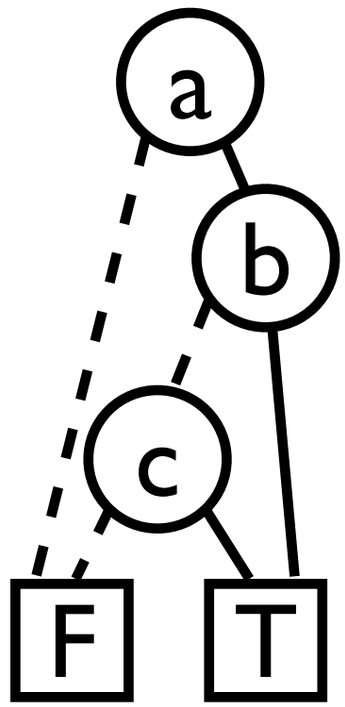
disjoint sum

yes

$(a \wedge b)$
$\vee(a \wedge c)$

$(a \wedge b \wedge (c \vee \neg c))$
$\vee(a \wedge c \wedge (b \vee \neg b))$

no



filter

probability

expand

neutral sum

min-sum

yes

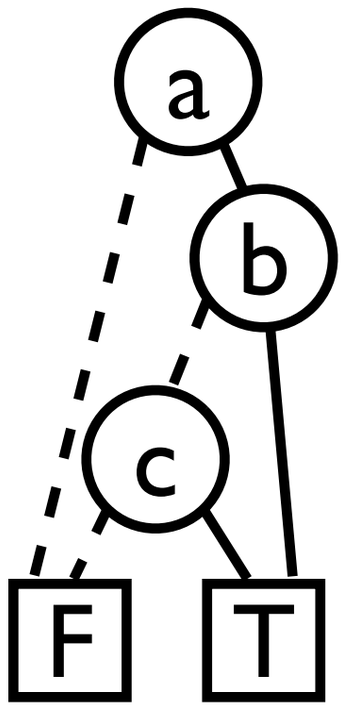
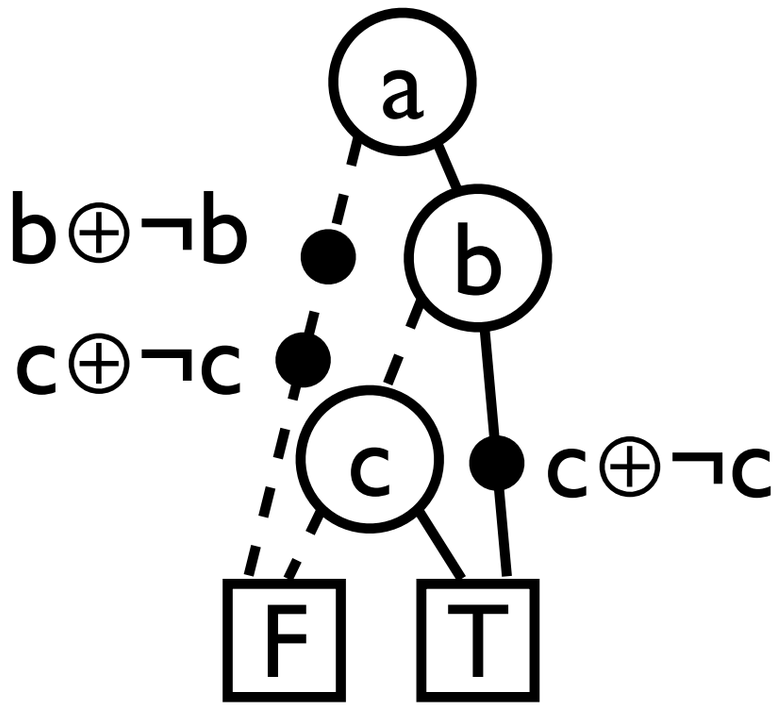
no

most likely

disjoint sum

yes

no

$(a \wedge b)$ $\vee(a \wedge c)$	$(a \wedge b \wedge (c \vee \neg c))$ $\vee(a \wedge c \wedge (b \vee \neg b))$
	

filter

probability

expected cost

expand

Approximate Inference

- Lower and upper bounds

$$\phi_L \models \phi \models \phi_U$$

$$P(\phi_L) \leq P(\phi) \leq P(\phi_U)$$

- Sampling

Approximate Inference

also see Vlasselaer et al,
Tue 9:40

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Tue 9:40

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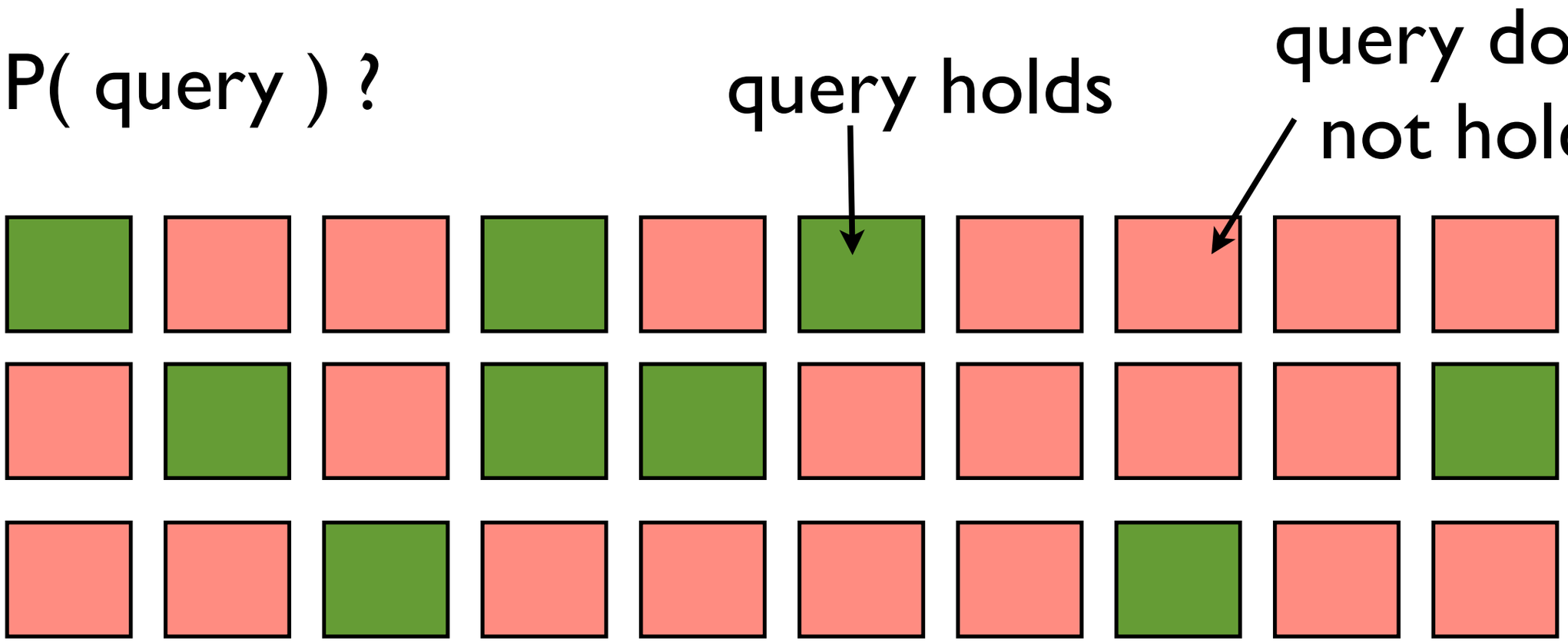
Sampling

- P(query) ?



$$P(\text{query}) \approx \frac{\#\text{ query holds}}{\#\text{ worlds sampled}}$$

Rejection Sampling

- P(query | evidence) ?

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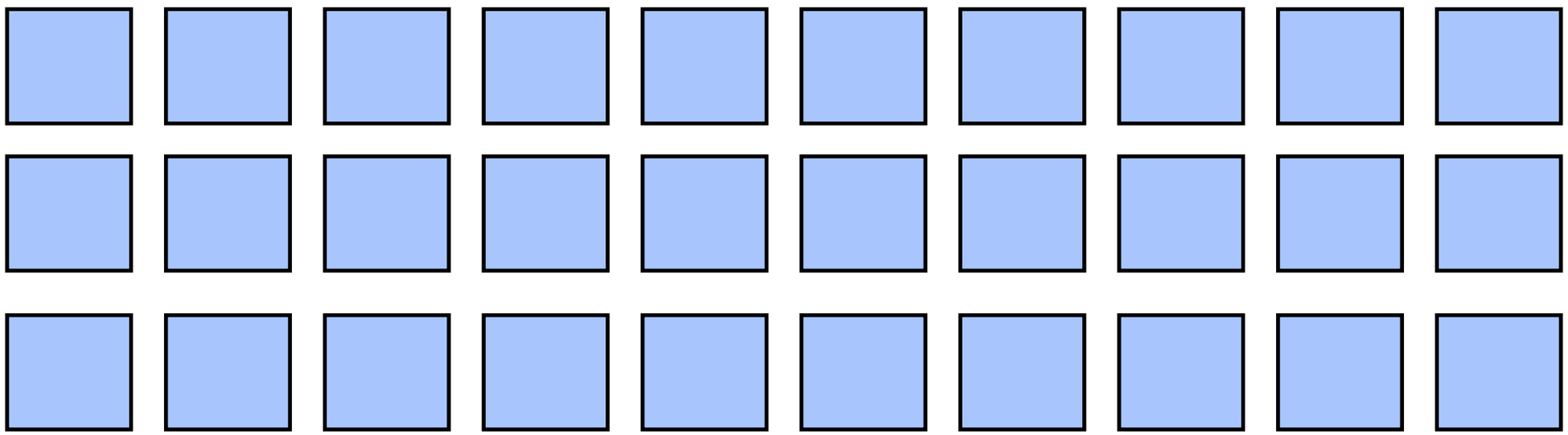

Rejection Sampling

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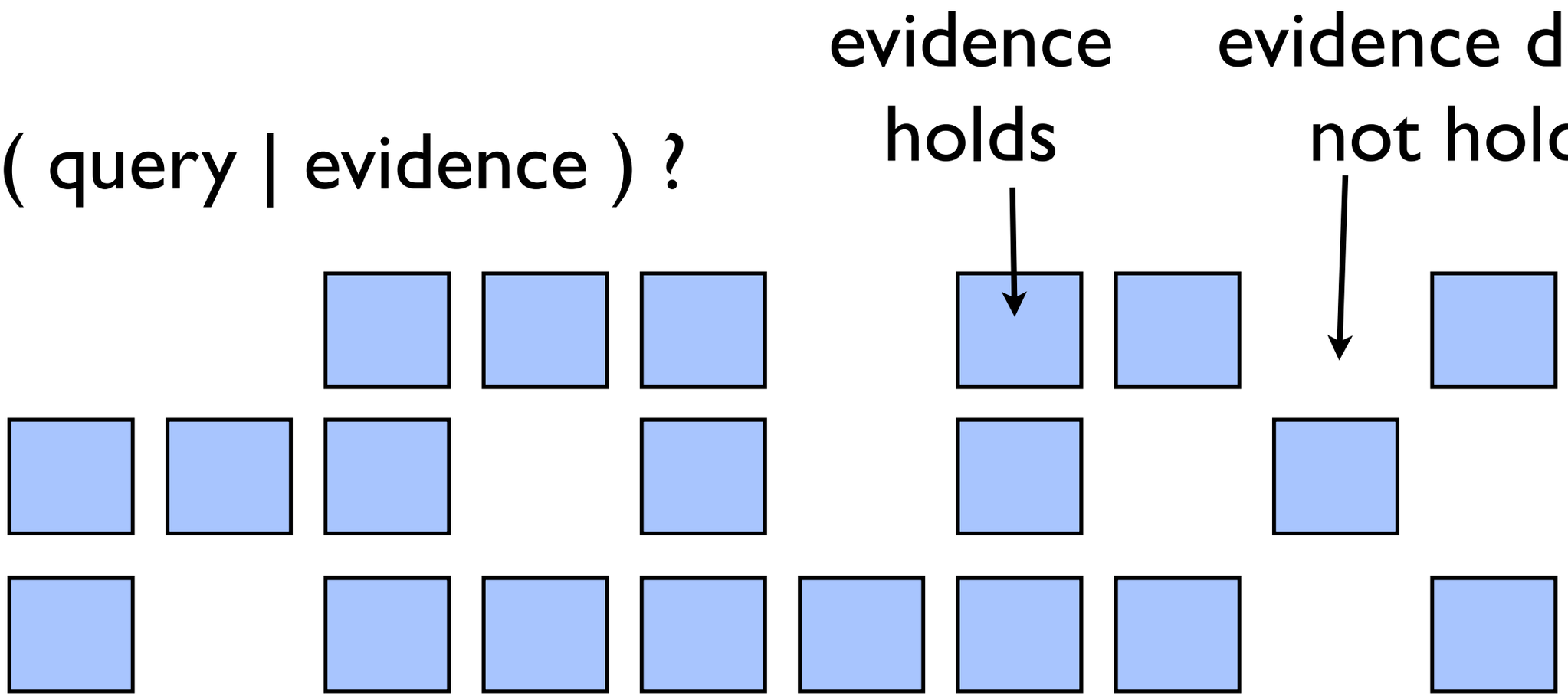
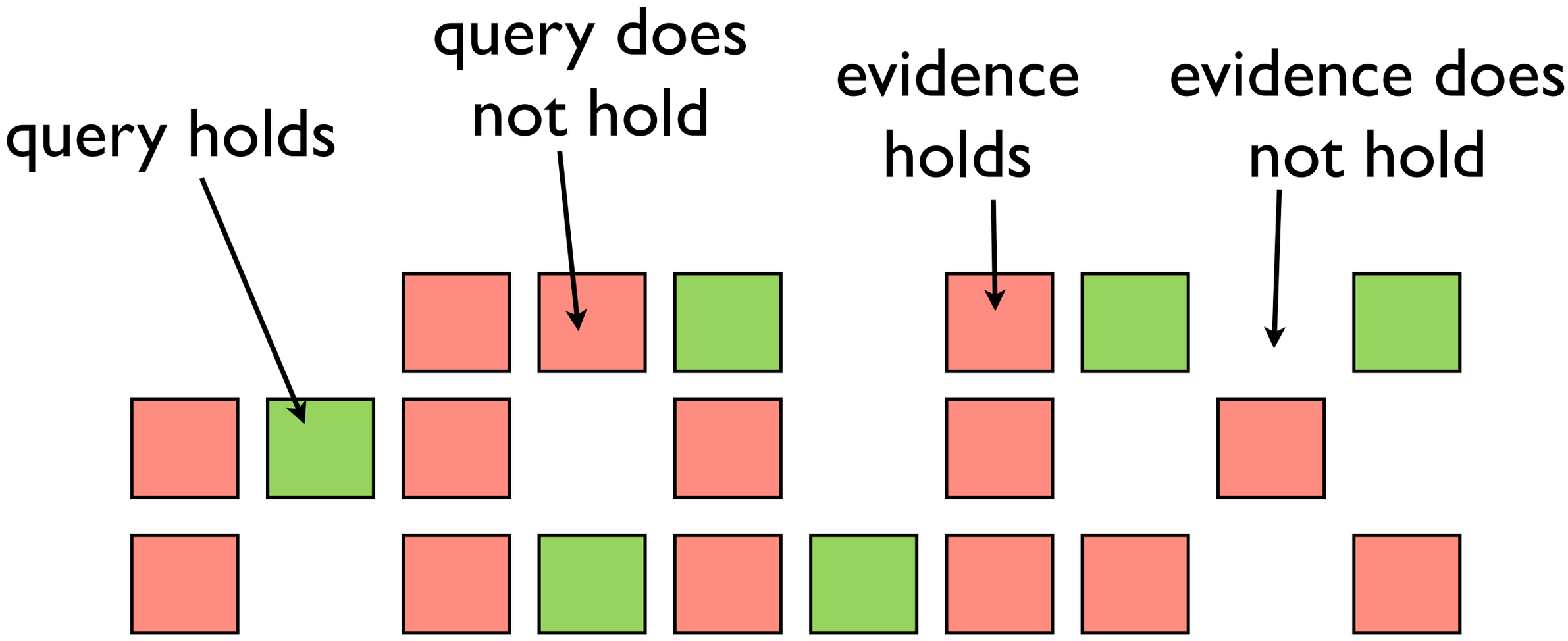


Rejection Sampling



$$P(\text{query} \mid \text{evidence}) \approx \frac{\#\ \text{query \& evidence holds}}{\#\ \text{evidence holds}}$$

Likelihood Weighting

- P(query | evidence) ?

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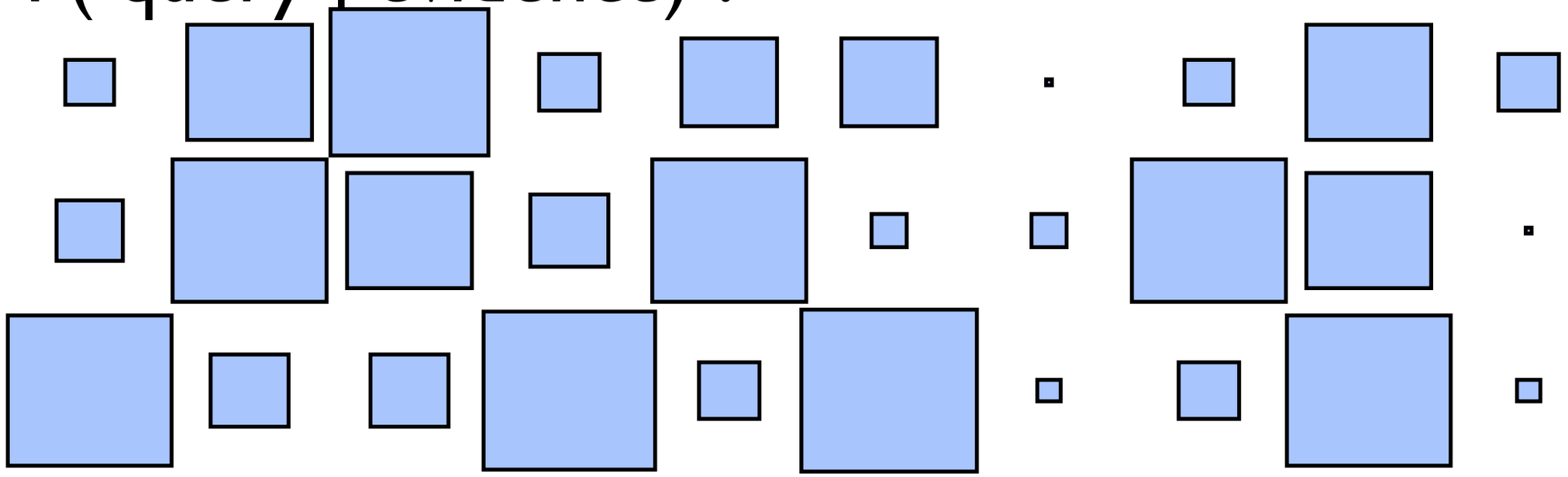
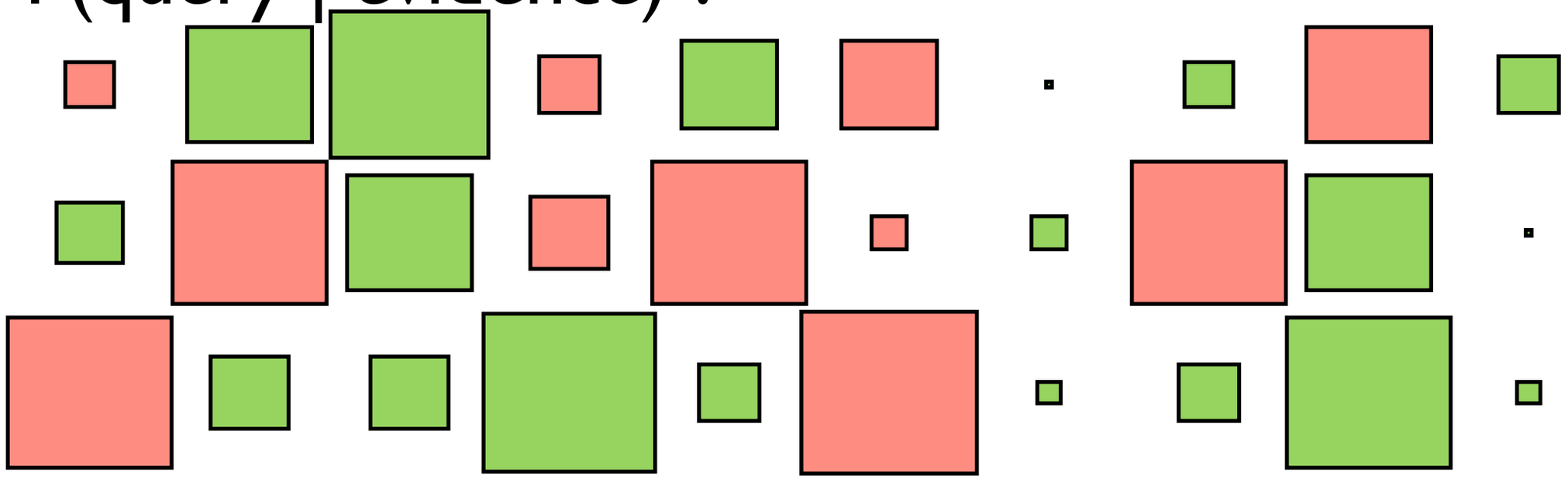


Likelihood Weighting

- P(query | evidence) ?



Markov Chain Monte Carlo (MCMC)

- Generate next sample by modifying current one
- Most common inference approach for PP languages such as Church, BLOG, ...
- Also considered for PRISM and ProbLog

Key challenges:
- how to propose next sample
- how to handle evidence

Roadmap

- Modeling
- Inference
- Learning & Dynamics
- Advanced Inference and KBMC

... with some detours on the way

Part III : Learning and Dynamics

Parameter Learning

e.g., webpage classification model

for each ***CLASS1***, ***CLASS2*** and each ***WORD***

?? :: link_class(Source,Target,***CLASS1***,***CLASS2***).
?? :: word_class(***WORD***,***CLASS***).

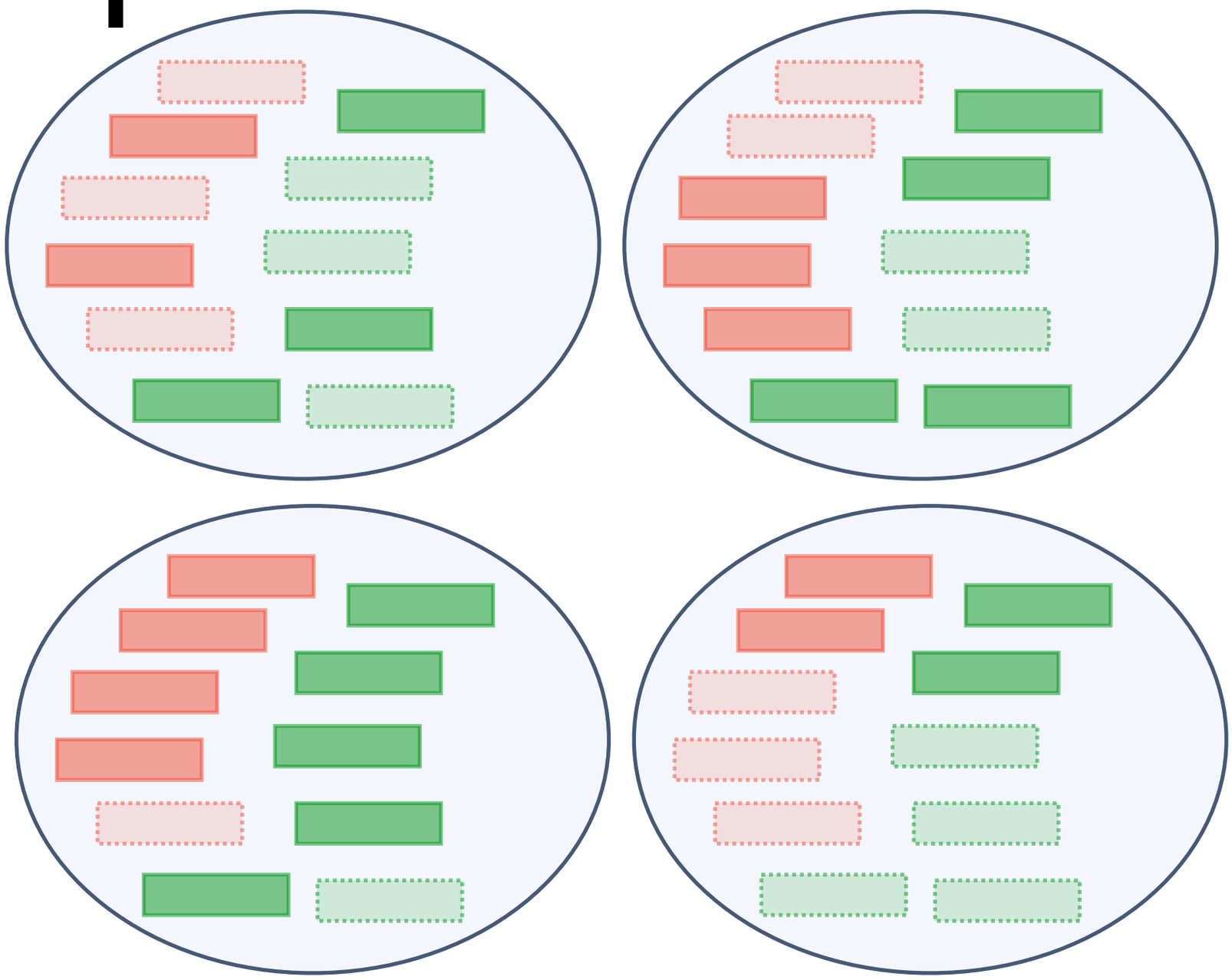
```
class(Page,C)    :-  has_word(Page,W), word_class(W,C).

class(Page,C)    :-  links_to(OtherPage,Page),
                     class(OtherPage,OtherClass),
                     link_class(OtherPage,Page,OtherClass,C).
```

Sampling Interpretations



Sampling Interpretations

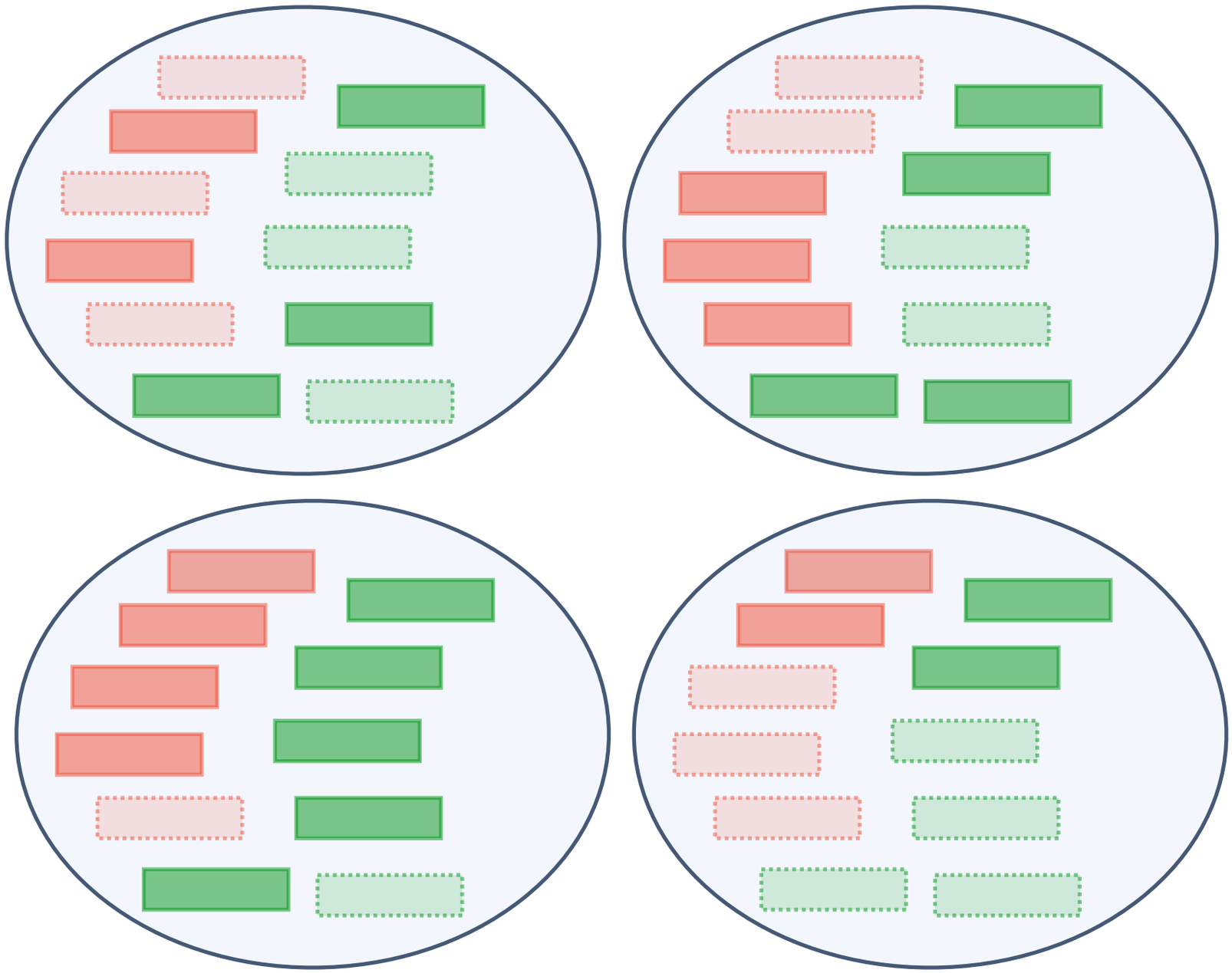




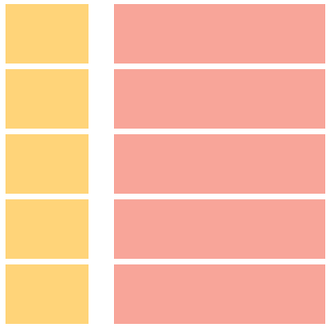
III

Parameter Estimation

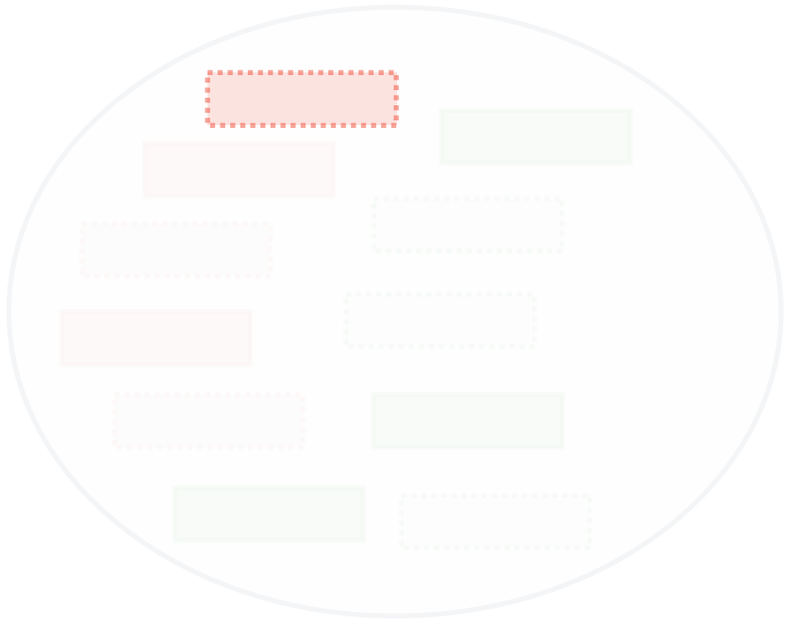


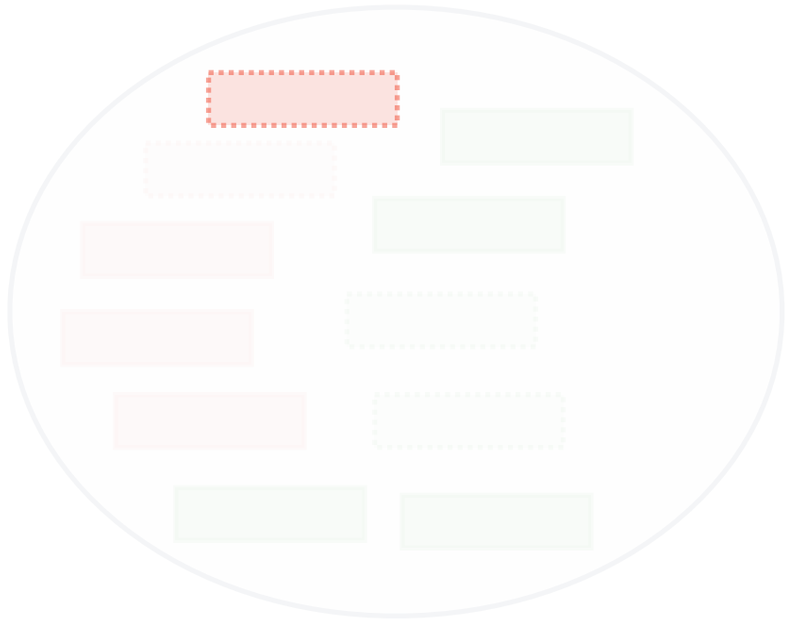


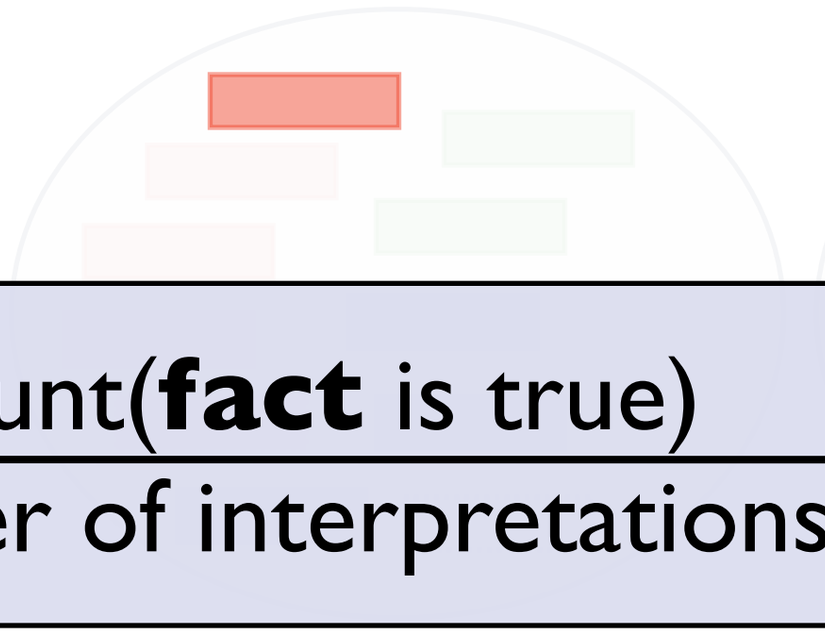
Parameter Estimation







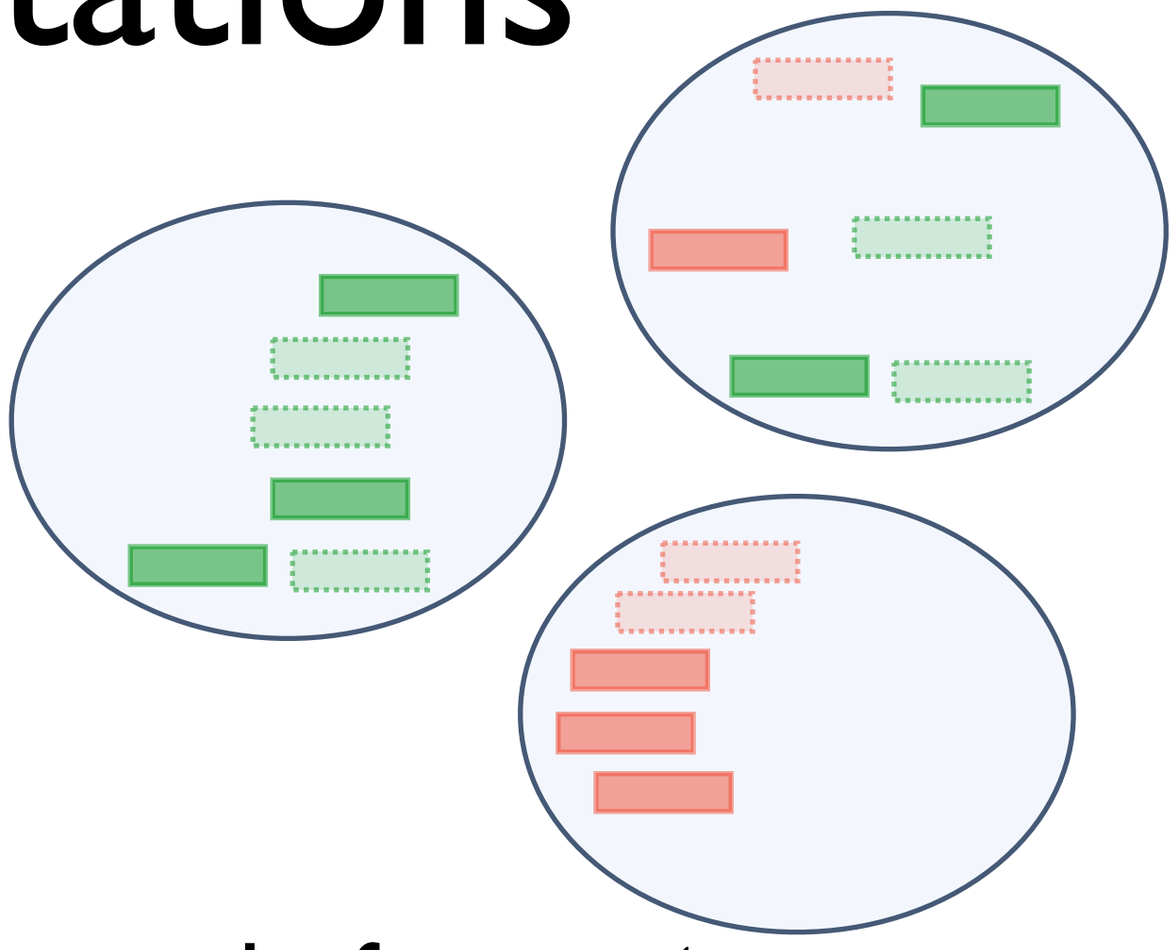






$$p(\textbf{fact}) = \frac{\text{count}(\textbf{fact} \text{ is true})}{\text{Number of interpretations}}$$

Learning from partial interpretations



- Not all facts observed
- Soft-EM
- use **expected count** instead of **count**
- **P(Q |E) -- conditional queries !**

 [Gutmann et al, ECML 11; Fierens et al, TPLP 14]

Learning from single facts / entailment



- Only true facts are given; e.g. as in HMM
  - key setting in PRISM, also in ProbLog
- EM-based, variations exist
- use **expected count** instead of **count**
- **P(Q |E) -- conditional queries !**

[Gutmann et al, ECML 08; Sato 95]

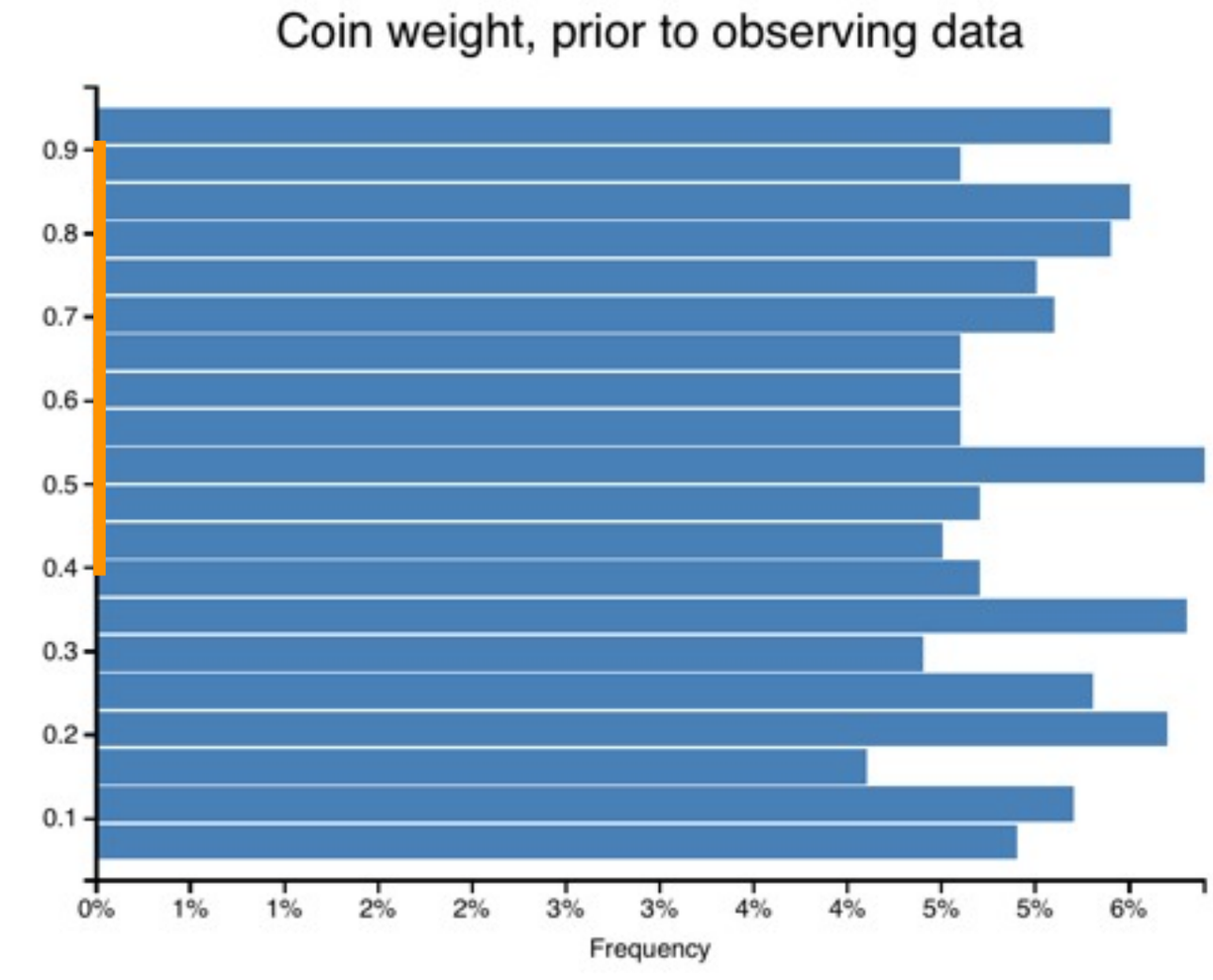
Bayesian Parameter Learning

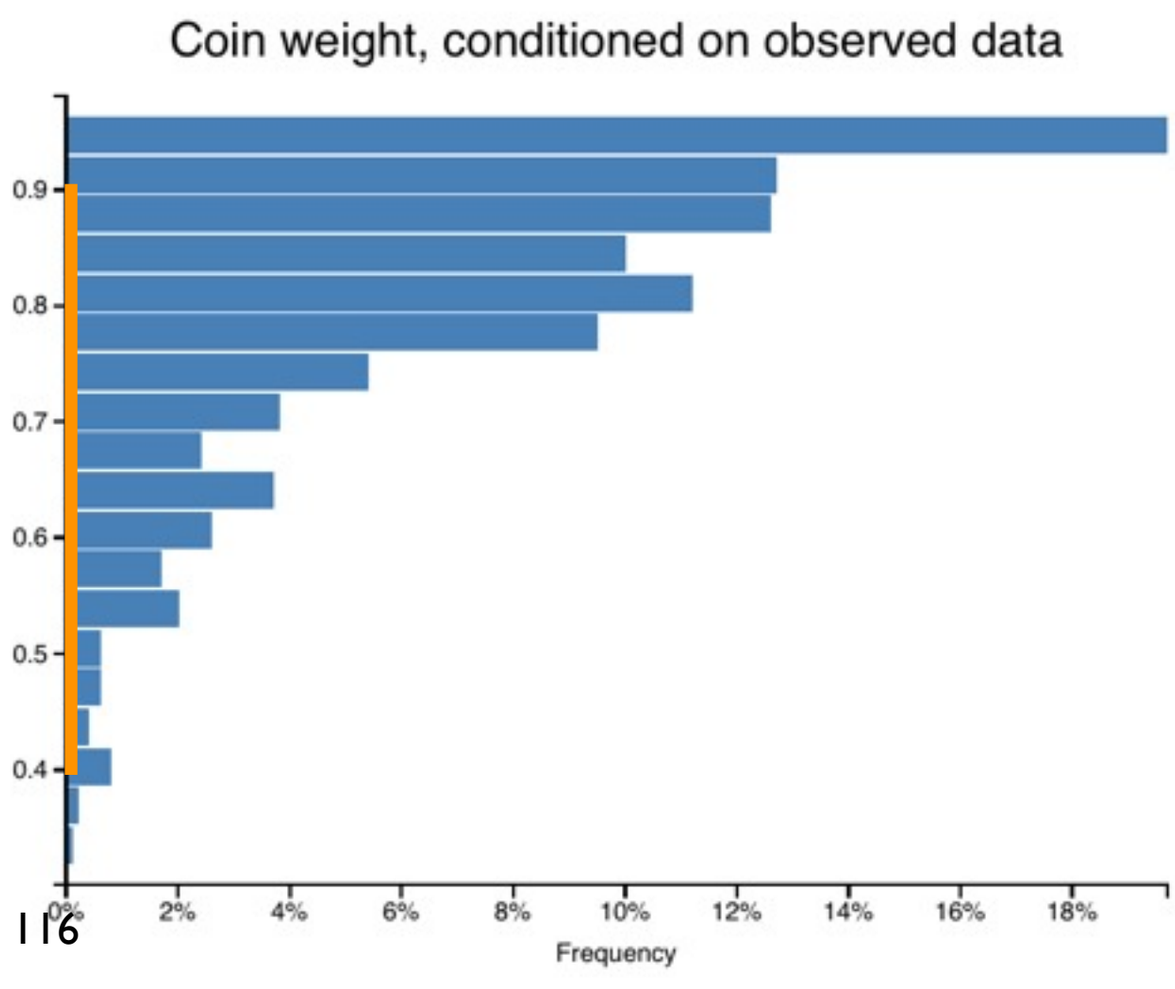
- Learning as inference (e.g., Church)
- Prior distributions for parameters
- Given data, find most likely parameter values

Example

[from probmods.org]

- Flipping a coin with unknown weight
- Prior: uniform distribution on [0,1]
- Observation: 5x heads in a row
- Sampling from Church model:





ProbLog Example

prior

```
0.05::weight(C,0.1); 0.2::weight(C,0.3); 0.5::weight(C,0.5);
                  0.2::weight(C,0.7); 0.05::weight(C,0.9) :- coin(C).
```

```
Param::toss(_,Param,_).
heads(C,R) :- weight(C,Param),toss(C,Param,R).
tails(C,R) :- weight(C,Param),\+toss(C,Param,R).

data(C,[]).
data(C,[h|R]) :- heads(C,R), data(C,R).
data(C,[t|R]) :- tails(C,R), data(C,R).
```

```
coin(c1).
coin(c2).
param(0.1).
param(0.3).
param(0.5).
param(0.7).
param(0.9).
```

```
query(weight(C,X)) :- coin(C),param(X).
```

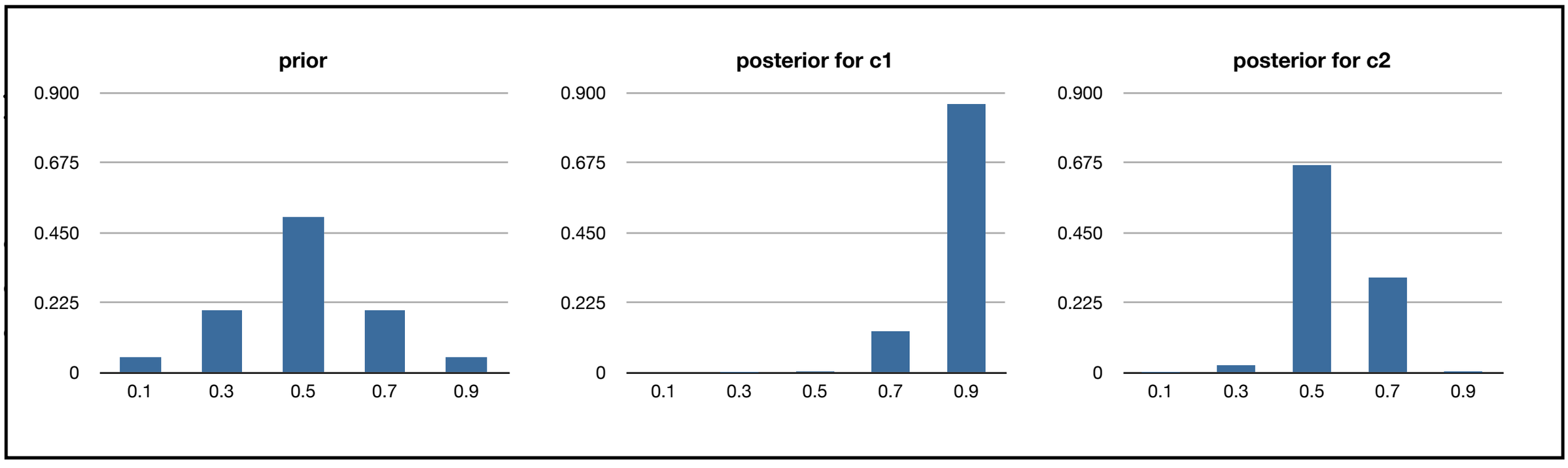
ask for posterior

```
evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h,h,h,h]),true).
evidence(data(c2,[h,t,h,h,h,h,h,t,t,h,t,t,h]),true).
```

data

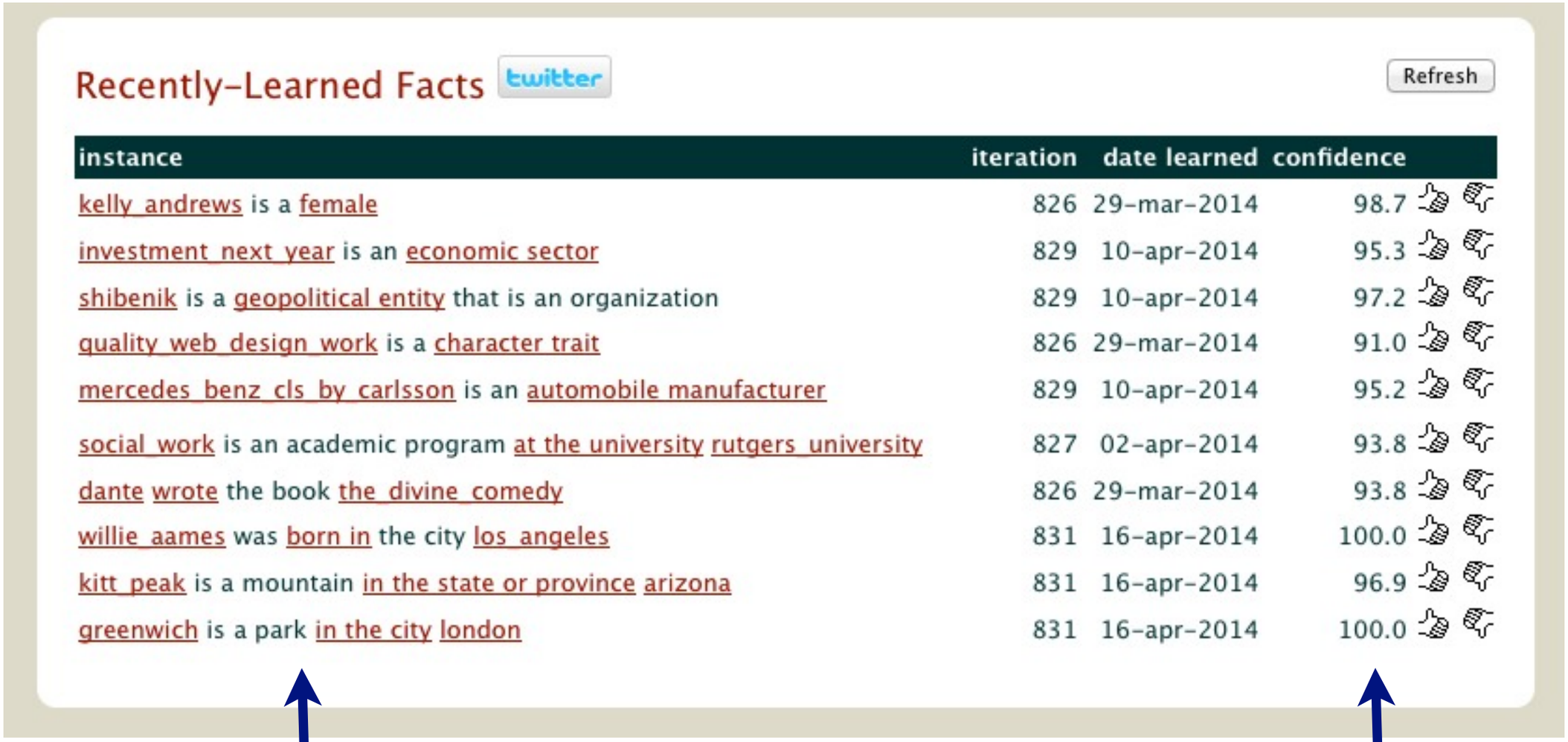
ProbLog Example

```
0.05::weight(C,0.1); 0.2::weight(C,0.3); 0.5::weight(C,0.5);
                     0.2::weight(C,0.7); 0.05::weight(C,0.9) :- coin(C).
```




```
query(weight(C,X)) :- coin(C),param(X).

evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h,h,h,h]),true).
evidence(data(c2,[h,t,h,h,h,h,h,t,t,h,t,t,h]),true).
```

Information Extraction in NELL



Recently-Learned Facts twitter

Refresh

instance	iteration	date learned	confidence
kelly_andrews is a female	826	29-mar-2014	98.7
investment_next_year is an economic sector	829	10-apr-2014	95.3
shibenik is a geopolitical entity that is an organization	829	10-apr-2014	97.2
quality_web_design_work is a character trait	826	29-mar-2014	91.0
mercedes_benz_cls_by_carlsson is an automobile manufacturer	829	10-apr-2014	95.2
social_work is an academic program at the university rutgers_university	827	02-apr-2014	93.8
dante wrote the book the_divine_comedy	826	29-mar-2014	93.8
willie_aames was born in the city los_angeles	831	16-apr-2014	100.0
kitt_peak is a mountain in the state or province arizona	831	16-apr-2014	96.9
greenwich is a park in the city london	831	16-apr-2014	100.0

instances for many different relations

degree of certainty

NELL: http://rtw.ml.cmu.edu/rtw/

Rule learning in NELL

- Original approach
  - Make probabilistic data deterministic
  - run classic rule-learner (variant of FOIL)
  - re-introduce probabilities on learned rules and predict

Probabilistic Rule Learning

- Learn the rules directly in a PLP setting
- Generalize relational learning and inductive logic programming directly towards probabilistic setting
- Traditional rule learning/ILP as a special case
- Apply to probabilistic databases like NELL
- Approach in PPR (Cohen et al) and in ProbLog (this IJCAI)

NELL

Table 5: Number of facts per predicate (NELL athlete dataset)

athletecoach(person,person)	18	athleteplaysforteam(person,team)	721
athleteplayssport(person,sport)	1921	teamplaysinleague(team,league)	1085
athleteplaysinleague(person,league)	872	athletealsoknownas(person,name)	17
coachesinleague(person,league)	93	coachesteam(person,team)	132
teamhomestadium(team,stadium)	198	teamplayssport(team,sport)	359
athleteplayssportsteamposition(person,position)	255	athletehomestadium(person,stadium)	187
athlete(person)	1909	attraction(stadium)	2
coach(person)	624	female(person)	2
male(person)	7	hobby(sport)	5
organization(league)	1	person(person)	2
personafrica(person)	1	personasia(person)	4
personaustralia(person)	22	personcanada(person)	1
personeurope(person)	1	personmexico(person)	108
personus(person)	6	sport(sport)	36
sportsleague(league)	18	sportsteam(team)	1330
sportsteamposition(position)	22	stadiumoreventvenue(stadium)	171

athleteplaysforteam

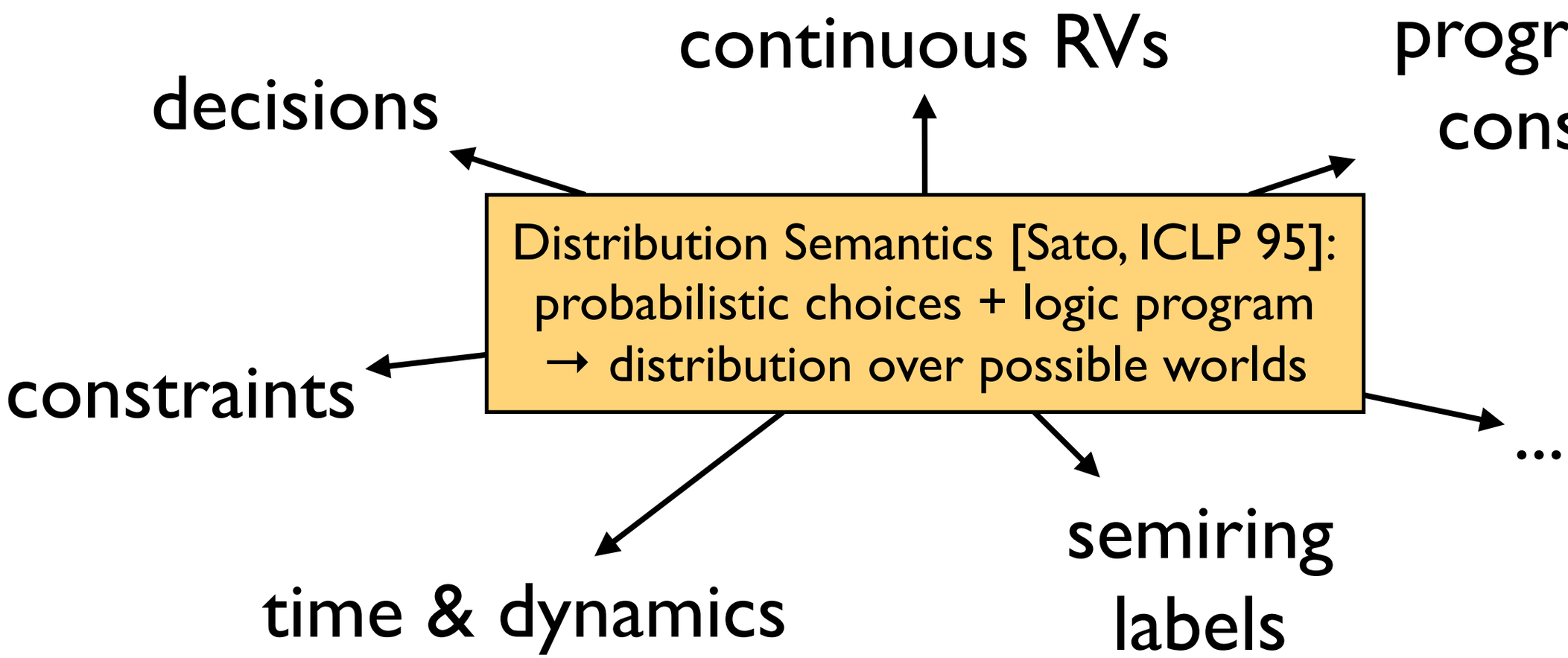
athleteplaysforteam(A,B) :- coachesteam(A,B).
0.875::athleteplaysforteam(A,B) :- teamhomestadium(B,C), athletehomestadium(A,C).
0.99080::athleteplaysforteam(A,B) :- teamhomestadium(B,_), male(A), athleteplayssport(A,_).
0.75::athleteplaysforteam(A,B) :- teamhomestadium(B,_), athleteplaysinleague(A,C), teamplaysinleague(B,C), athlete(A).
0.75::athleteplaysforteam(A,B) :- teamplayssport(B,C), athleteplayssport(A,C), coach(A), teamplaysinleague(B,_).
0.97555::athleteplaysforteam(A,B) :- personus(A), teamplayssport(B,_).
0.762::athleteplaysforteam(A,B) :- teamplayssport(B,C), athleteplayssport(A,C), personmexico(A), teamplaysinleague(B,_).
0.52571::athleteplaysforteam(A,B) :- teamplayssport(B,C), athleteplayssport(A,C), athleteplaysinleague(A,_), teamplaysinleague(B,_), athlete(A), teamplayssport(B,C).
0.50546::athleteplaysforteam(A,B) :- teamplayssport(B,_), teamplaysinleague(B,C), athleteplaysinleague(A,C), athleteplayssport(A,_).
0.50::athleteplaysforteam(A,B) :- teamplayssport(B,_), teamplaysinleague(B,C), athleteplaysinleague(A,C).
0.52941::athleteplaysforteam(A,B) :- teamplayssport(B,_), teamhomestadium(B,_), coach(A), teamplaysinleague(B,_).
0.55287::athleteplaysforteam(A,B) :- teamplayssport(B,_), teamplaysinleague(B,C), athleteplaysinleague(A,C), athlete(A).
0.46875::athleteplaysforteam(A,B) :- teamplayssport(B,_), teamplaysinleague(B,_), coach(A), teamhomestadium(B,_).

Roadmap

- Modeling
- Inference
- Learning & Dynamics
- Advanced Inference and KBMC

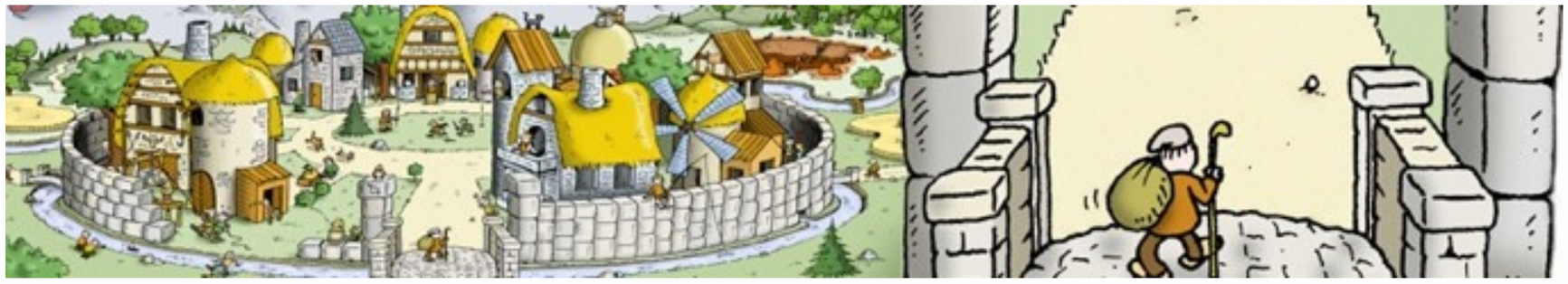
... with some detours on the way

Extensions of basic PLP



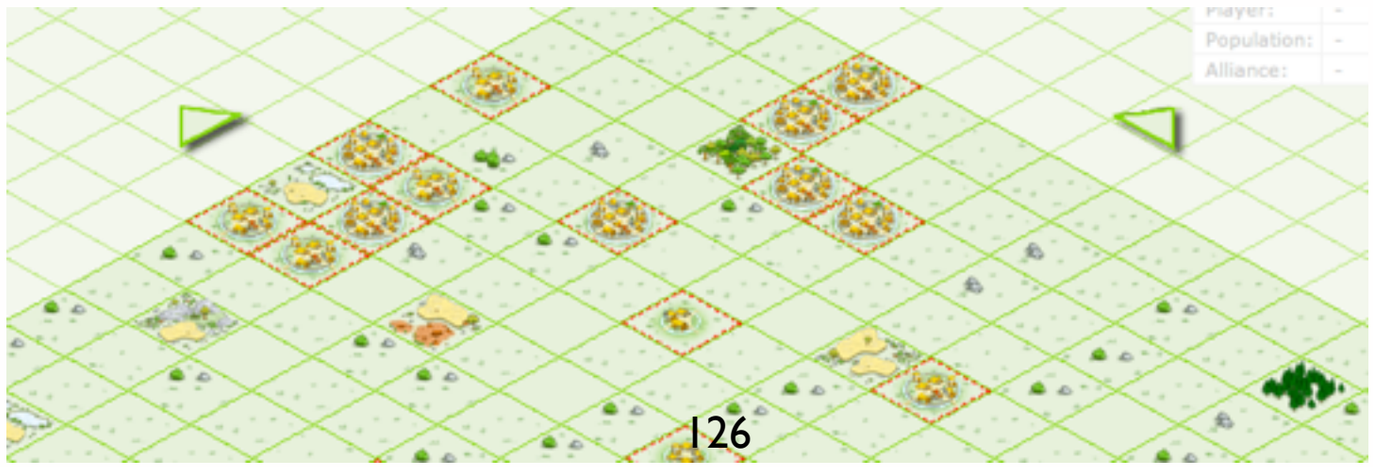
Dynamics

Dynamics: Evolving Networks



- *Travian*: A massively multiplayer real-time strategy game
- Commercial game run by TravianGames GmbH
- ~3.000.000 players spread over different "worlds"
- ~25.000 players in one world

[Thon et al. ECML 08]



World Dynamics

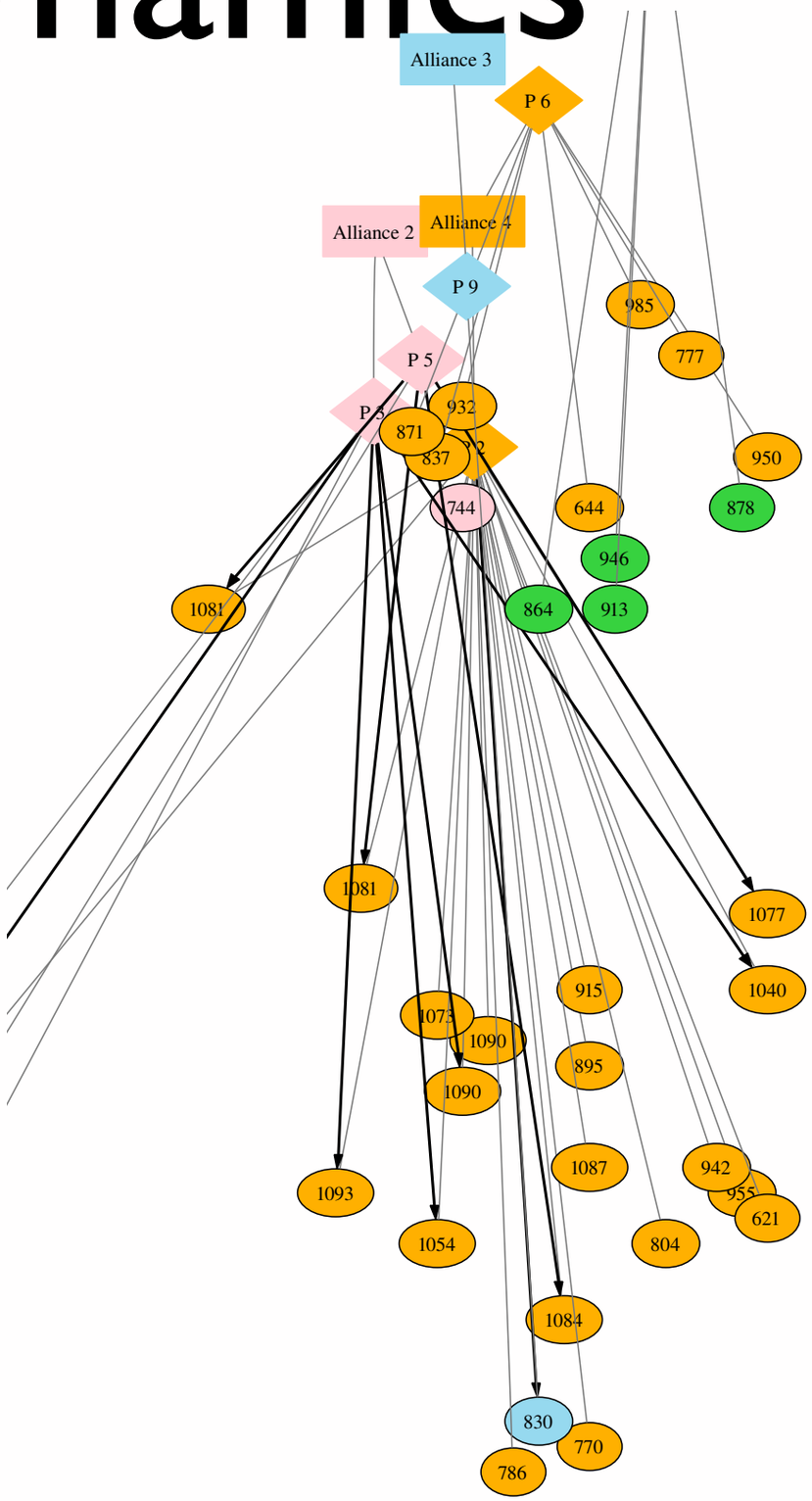
Fragment of world with

~10 alliances
~200 players
~600 cities

alliances color-coded

Can we build a model
of this world ?
Can we use it for playing
better ?

[Thon, Landwehr, De Raedt, ECML08]



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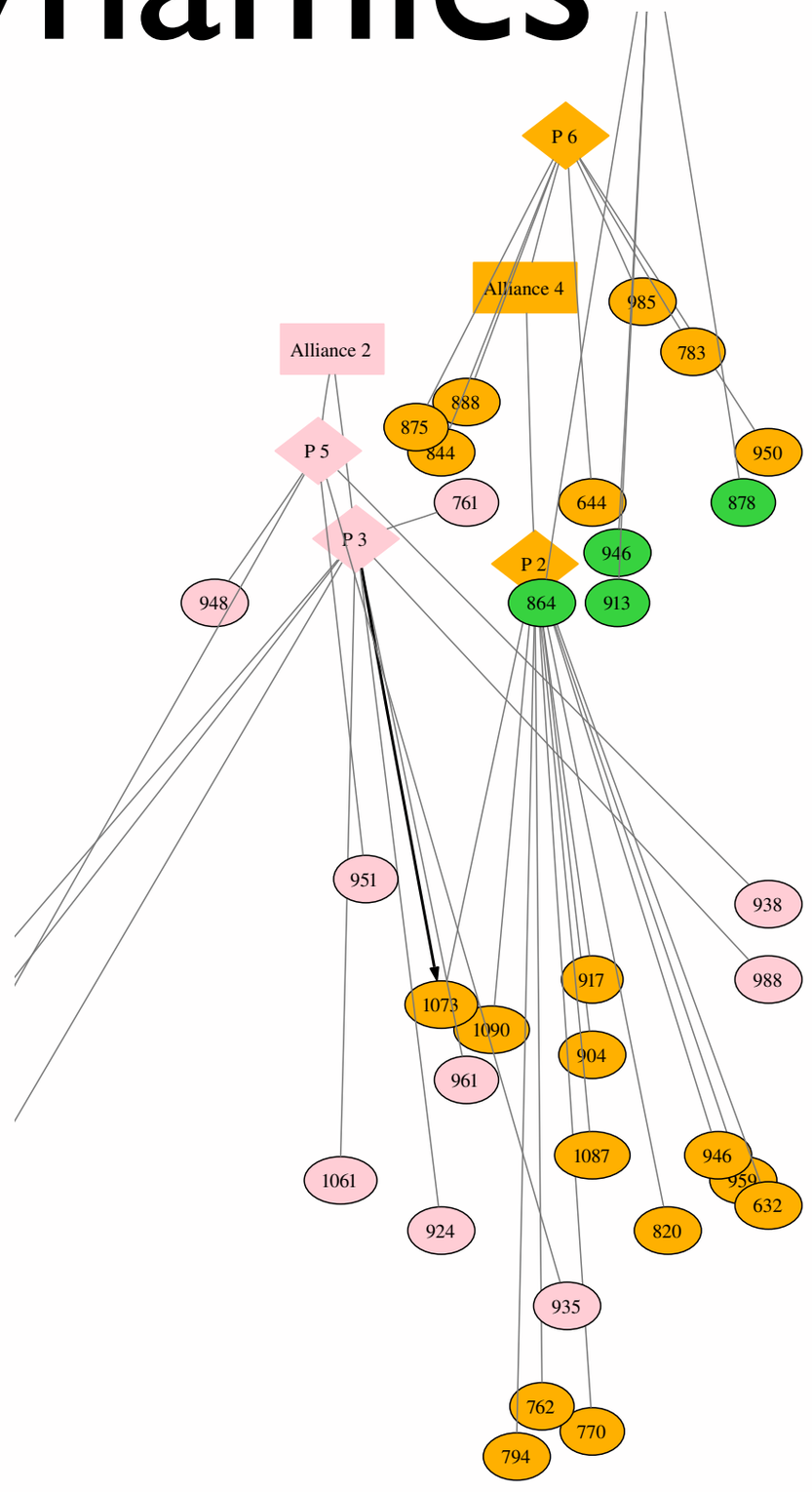

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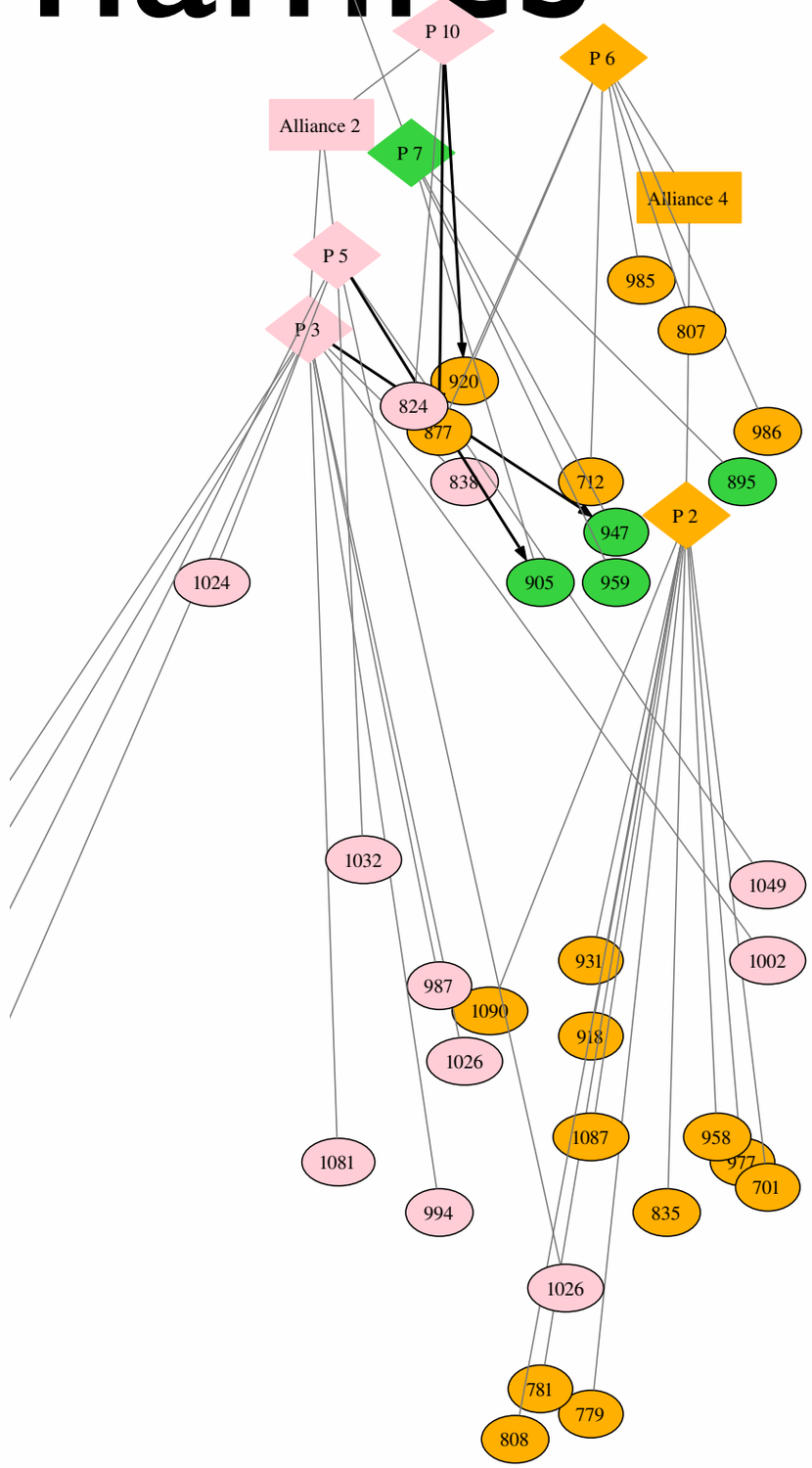

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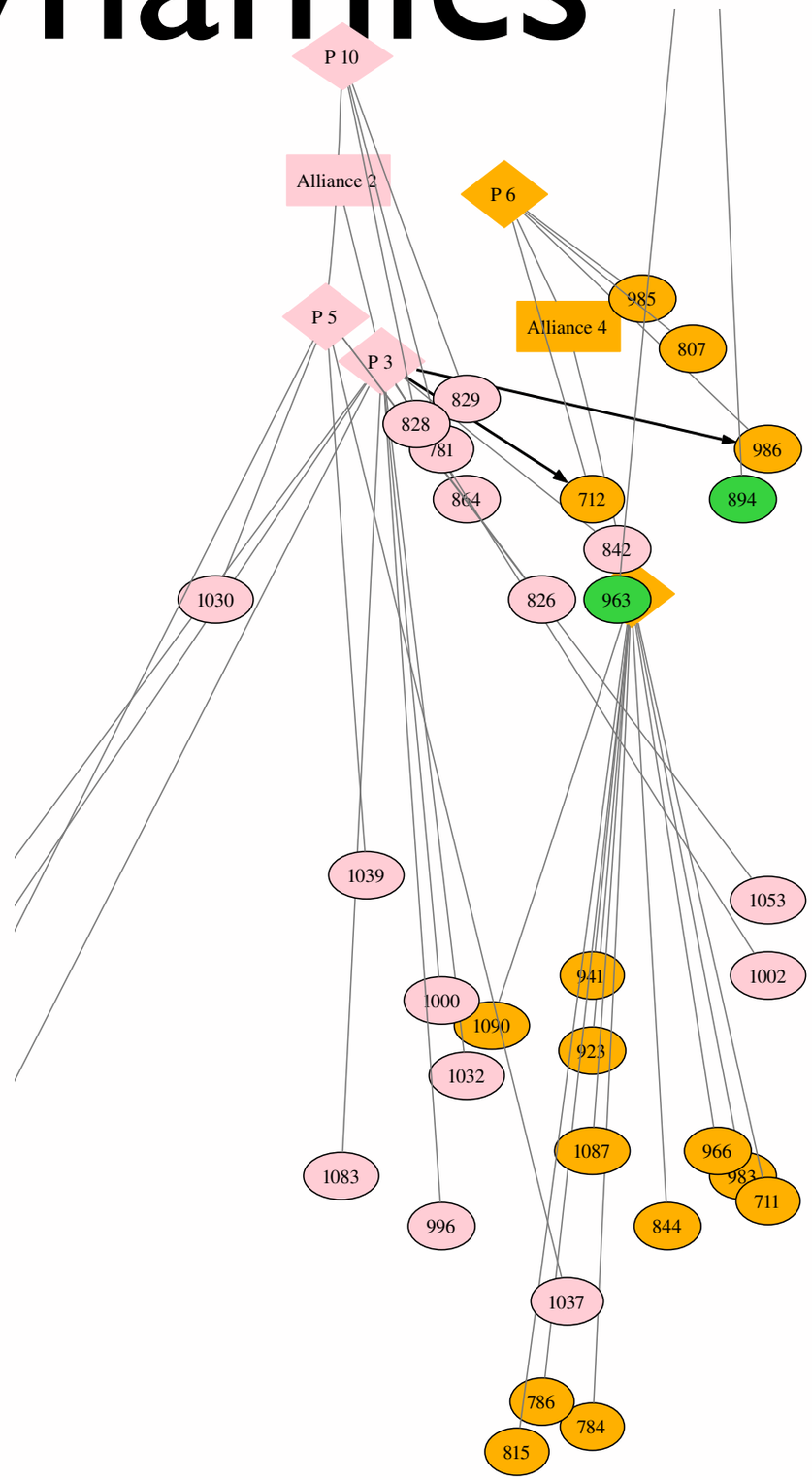

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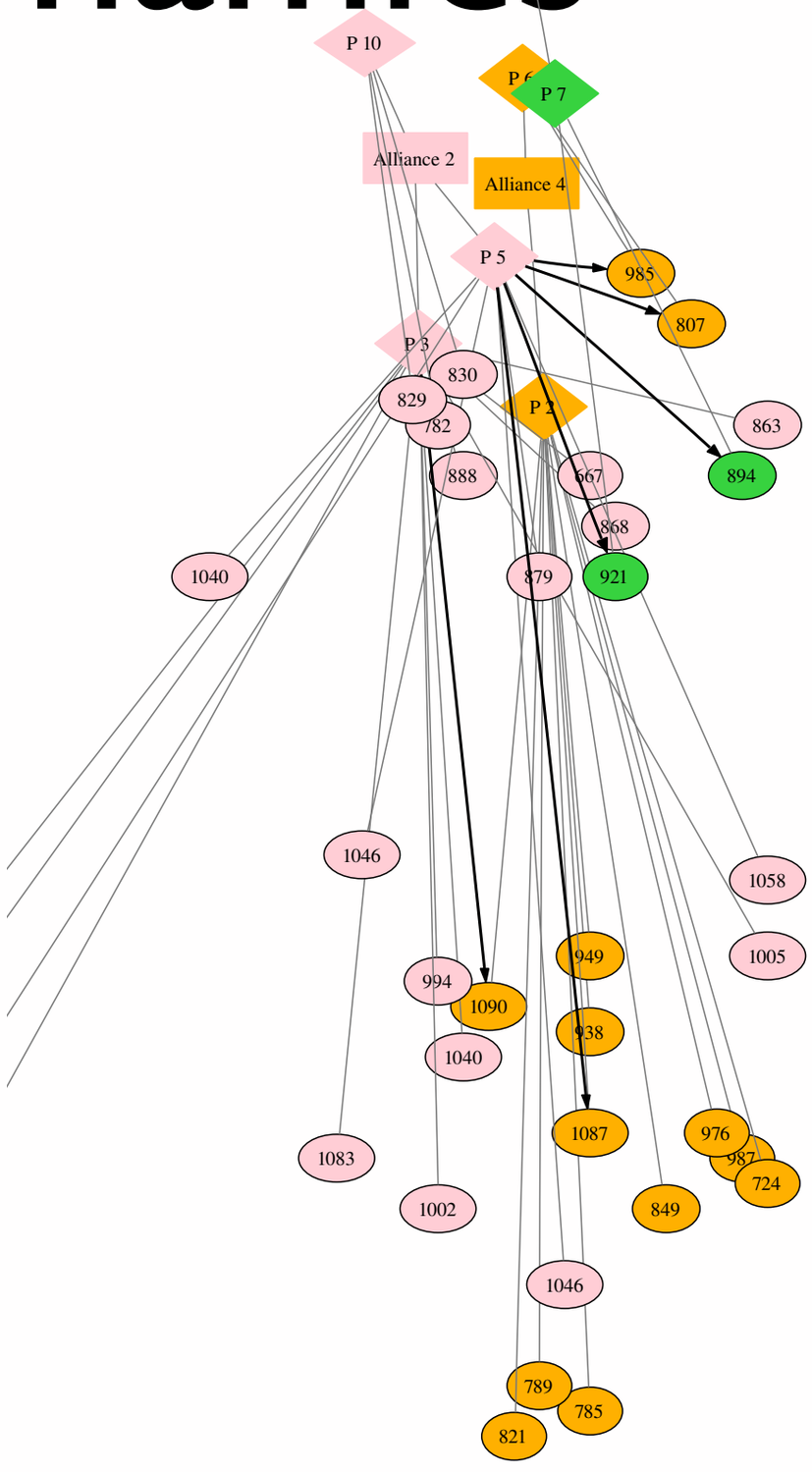

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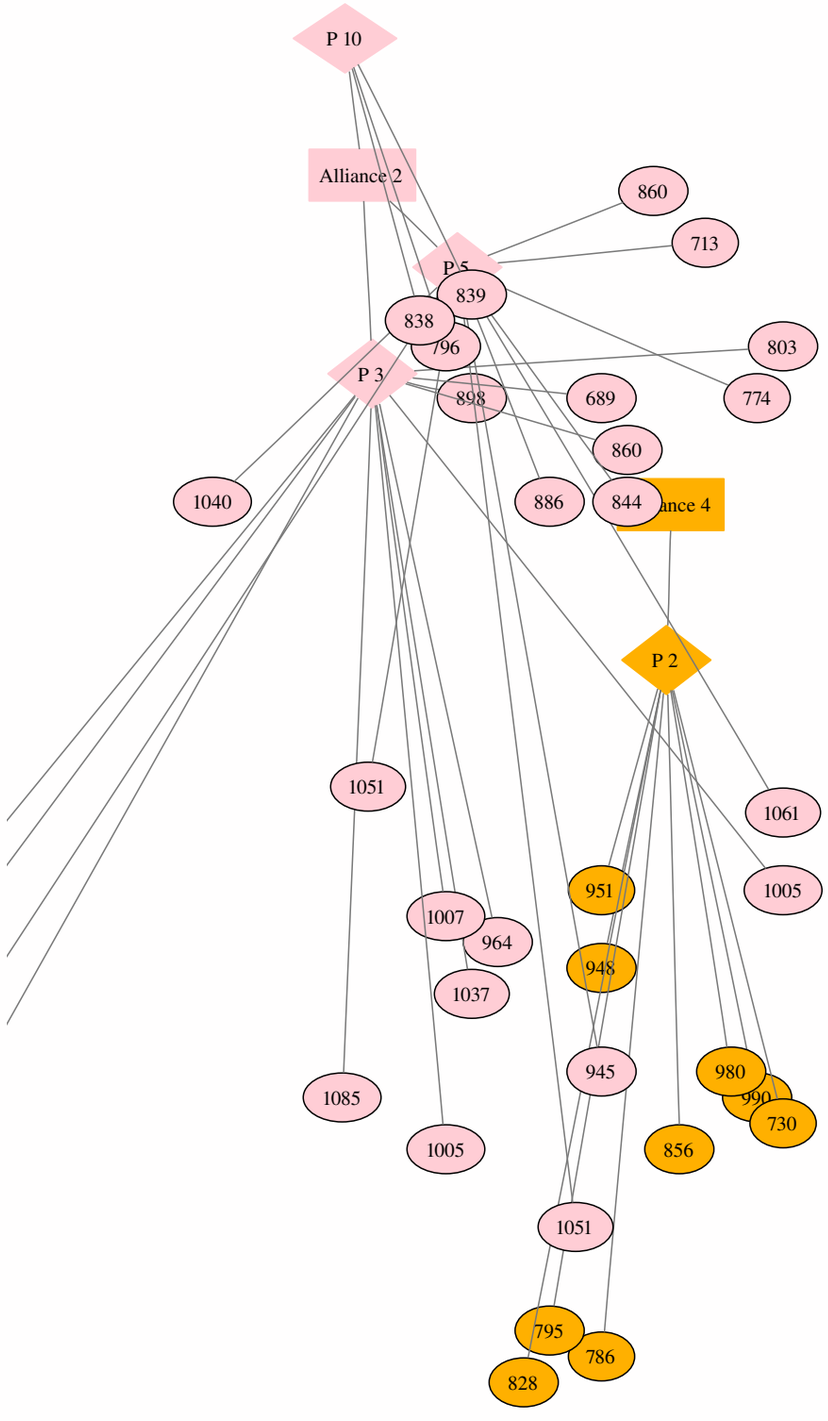
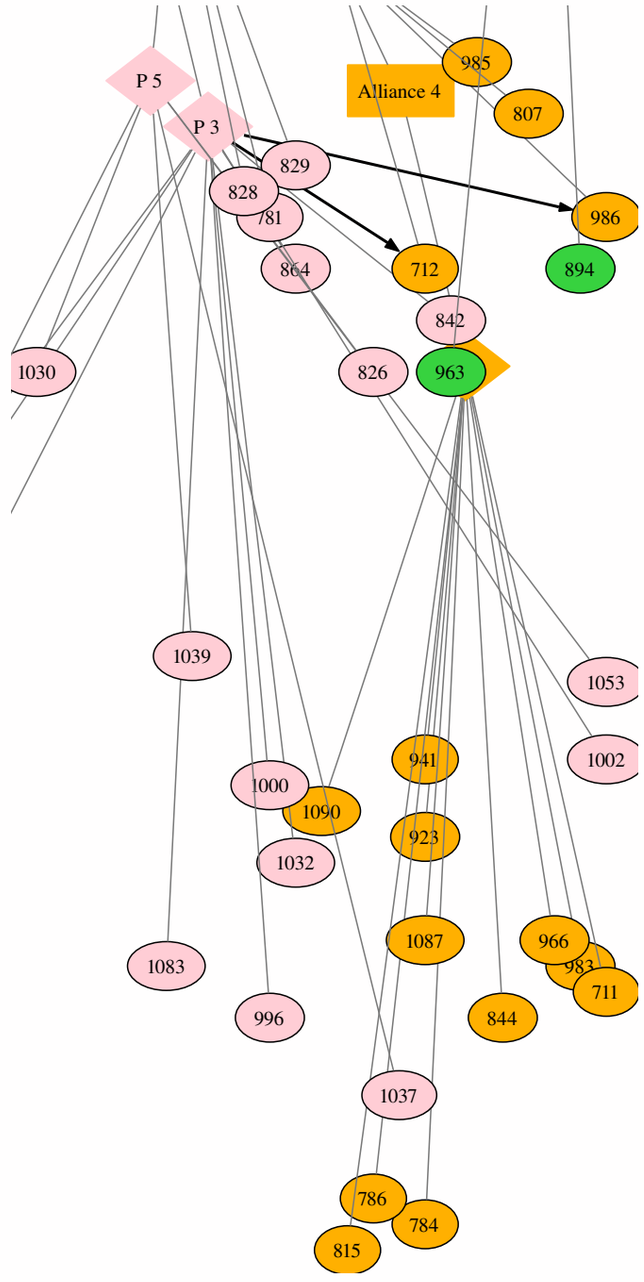


CPT-Rules

$$\frac{b_1, \ldots b_n}{\text{cause}} \rightarrow \frac{h_1 : p_1 \vee \ldots \vee h_m : p_m}{\text{effect}}$$

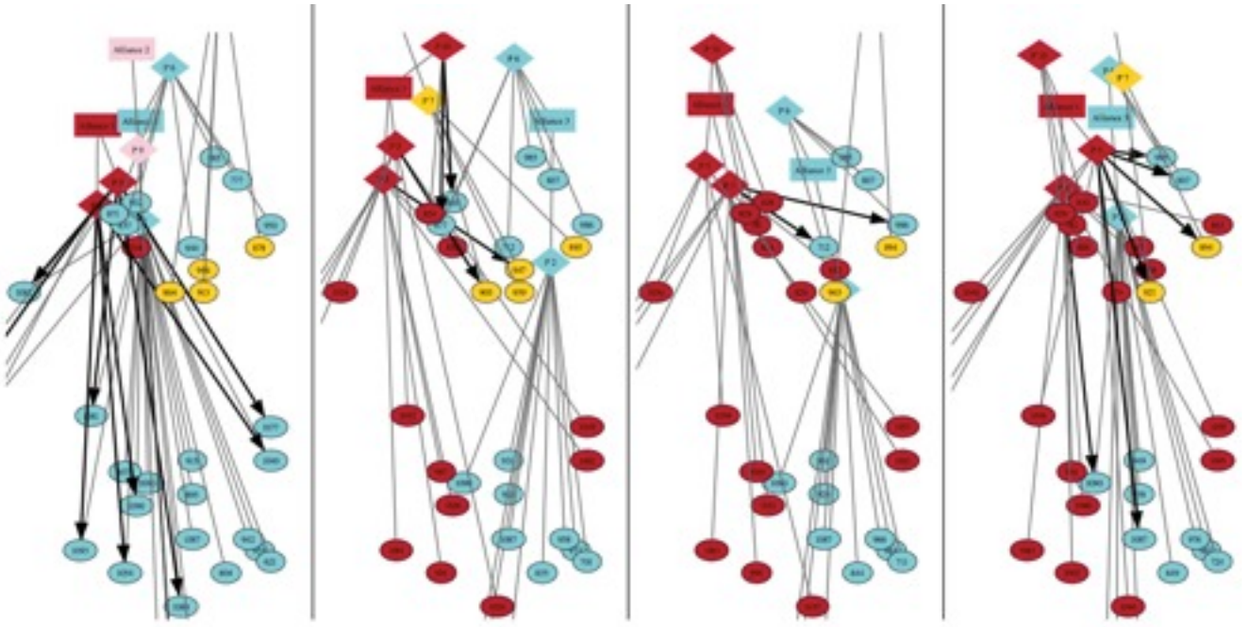
$$city(C, Owner), city(C2, Attacker), close(C, C2) \rightarrow$$
$$conquest(Attacker, C2) \ : \ p \ \vee \ nil \ : \ (1-p)$$

conquer a city which is close
P(conquest(),Time+5) ?
learn parameters



Thon et al. MLJ 11

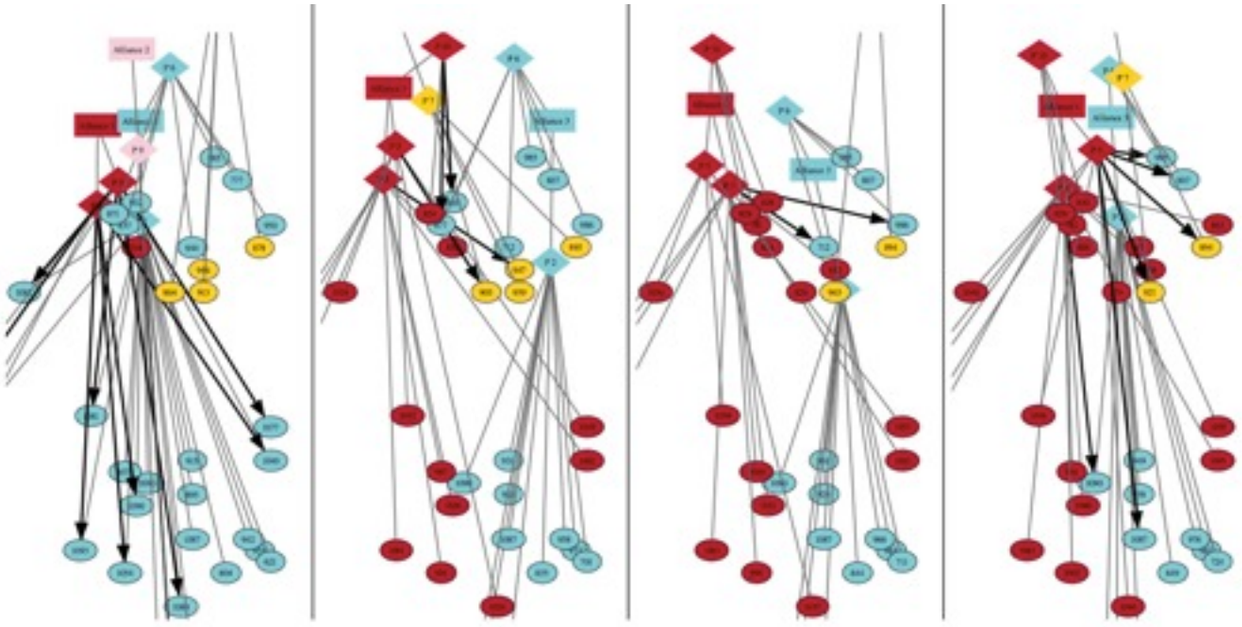
Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



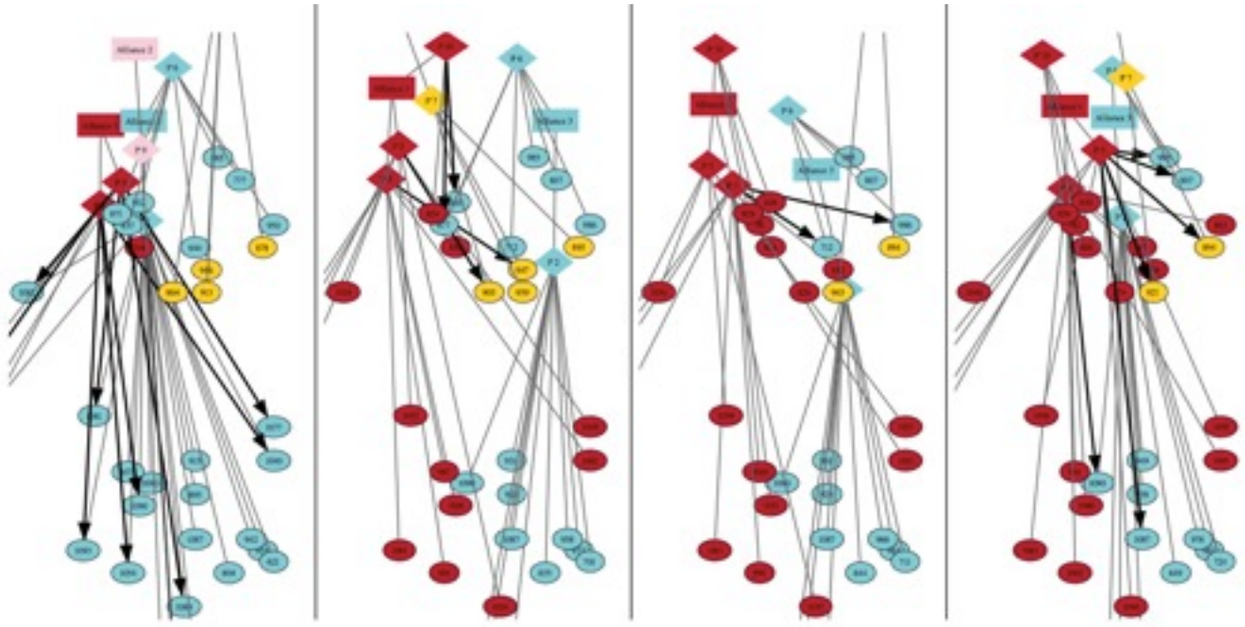
how does the world change over time?

```
0.4::conquest(Attacker,C); 0.6::nil :-
        city(C,Owner),city(C2,Attacker),close(C,C2).
```

if **cause** holds at time T

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

one of the **effects** holds at time T+1

```
0.4::conquest(Attacker,C); 0.6::nil :-
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```
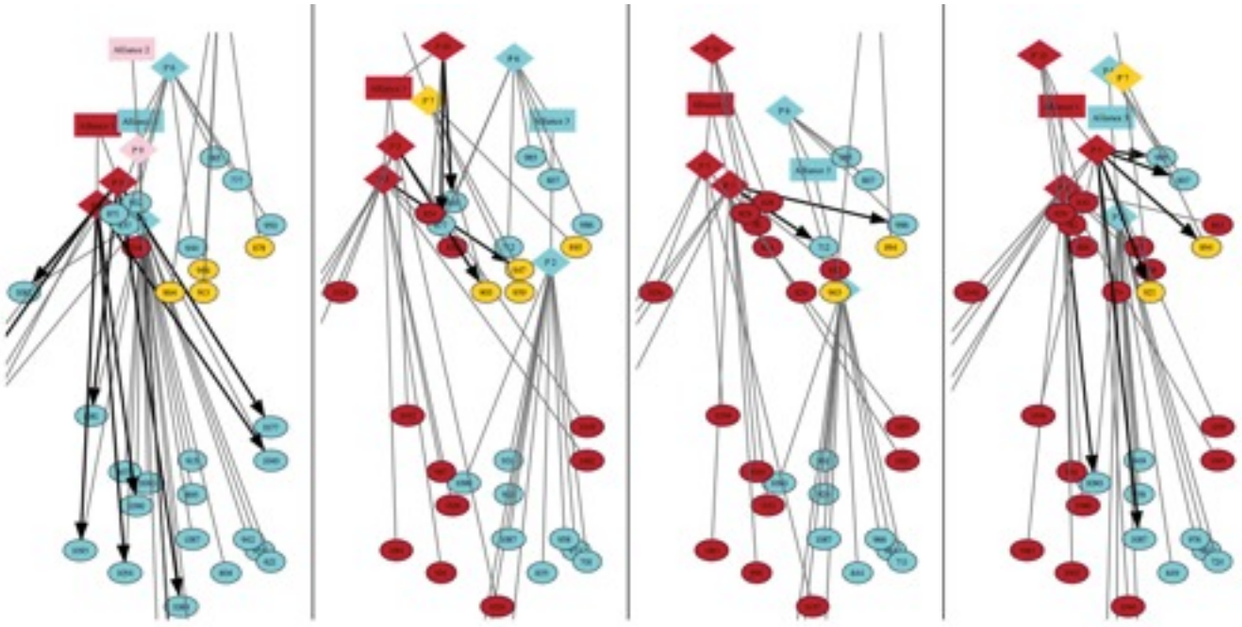

if **cause** holds at time T

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



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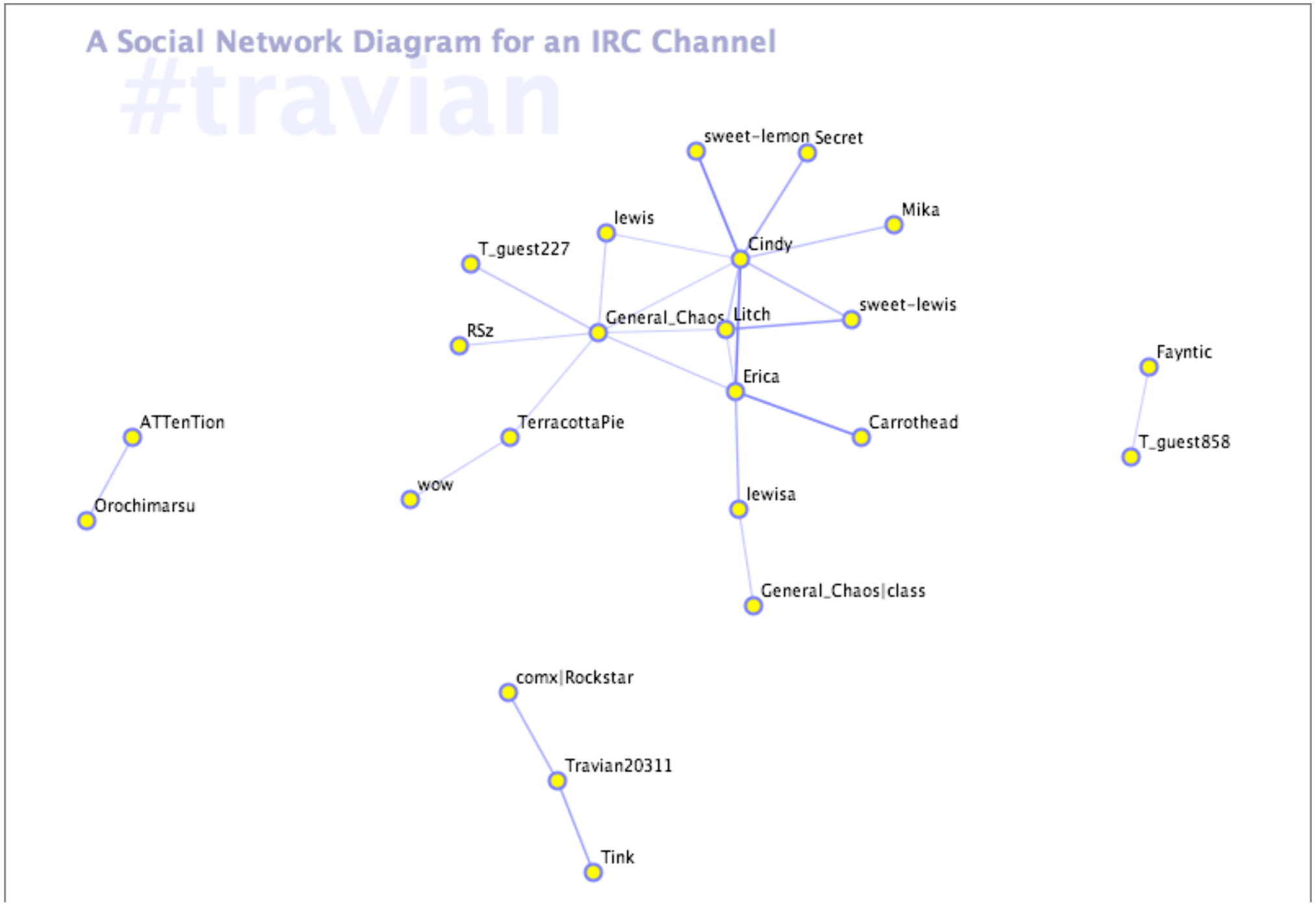
                    city(C,Owner),city(C2,Attacker),close(C,C2).
```

if **cause** holds at time T

[Thon et al, MLJ 11]

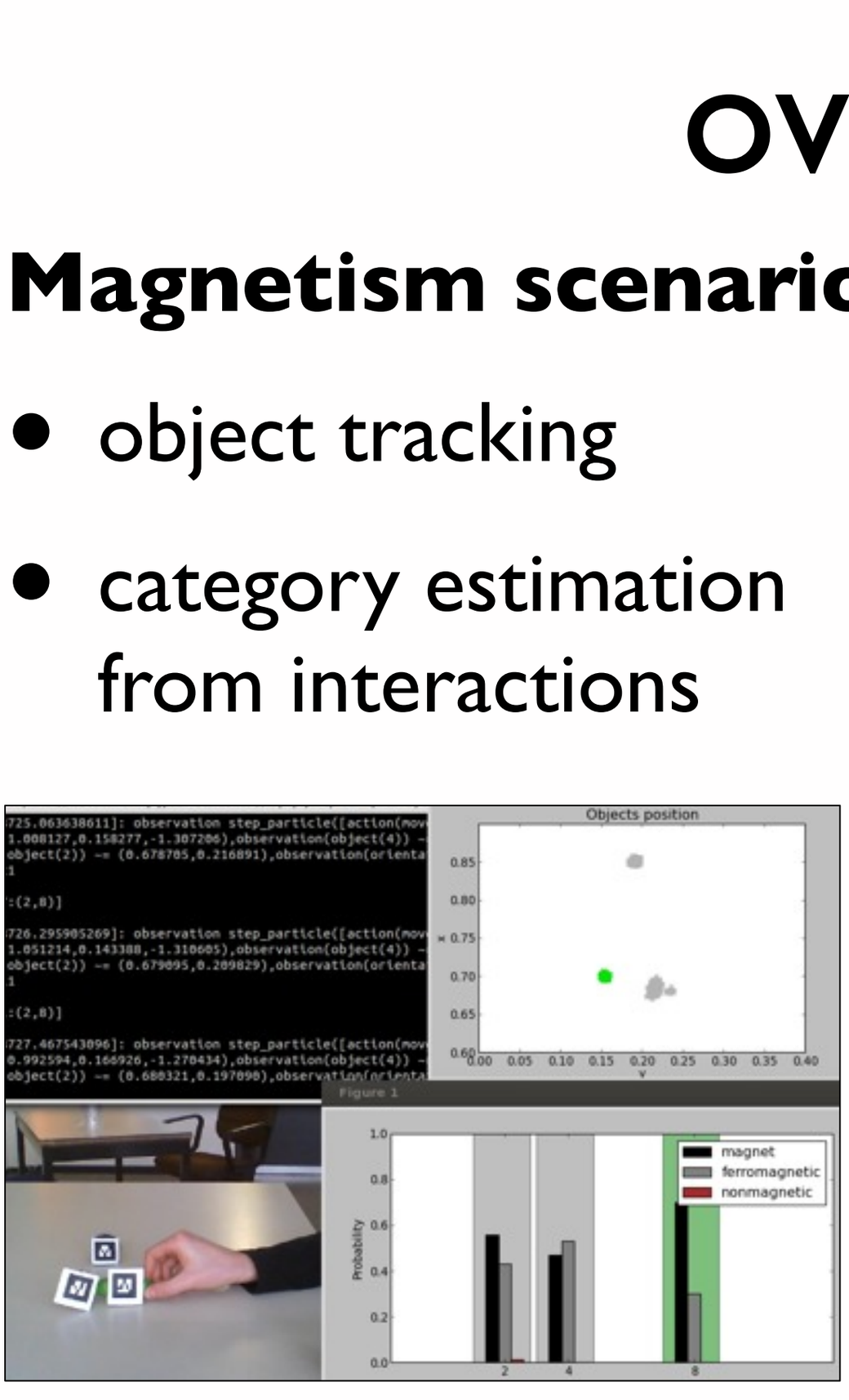
Social Network of Chats



Relational State Estimation over Time

Magnetism scenario

- object tracking
- category estimation from interactions



[Nitti et al, IROS 13]

Relational State Estimation over Time

Magnetism scenario

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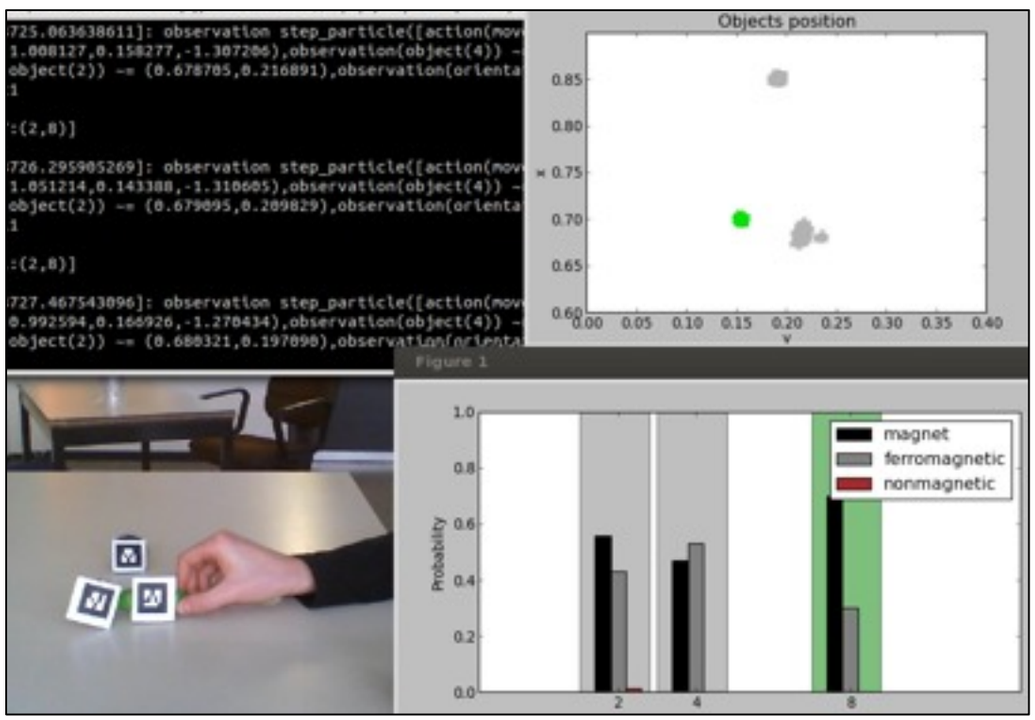
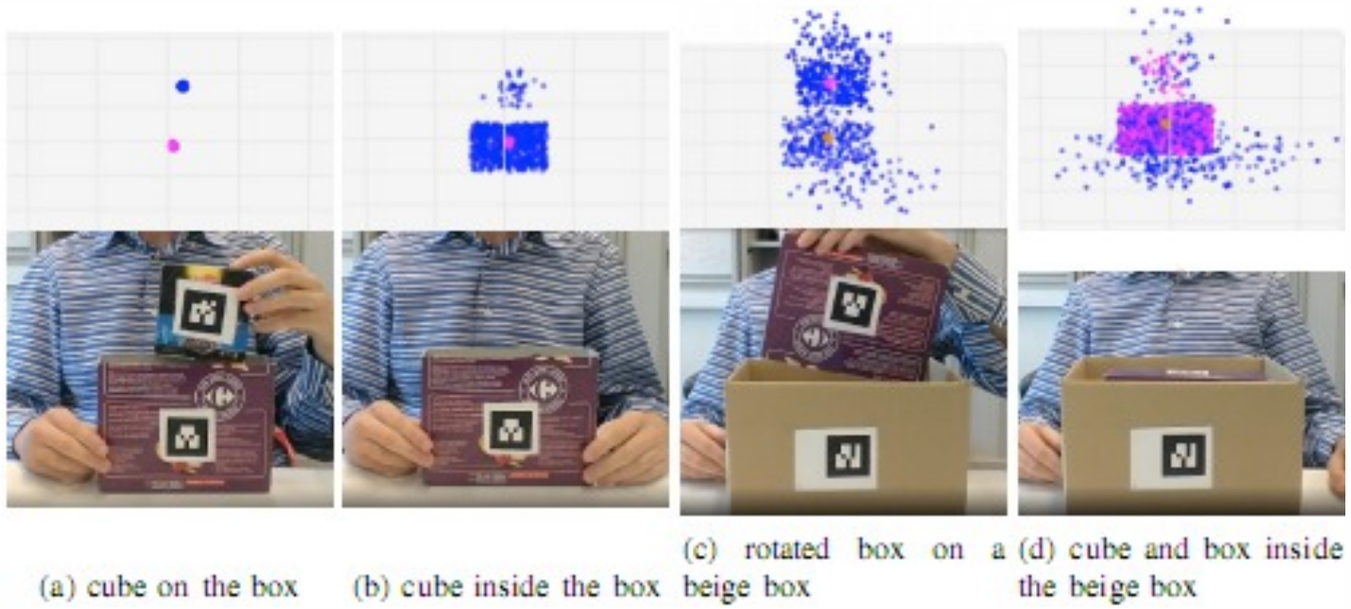


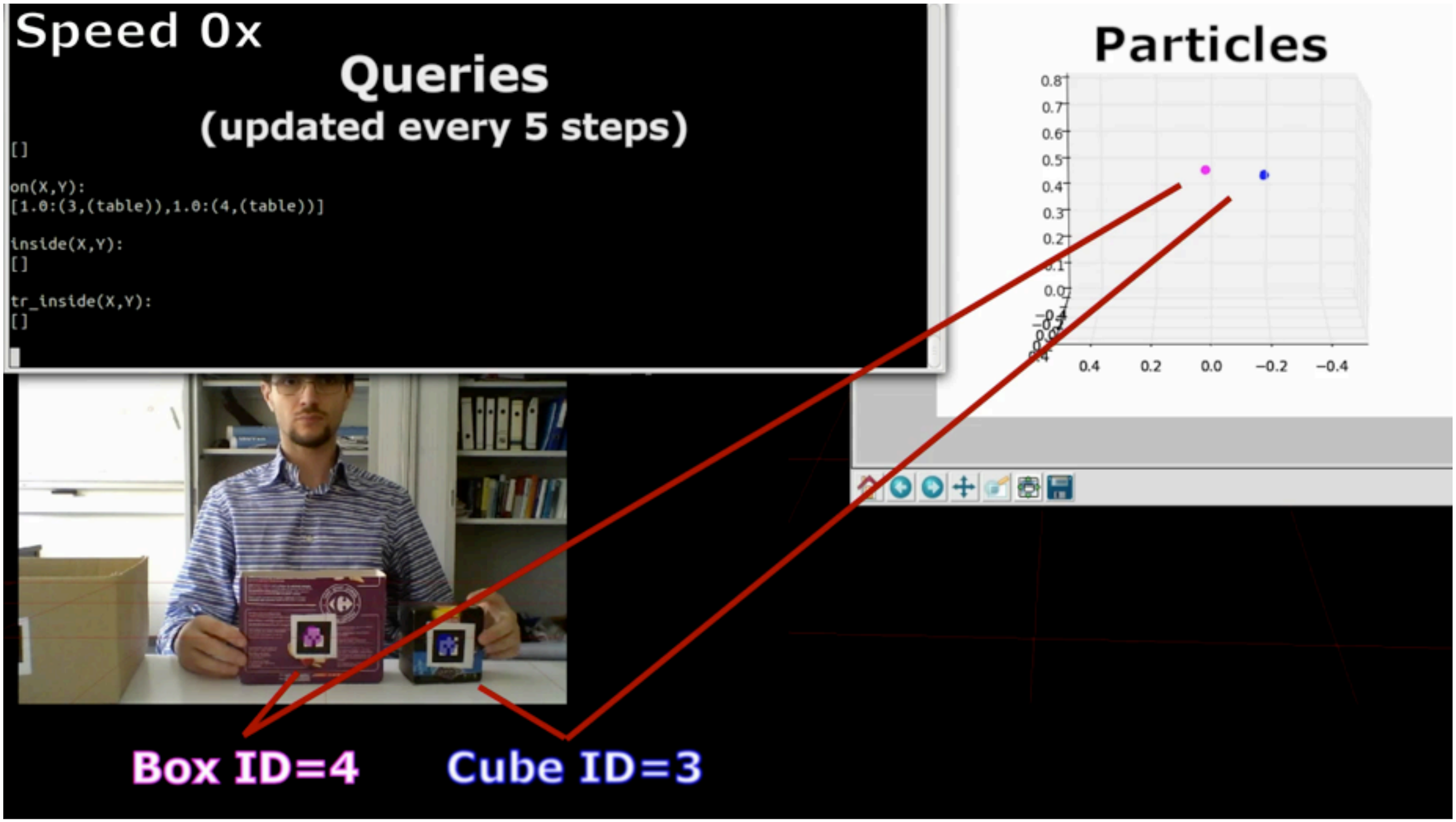


(a) cube on the box (b) cube inside the box (c) rotated box on a beige box (d) cube and box inside the beige box

Box scenario

- object tracking even when invisible
- estimate spatial relations

[Nitti et al, IROS 13]


Speed 0x
Queries
(updated every 5 steps)
[]
on(X,Y):
[1.0:(3,(table)),1.0:(4,(table))]
inside(X,Y):
[]
tr_inside(X,Y):
[]
Particles
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.4
0.2
0.0
−0.2
−0.4
Box ID=4
Cube ID=3

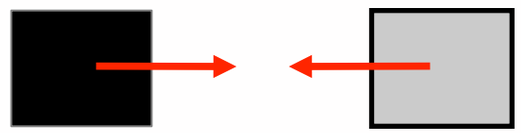
IROS 13

Magnetic scenario

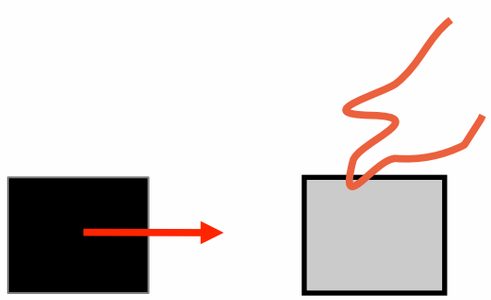
- 3 object types: magnetic, ferromagnetic, nonmagnetic

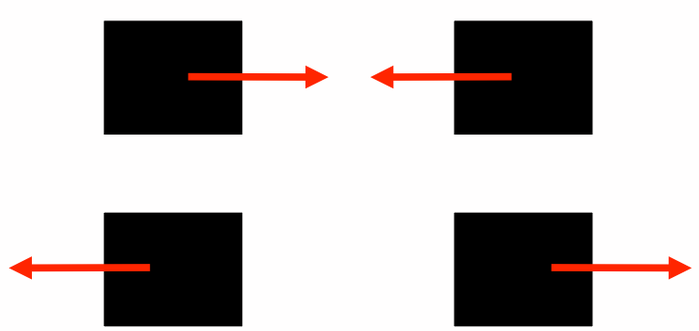


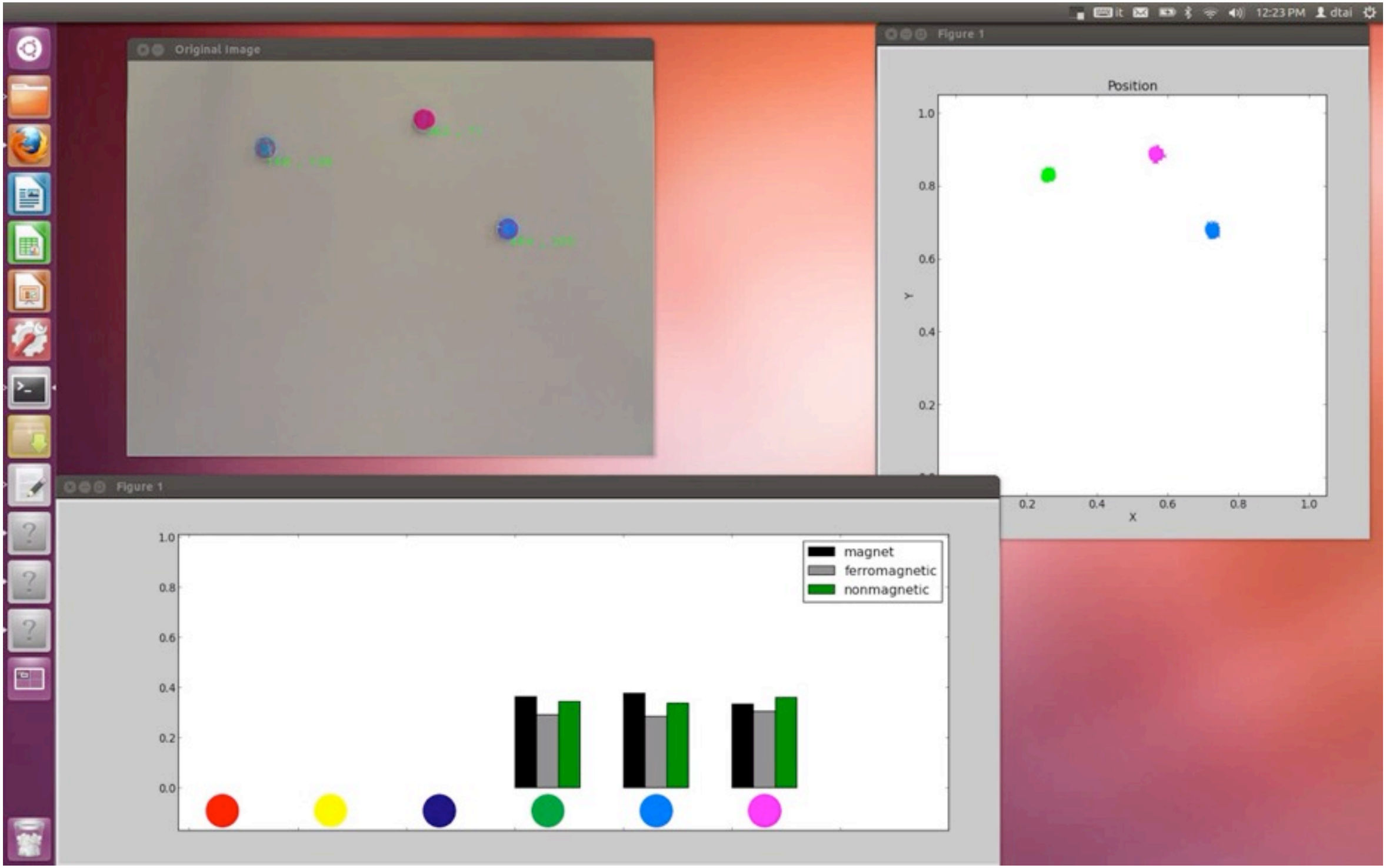
- Nonmagnetic objects do not interact
- A magnet and a ferromagnetic object attract each other



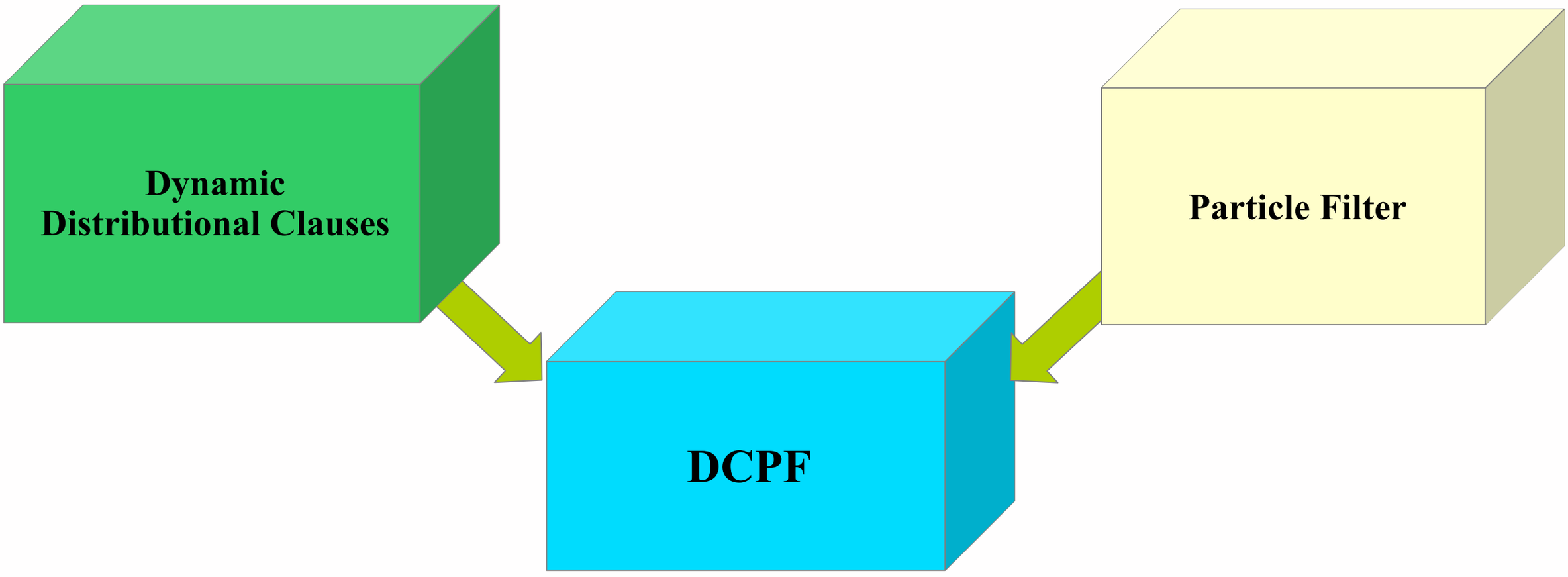
- Magnetic force that depends on the distance
- If an object is held magnetic force is compensated.






Original image
Figure 1
Position
1.0
0.8
0.6
0.4
0.2
Y
0.2
0.4
0.6
0.8
1.0
X
Figure 1
1.0
0.8
0.6
0.4
0.2
0.0
magnet
ferromagnetic
nonmagnetic

DC Particle Filter (DCPF)



Goal

Flexible (relational) state representation

Fast inference (state estimation) in general models

"A particle filter for hybrid relational domains" IROS 2013
D. Nitti, T. De Laet, L. De Raedt

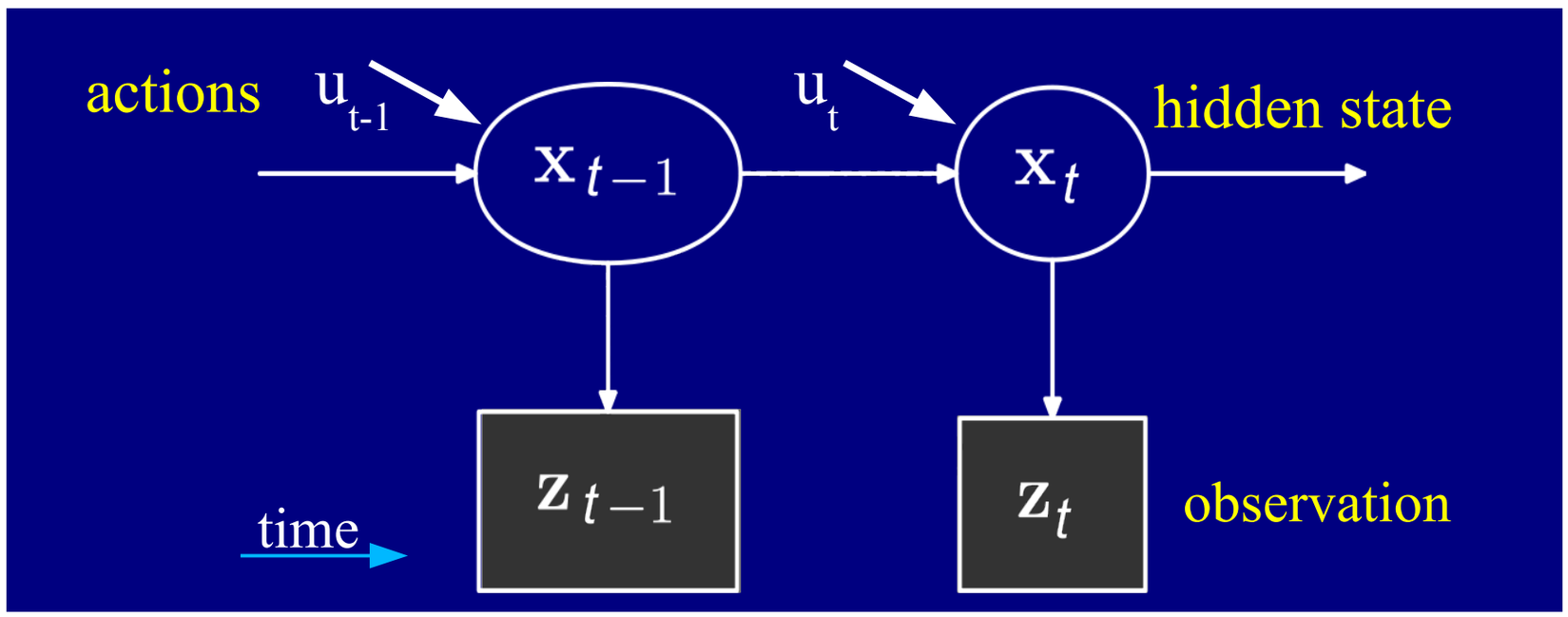
Dynamic Distributional Clauses

Prior distribution $p(x_0)$

State transition model $p(x_t|x_{t-1},u_t)$

Measurement model $p(z_t|x_t)$

Other rules: $p(x'_t|x''_t)$



Magnetic scenario

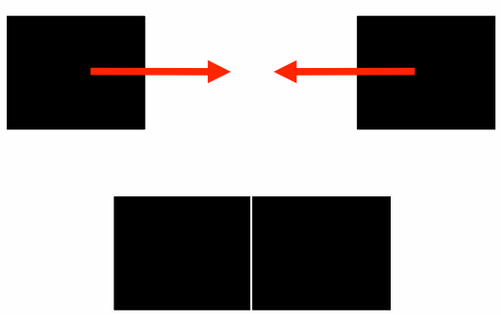
- 3 object types: magnetic, ferromagnetic, nonmagnetic

$$type(X)_t \sim finite([1/3:magnet, 1/3:ferromagnetic, 1/3:nonmagnetic]) \leftarrow object(X).$$

- 2 magnets attract or repulse

$$interaction(A,B)_t \sim finite([0.5:attraction, 0.5:repulsion]) \leftarrow object(A), object(B), A<B, type(A)_t = magnet, type(B)_t = magnet.$$

- Next position after attraction

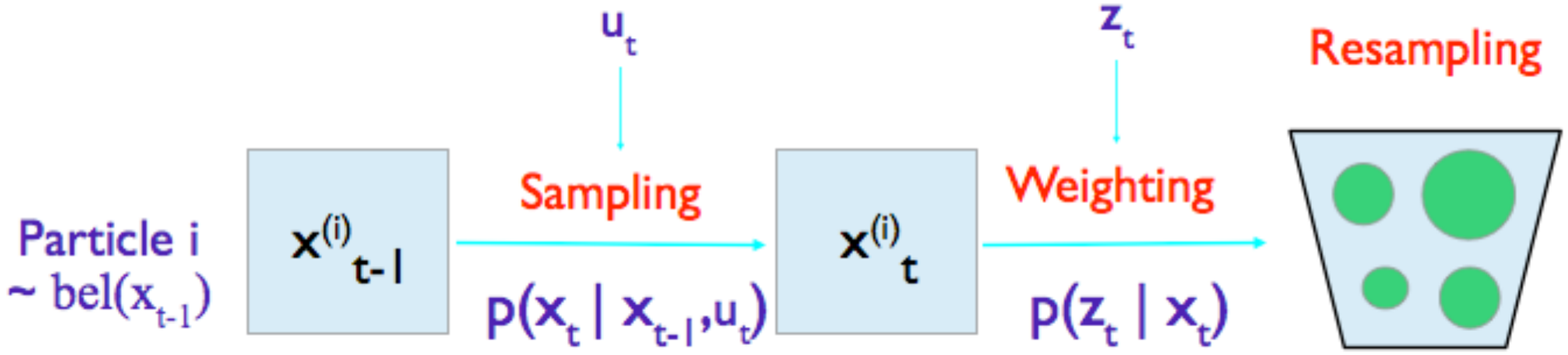


$$pos(A)_{t+1} \sim gaussian(middlepoint(A,B)_t, Cov) \leftarrow near(A,B)_t, not(held(A)), not(held(B)), interaction(A,B)_t = attr, c/dist(A,B)_t^2 > friction(A)_t.$$

$$pos(A)_{t+1} \sim gaussian(pos(A)_t, Cov) \leftarrow not(\ attraction(A,B)\ ).$$

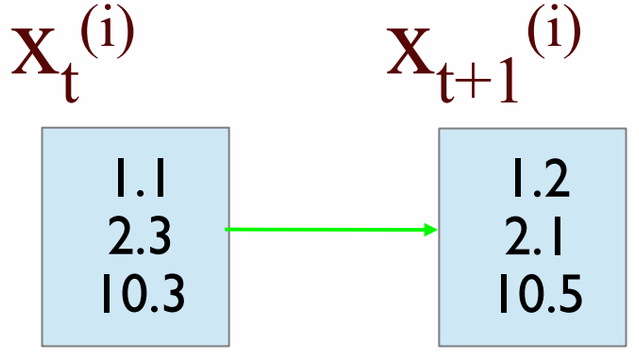
Particle Filter
(Sequential Monte Carlo)

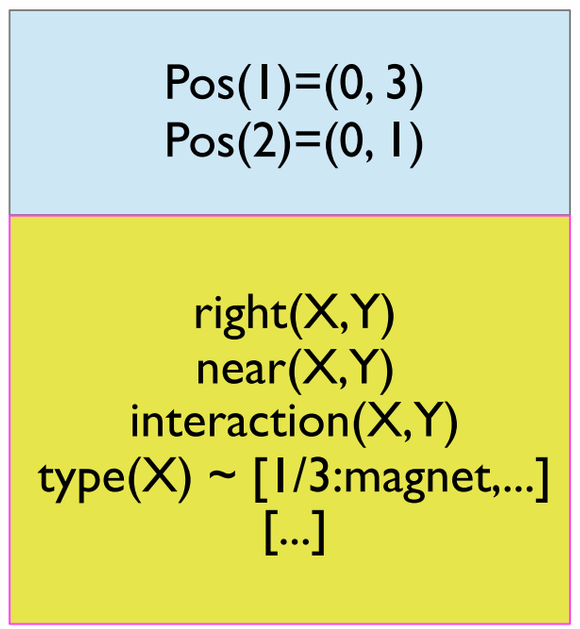
- Based on sampling → approximate inference
- Particles (samples) to represent bel(x_t)



Classical Particle Filter vs DCPF

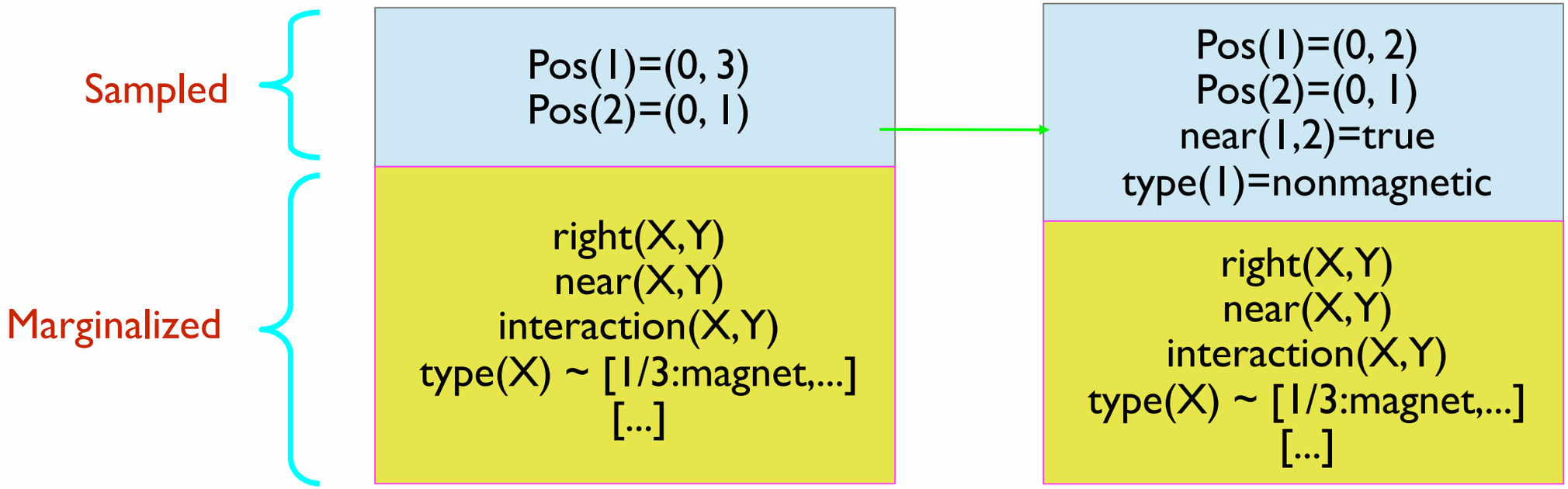
- Classical PF
  - Fixed set of random variables
  - Update the entire state
- DCPF
  - **Adaptive state** (particle): the number of facts / random variables can change over time
  - Particles are partial interpretations
  - Expressive language



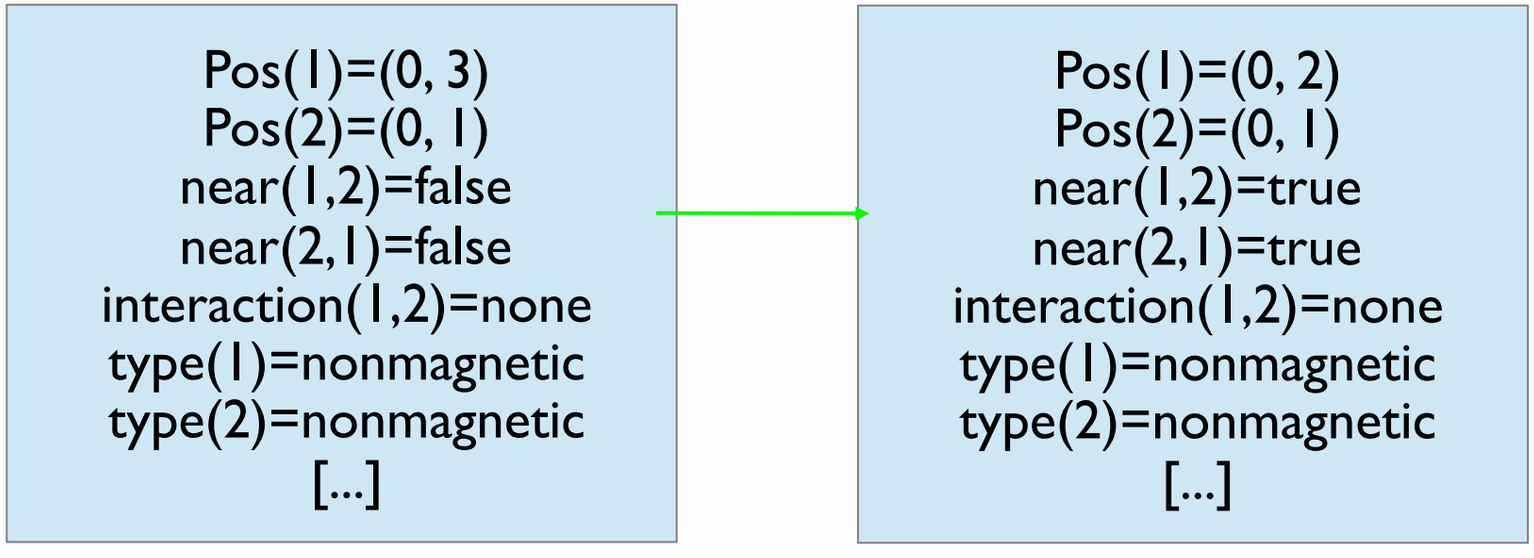


Optimized inference: partial state

Distributional Clauses Particle Filter (DCPF)

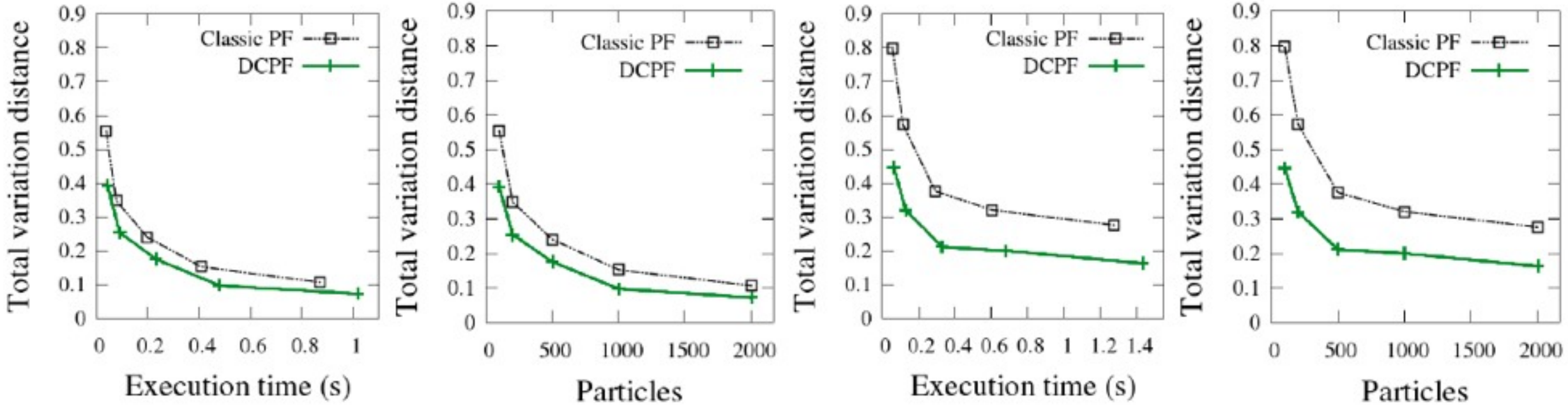


Classical particle filter



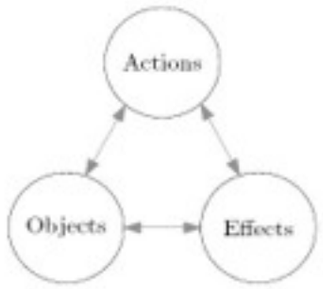
Experiments

- Particles are partial state, remaining variables are marginalized
- Better performance in bigger models



Learning relational affordances

Learn probabilistic model



Inputs	Outputs	Function
(O, A)	E	Effect prediction
(O, E)	A	Action recognition/planning
(A, E)	O	Object recognition/selection

Learning relational affordances between two objects (learnt by experience)

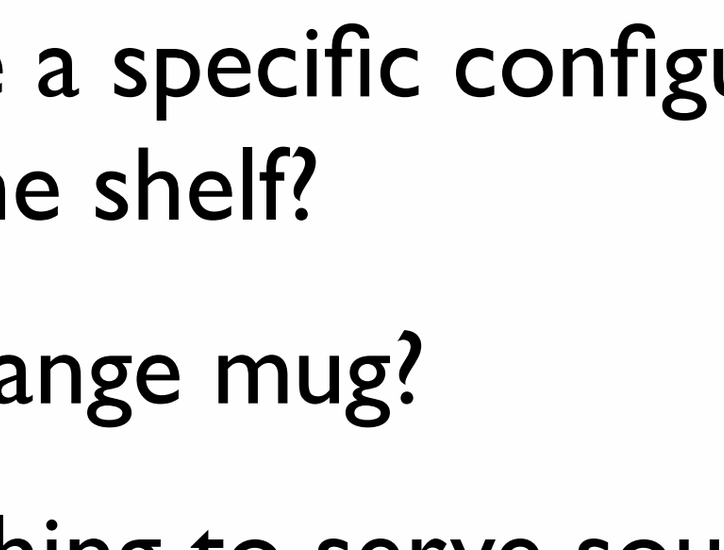
From two object interactions

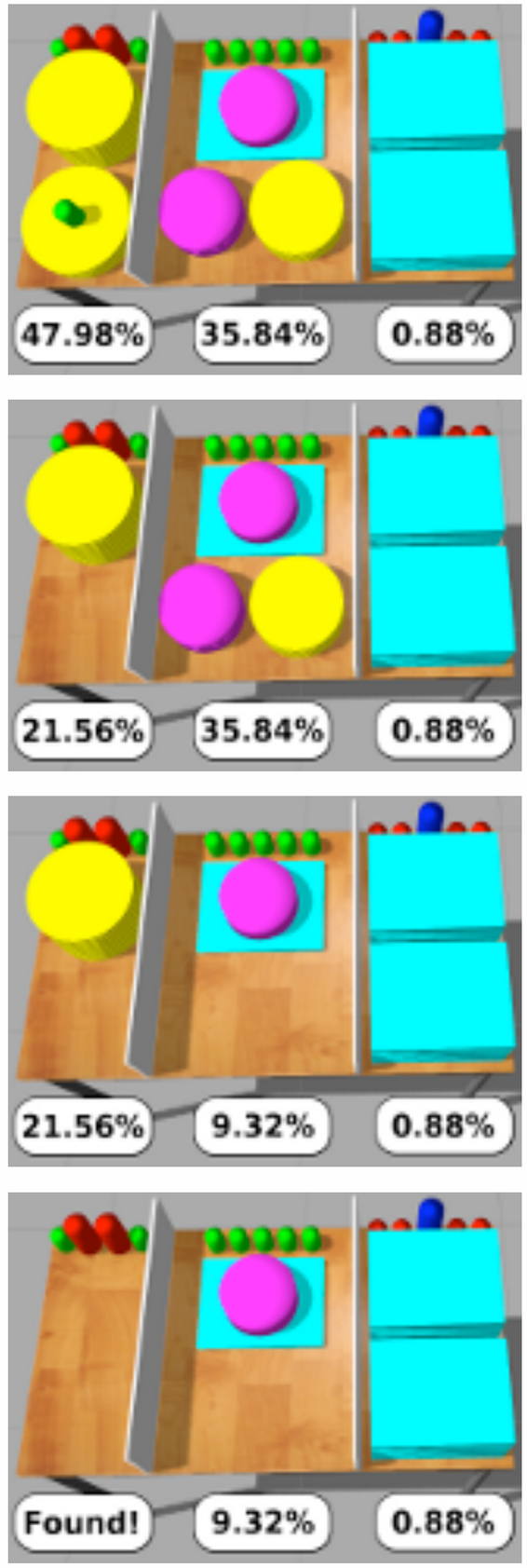
Generalize to N

Moldovan et al. ICRA 12, 13, 14

Occluded Object Search

- How to achieve a specific configuration of objects on the shelf?
- Where's the orange mug?
- Where's something to serve soup in?
- Models of objects and their spatial arrangement





[Moldovan *et al. 14*]

ProbLog for activity recognition from video



CAVIAR-INRIA human activity dataset

28 videos
≈ 26.500 frames

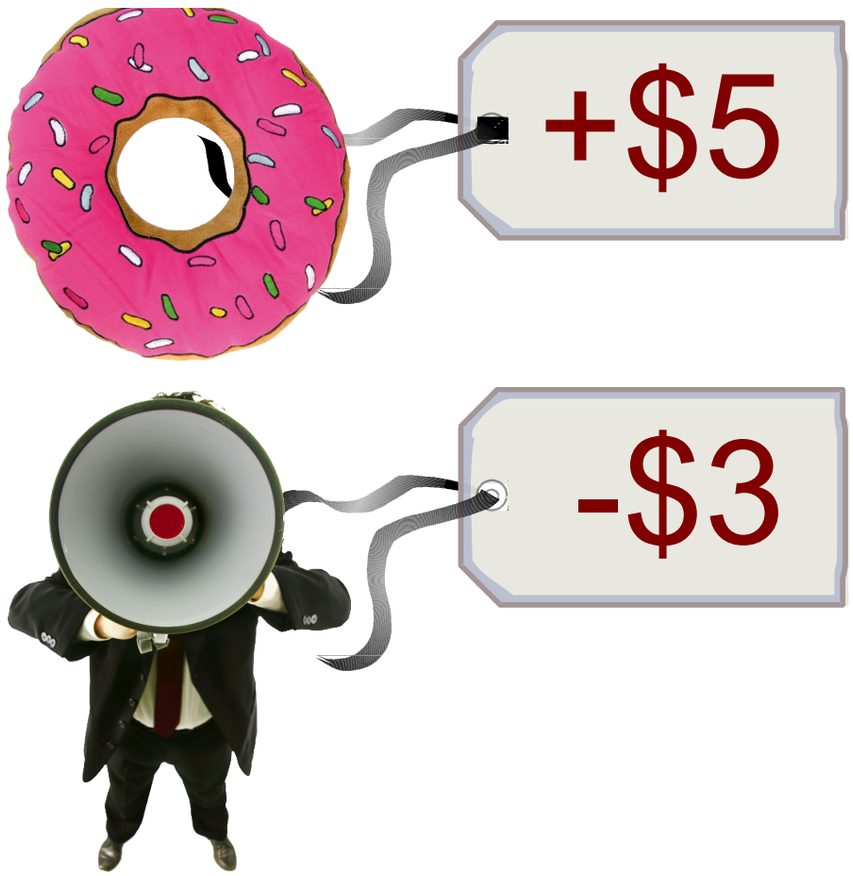
- Separation between low-level events (LLE) and high-level events (HLE)
  - LLE: *walking, running, active, inactive, abrupt*
  - HLE: *meeting, moving, fighting, leaving_object*
- Probabilistic Logic approach: *Event Calculus in ProbLog* (Prob-EC) to infer the high-level events from an **algebra** of low-level events.
- Example:

$$initiatedAt(fighting(P_1, P_2) = true, T) \leftarrow$$
$$happensAt(abrupt(P_1), T),$$
$$holdsAt(close(P_1, P_2, 44) = true, T),$$
$$not\ happensAt(inactive(P_2), T).$$

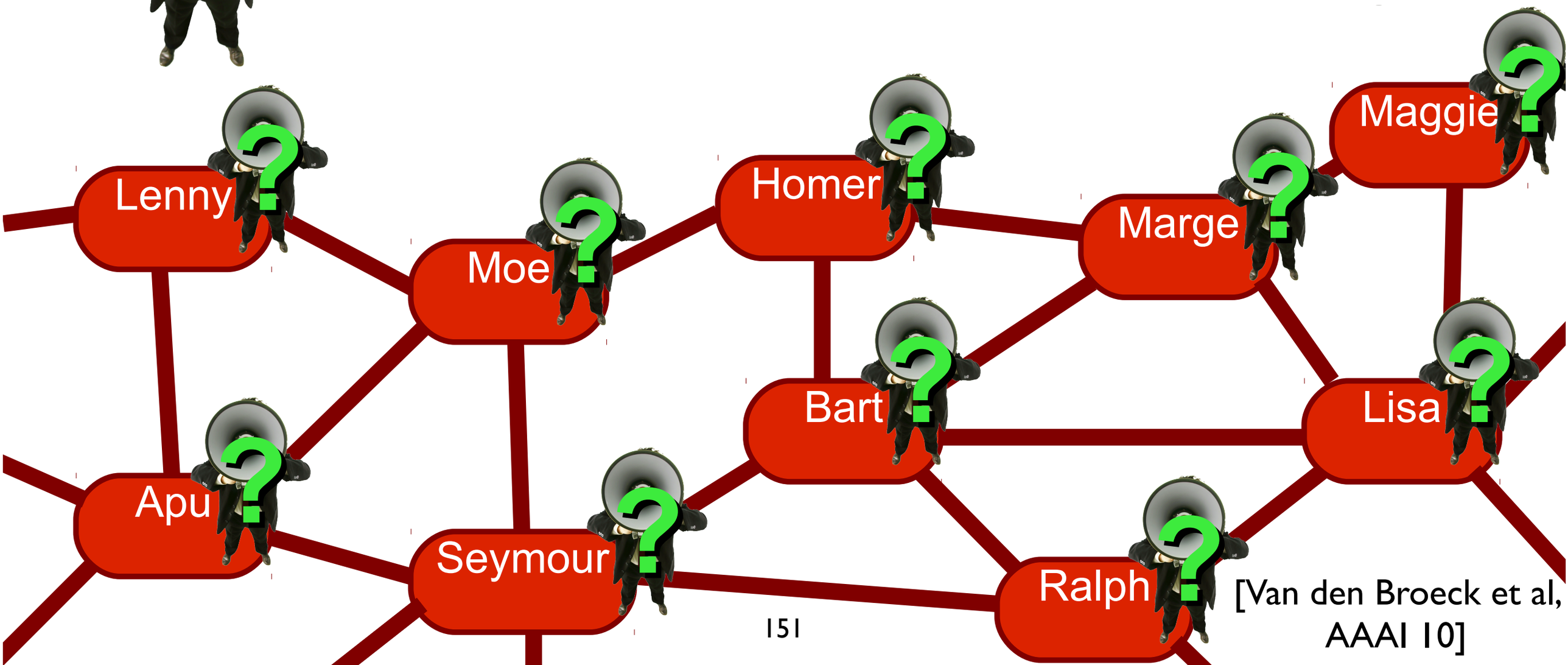
[Skarlatidis et al, TPLP 13]

decisions

Viral Marketing

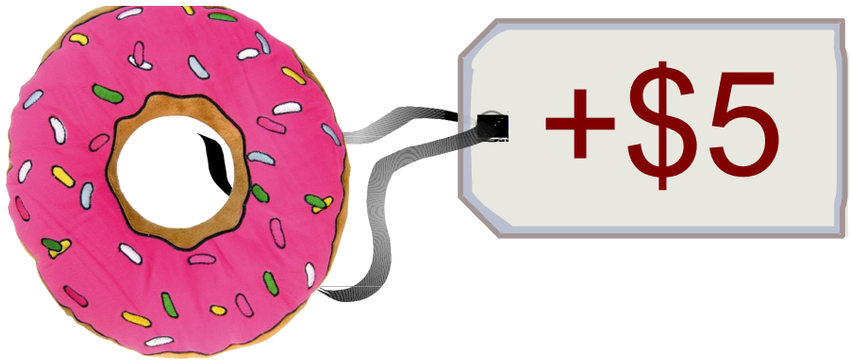


Which advertising strategy maximizes expected profit?



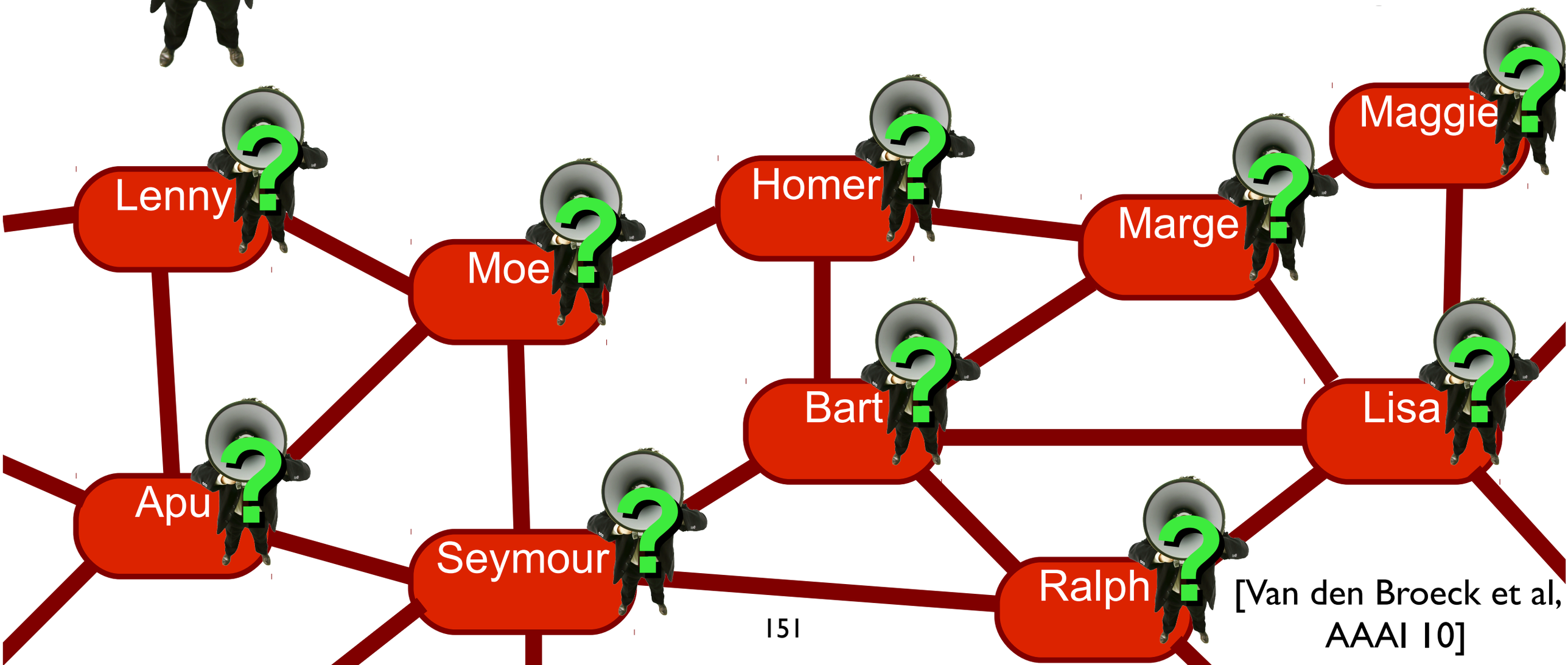
[Van den Broeck et al, AAAI 10]

Viral Marketing



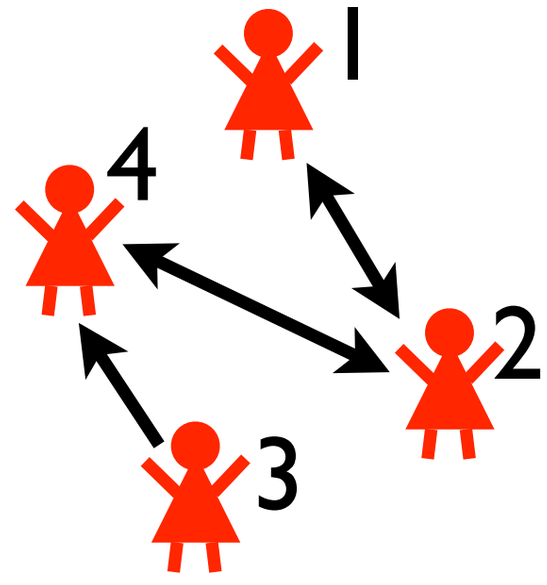


decide truth values of some atoms



[Van den Broeck et al, AAAI 10]

DTProbLog



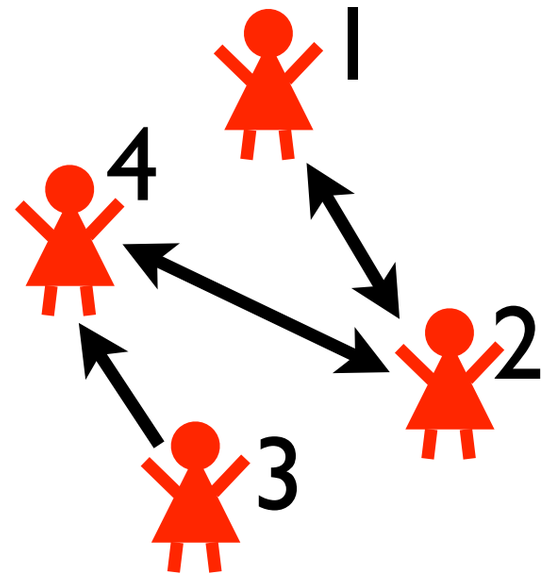

```
person(1).
person(2).
person(3).
person(4).
```

```
friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

DTProbLog

```
? :: marketed(P) :- person(P).
```

decision fact: true or false?

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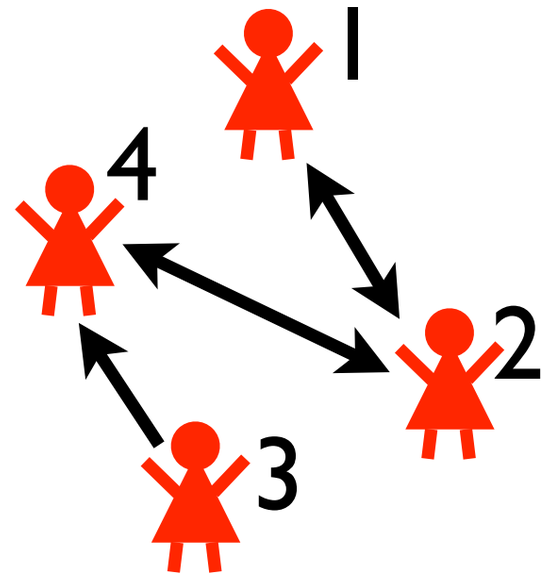
probabilistic facts
+ logical rules

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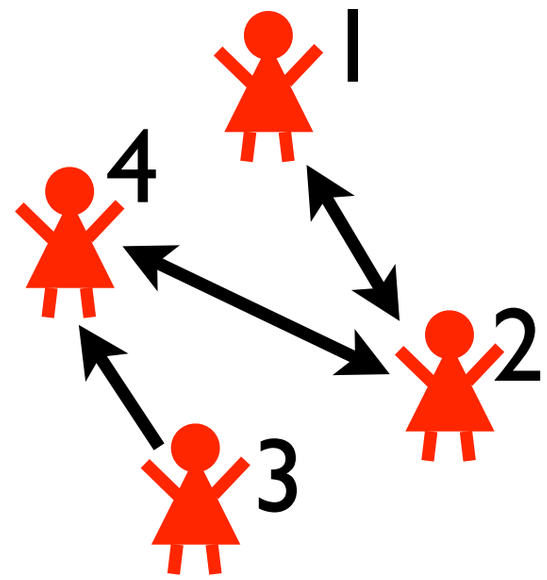


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```
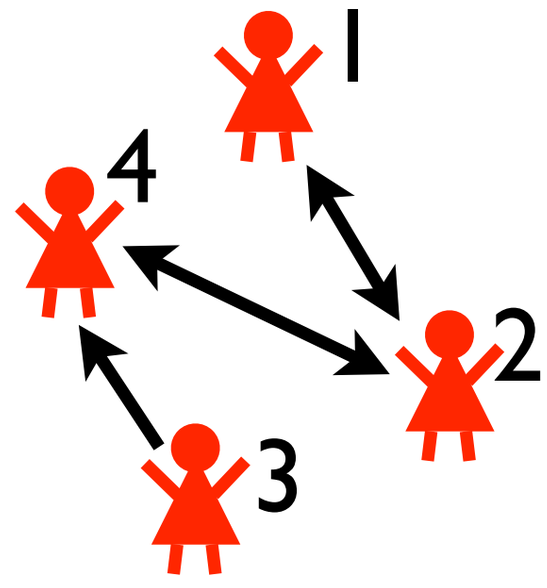

utility facts: cost/reward if true

DTProbLog

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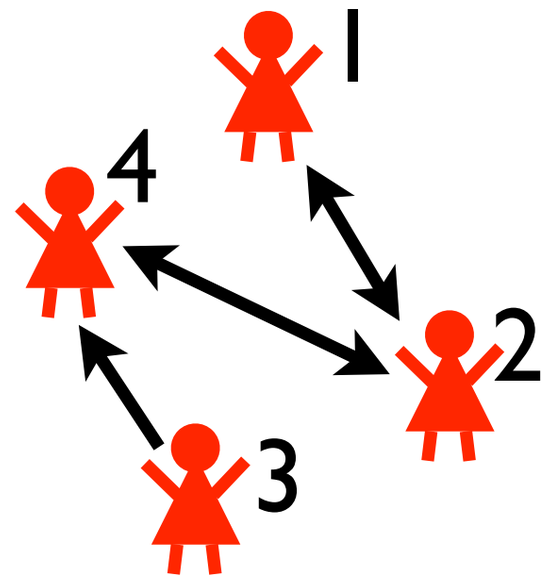
DTProbLog

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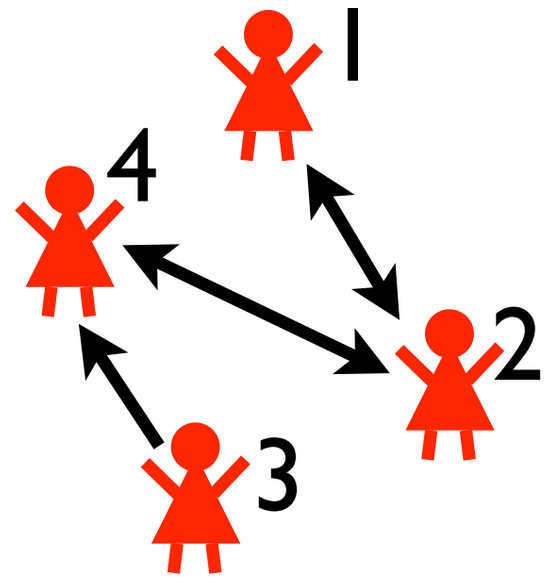


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```

```
marketed(1)        marketed(3)
```

DTProbLog

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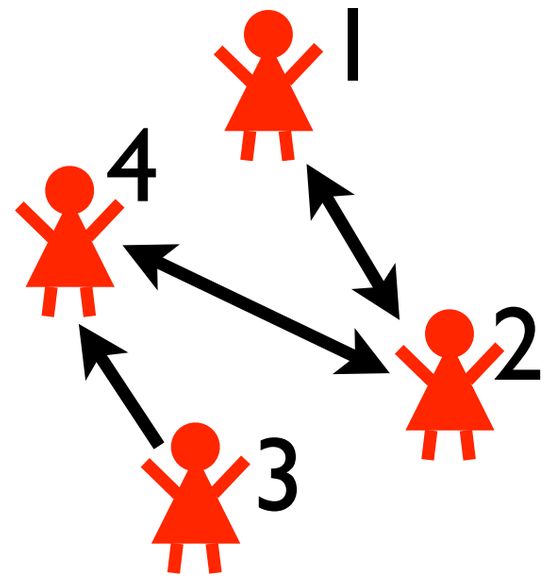


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```
marketed(1)        marketed(3)

    bt(2,1)   bt(2,4)          bm(1)
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DTProbLog

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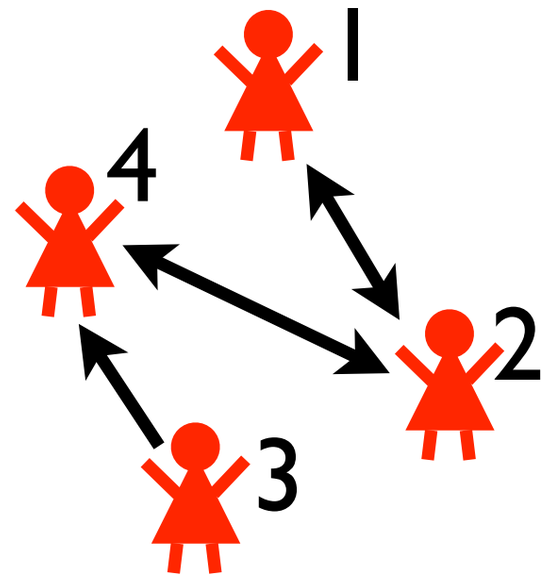


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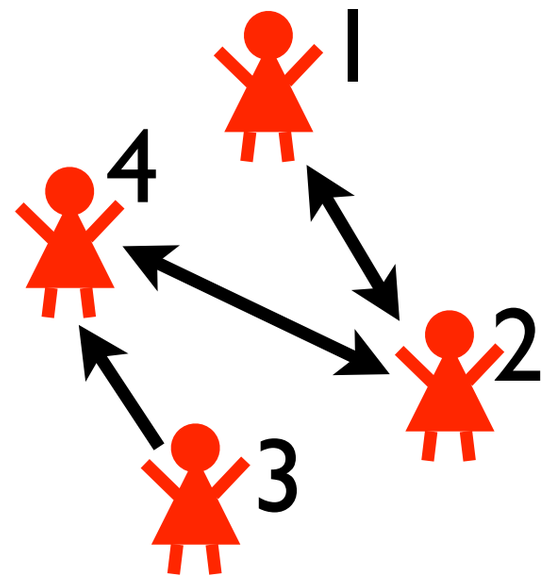
```
marketed(1)        marketed(3)

    bt(2,1)   bt(2,4)        bm(1)

buys(1)    buys(2)
```

DTProbLog

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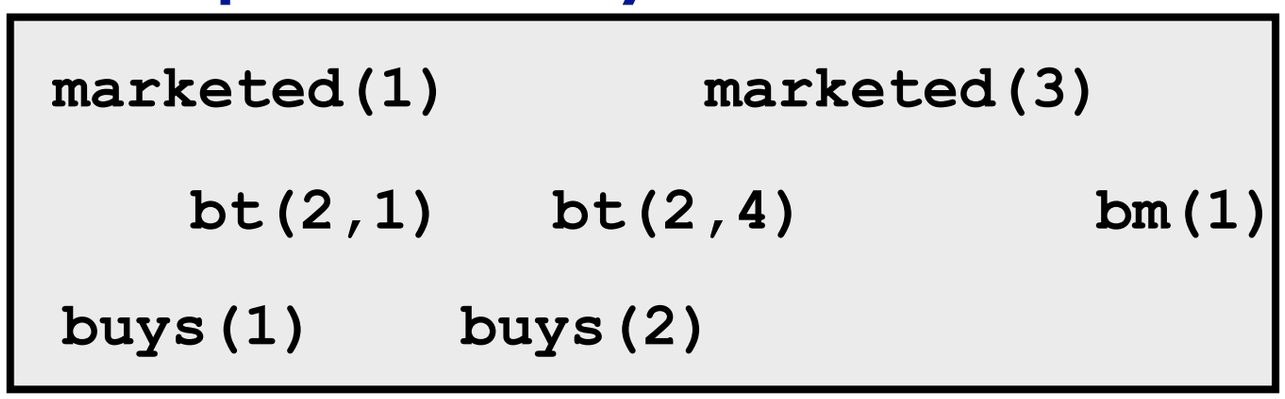
utility = −3 + −3 + 5 + 5 = 4

probability = 0.0032



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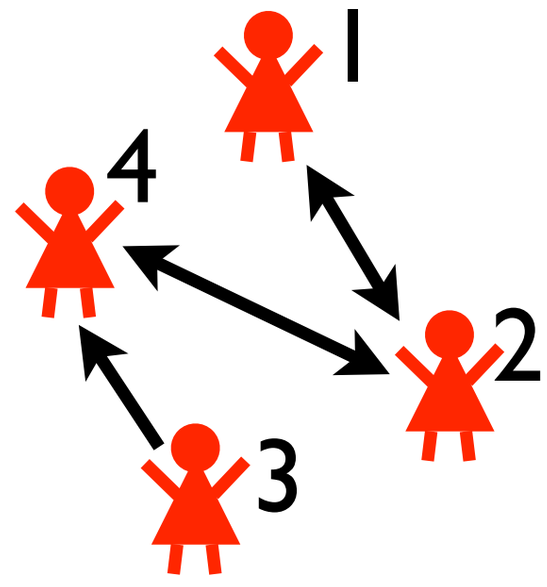


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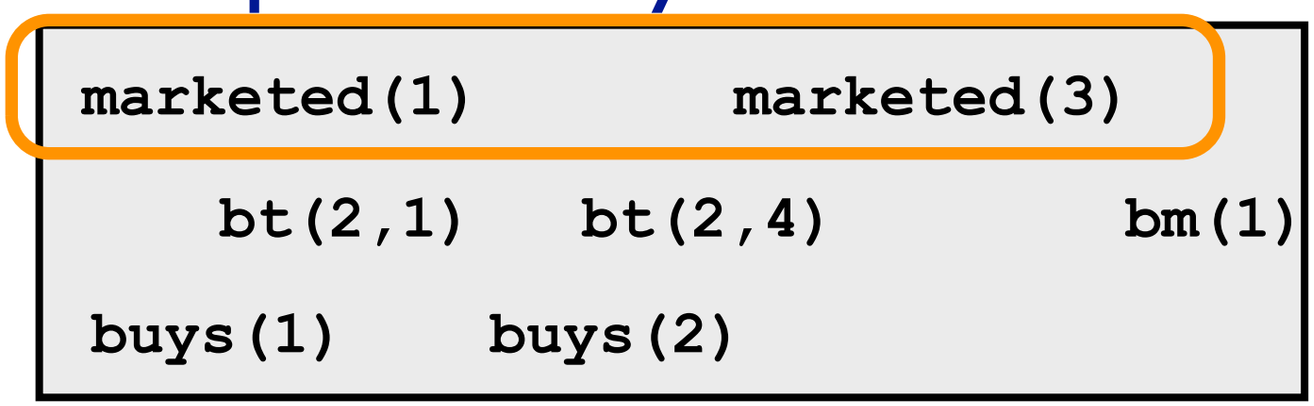
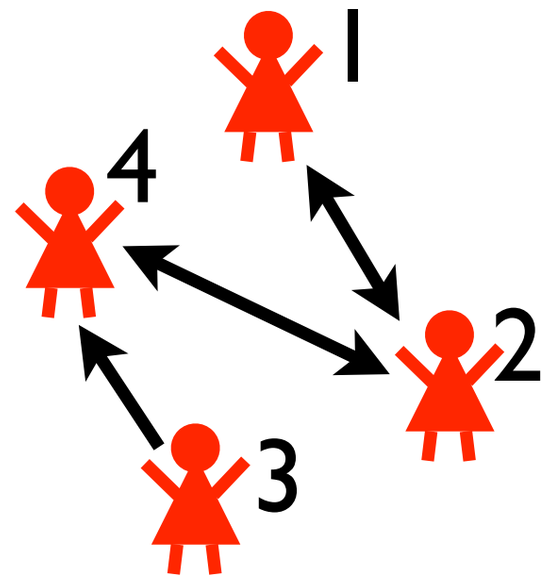


world contributes 0.0032×4 to expected utility of strategy

DTProbLog

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```

task: find strategy that maximizes expected utility
solution: using ProbLog technology

Phenetic

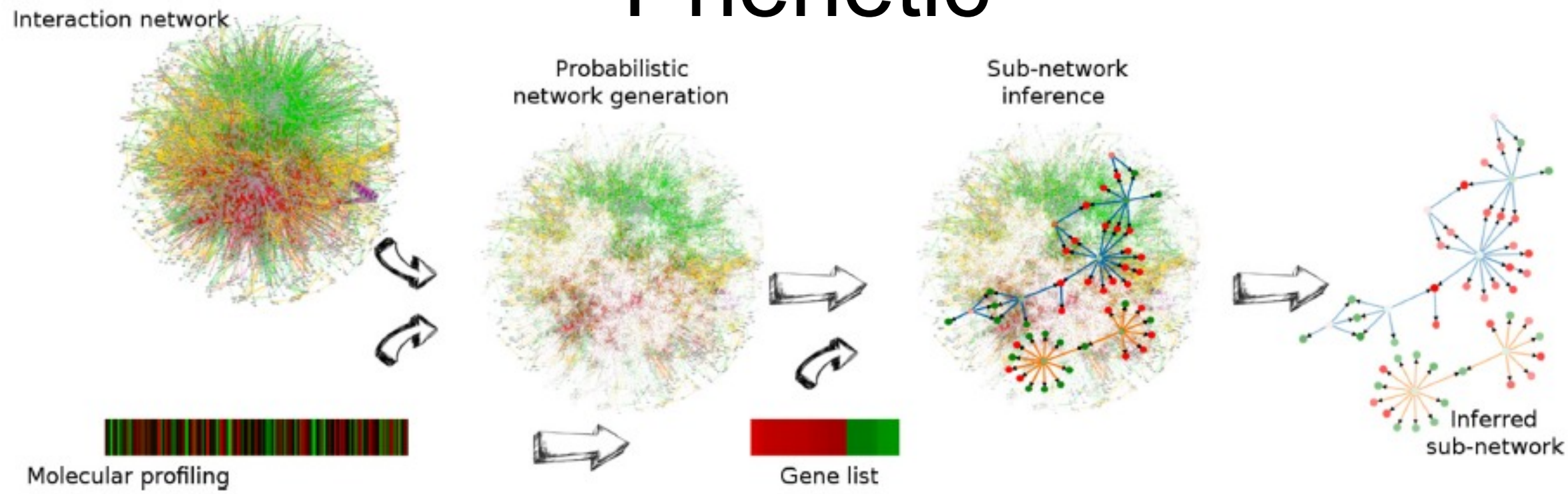

Figure 1. Overview of PheNetic, a web service for network-based interpretation of 'omics' data. The web service uses as input a genome wide interaction network for the organism of interest, a user generated molecular profiling data set and a gene list derived from these data. Interaction networks for a wide variety of organisms are readily available from the web server. Using the uploaded user-generated molecular data the interaction network is converted into a probabilistic network: edges receive a probability proportional to the levels measured for the terminal nodes in the molecular profiling data set. This probabilistic interaction network is used to infer the sub-network that best links the genes from the gene list. The inferred sub-network provides a trade-off between linking as many genes as possible from the gene list and selecting the least number of edges.

- Causes: Mutations
  - All related to similar phenotype
- Effects: Differentially expressed genes
- 27 000 cause effect pairs

- Interaction network:
  - 3063 nodes
    - Genes
    - Proteins
  - 16794 edges
    - Molecular interactions
    - Uncertain

- Goal: connect causes to effects through common subnetwork
  - = Find mechanism
- Techniques:
  - DTProbLog
  - Approximate inference

[De Maeyer et al., Molecular Biosystems 13, NAR 15]

Roadmap

- Modeling
- Inference
- Learning & Dynamics
- Advanced Inference and KBMC

... with some detours on the way

Part IV: Advanced Inference and KBMC

Complexity of Querying

Complexity of querying

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.96x0.99=0.95

0.9x0.93=0.83

0.87x0.93=0.80

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

Complexity of querying

ProducesProduct

Company	Product	P
...	...	...

HeadquarteredIn

Company	City	P
...	...	...

```
select distinct x.Product, x.Company,
                x.P * y.P as P
from ProducesProduct x, HeadquarteredIn y
where x.Company = y.Company
      and y.City = 'san_jose'
order by P desc
```

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.96x0.99=0.95

0.9x0.93=0.83

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Product	Company	P
personal_computer	ibm	0.95
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```
result(Product,Company) :-
   producesProduct(Company,Product),
   headquarteredIn(Company,san_jose).
query(result(_,_)).
```

Complexity of querying

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sony	walkman	0.96
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```
result(Product,Company) :-
    producesProduct(Company,Product),
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query(result(_,_)).
```

each ground query has a **single proof**

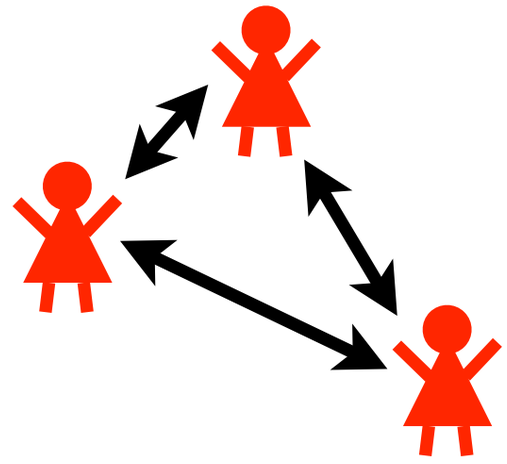
→ no disjoint-sum-problem,

easy evaluation

Complexity of querying in probabilistic databases

- queries have fixed size (no recursion)
- size of query << size of database
- complexity of evaluating given query measured in size of database (data complexity)
- previous example: polynomial
  (as all standard relational database queries)

How hard is evaluating q?



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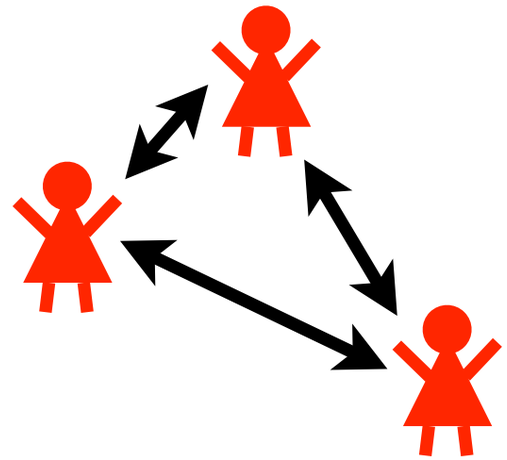

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proofs

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s(2),i(2,3)
s(3),i(3,1)
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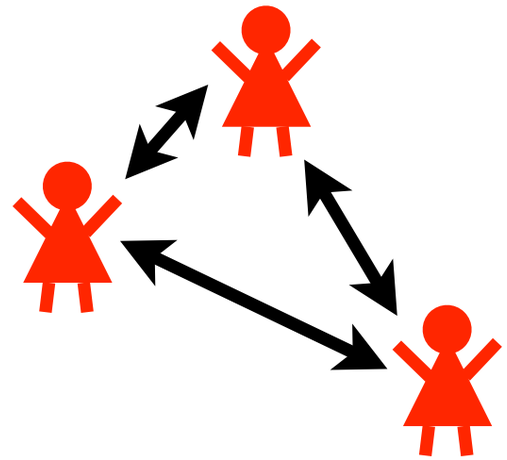

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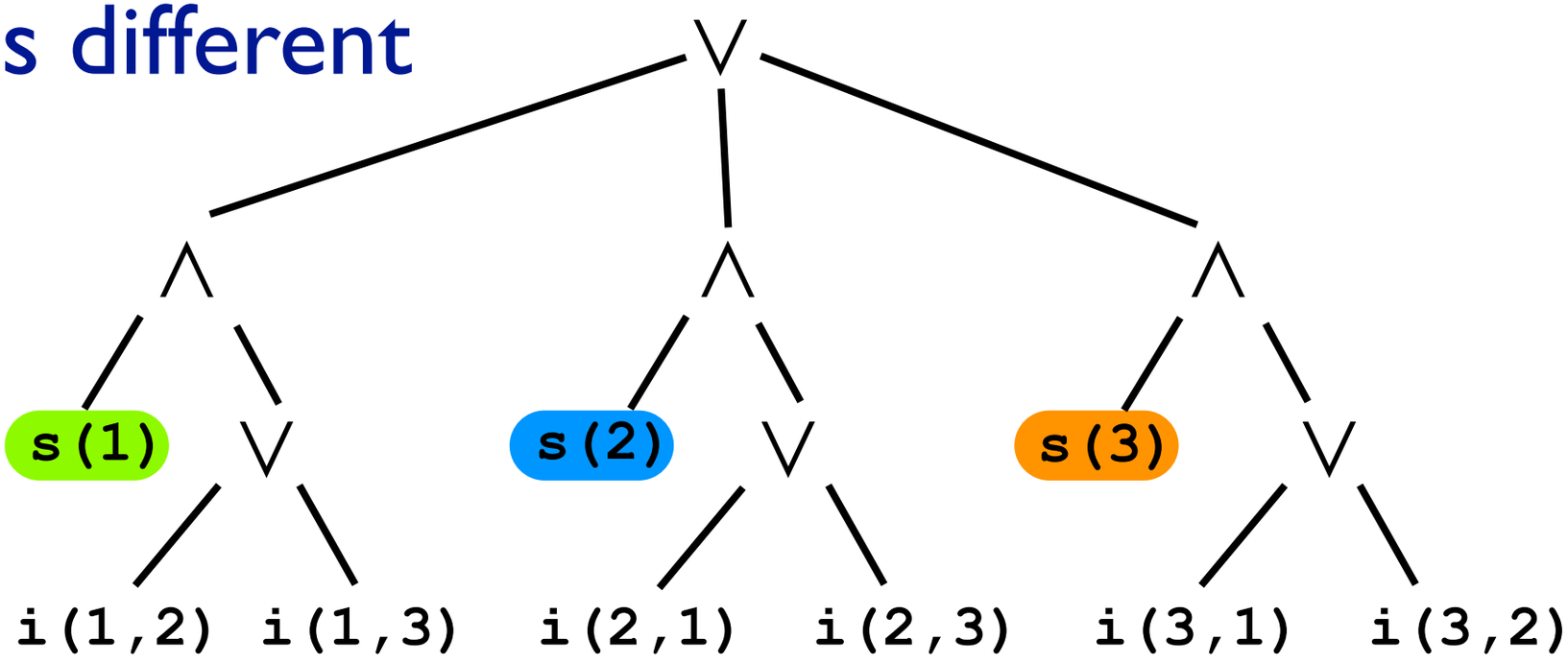
tree structure, all leaves different



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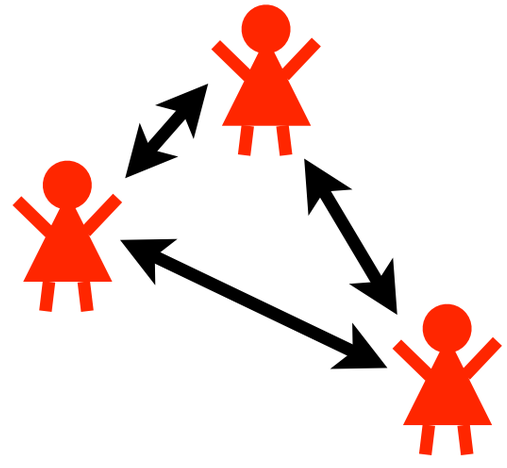

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- s(1),i(1,2)
- s(1),i(1,3)
- s(2),i(2,1)
- s(2),i(2,3)
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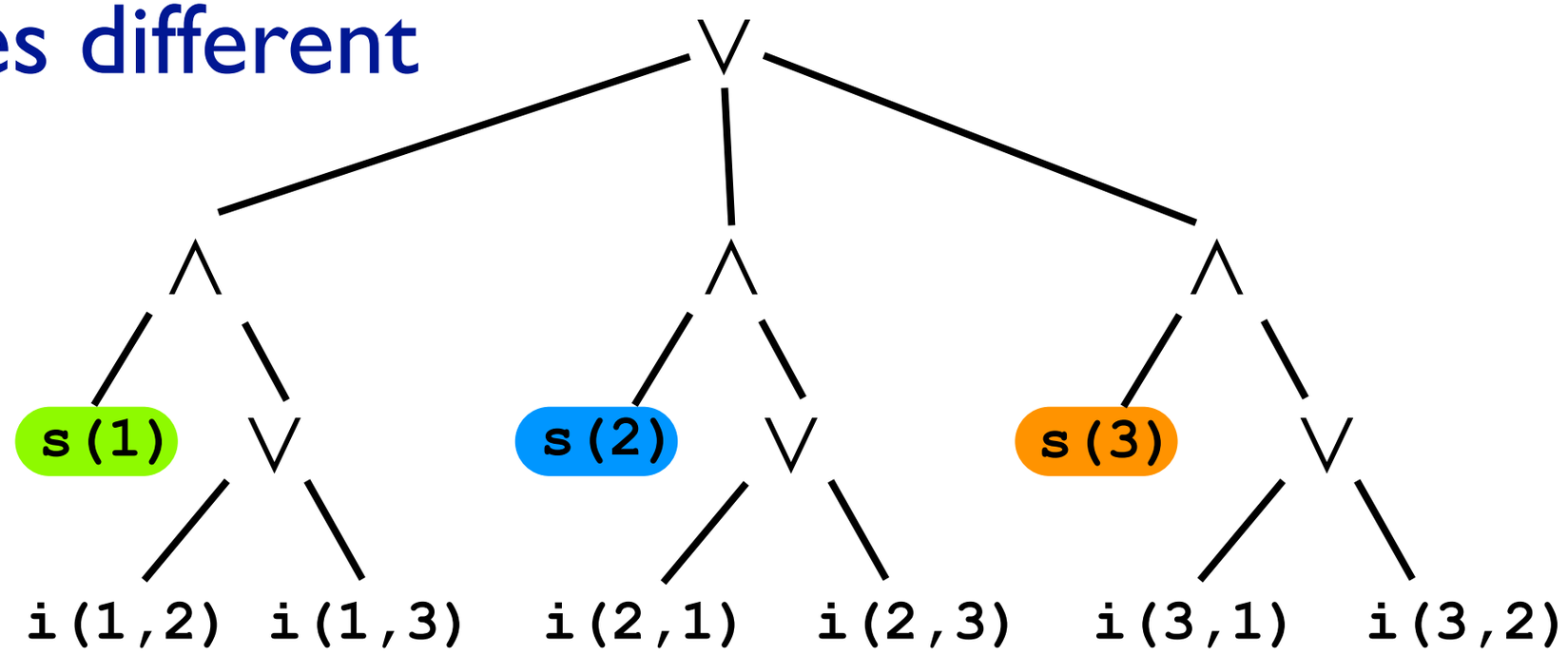
tree structure, all leaves different

$$P(\varphi_1 \wedge \varphi_2) = P(\varphi_1) \cdot P(\varphi_2)$$
$$P(\varphi_1 \vee \varphi_2) = 1 - (1 - P(\varphi_1)) \cdot (1 - P(\varphi_2))$$



$$P(\mathtt{q}) = 1 - \prod_{j=1..n} (1 - \left(P(\mathtt{s}(\mathtt{X_j}))(1 - \prod_{k=1..n, k \neq j} (1 - P(\mathtt{i}(\mathtt{X_j}, \mathtt{Y_k})))) \right))$$

polynomial in database size / number of persons n

proofs

`s(1)`,`i(1,2)`

`s(1)`,`i(1,3)`

`s(2)`,`i(2,1)`

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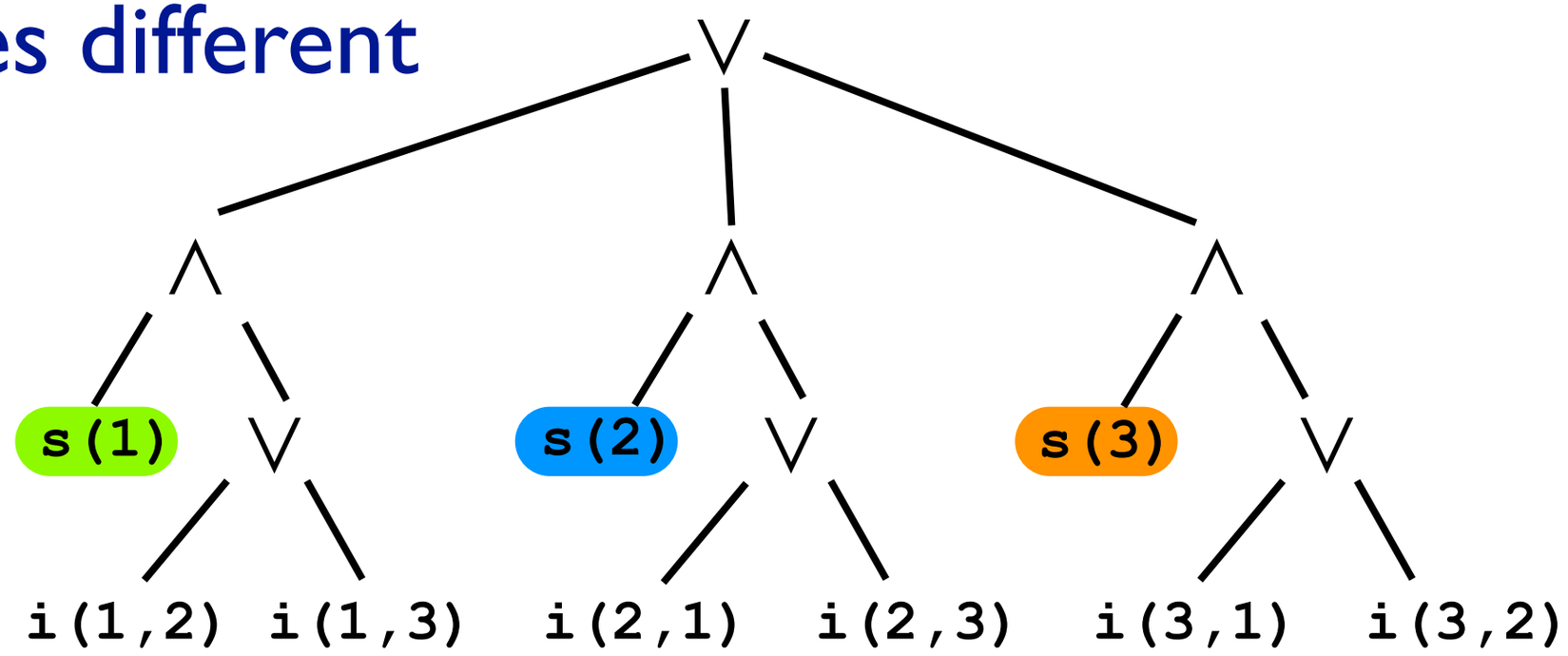

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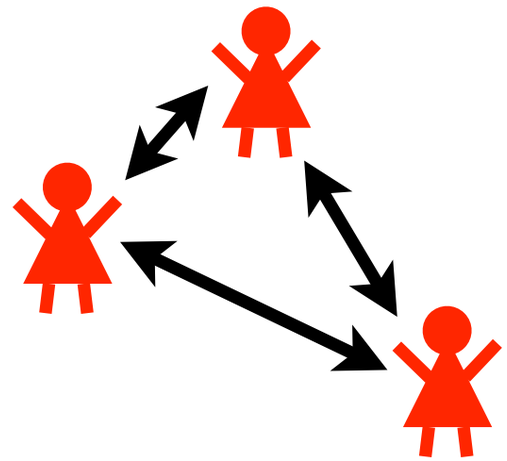

```
q2 :- stress(X), influences(X,Y), male(Y).
```

```
0.3::male(1).
0.8::male(2).
0.9::male(3).
```

```
0.7::stress(1).
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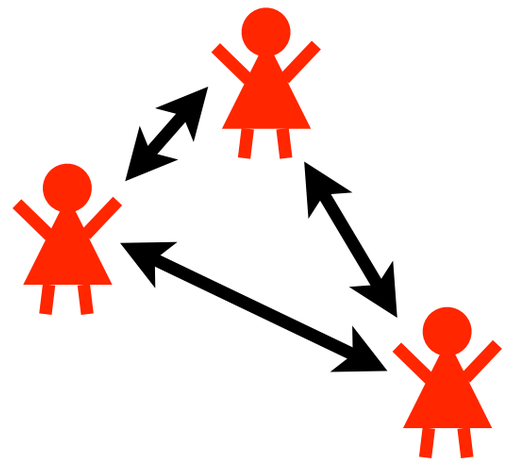

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```
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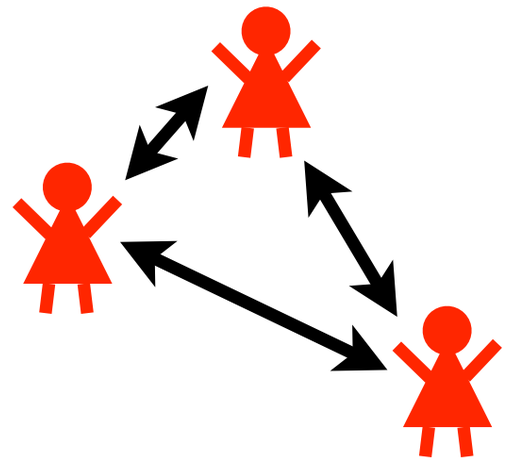

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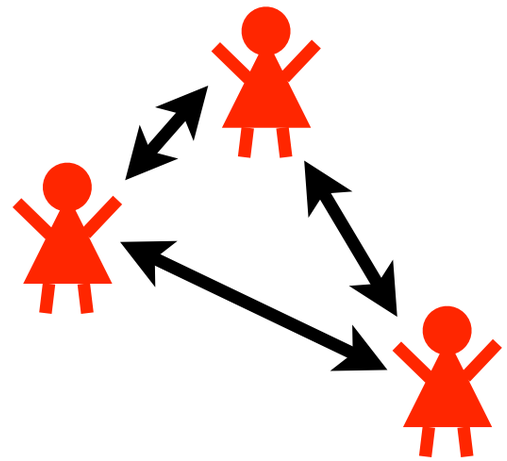

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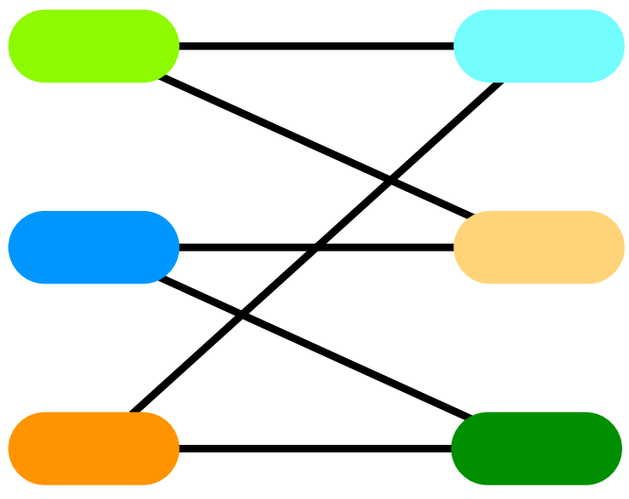

cannot build tree structure with all leaves different

Read-Once Formulas

- Propositional formulas that can be rewritten such that each variable occurs at most once (= as tree with all leaves different)

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- Can be evaluated in polynomial time

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- Unate formula: read once ↔ P4-free & normal

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- Propositional formulas that can be rewritten such that each variable occurs at most once (= as tree with all leaves different)
- Can be evaluated in polynomial time
- Unate formula: read once ↔ P4-free & normal



no variable appears both pos & neg

$X \vee Y$ is unate

$(X \vee Y) \wedge (\neg X \vee Z)$ not

P4-free and normal

- represent DNF formula as graph:
  - a node for each variable
  - edge (X,Y) ↔ X,Y in same conjunct

P4-free and normal

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- **normal**: each clique is part of a conjunct

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  - a node for each variable
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- **P4-free**: no induced 4 node subgraph is a path

Our first example

```
s(1),i(1,2)
s(1),i(1,3)
s(2),i(2,1)
s(2),i(2,3)
s(3),i(3,1)
s(3),i(3,2)
```

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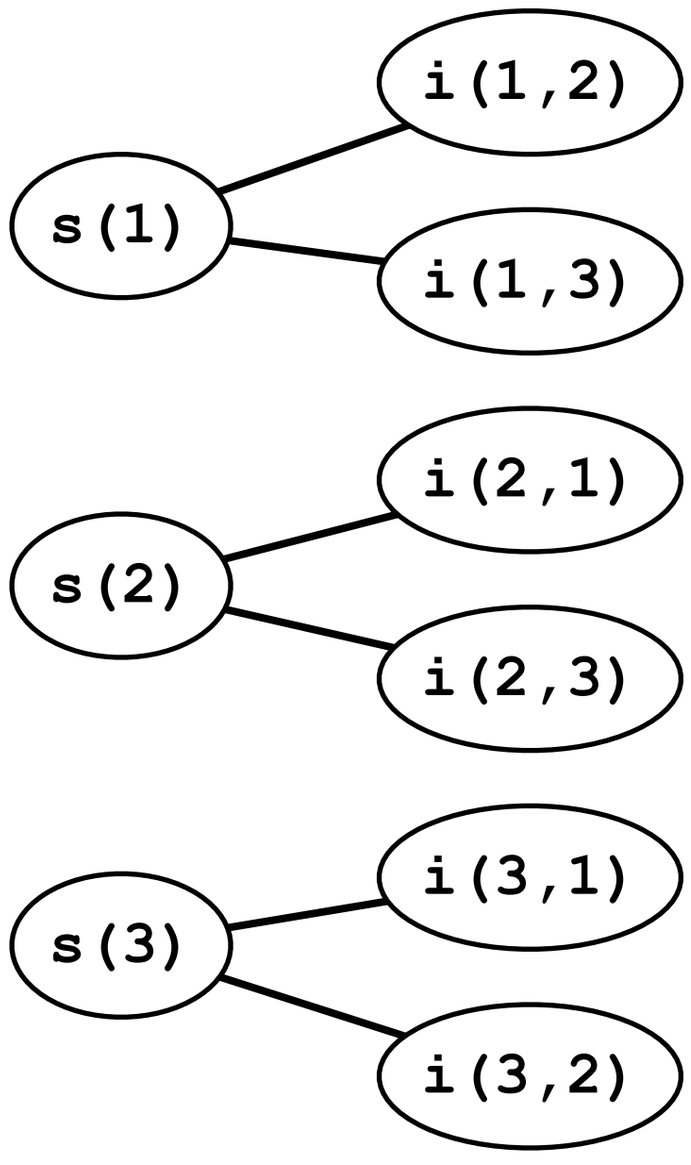

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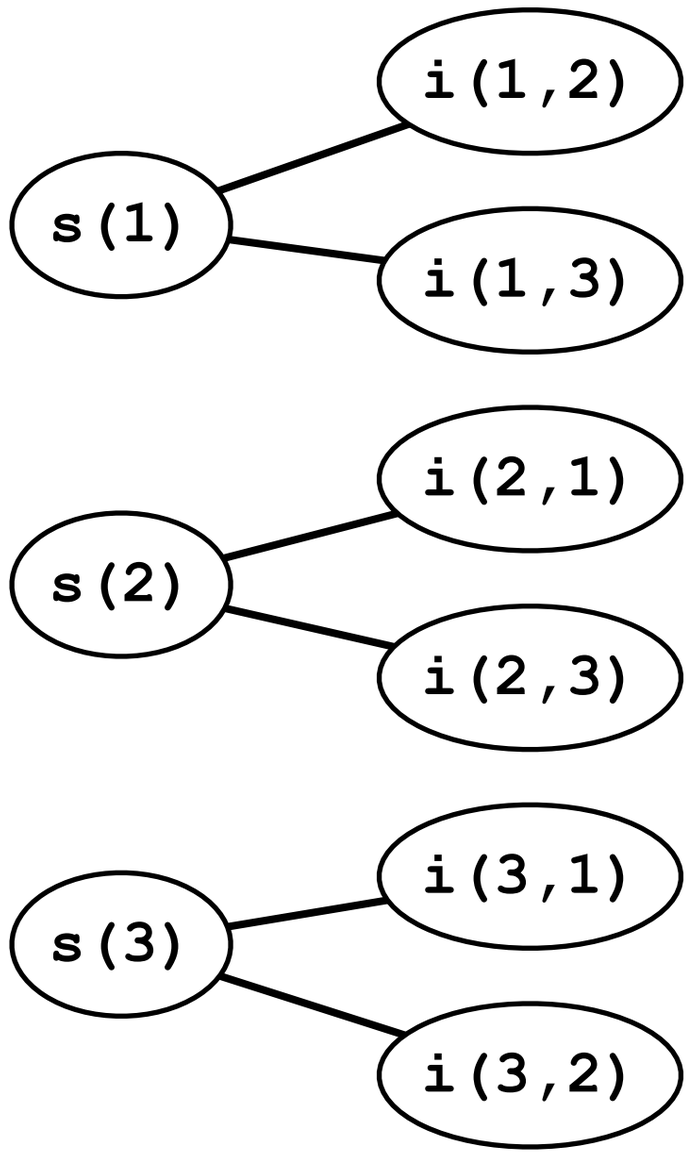

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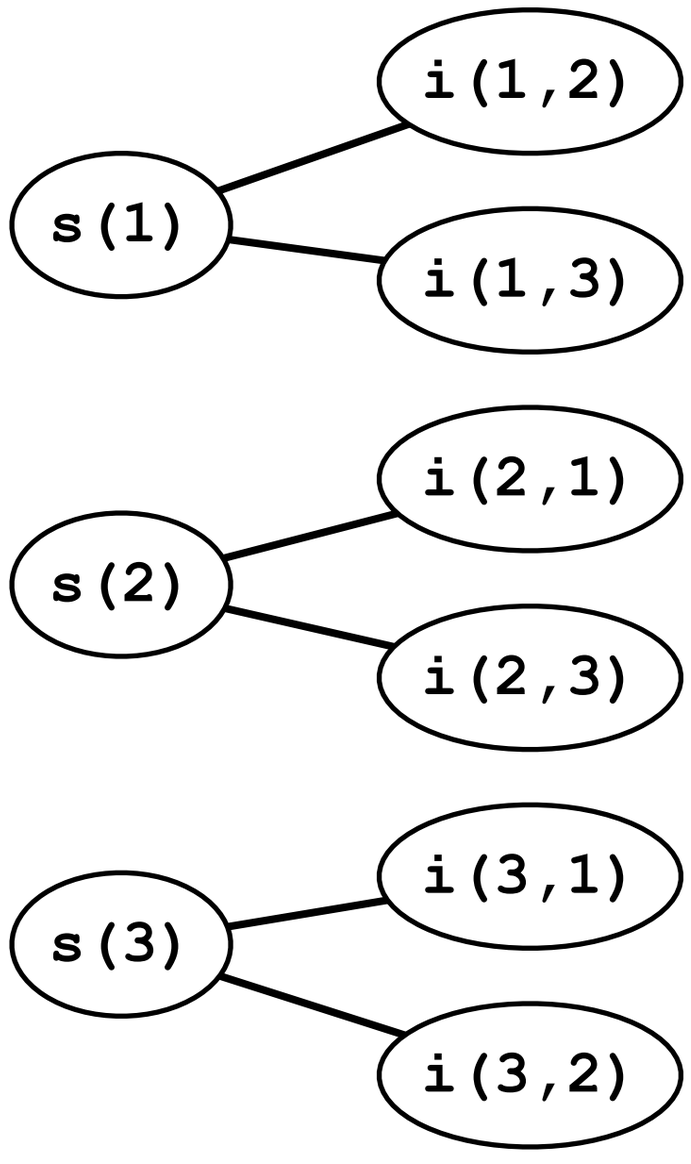

- **normal**: each clique is part of a conjunct ✔
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read-once

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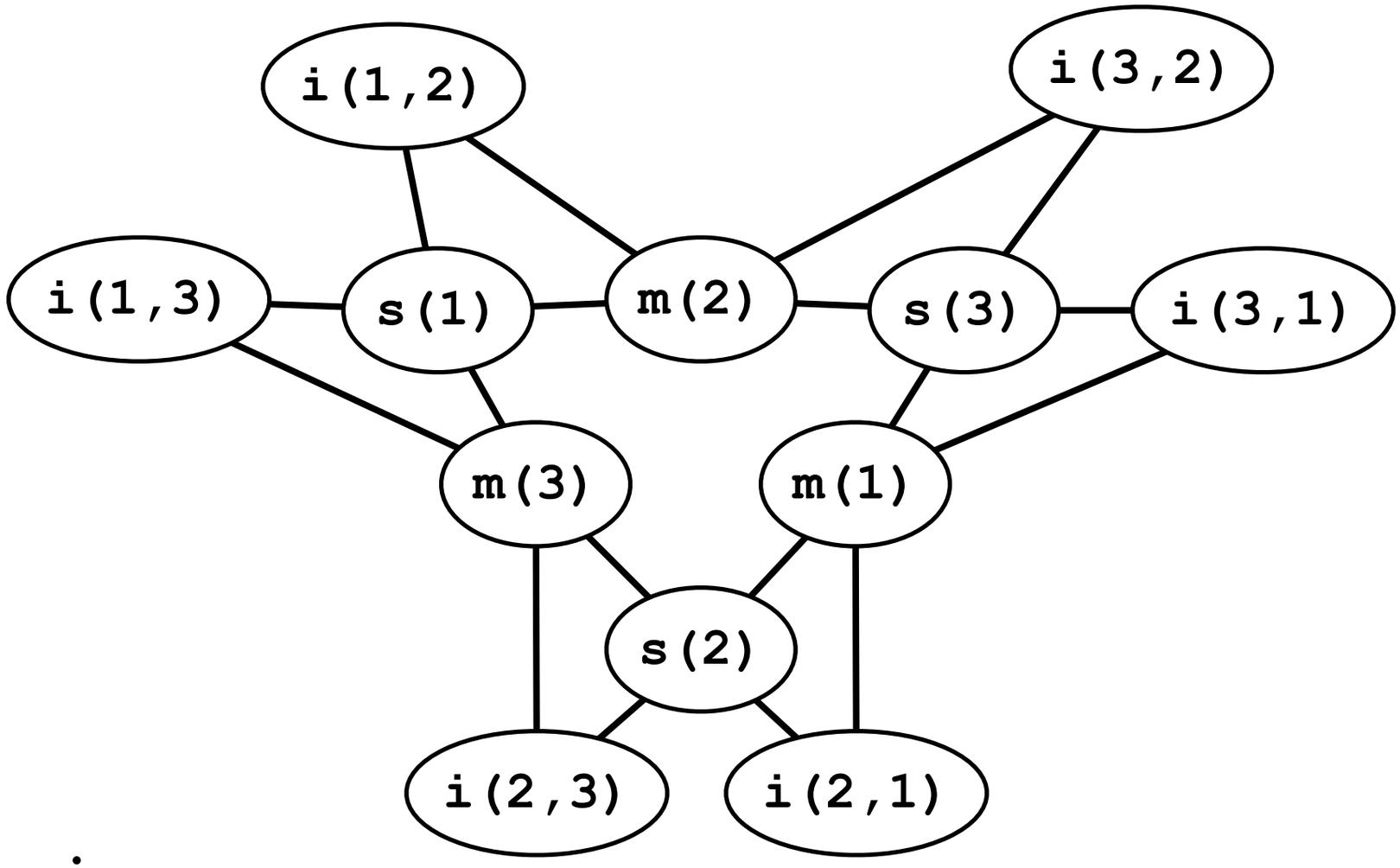

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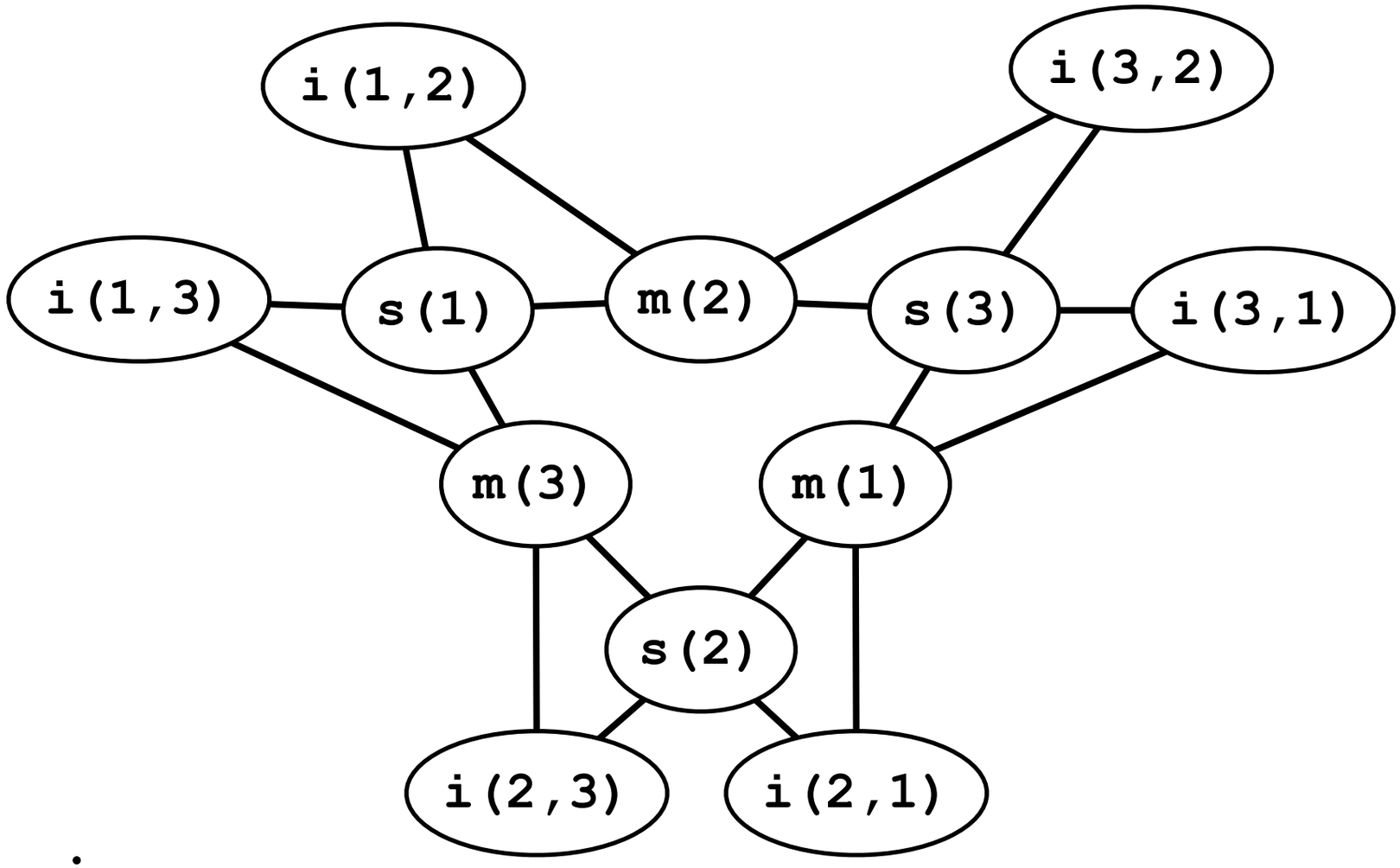

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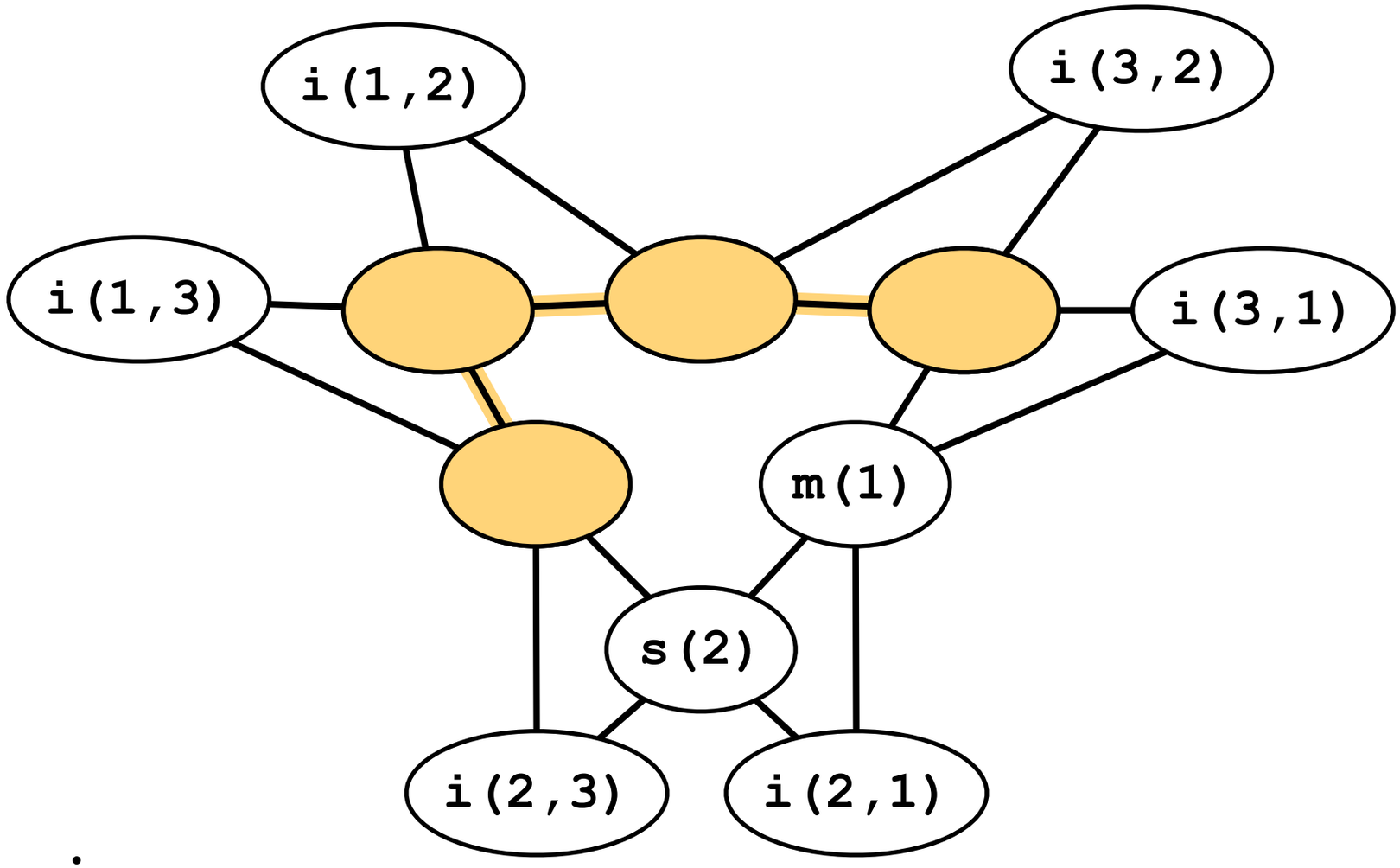

- **normal**: each clique is part of a conjunct ✔
- **P4-free**: no induced 4 node subgraph is a path ✗

not read-once
(and in fact known to be #P-hard, cf. PDB-book)

Dichotomy of UCQ Evaluation

- **U**nion of **C**onjunctive **Q**ueries
  ≈ Datalog without recursion and negation
- Theorem: UCQ evaluation is either polynomial in database size or #P-hard

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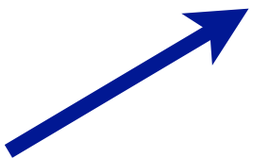
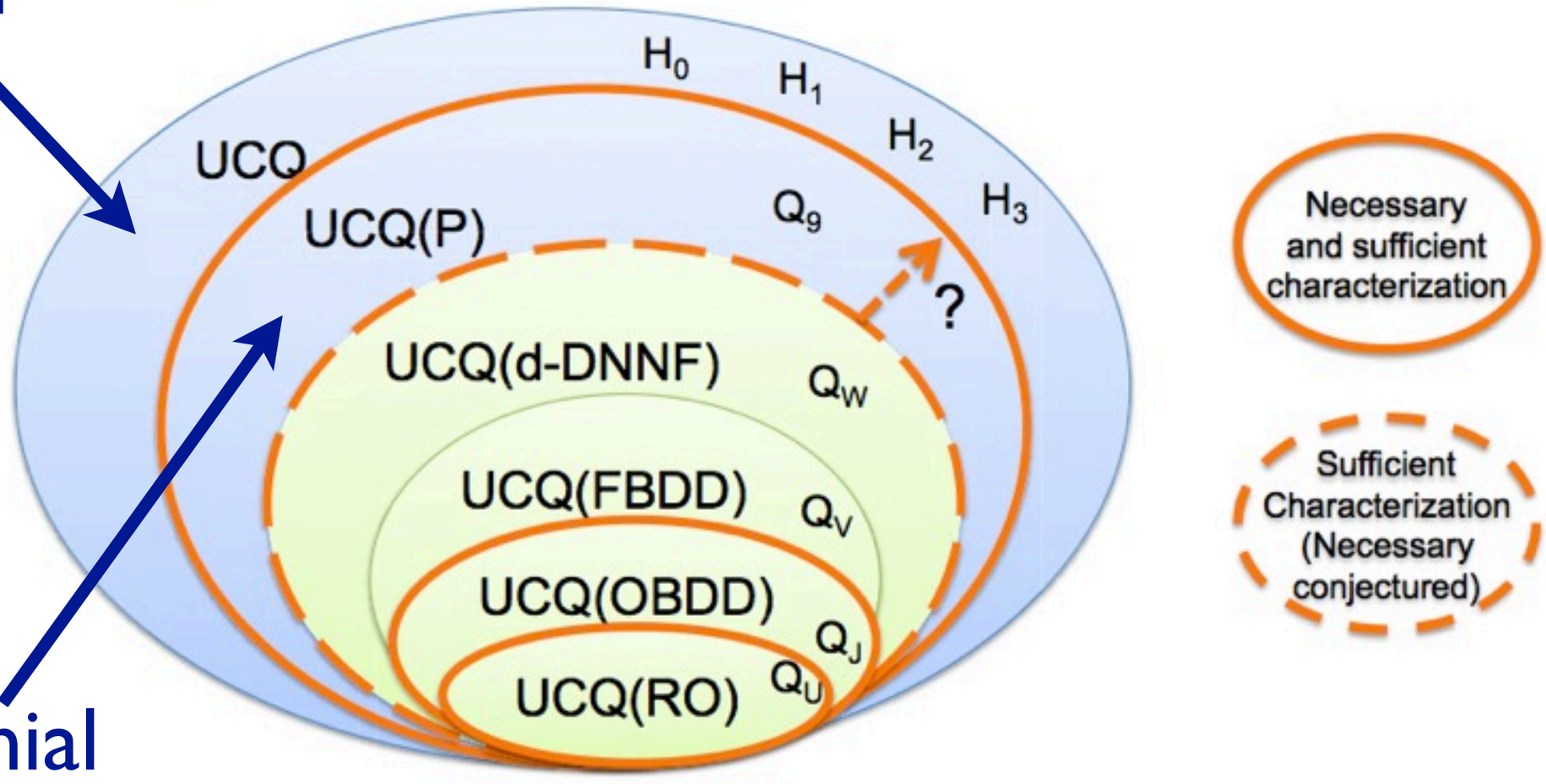


counting version of NP decision problems, e.g., model counting

#P-hard

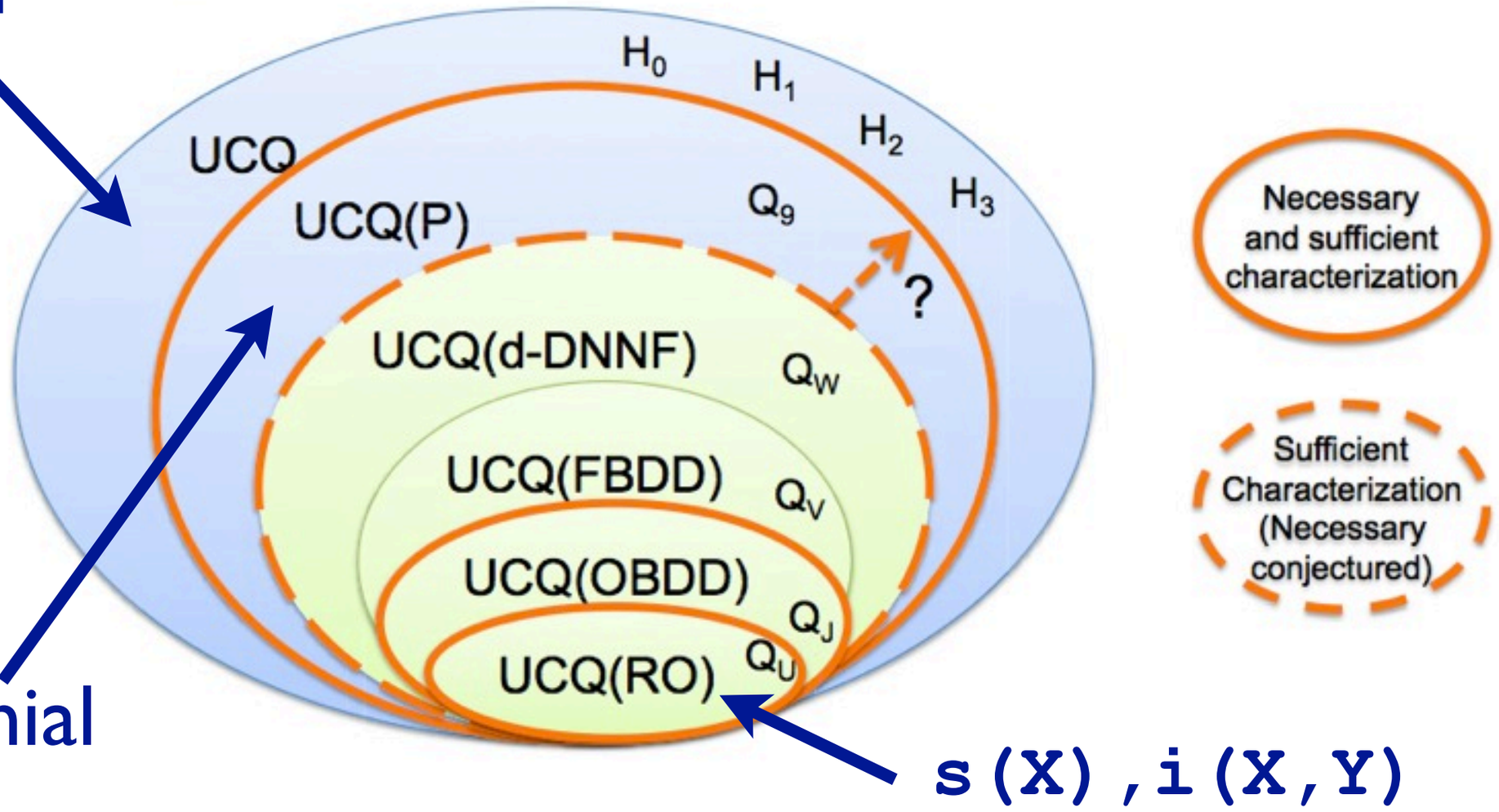


polynomial

Figure 5.4: The query compilation hierarchy for Unions of Conjunctive Queries (UCQ).

Fig. from [Suciu et al 2011]

#P-hard



polynomial

s(X),i(X,Y)

Figure 5.4: The query compilation hierarchy for Unions of Conjunctive Queries (UCQ).

Fig. from [Suciu et al 2011]

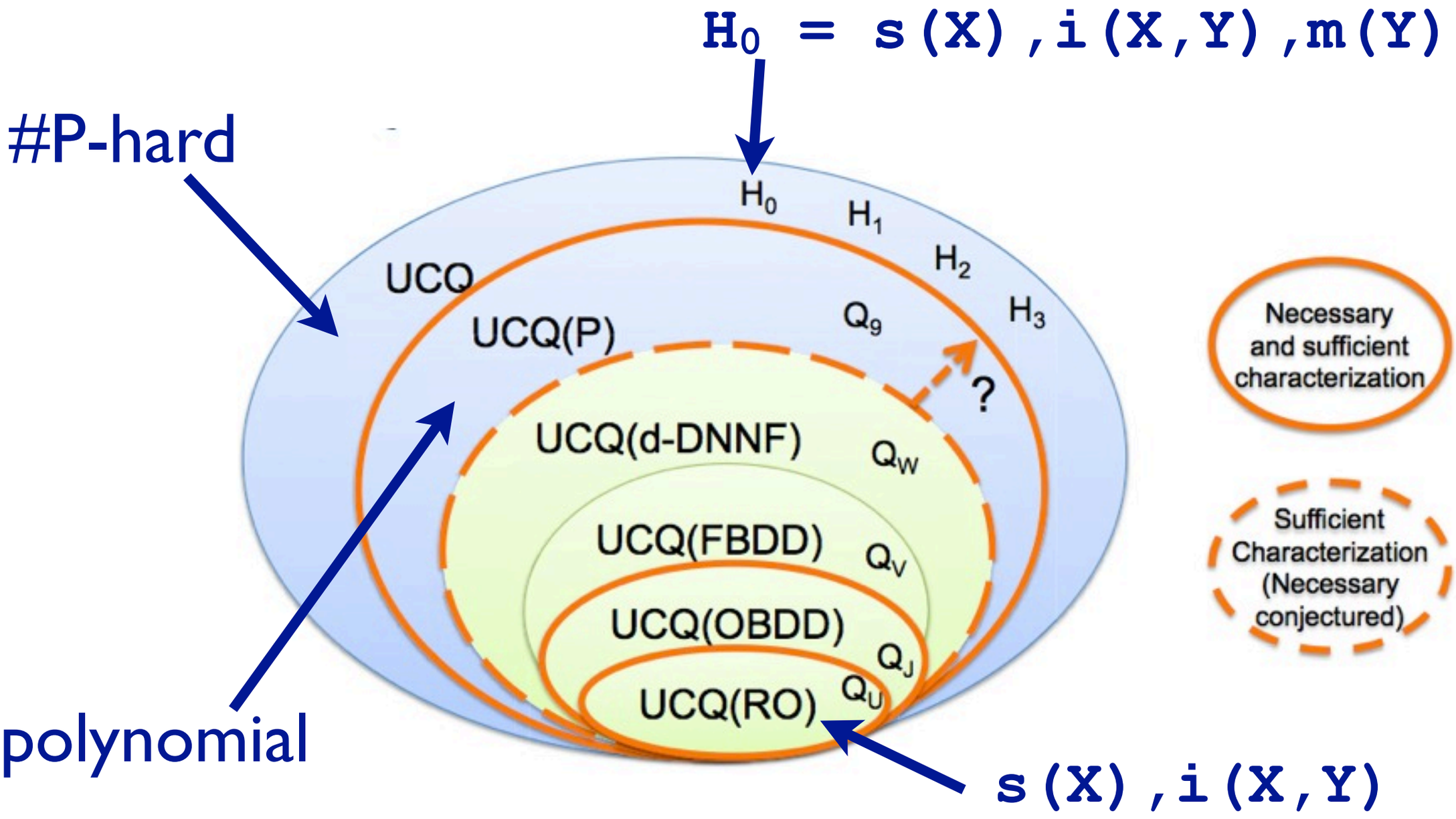


Figure 5.4: The query compilation hierarchy for Unions of Conjunctive Queries (UCQ).

Fig. from [Suciu et al 2011]

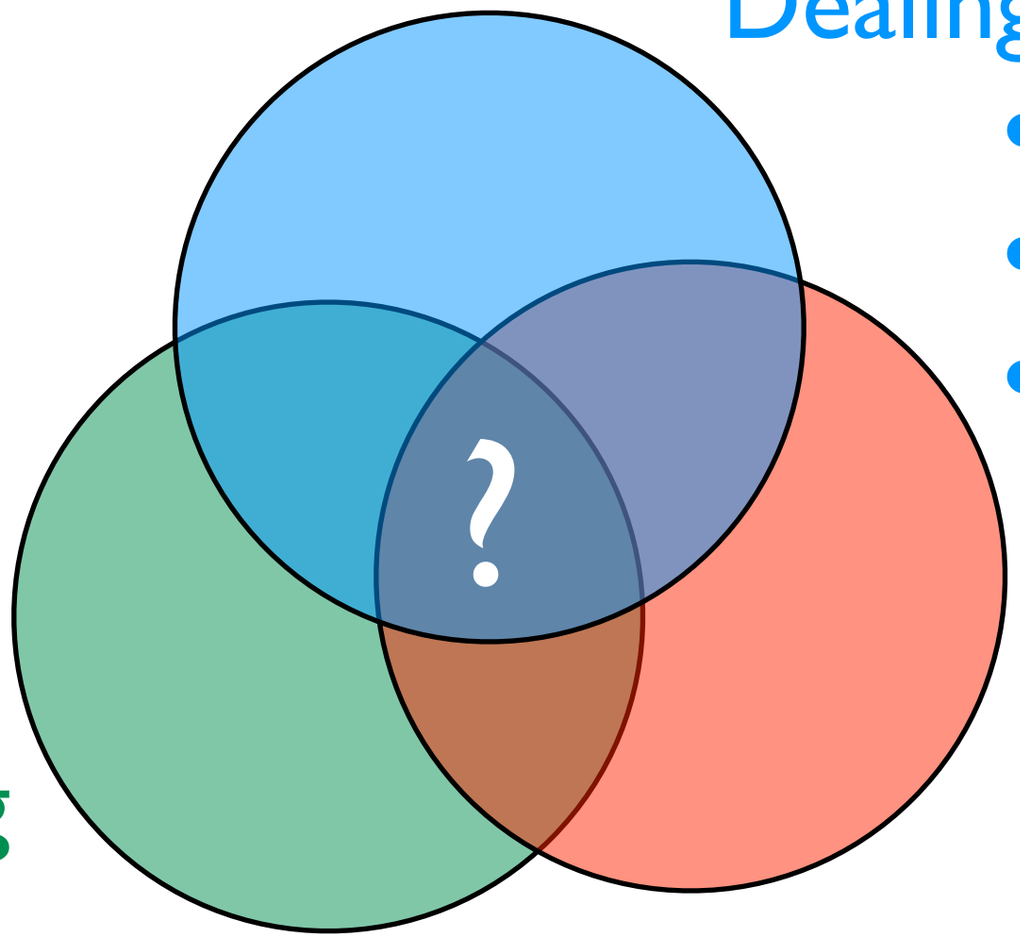
A key question in AI:

Dealing with uncertainty

- probability theory
- graphical models
- ...

Reasoning with relational data

- logic
- databases
- programming
- ...



Learning

- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

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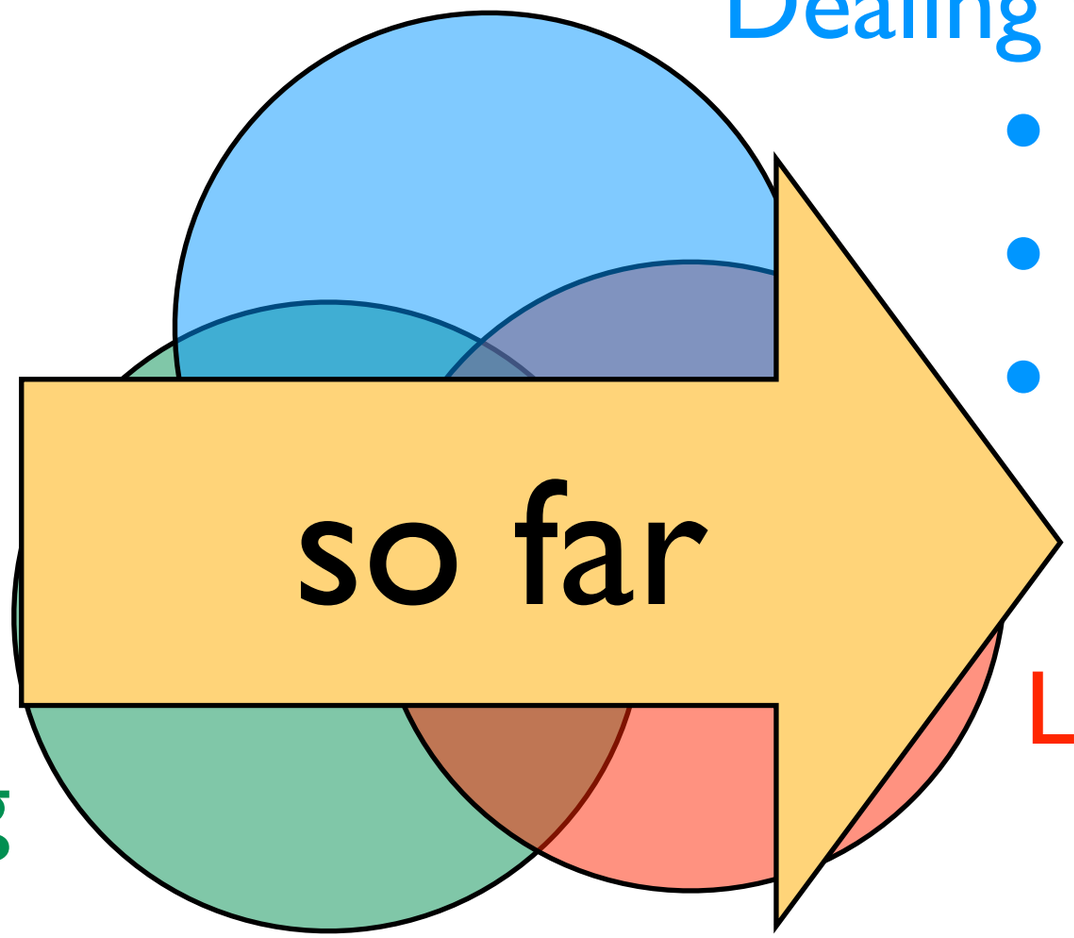

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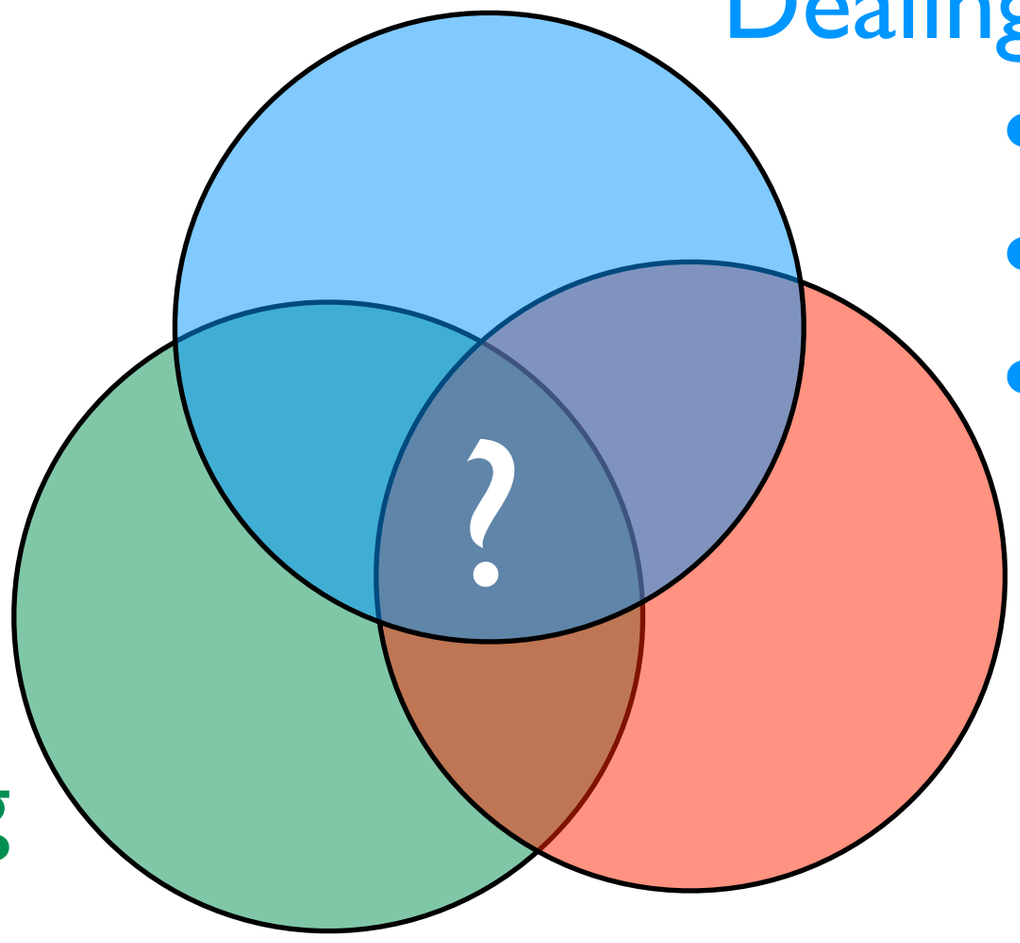

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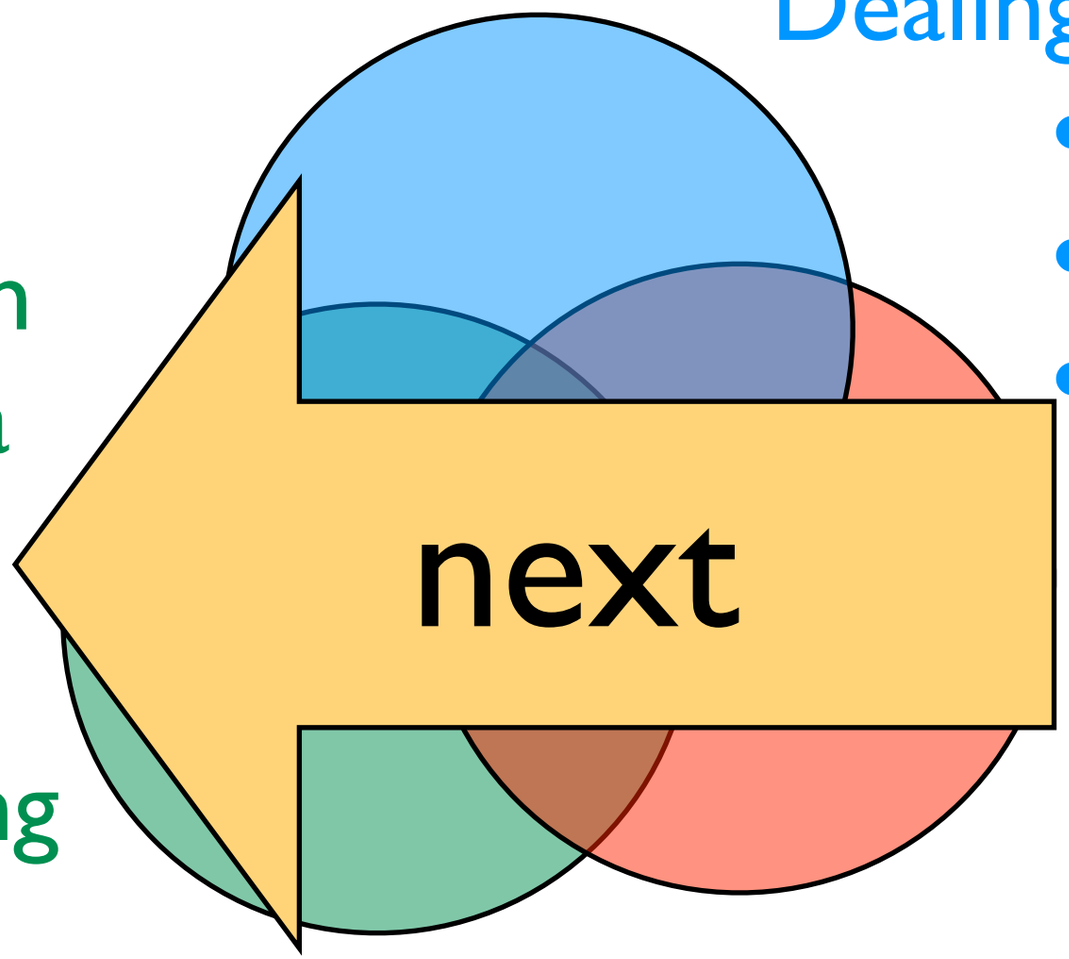


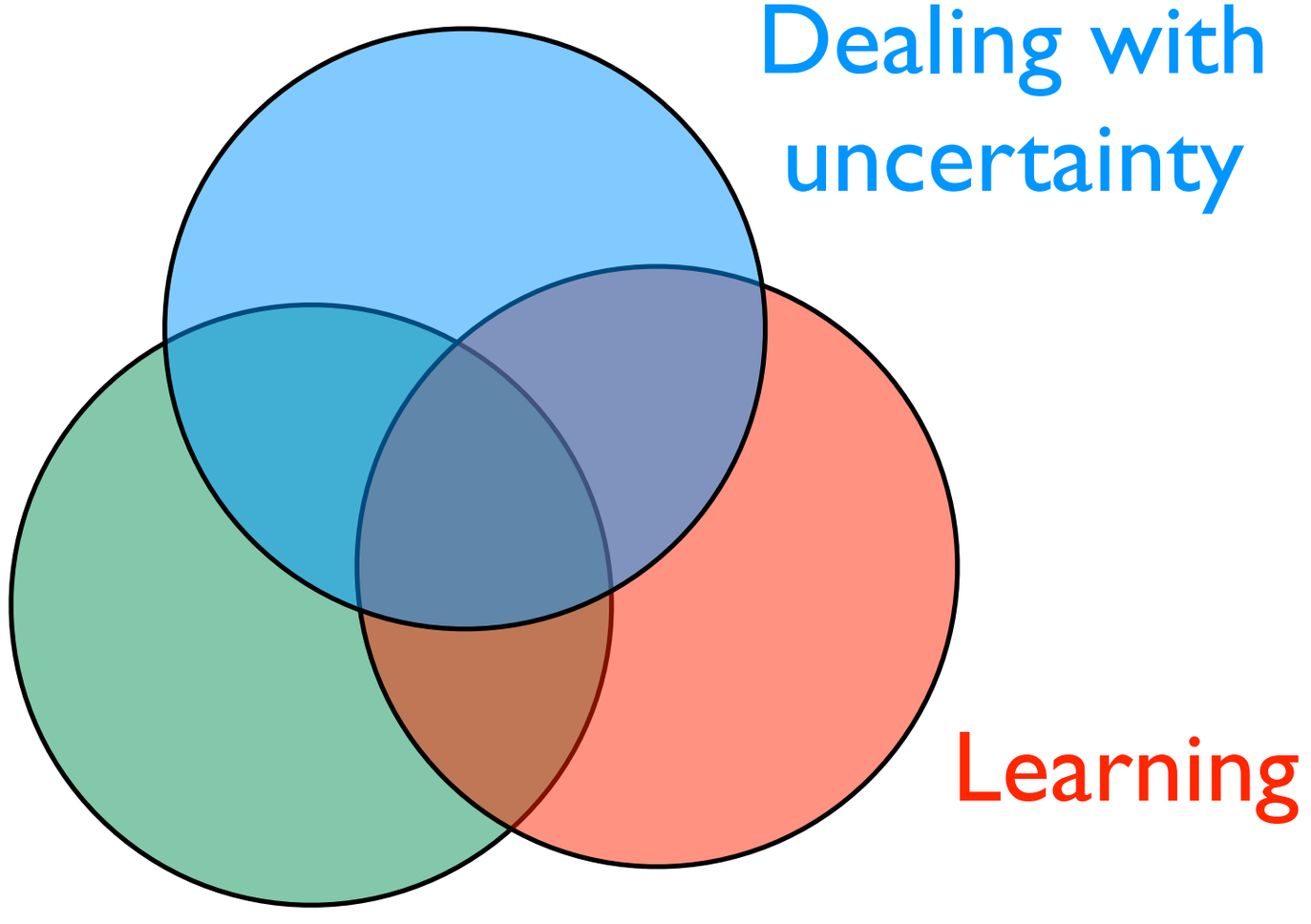
Learning

- parameters
- structure

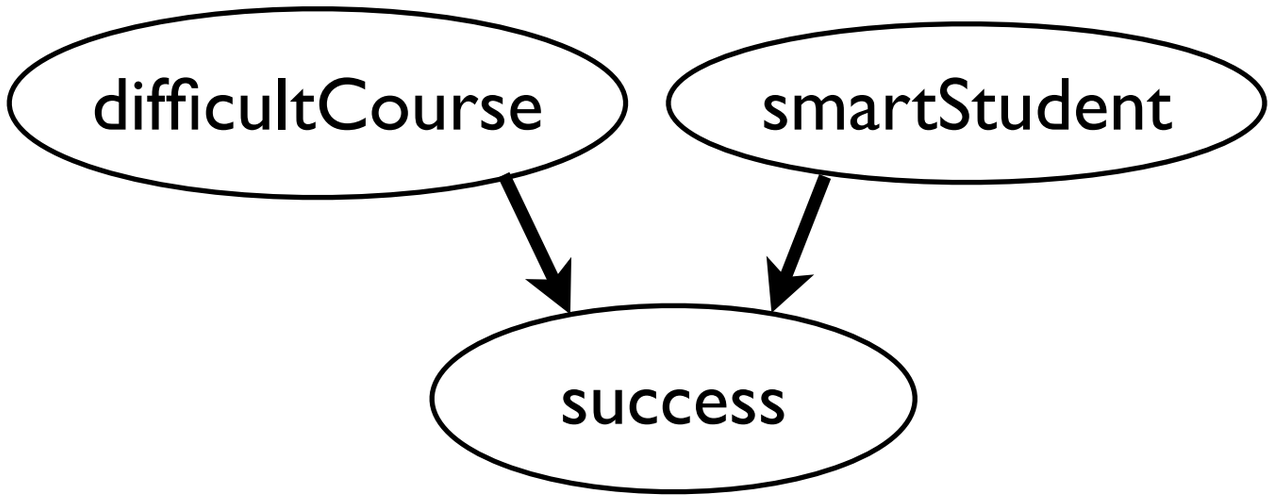
Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

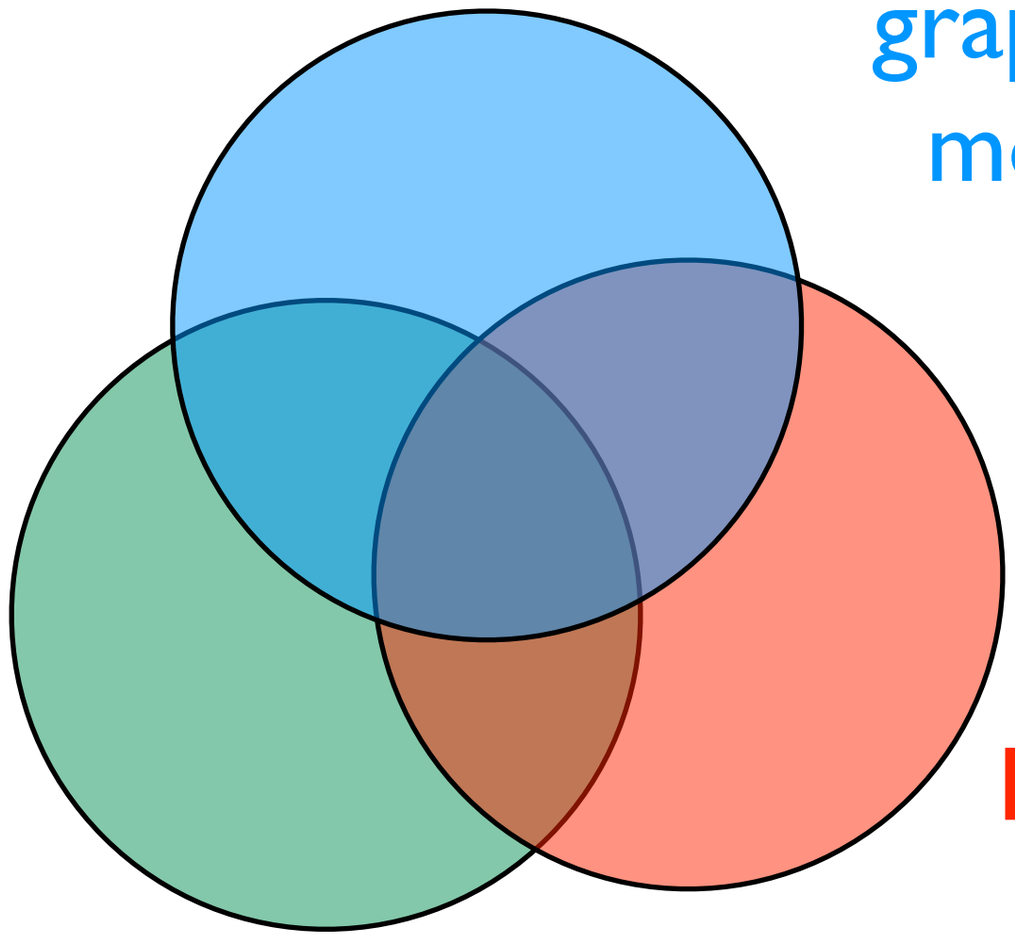
lifted graphical models

Lifted graphical models



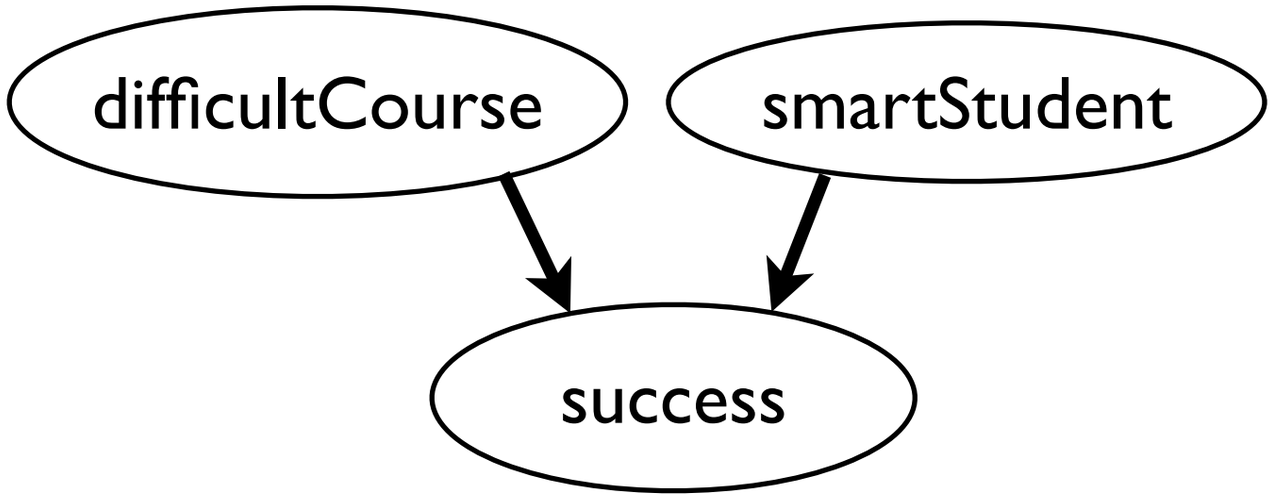
Lifted graphical models

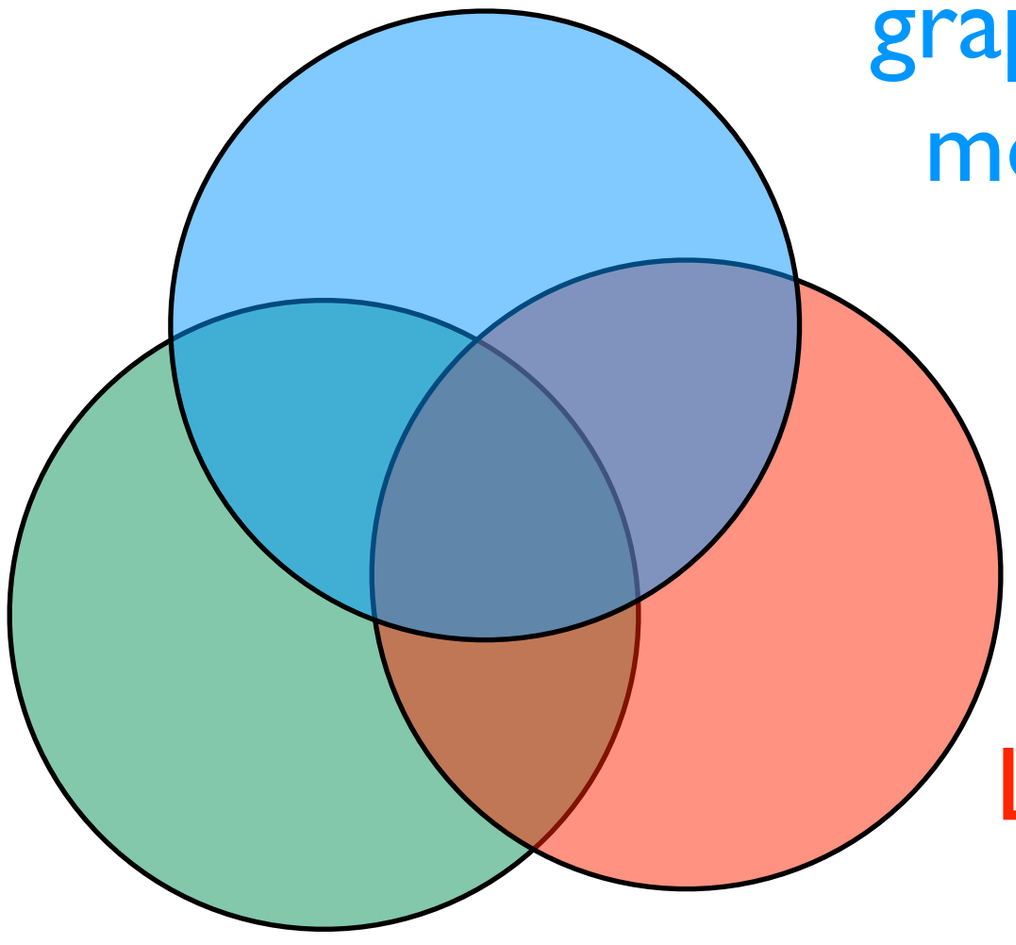




Lifted graphical models

fixed set of random variables





Lifted graphical models

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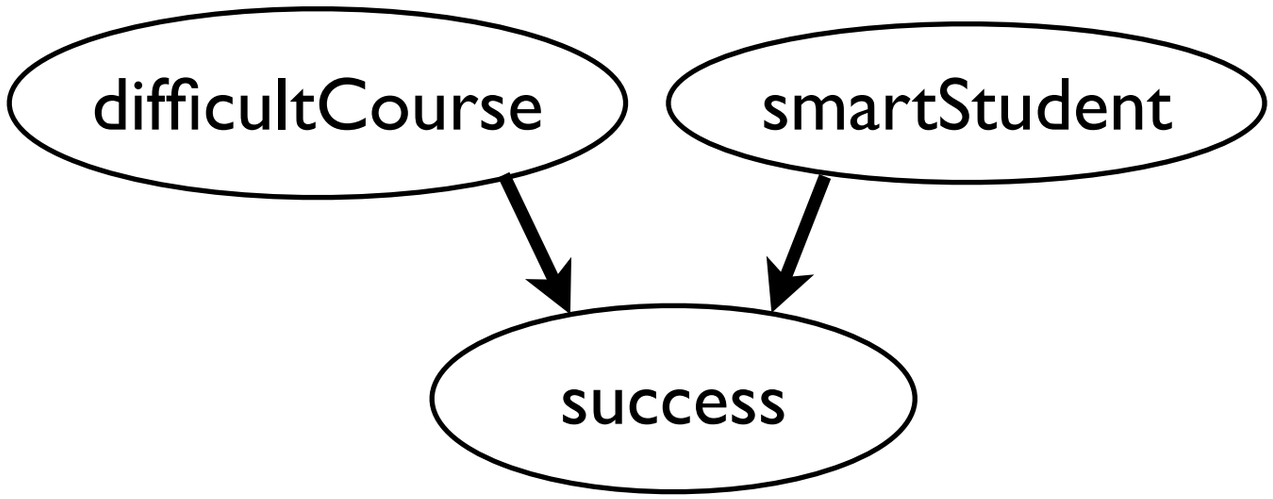
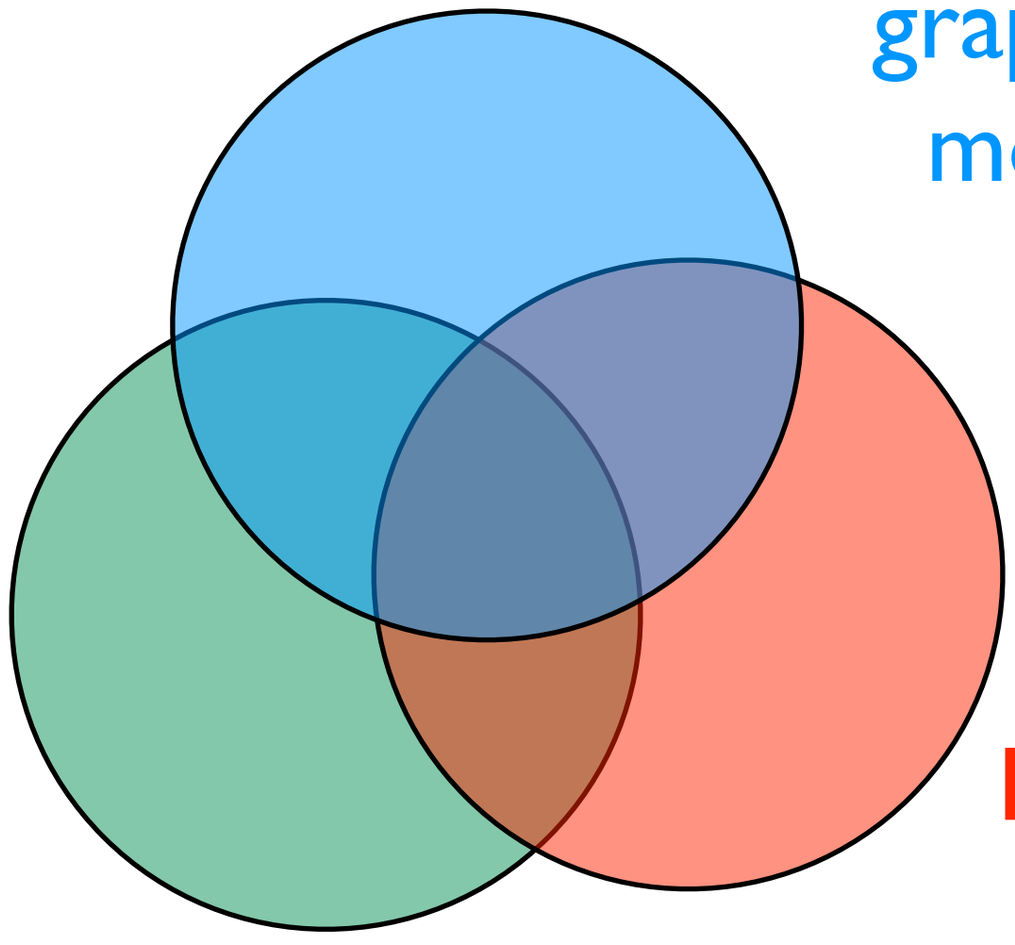
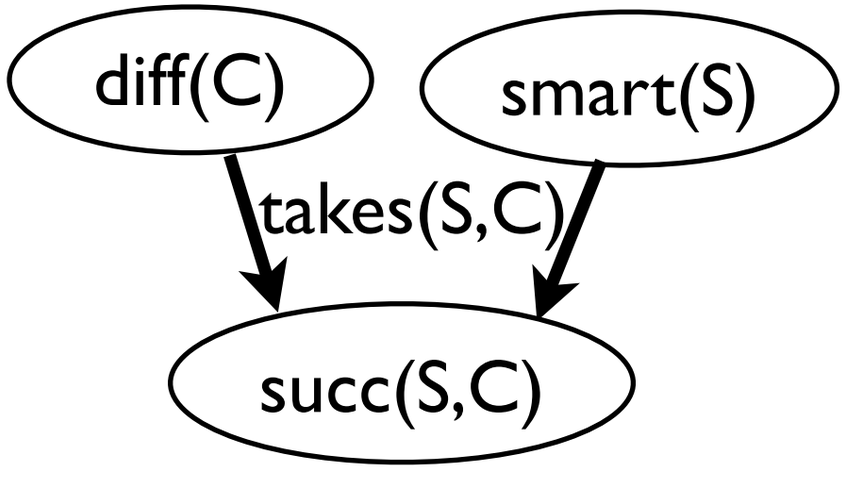


graphical model



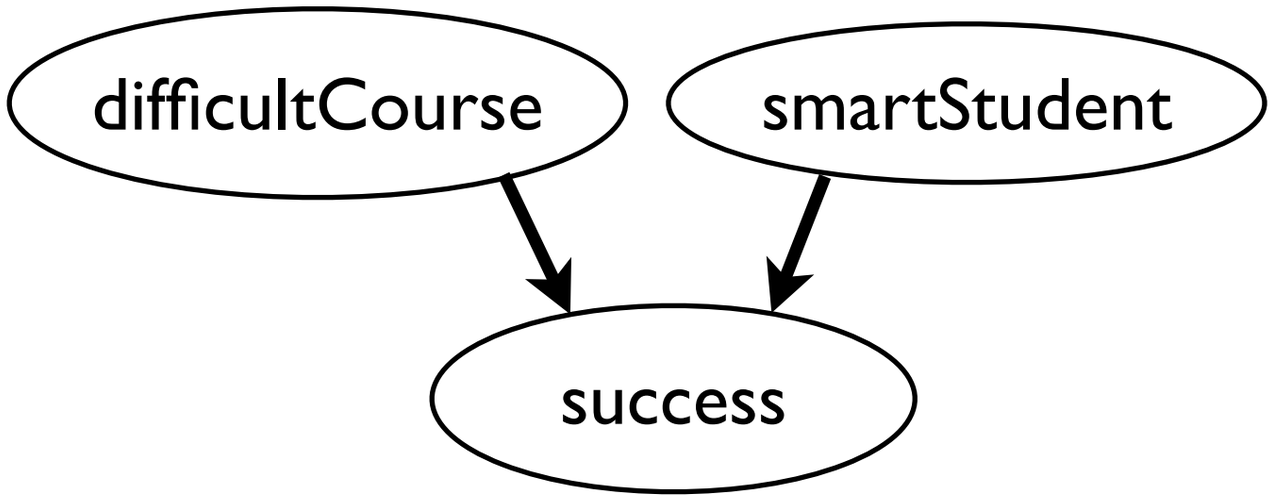
Learning

relational definition of graphical model

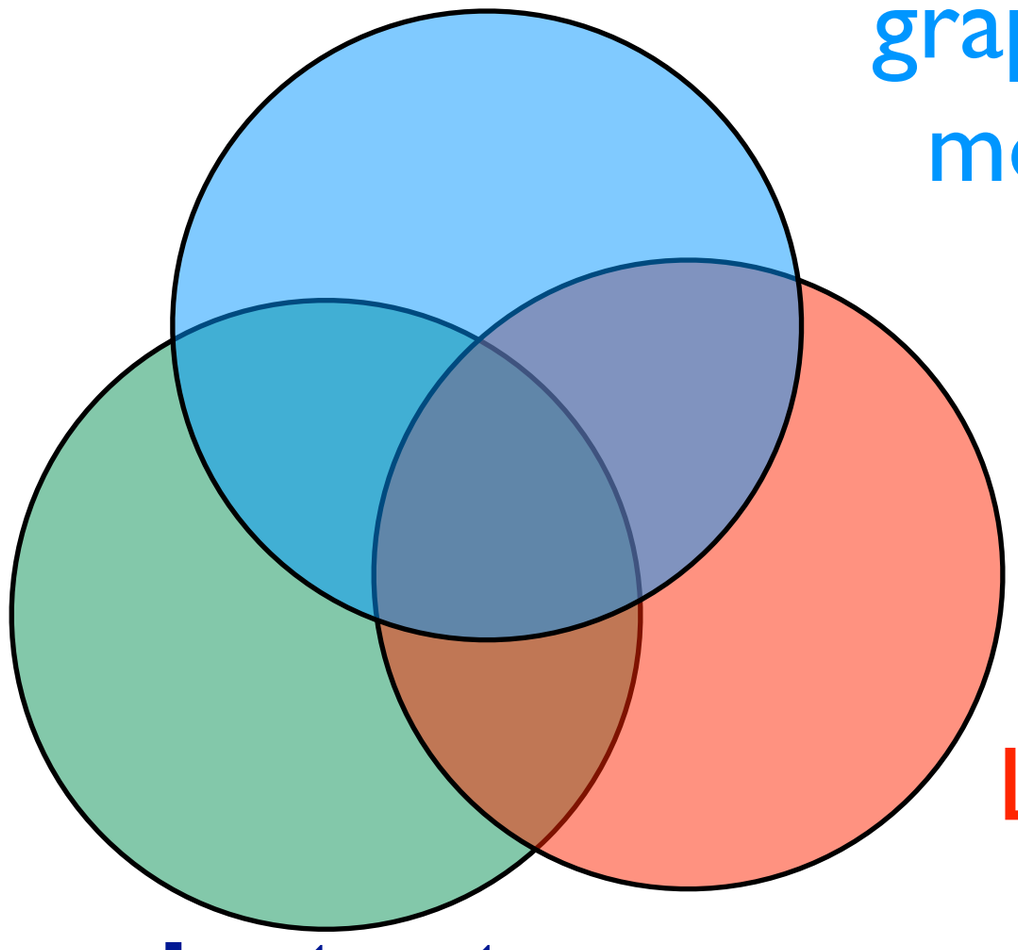


Lifted graphical models

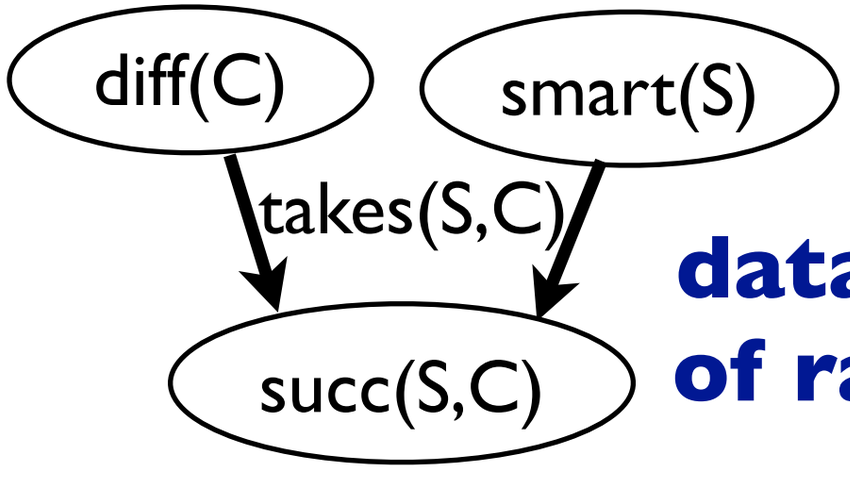
fixed set of random variables



graphical model



relational definition of graphical model

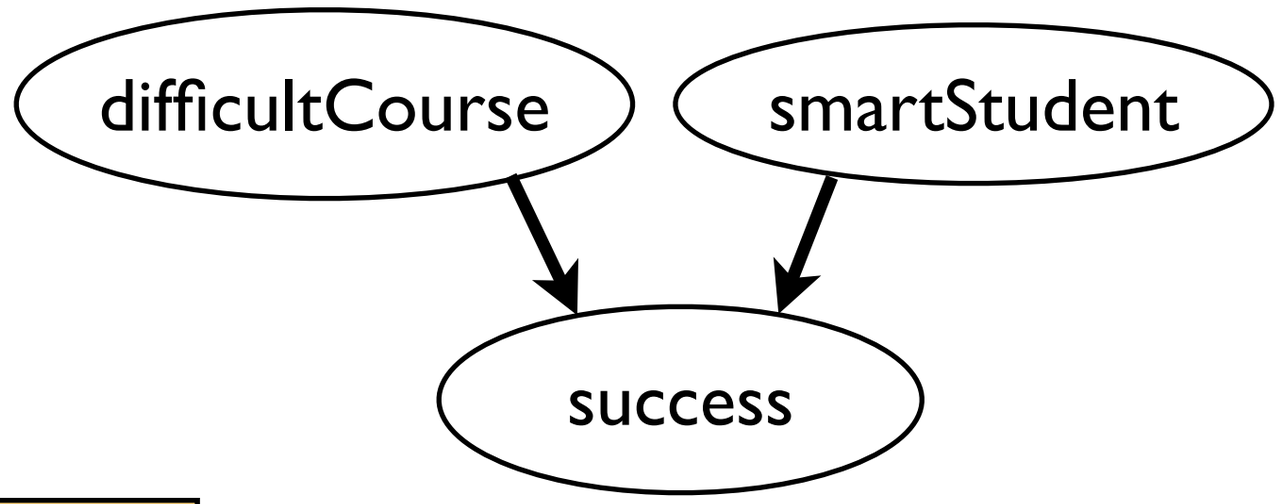


Learning

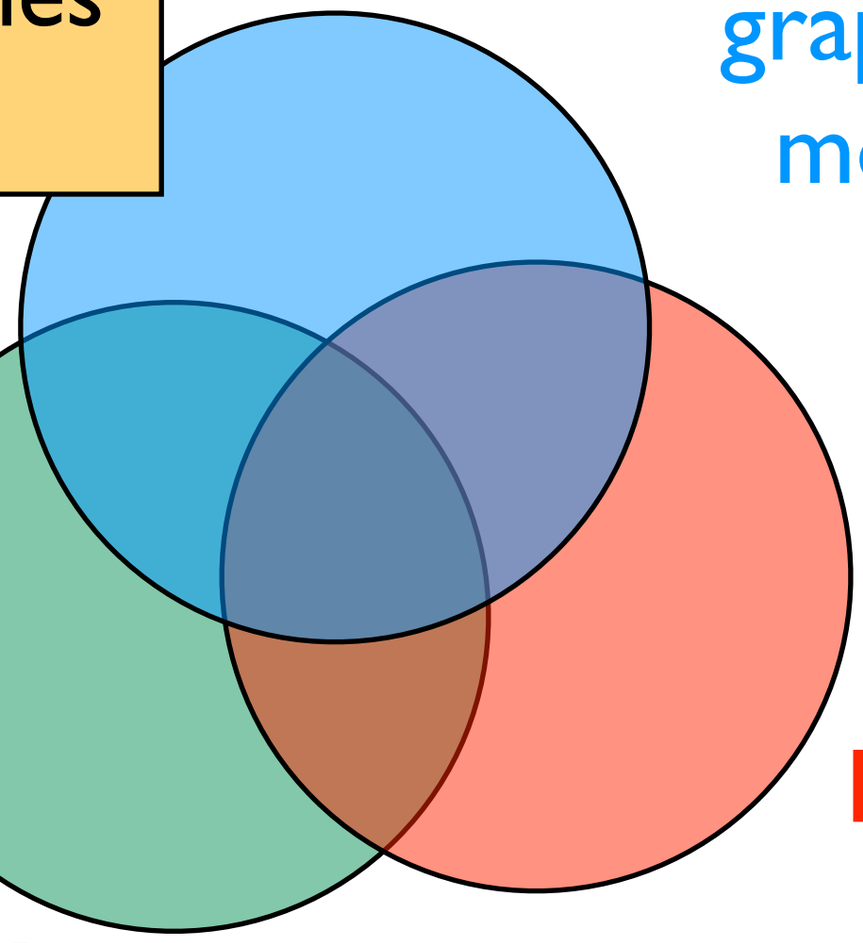
data-dependent set of random variables

Lifted graphical models

fixed set of random variables



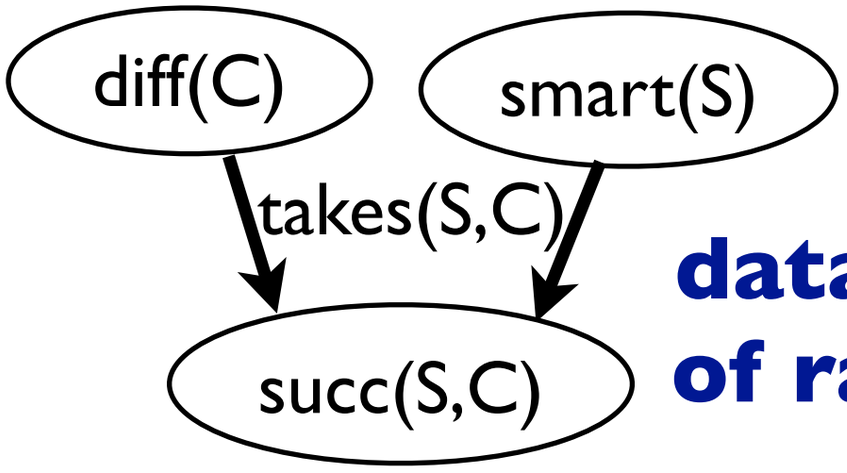
relational language defines graphical model



graphical model

relational definition of graphical model

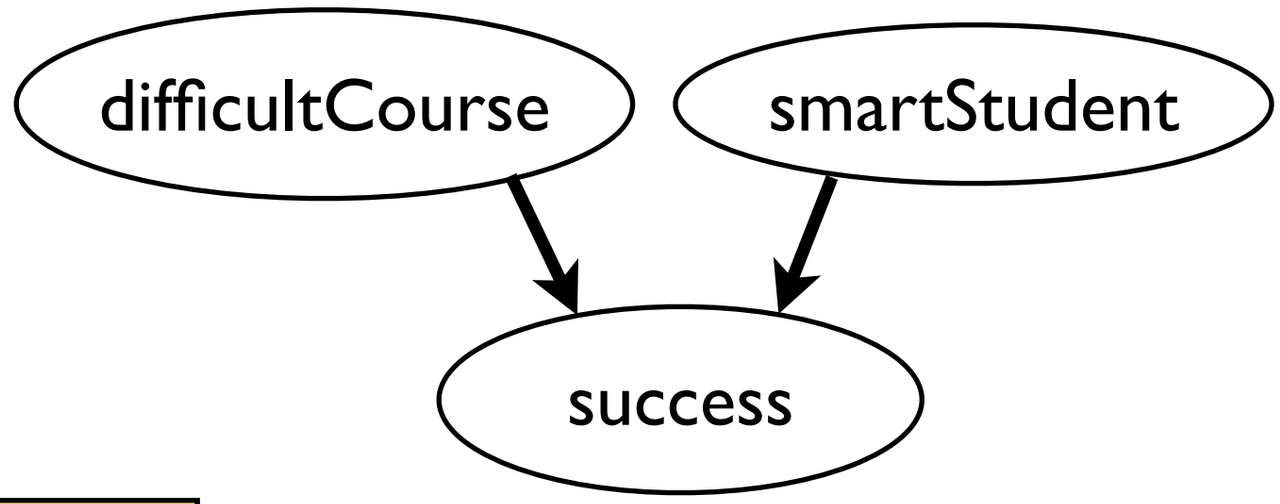
Learning



data-dependent set of random variables

Lifted graphical models

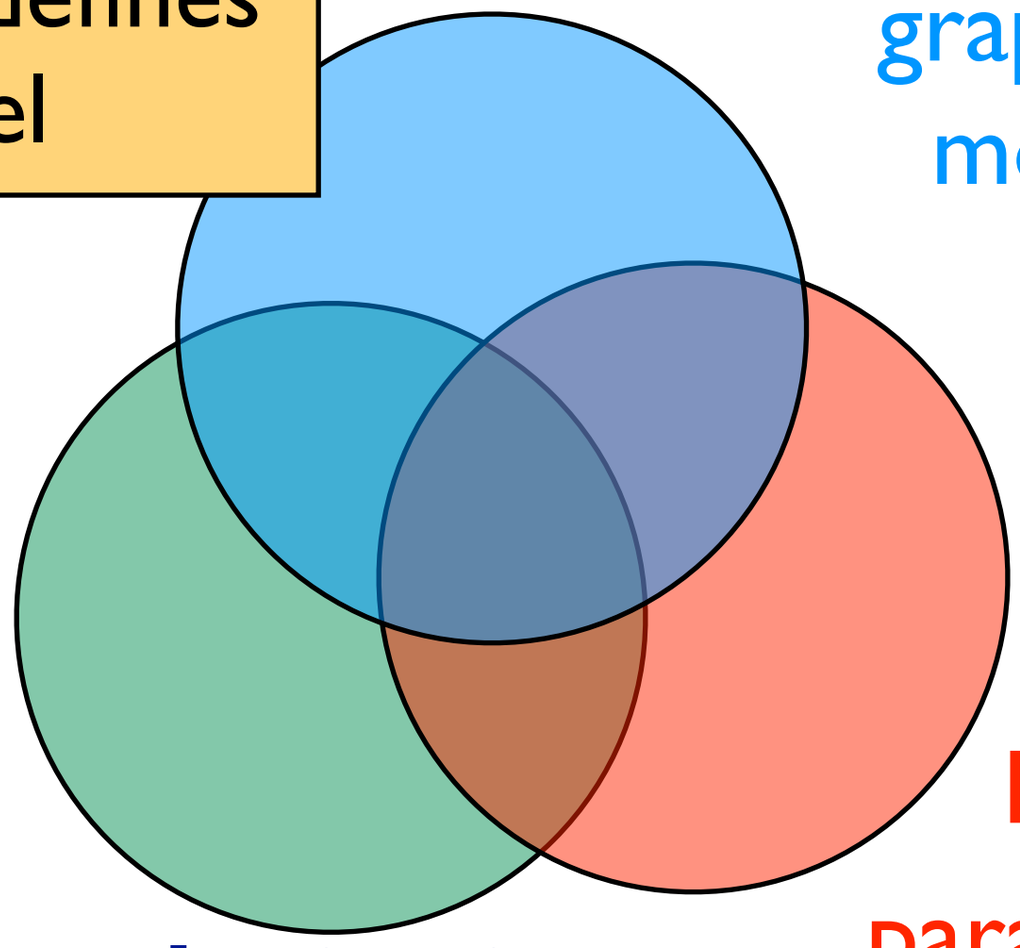
fixed set of random variables



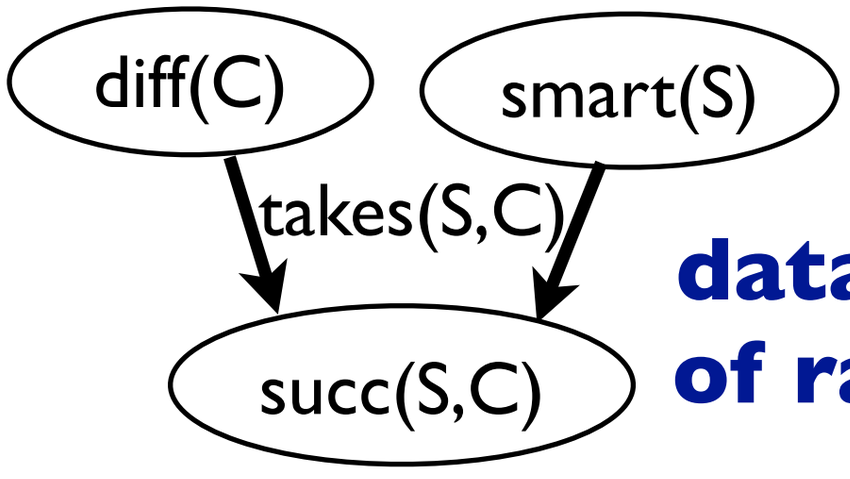
relational language defines graphical model

graphical model

relational definition of graphical model

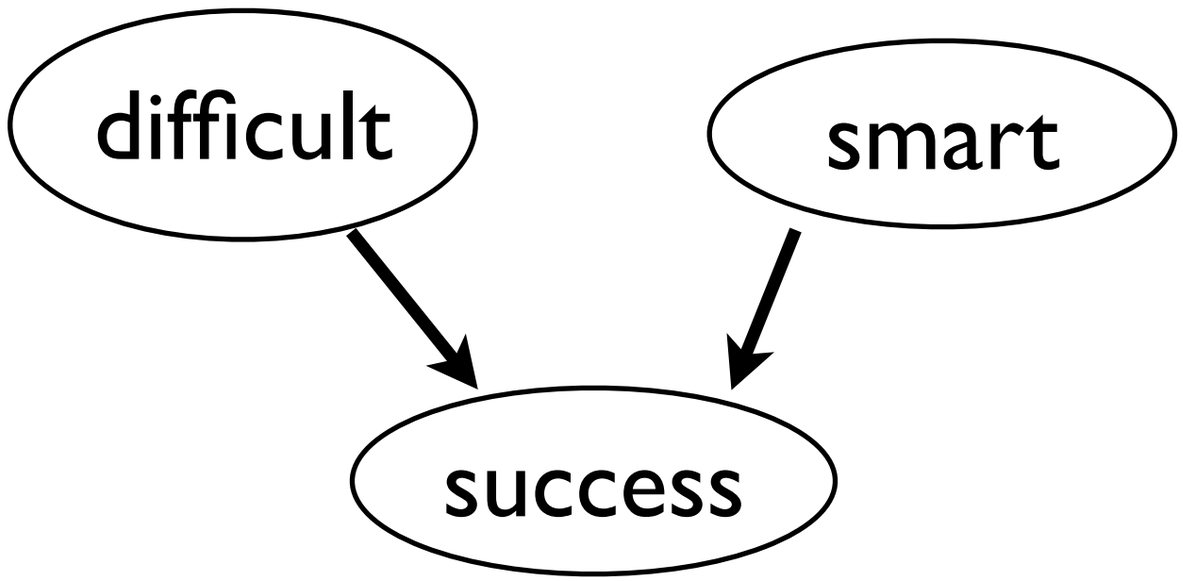


Learning parameters & structure



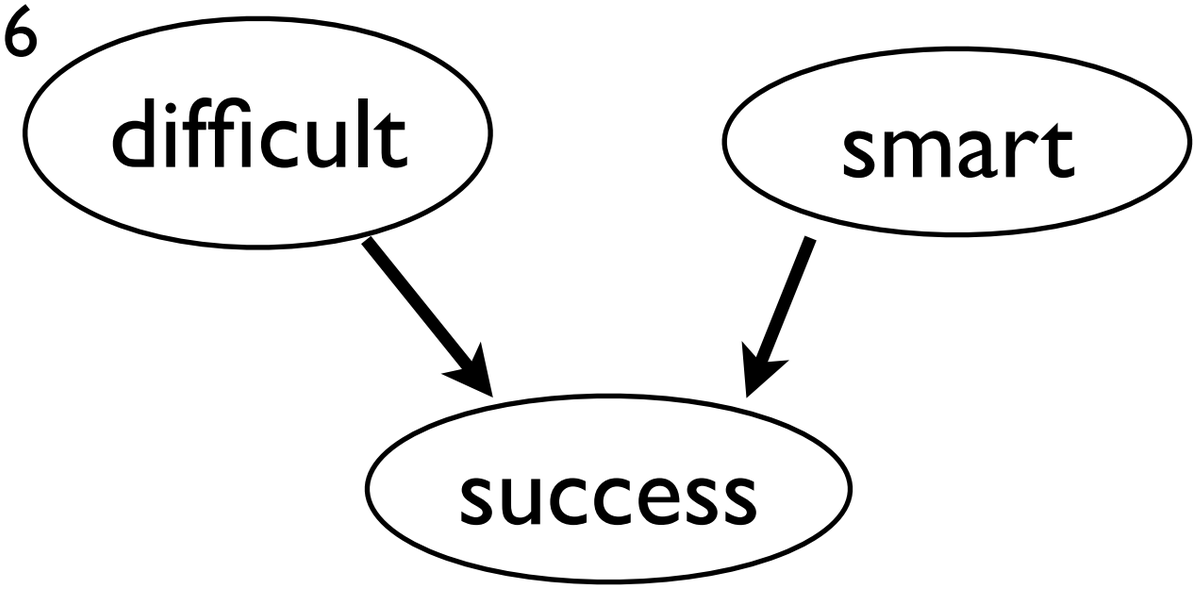
data-dependent set of random variables

Bayesian Network



Bayesian Network

P(difficult=T) = 0.6



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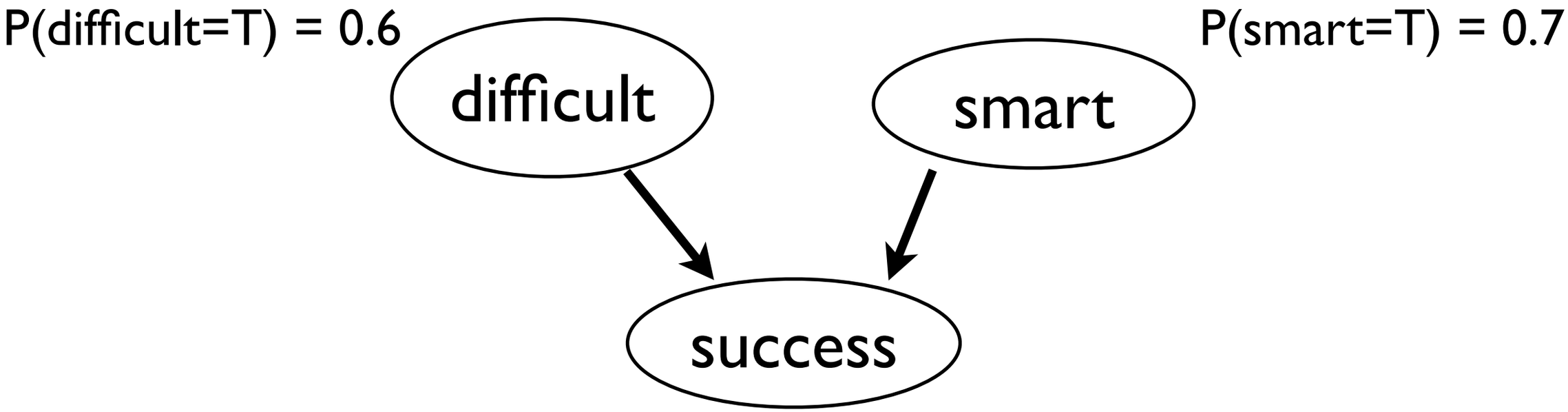
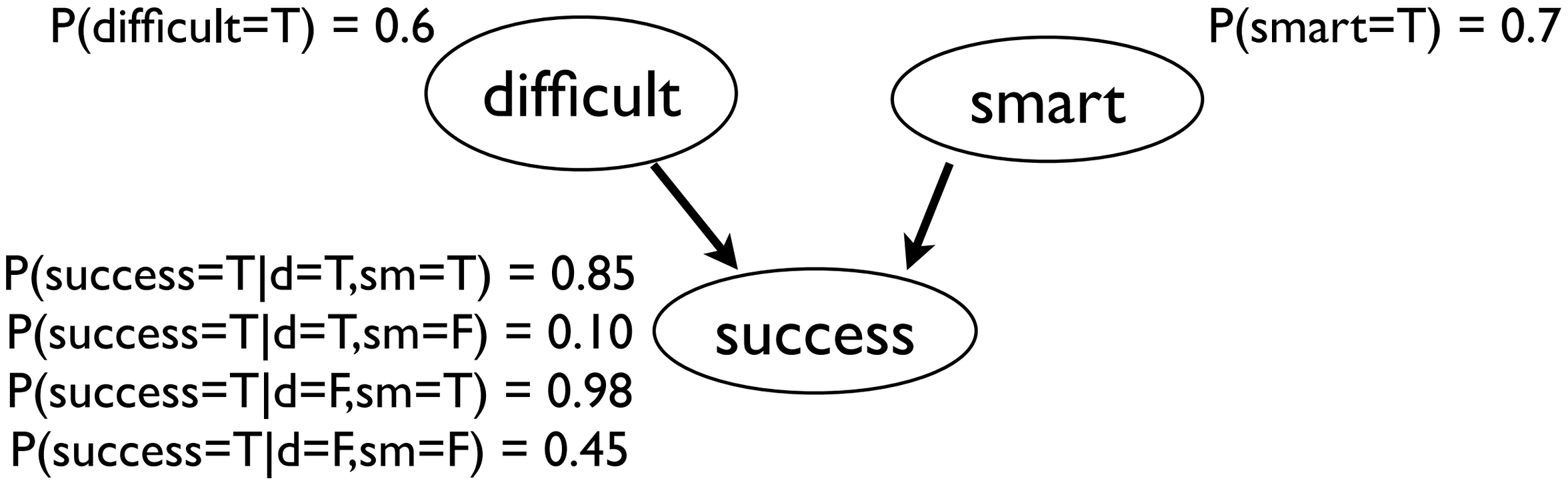


Bayesian Network



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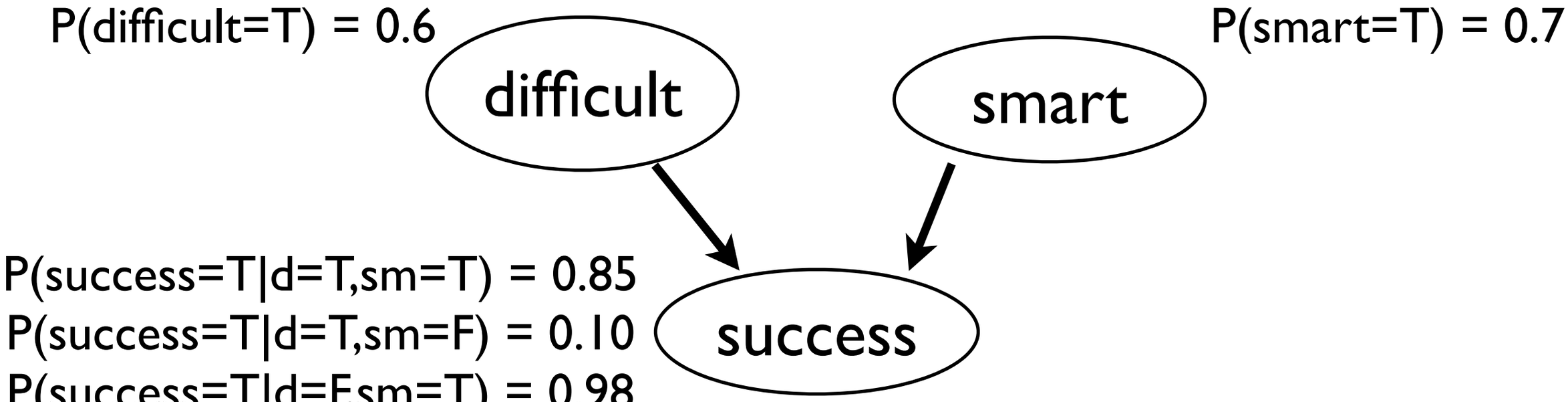
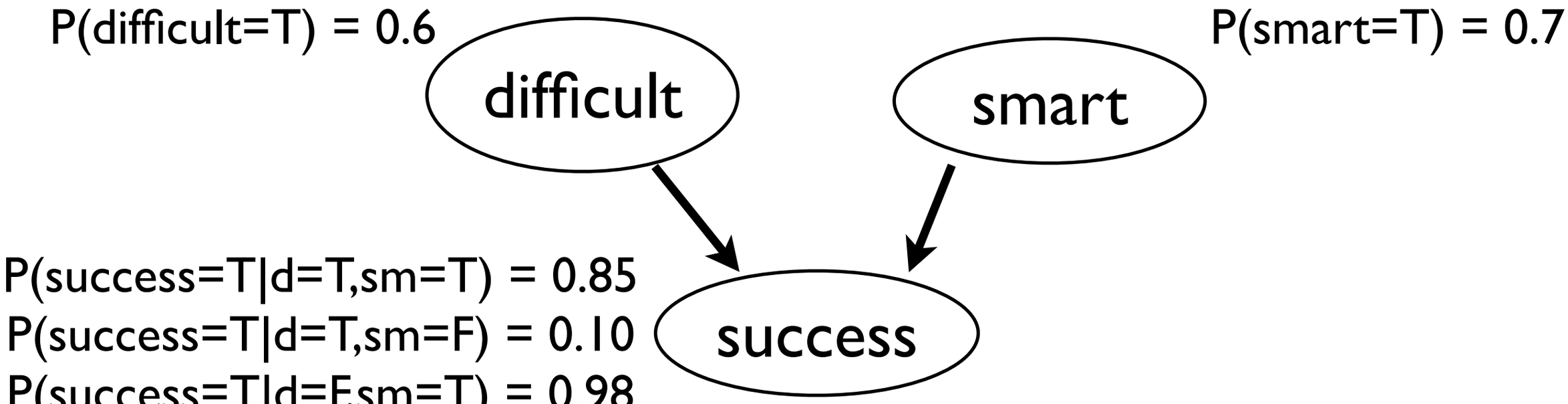


joint distribution

$$P(difficult) \times P(smart) \times P(success|difficult,smart)$$

Bayesian Network



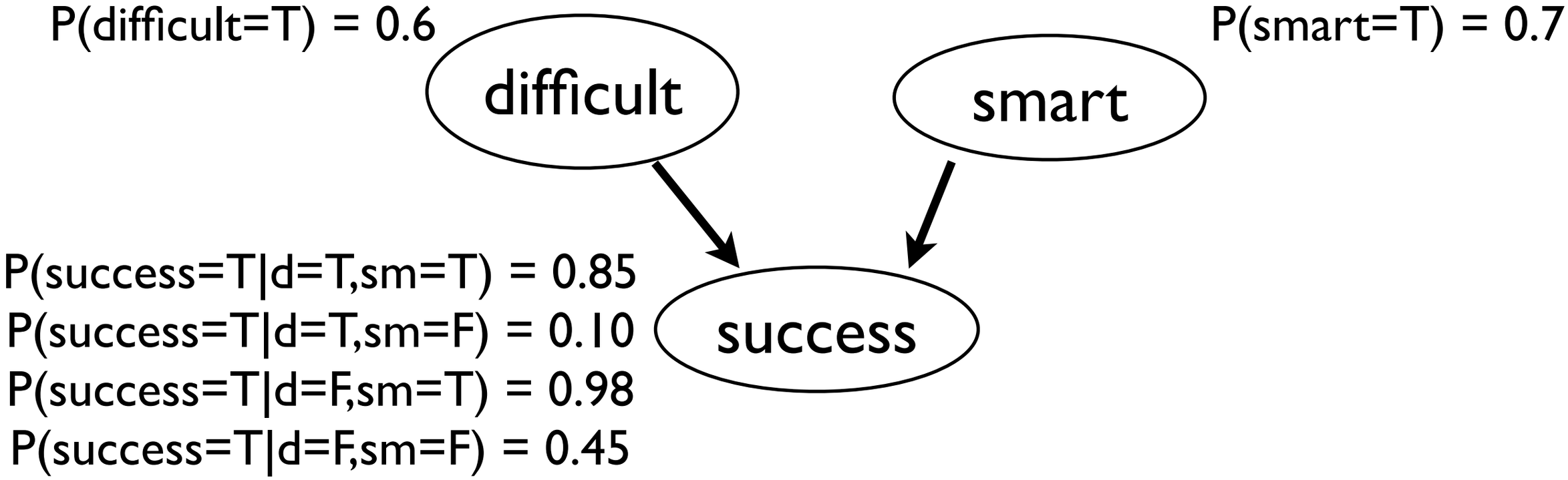
joint distribution

P(difficult)×P(smart)×P(success|difficult,smart)

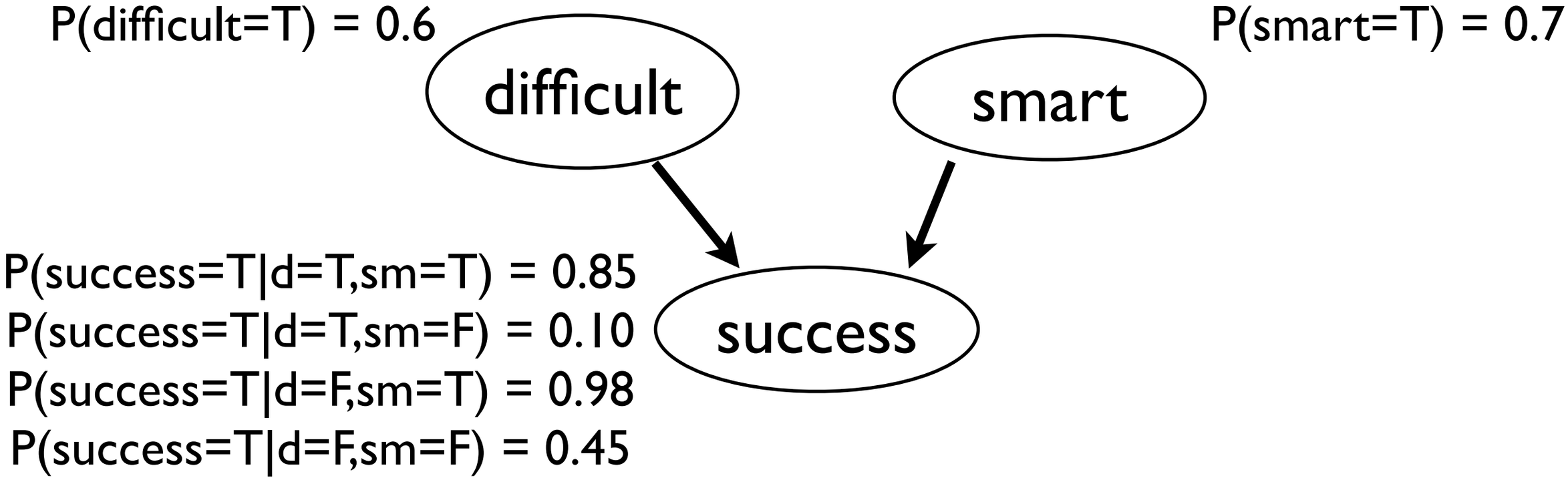
directed acyclic graph

$$P(X_1, \ldots, X_n) = \prod_{i=1,\ldots,n} P(X_i|\text{par}(X_i))$$

Bayesian Network in ProbLog?

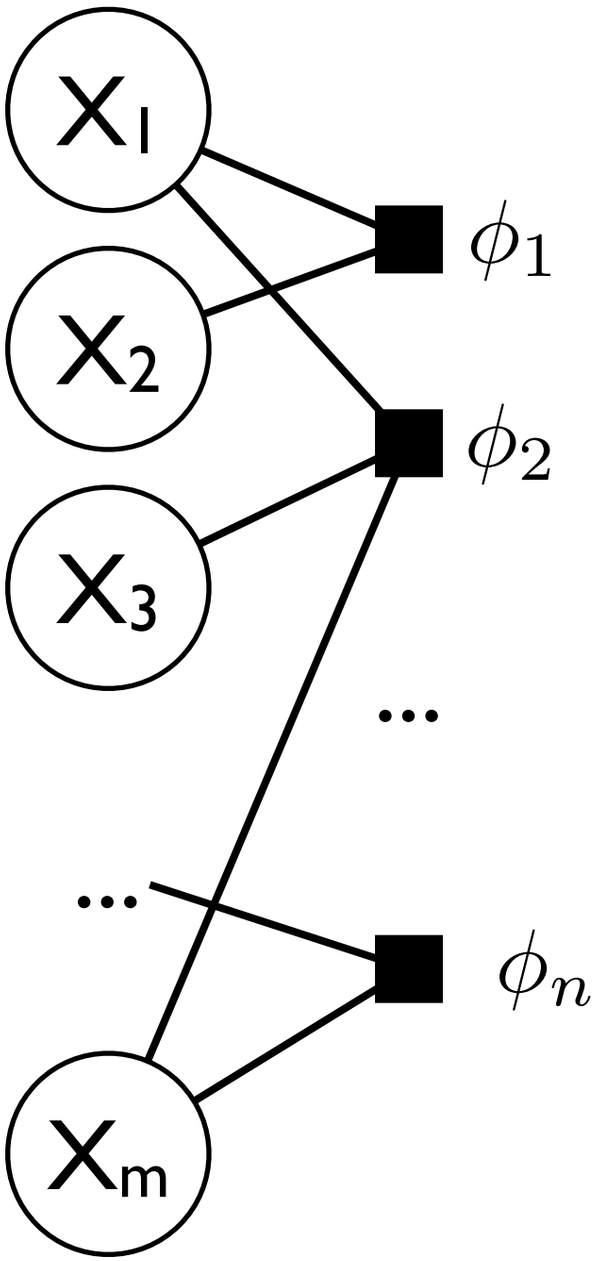


Bayesian Network in ProbLog?




```
0.6::difficult.
0.7::smart.
0.85::success :- difficult, smart.
0.10::success :- difficult, \+smart.
0.98::success :- \+difficult, smart.
0.45::success :- \+difficult, \+smart.
```

Factor Graph

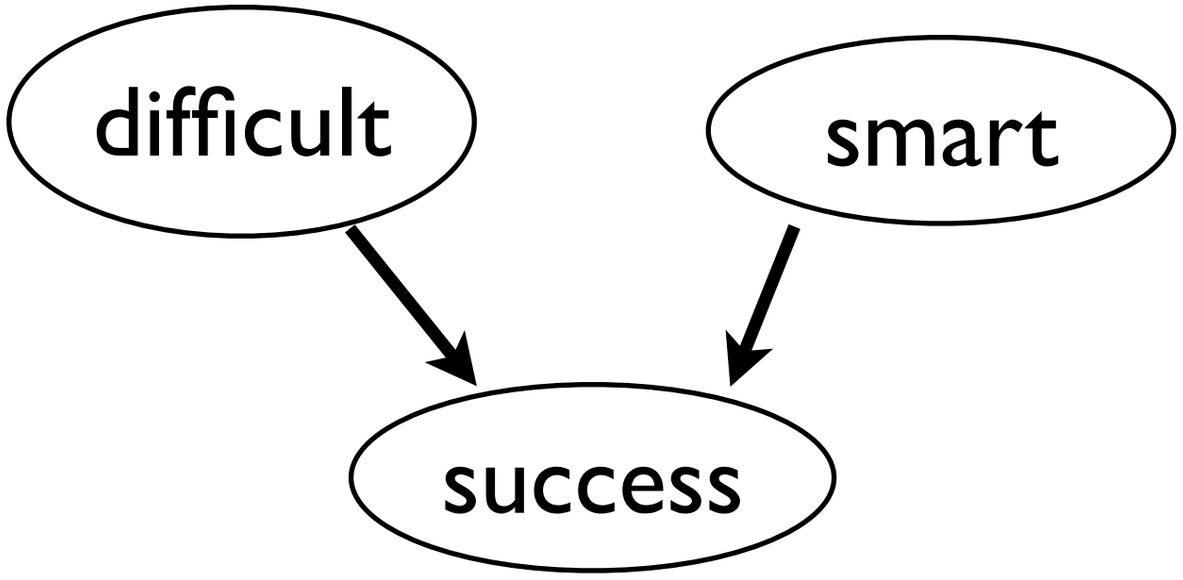


- bi-partite undirected graph
- variables $X_1, \ldots, X_m$
- factors $\phi_1, \ldots, \phi_n$ map interpretations of subsets of variables to non-negative reals
- joint distribution

$$P(\mathbf{X} = \mathbf{x}) = \frac{1}{Z} \prod_{i=1..n} \phi_i(\mathbf{X}_{\phi_i} = \mathbf{x}_{\phi_i})$$

$$Z = \sum_{\mathbf{x}'} \prod_{i=1..n} \phi_i(\mathbf{X}_{\phi_i} = \mathbf{x}'_{\phi_i})$$

BN as Factor Graph?



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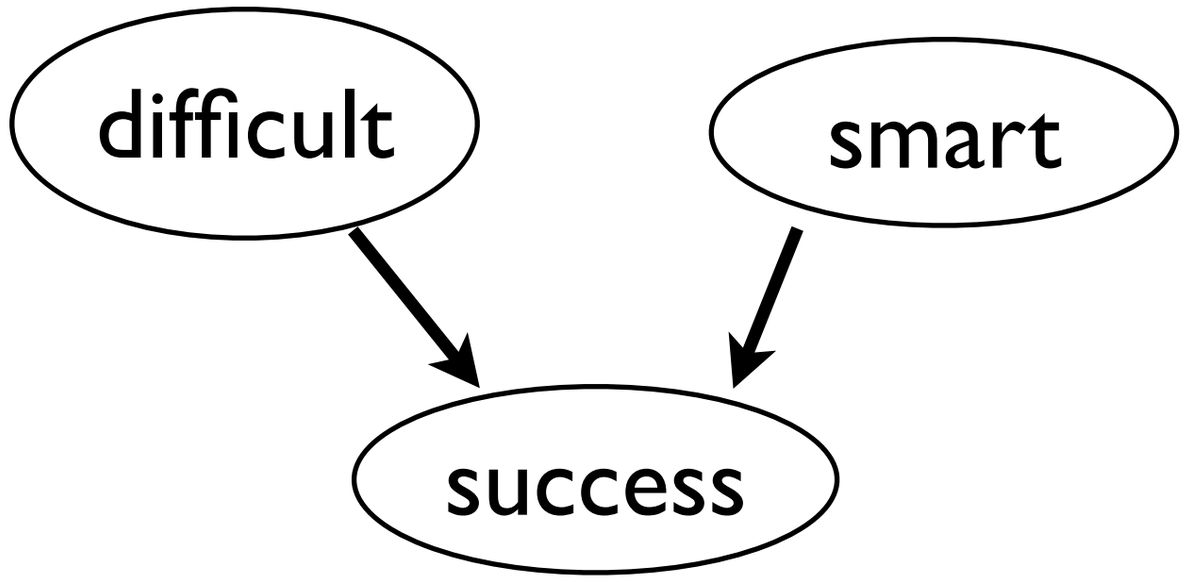
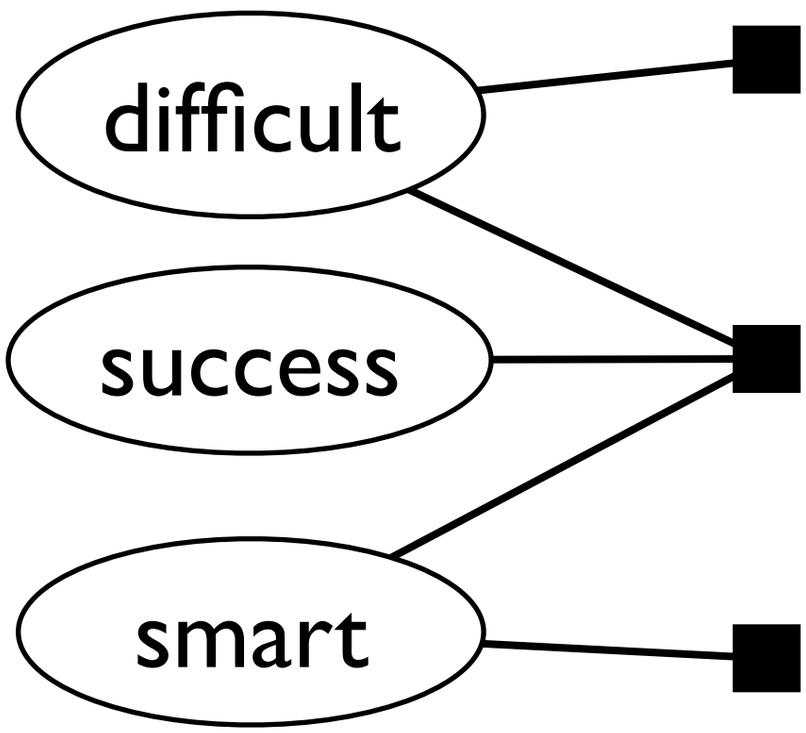




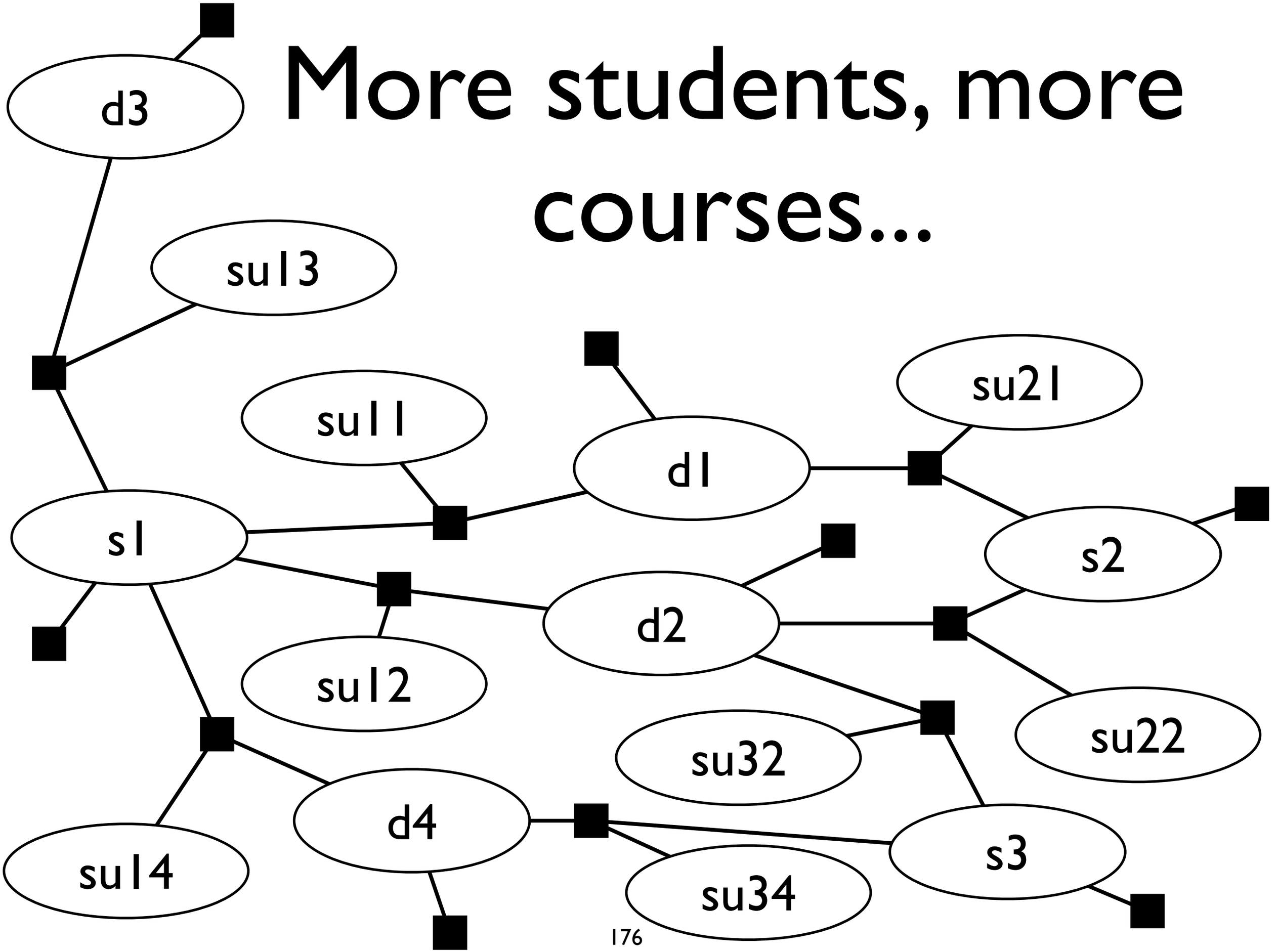
$$\phi_1(d) = P(d)$$

$$\phi_2(su, d, sm) = P(su|d, sm)$$

$$\phi_3(sm) = P(sm)$$

Z=1

More students, more courses...



More students, more

gets cumbersome quickly....
relational language to write template?

s(P)

su(P,C)

d(C)

d3

su13

s1

su12

su32

su22

d4

su14

su34

s3

Lots of proposals in the literature, e.g.

- relational Markov networks (RMNs) [Taskar et al 2002]
- Markov logic networks (MLNs) [Richardson & Domingos 2006]
- probabilistic soft logic (PSL) [Broecheler et al 2010]
- FACTORIE [McCallum et al 2009]
- Bayesian logic programs (BLPs) [Kersting & De Raedt 2001]
- relational Bayesian networks (RBNs) [Jaeger 2002]
- logical Bayesian networks (LBNs) [Fierens et al 2005]
- probabilistic relational models (PRMs) [Koller & Pfeffer 1998]
- Bayesian logic (BLOG) [Milch et al 2005]
- CLP(BN) [Santos Costa et al 2008]
- probabilistic programming languages such as ProbLog, PRISM, Church, ...
- and many more ...

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- Bay
- rela
- logi
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- and many more ...

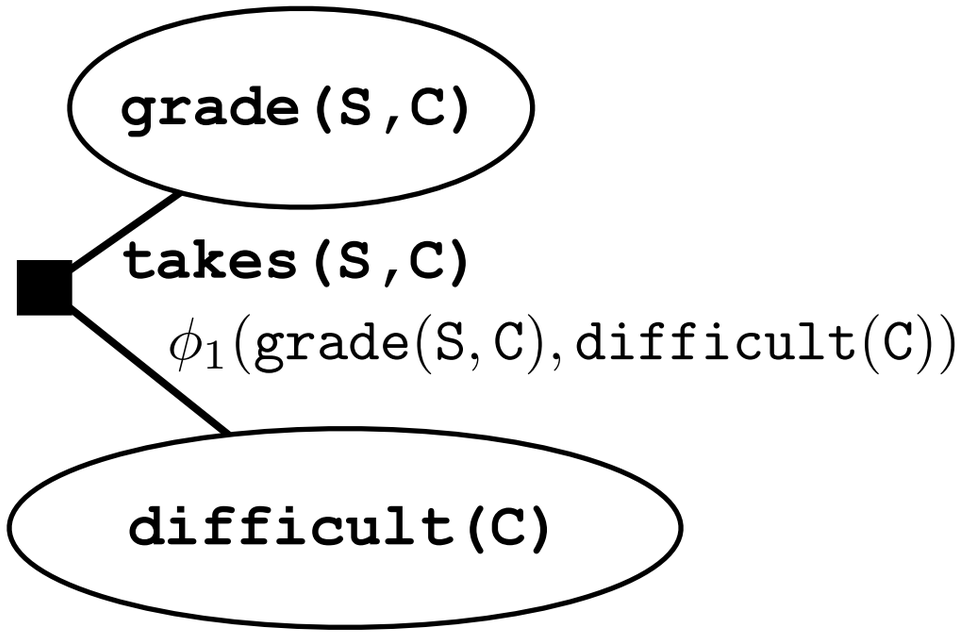
common principle:
parameterized factor graph
(par-factor graph)

Par-Factor Graph

[Poole 03]

Par-Factor Graph

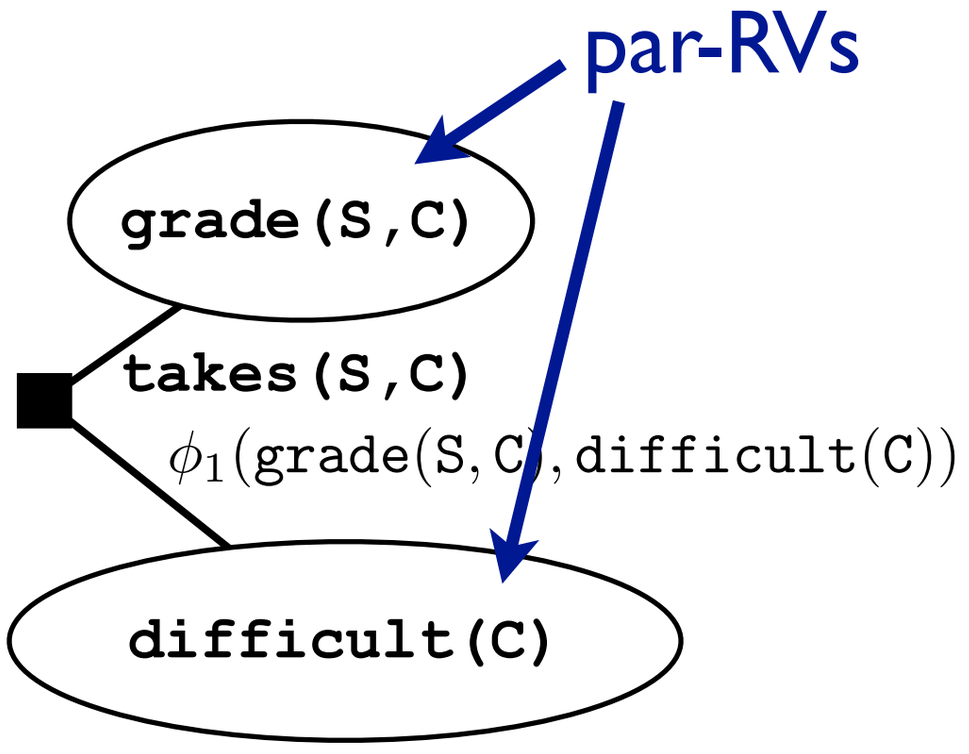
[Poole 03]



parameterized
factor (par-factor)

Par-Factor Graph

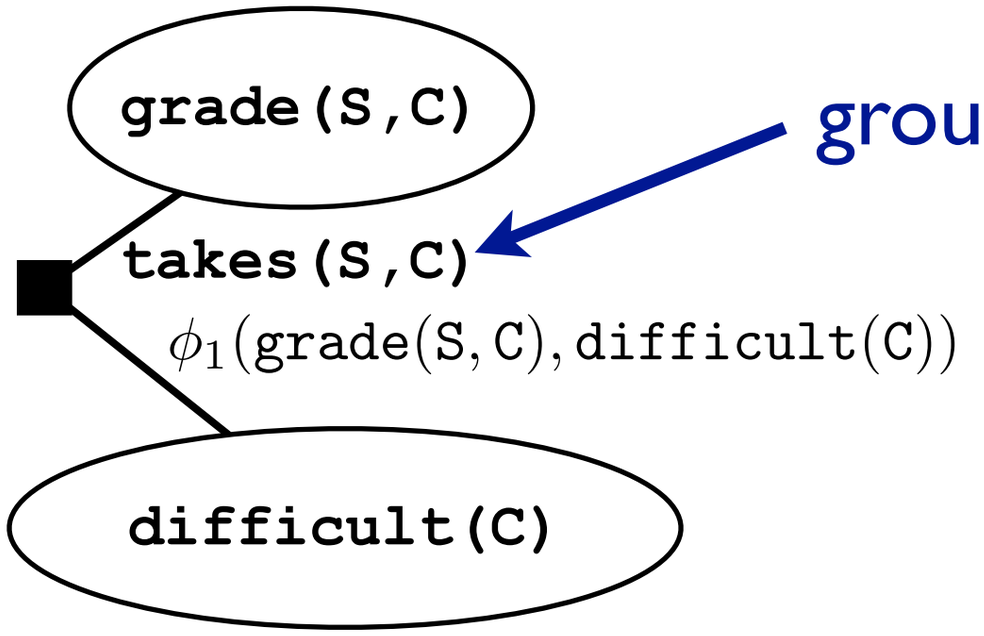
[Poole 03]



parameterized
factor (par-factor)

Par-Factor Graph

[Poole 03]



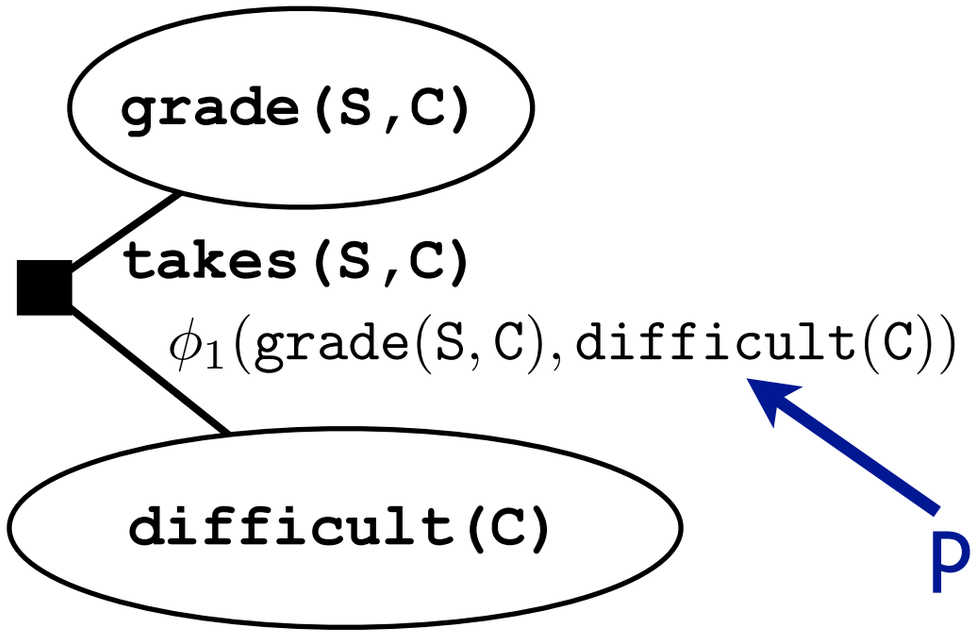
grounding constraint

parameterized

factor (par-factor)

Par-Factor Graph

[Poole 03]

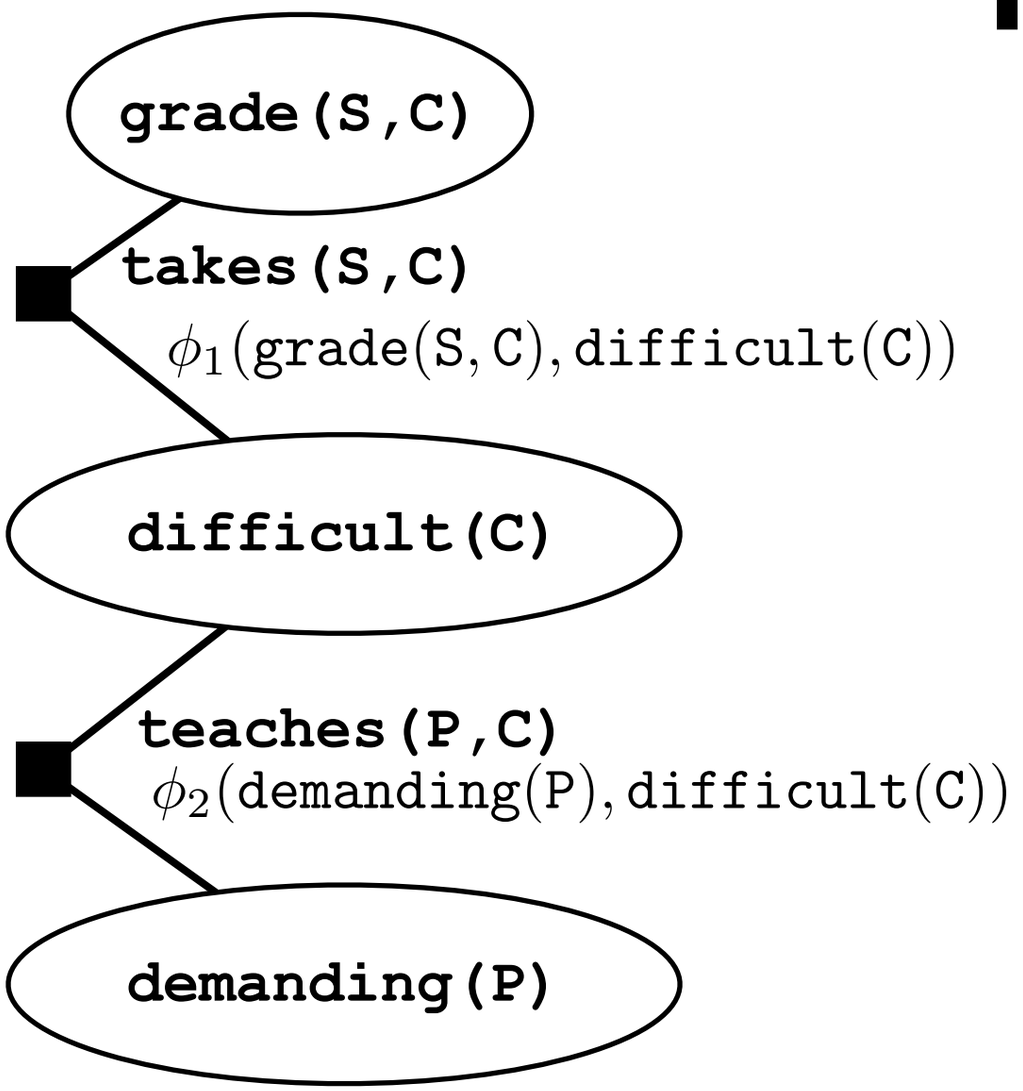


parameterized
factor (par-factor)

potential function

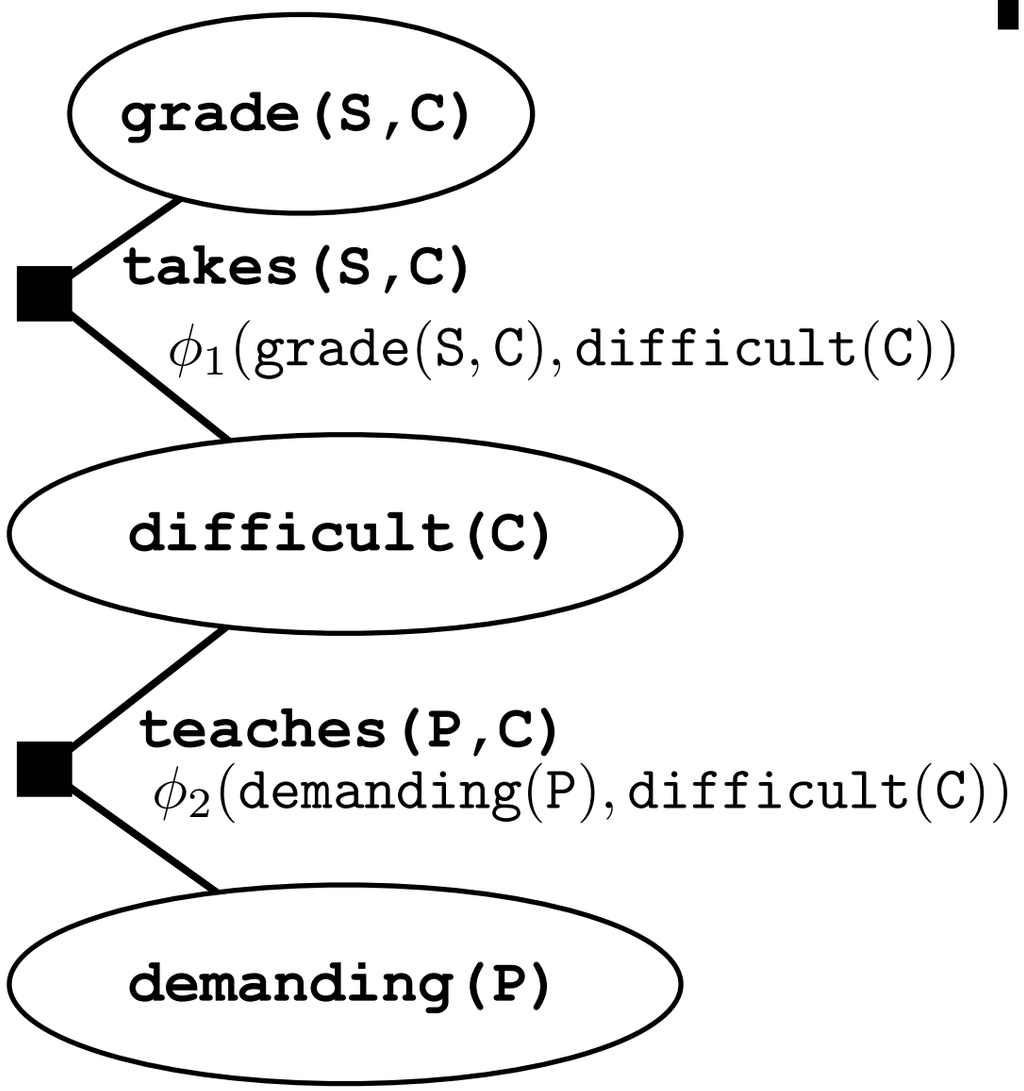
Par-Factor Graph

[Poole 03]



Par-Factor Graph

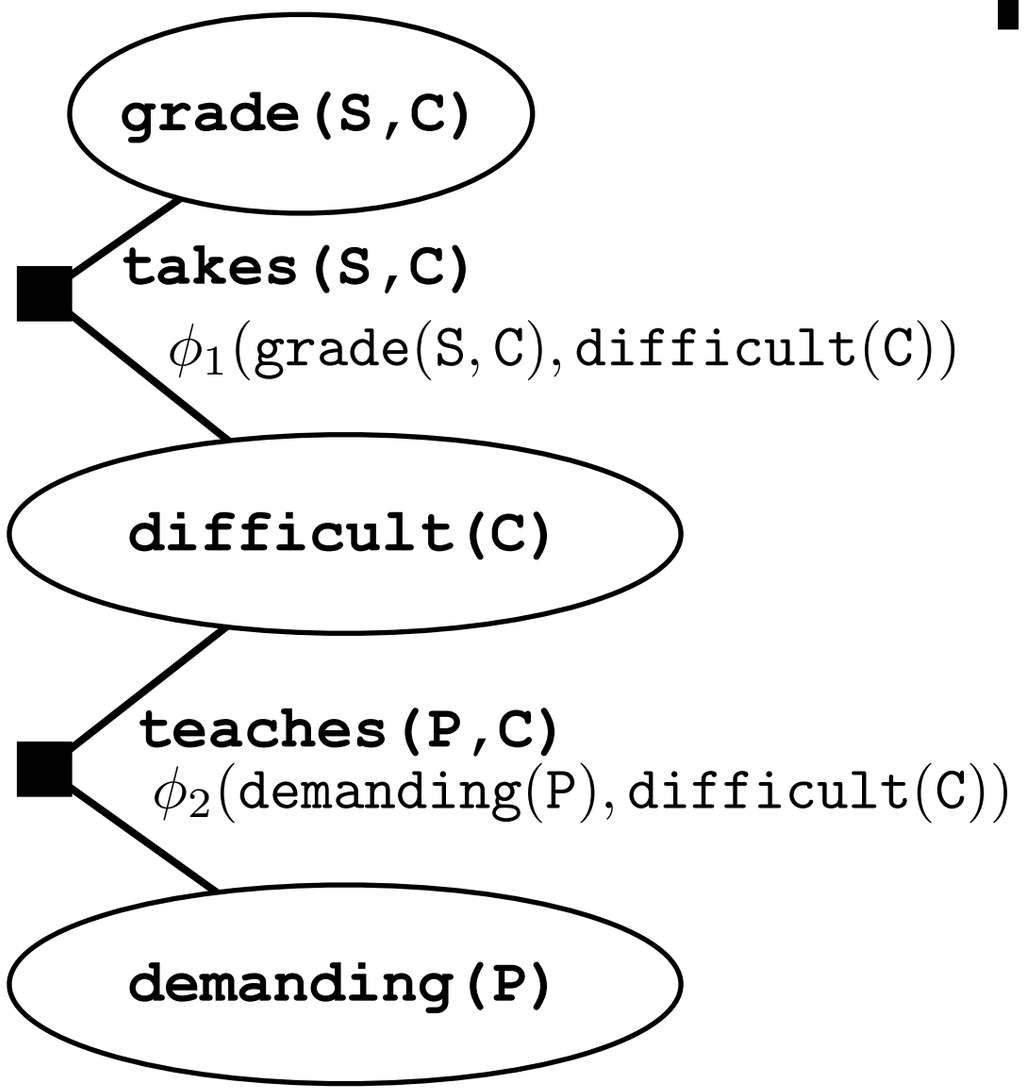
[Poole 03]



$$P(.) = \frac{1}{Z} \cdot \prod_{\mathtt{takes(s,c)}} \phi_1(\mathtt{grade(s,c)},\mathtt{difficult(c)}) \cdot \prod_{\mathtt{teaches(p,c)}} \phi_2(\mathtt{demanding(p)},\mathtt{difficult(c)})$$

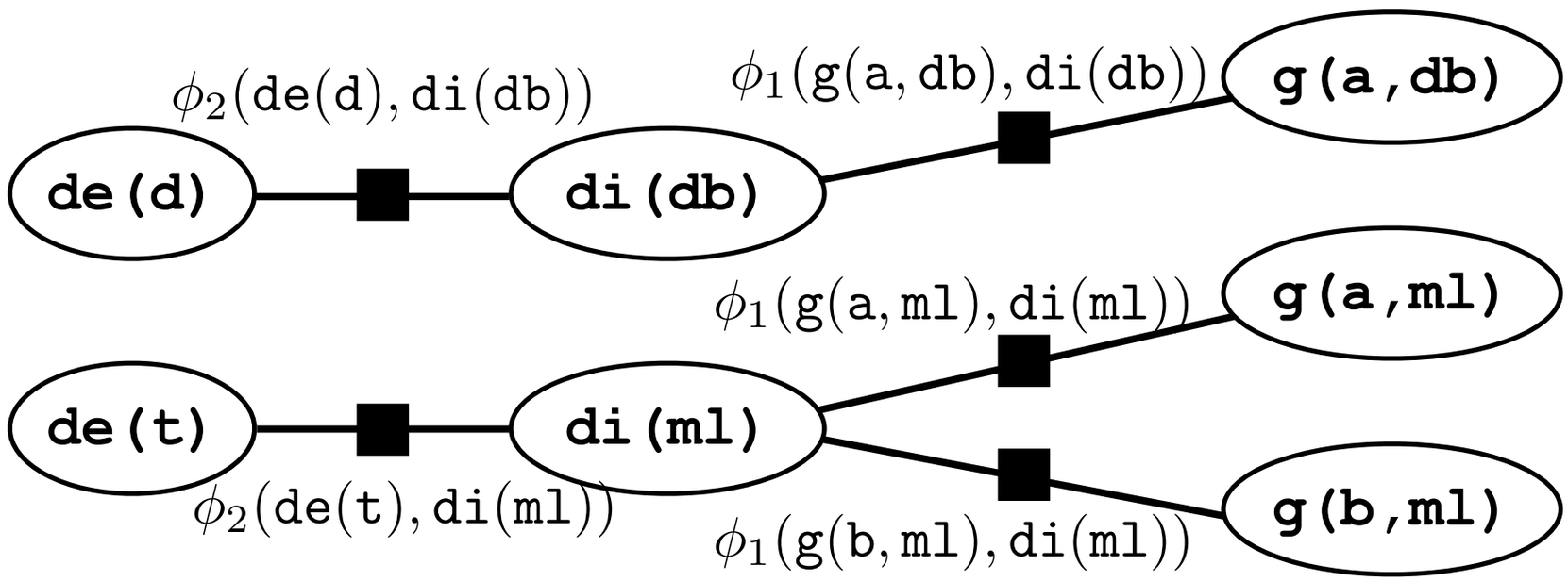
Par-Factor Graph

[Poole 03]

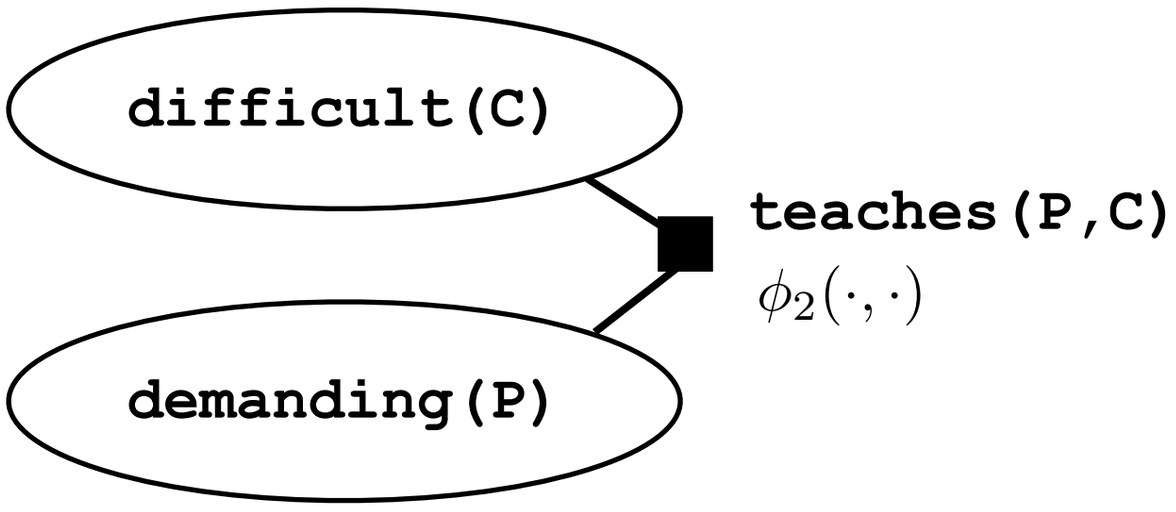


$$P(.) = \frac{1}{Z} \cdot \prod_{\mathtt{takes(s,c)}} \phi_1(\mathtt{grade}(\mathtt{s},\mathtt{c}),\mathtt{difficult}(\mathtt{c})) \cdot \prod_{\mathtt{teaches(p,c)}} \phi_2(\mathtt{demanding}(\mathtt{p}),\mathtt{difficult}(\mathtt{c}))$$

```
takes(ann,ml).
takes(bob,ml).
takes(ann,db).
teaches(dan,db).
teaches(tom,ml).
```

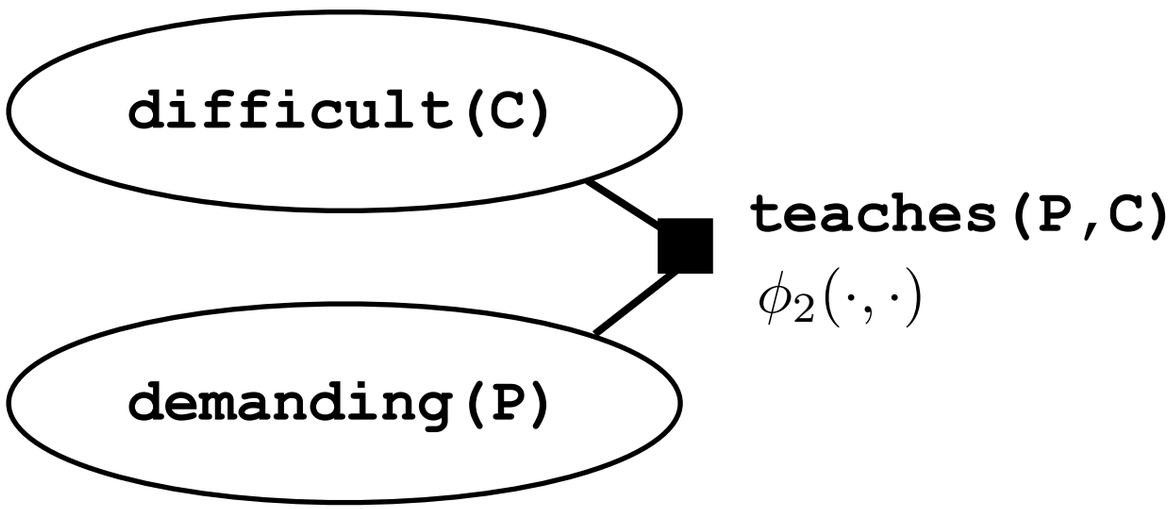


What if multiple professors teach the same course?




```
teaches(eve,db)
teaches(dan,db).
teaches(tom,ml).
teaches(bob,ai).
teaches(ann,ai).
teaches(ed,ai).
```

What if multiple professors teach the same course?

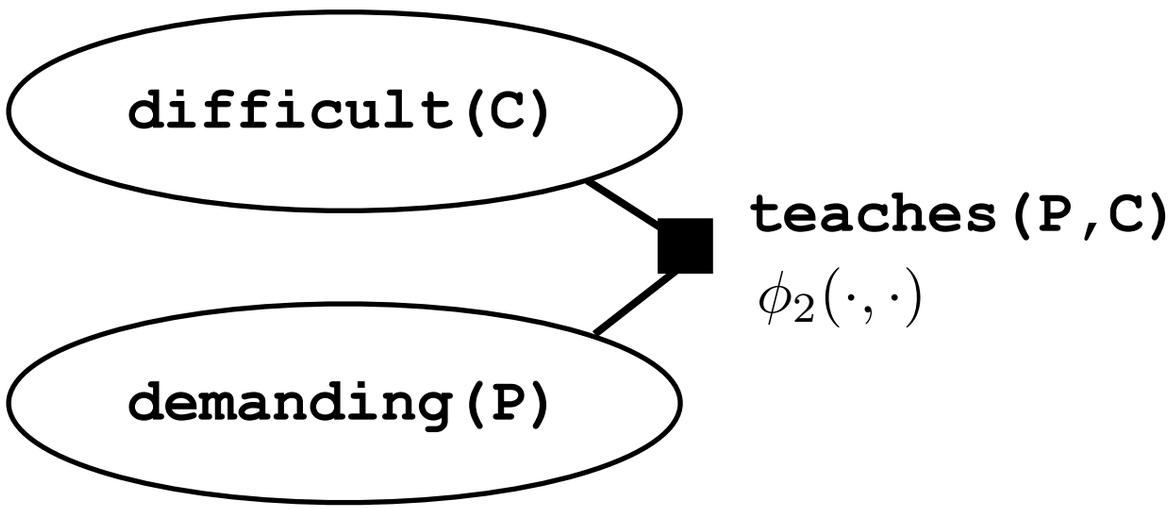



```
teaches(eve,db)
teaches(dan,db).
teaches(tom,ml).
teaches(bob,ai).
teaches(ann,ai).
teaches(ed,ai).
```

Option 1: aggregate values of RVs, then compute potential

$$\phi_2(\texttt{demanding(P)}, \texttt{difficult(C)}) = P(\texttt{difficult(C)}|\text{mean}\{\texttt{demanding(P)}\})$$

What if multiple professors teach the same course?




```
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teaches(dan,db).
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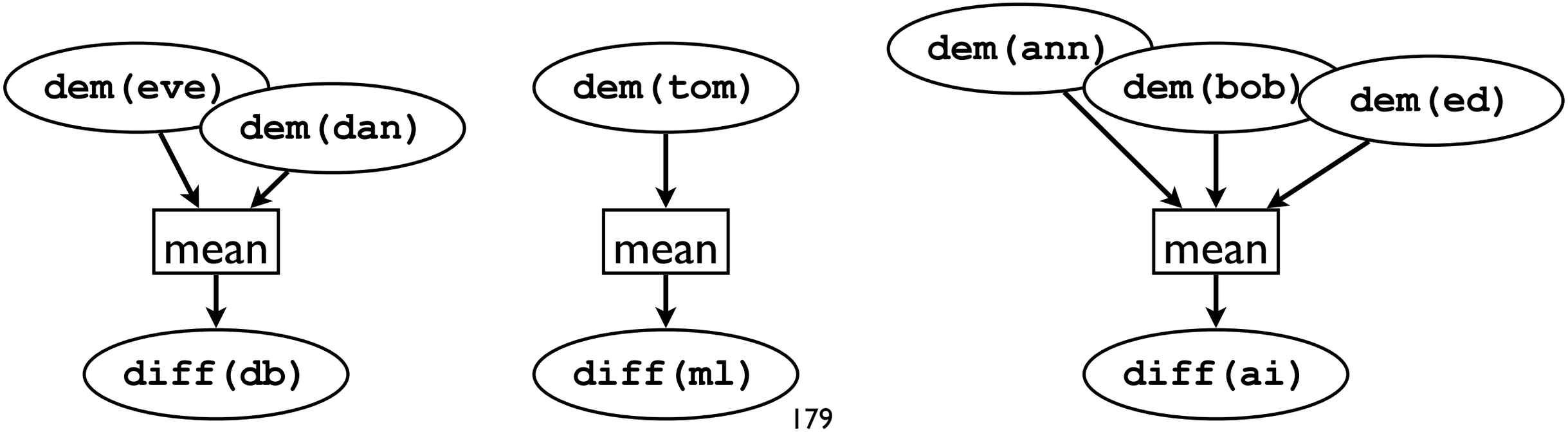
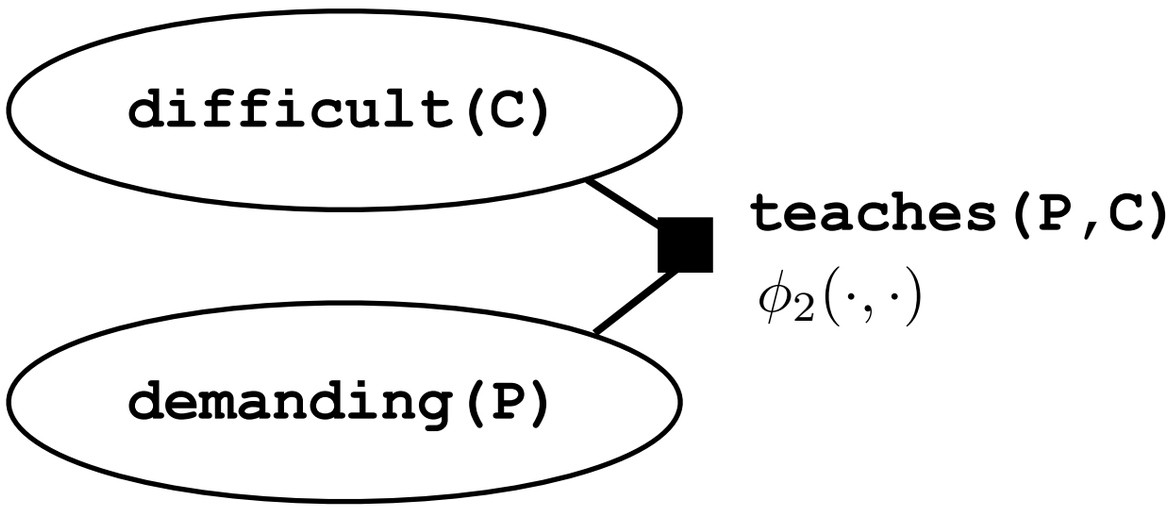
teaches(ed,ai).
```

Option 1: aggregate values of RVs, then compute potential

$$\phi_2(\texttt{demanding(P)}, \texttt{difficult(C)}) = P(\texttt{difficult(C)}|\text{mean}\{\texttt{demanding(P)}\})$$



What if multiple professors teach the same course?

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teaches(bob,ai).
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Option 2: compute potential for each RV, then combine

$$\phi_2(\texttt{demanding(P)}, \texttt{difficult(C)}) = 1 - \prod_{\texttt{P}} (1 - P(\texttt{difficult(C)} | \texttt{demanding(P)}))$$

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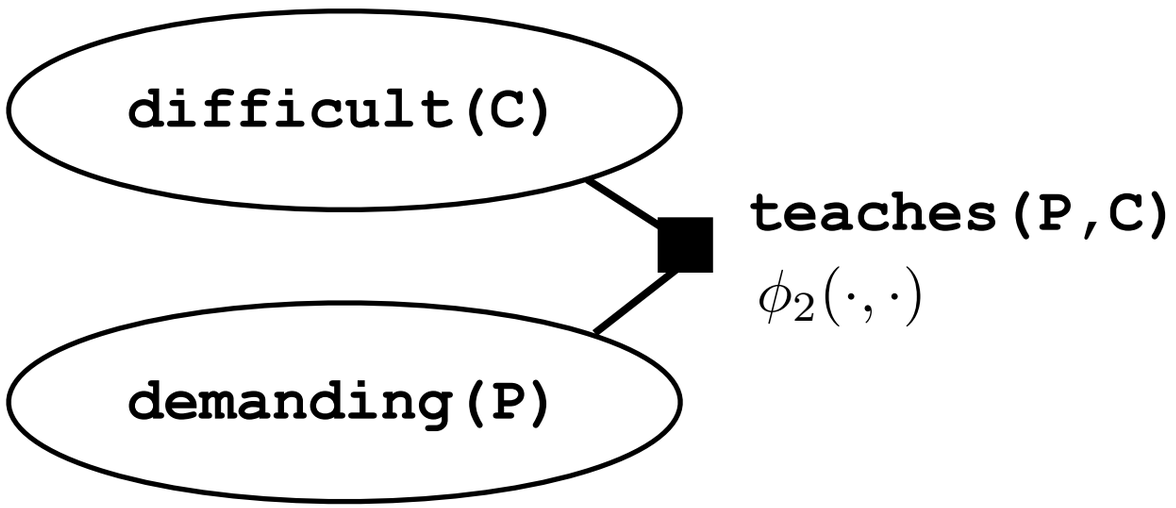


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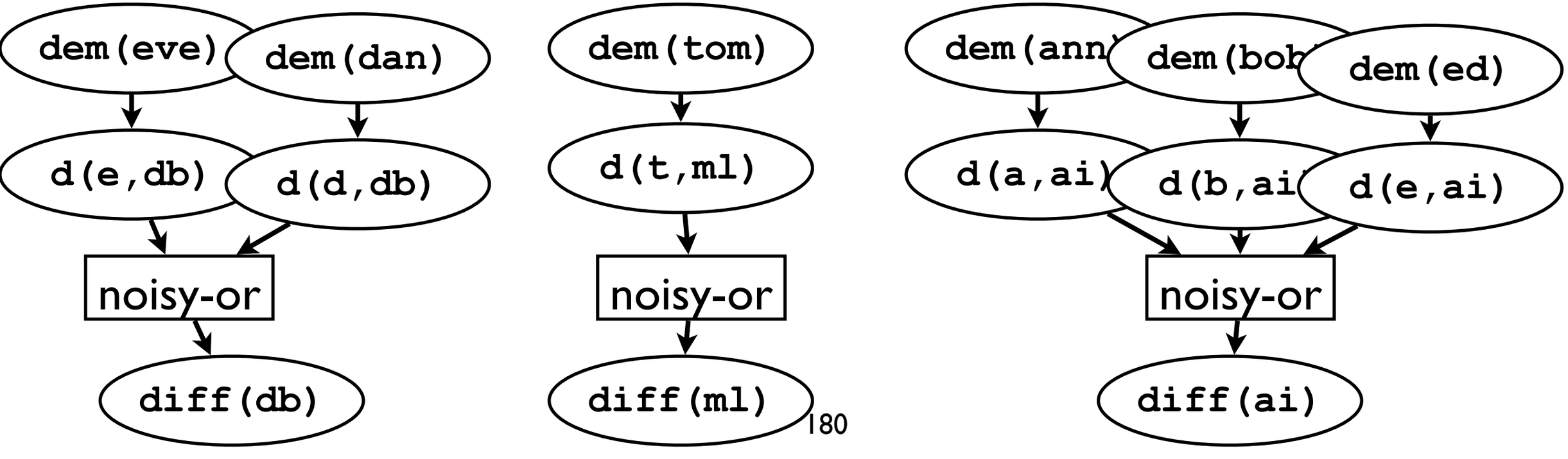


Example: CLP(BN)

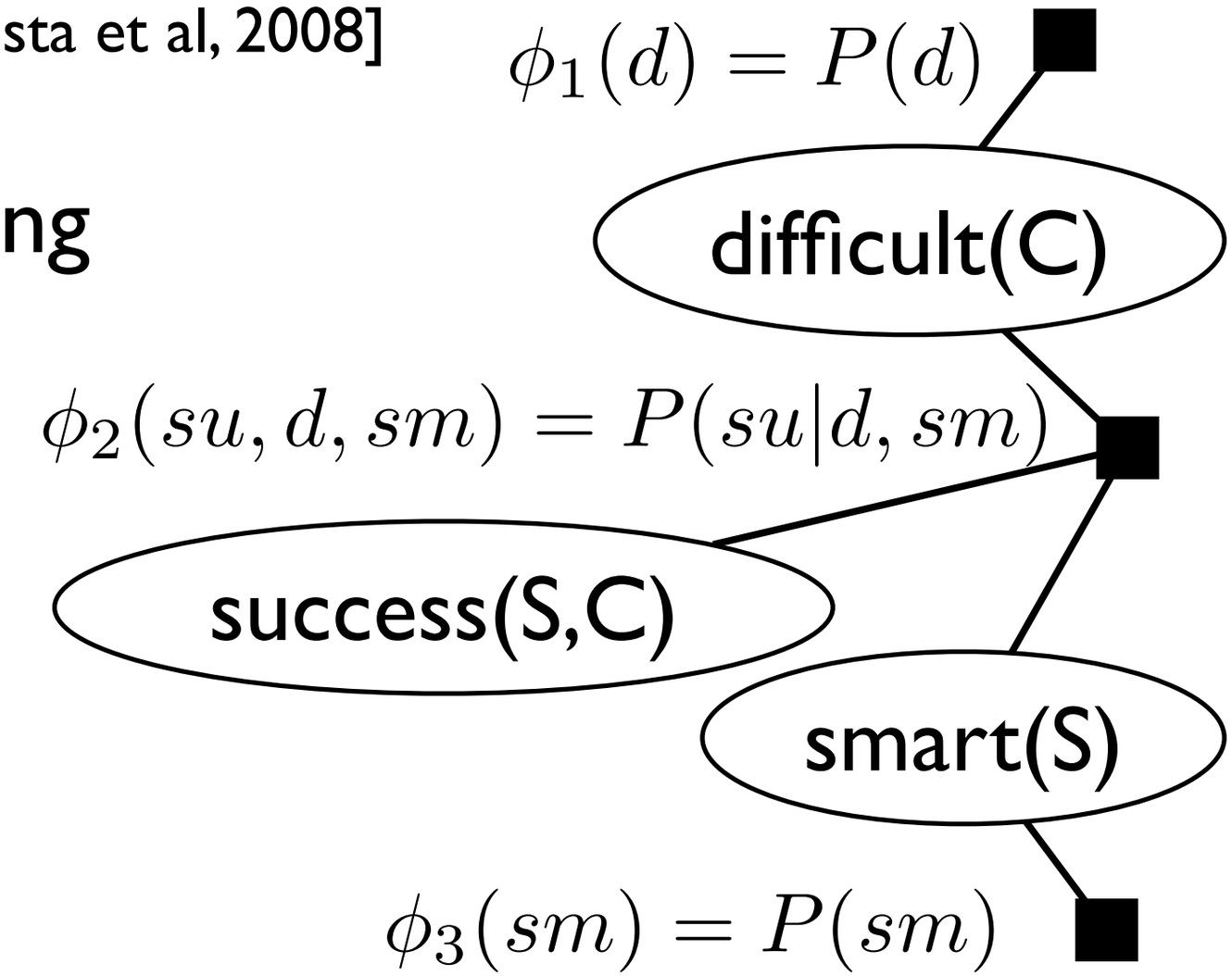
CLP(BN) [Santos Costa et al, 2008]

constraint logic programming
for Bayesian networks

CLP(BN)

[Santos Costa et al, 2008]

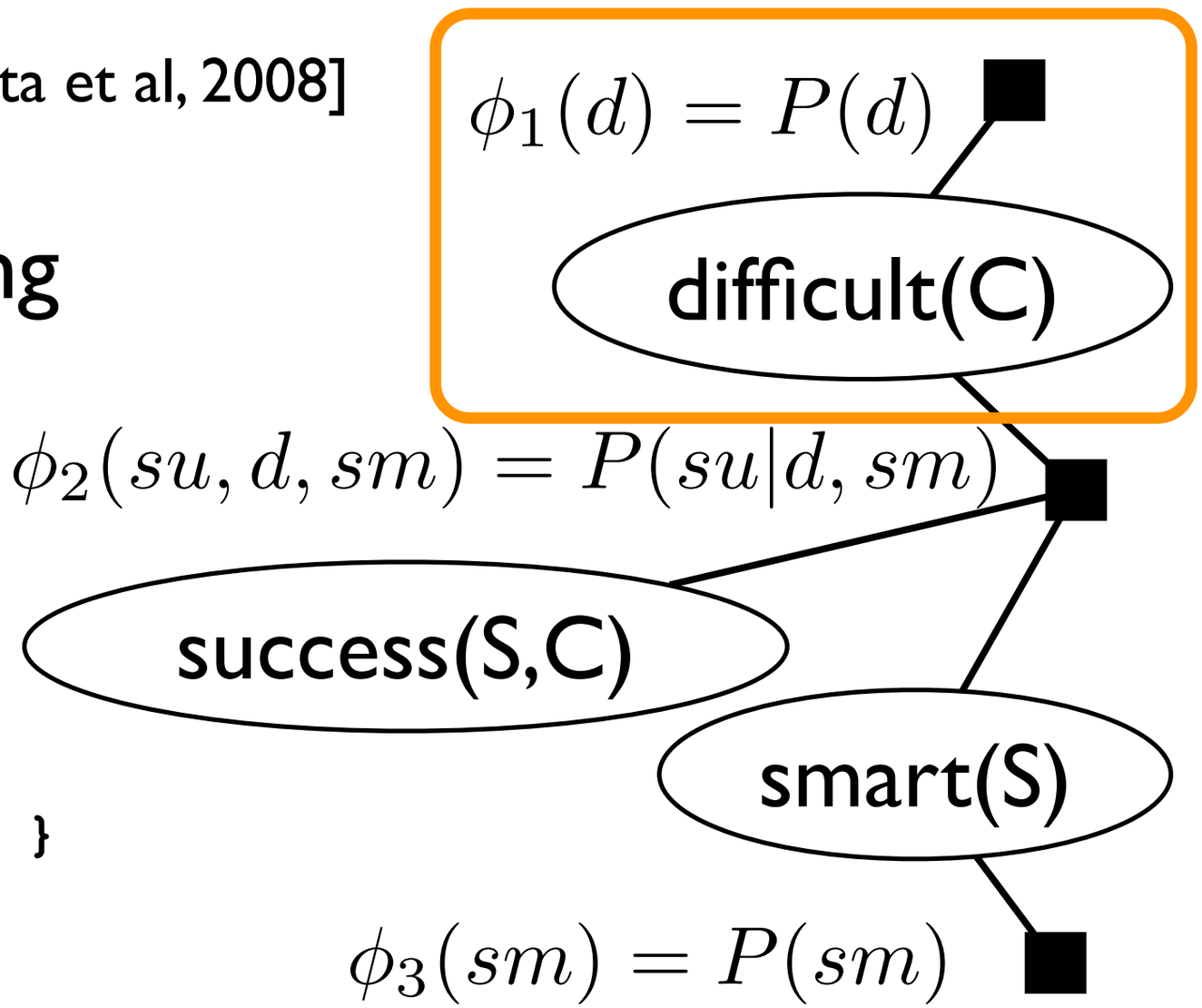
constraint logic programming
for Bayesian networks



CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks

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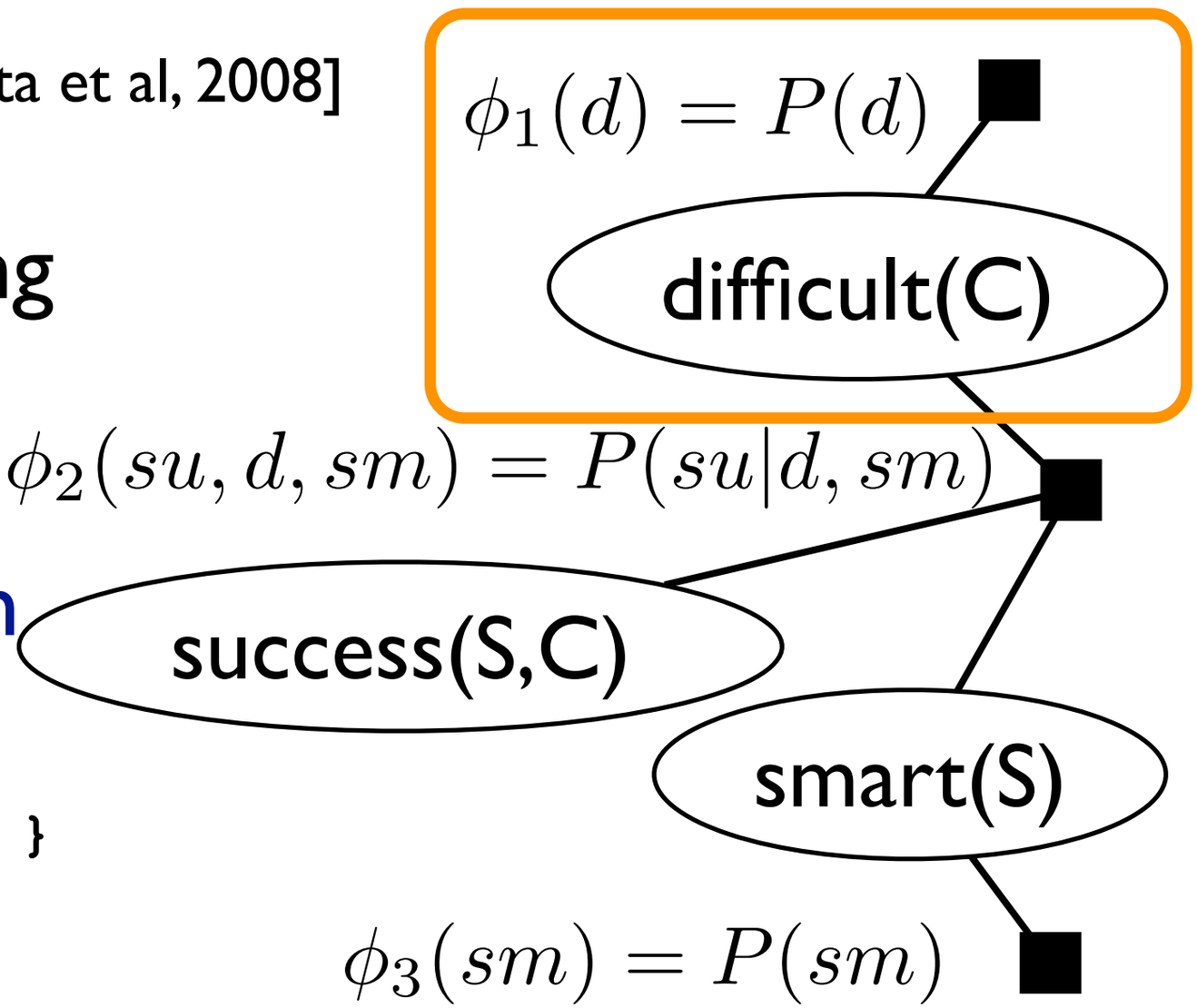
CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks



par-RV: unique skolem term

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CLP(BN)

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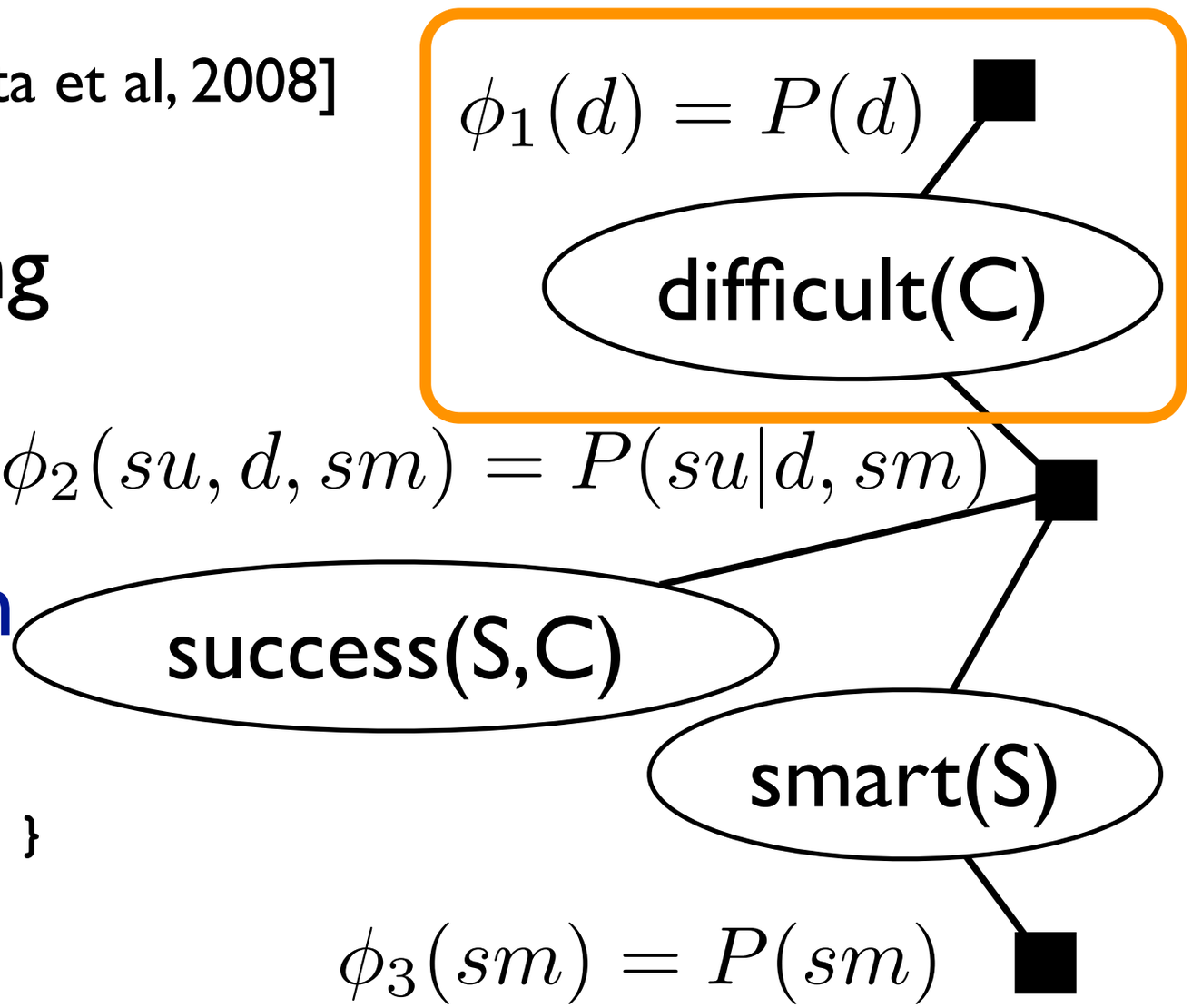
constraint logic programming for Bayesian networks



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possible values

CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming
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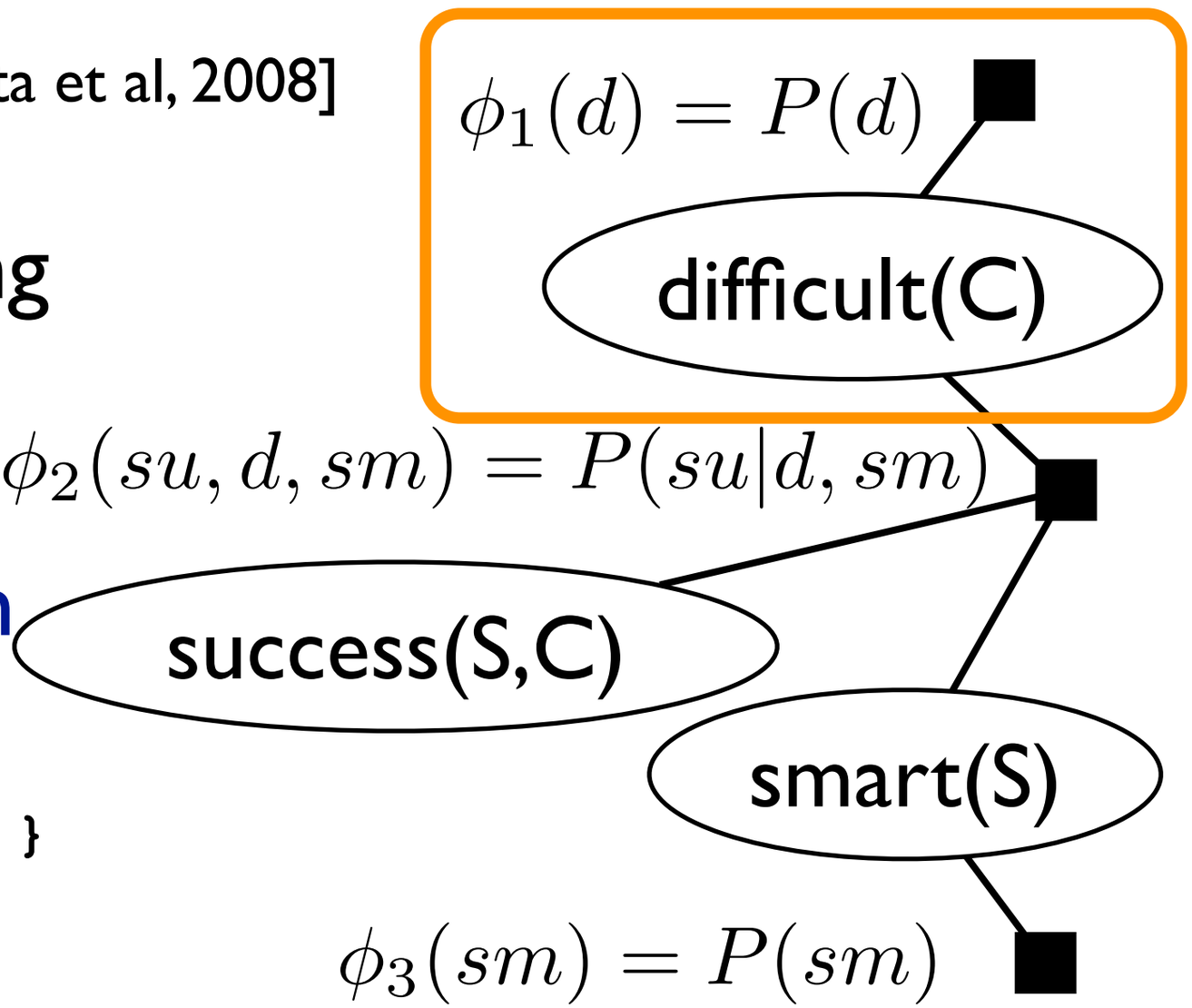

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possible values

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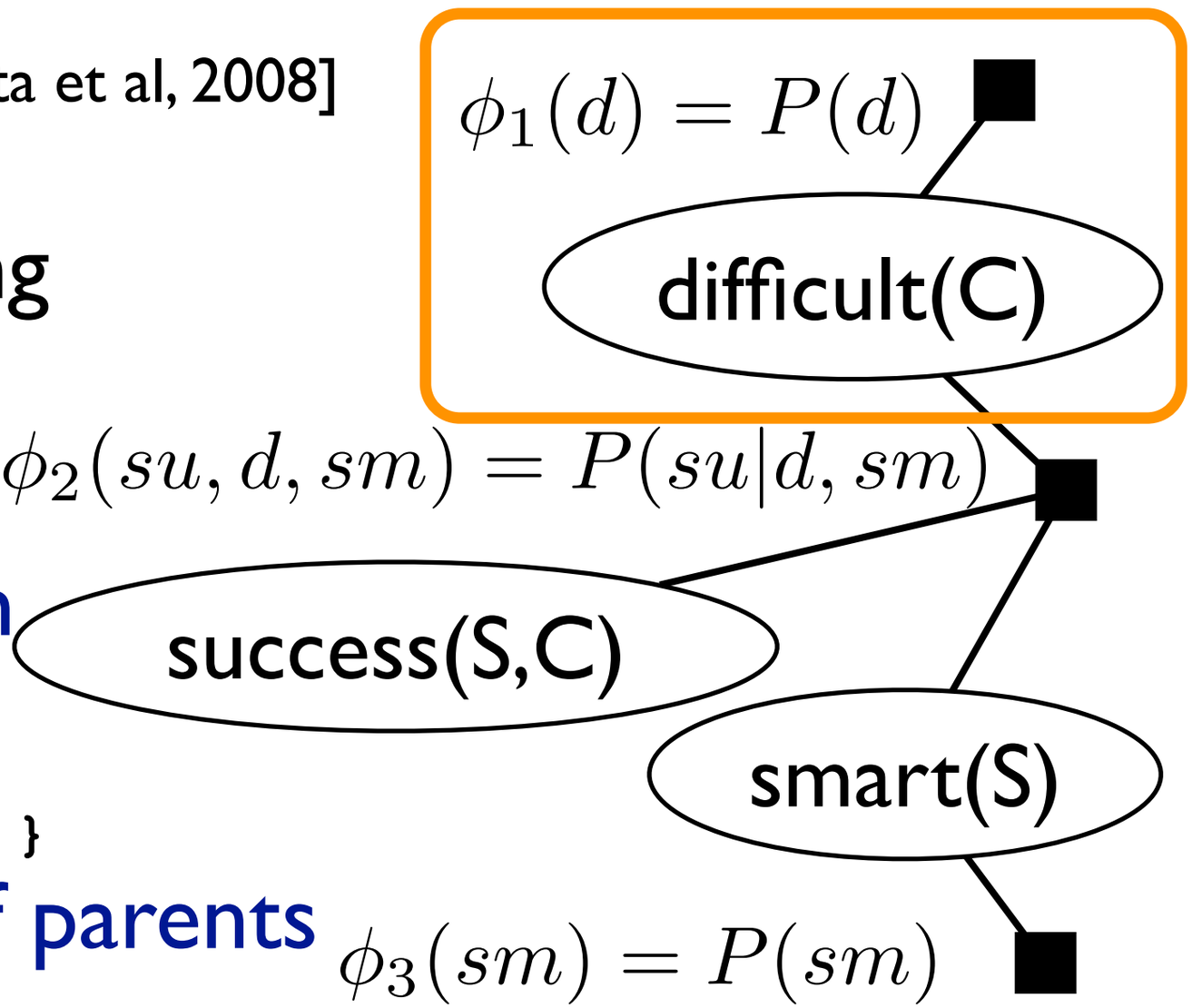
CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks



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possible values

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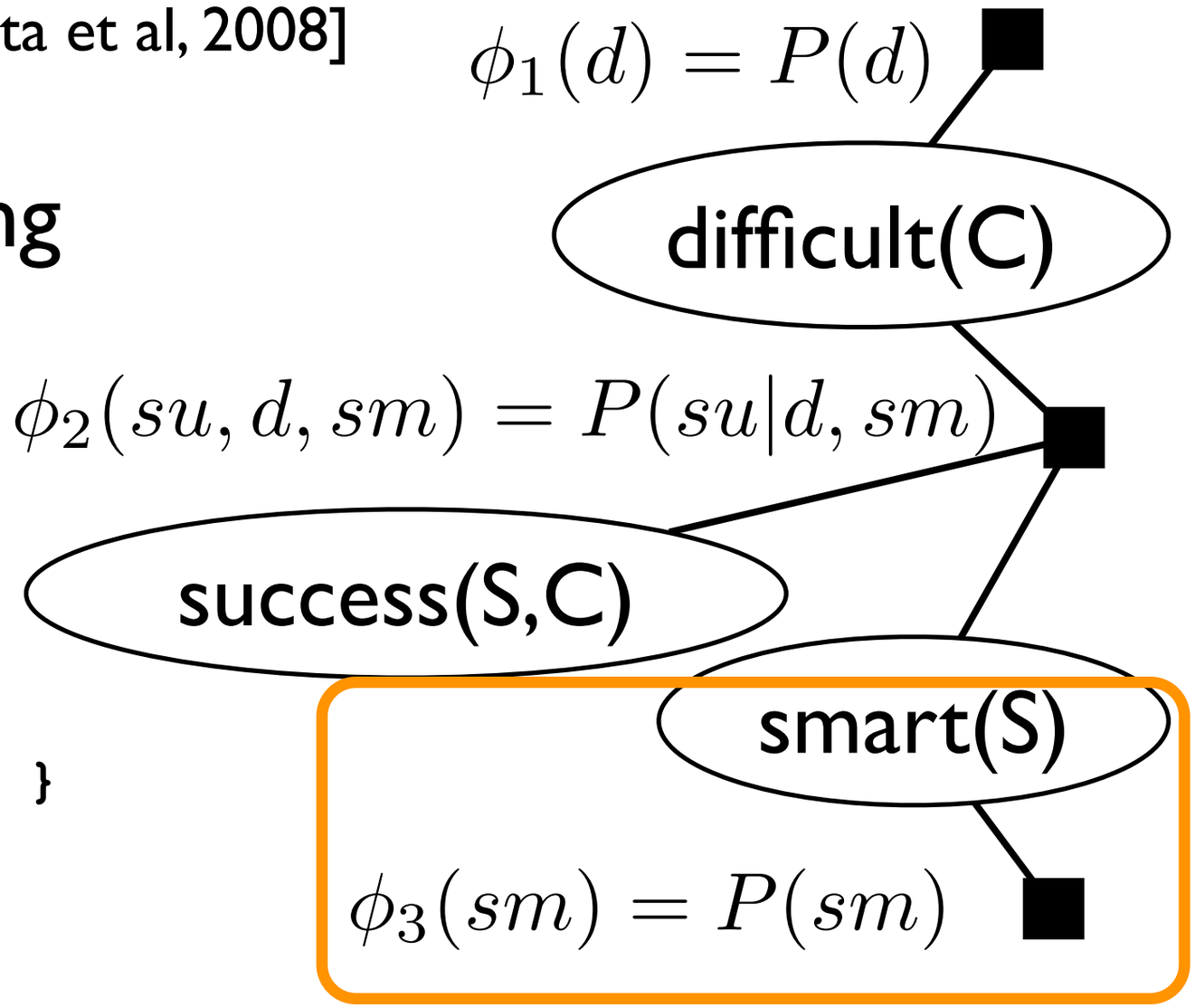
list of parents

CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks

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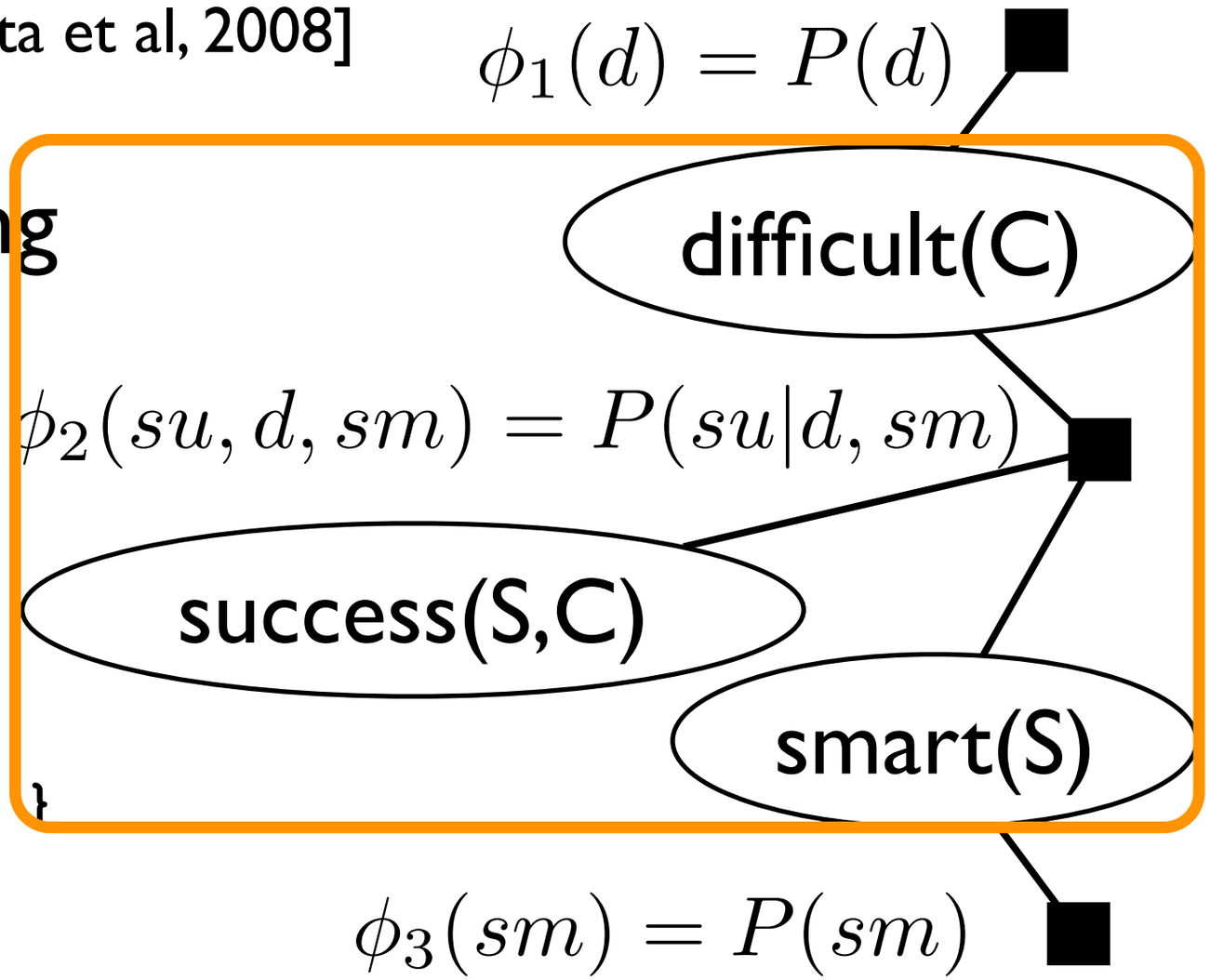
CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks

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CLP(BN)

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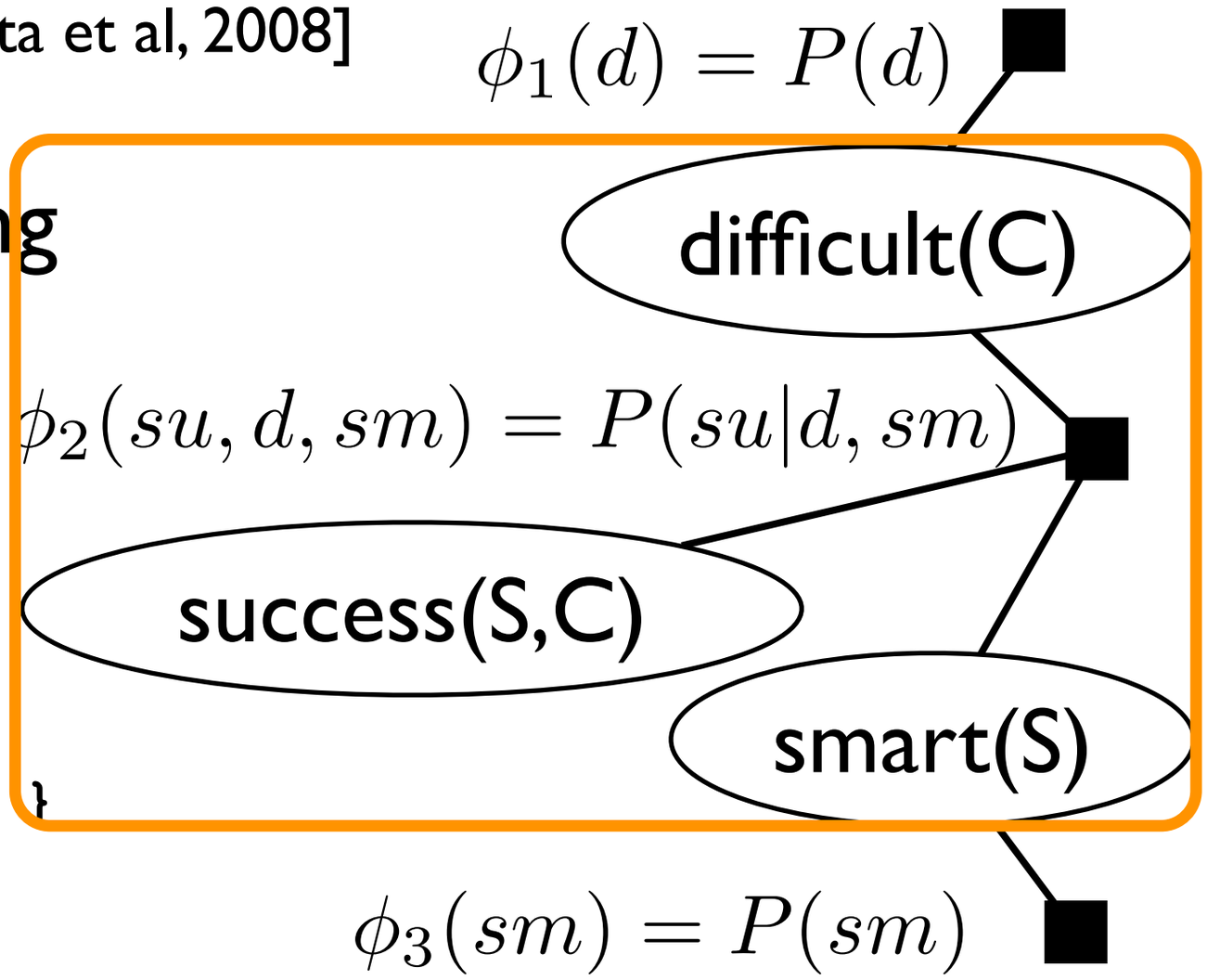
constraint logic programming for Bayesian networks

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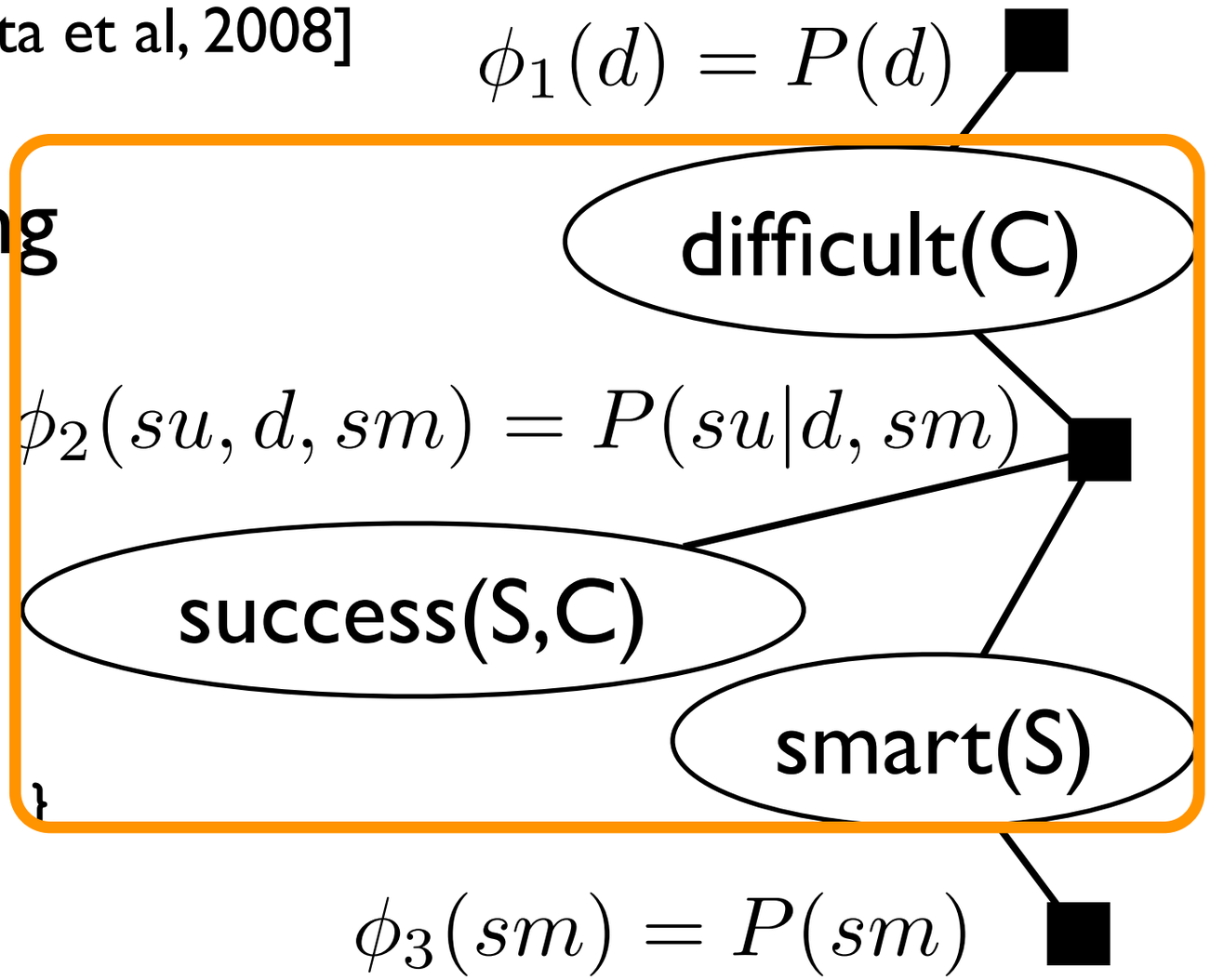
Prolog call constrains grounding

CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks

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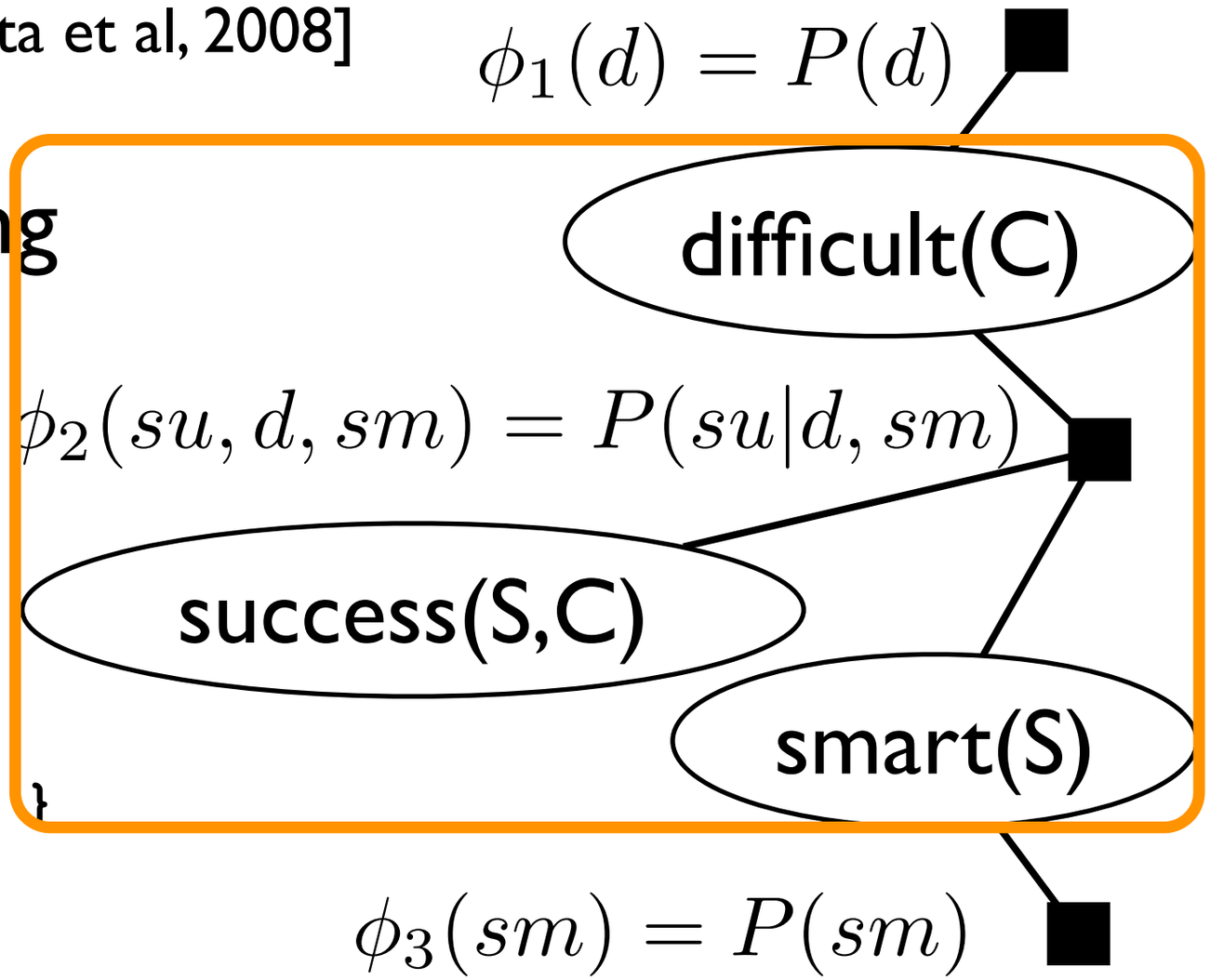
get parents

CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks

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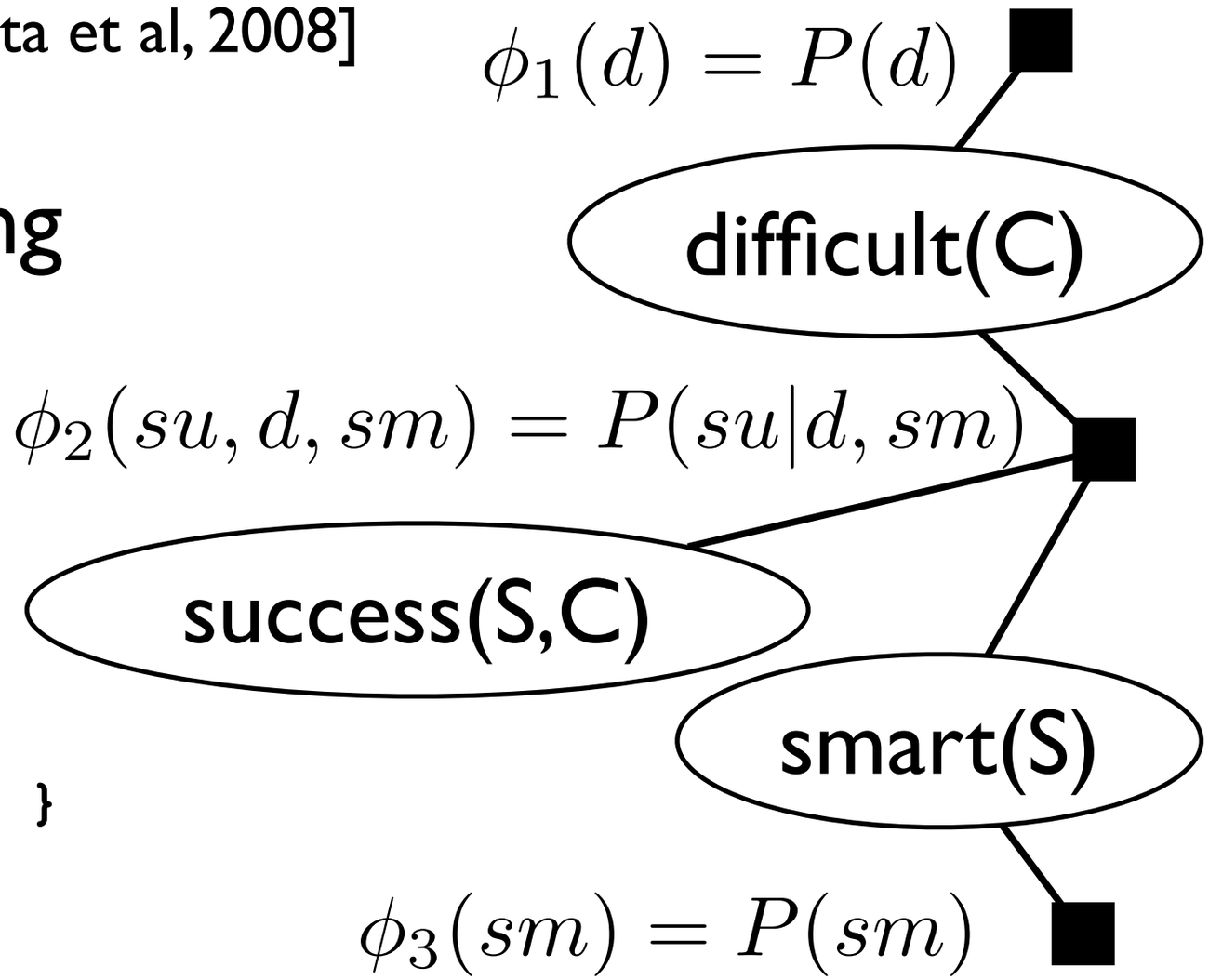
get parents

P(su|d=t,sm=t)

CLP(BN)

[Santos Costa et al, 2008]

constraint logic programming for Bayesian networks

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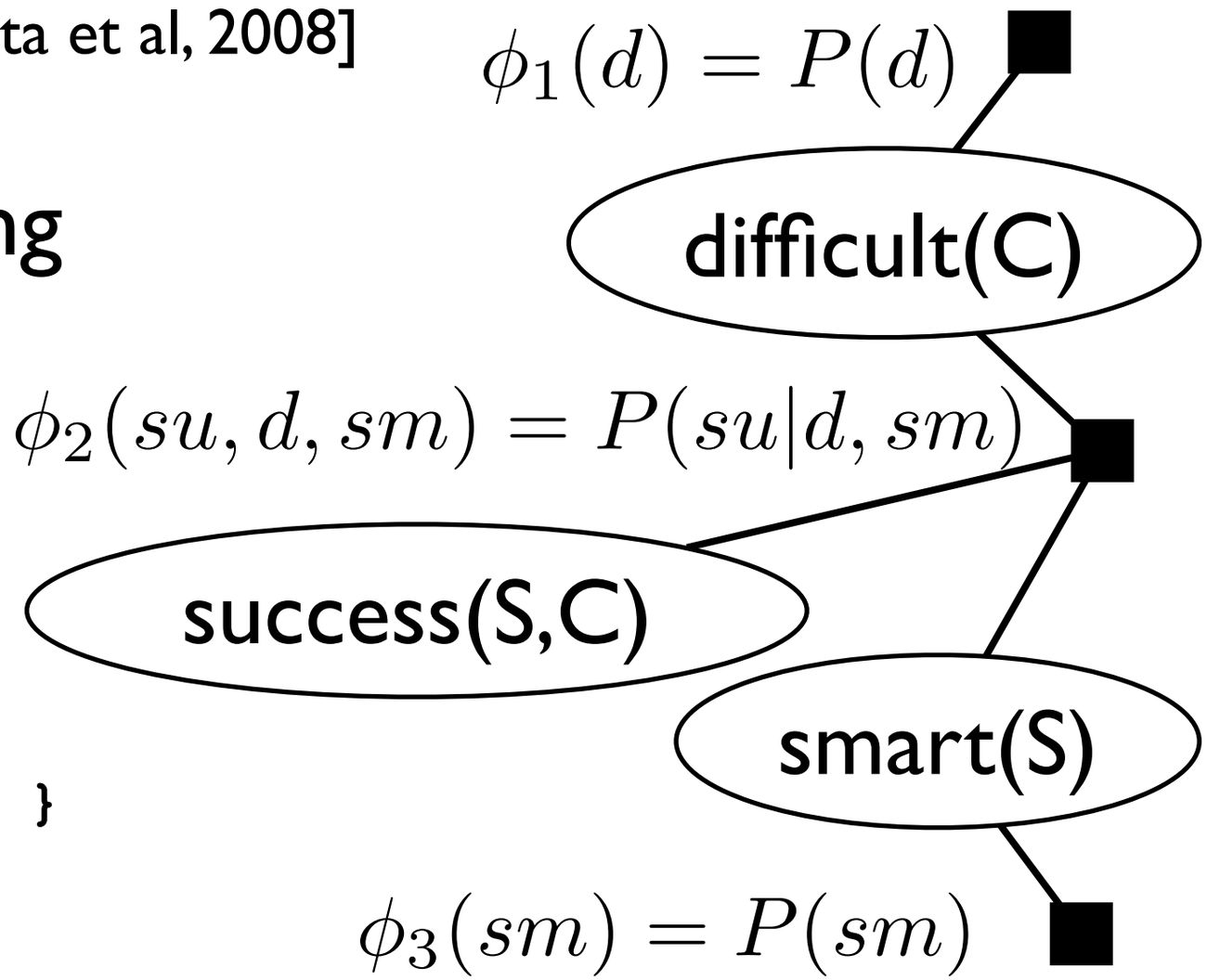


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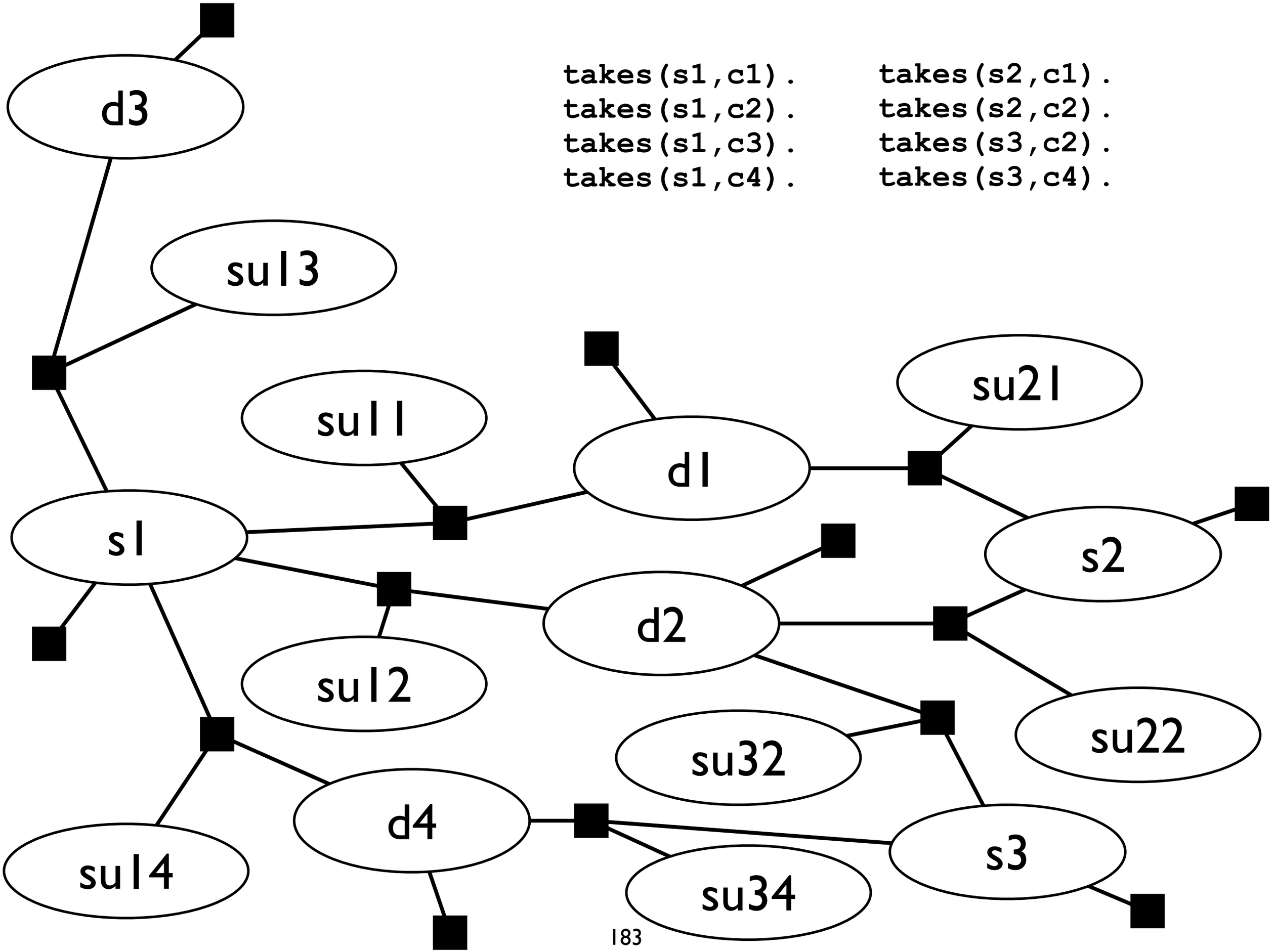

in ProbLog?

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```
0.6::difficult(C) :- takes(_,C).
0.7::smart(S) :- takes(S,_).
0.85::success(S,C) :- takes(S,C),difficult(C), smart(S).
0.10::success(S,C) :- takes(S,C),difficult(C), \+smart(S).
0.98::success(S,C) :- takes(S,C),\+difficult(C), smart(S).
0.45::success(S,C) :- takes(S,C),\+difficult(C), \+smart(S).
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Inference Example

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?-success(s2,c1,S).
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D=d(c1) with p(...,[])

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takes(s2,c1).
takes(s2,c2).
takes(s3,c2).
takes(s3,c4).
```

```
?-success(s2,c1,S).
```

Success=su(s2,c1) with p(...,[D,Sm])

Sm=sm(s2) with p(...,[])

D=d(c1) with p(...,[])

Inference Example

```
difficult(Course,Diff) :-
    { Diff = d(Course)
      with p([t,f],[0.6,0.4],[]) }

smart(Student,Smart) :-
    { Smart = sm(Student)
      with p([t,f],[0.7,0.3],[]) }

success(Student,Course,Success) :-
    takes(Student,Course),
    difficult(Course,D),
    smart(Student,Sm),
    { Success = su(Student,Course)
      with p([t,f],[0.85,0.1,0.98,0.45,
                    0.15,0.9,0.02,0.55],[D,Sm])}
```

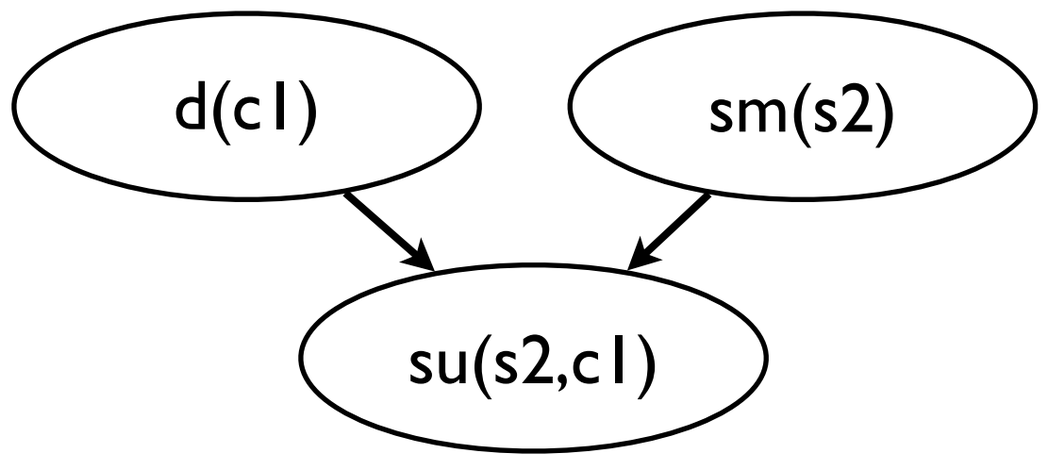
```
takes(s1,c1).
takes(s1,c2).
takes(s2,c1).
takes(s2,c2).
takes(s3,c2).
takes(s3,c4).
```

```
?-success(s2,c1,S).
```

Success=su(s2,c1) with p(...,[D,Sm])

Sm=sm(s2) with p(...,[])

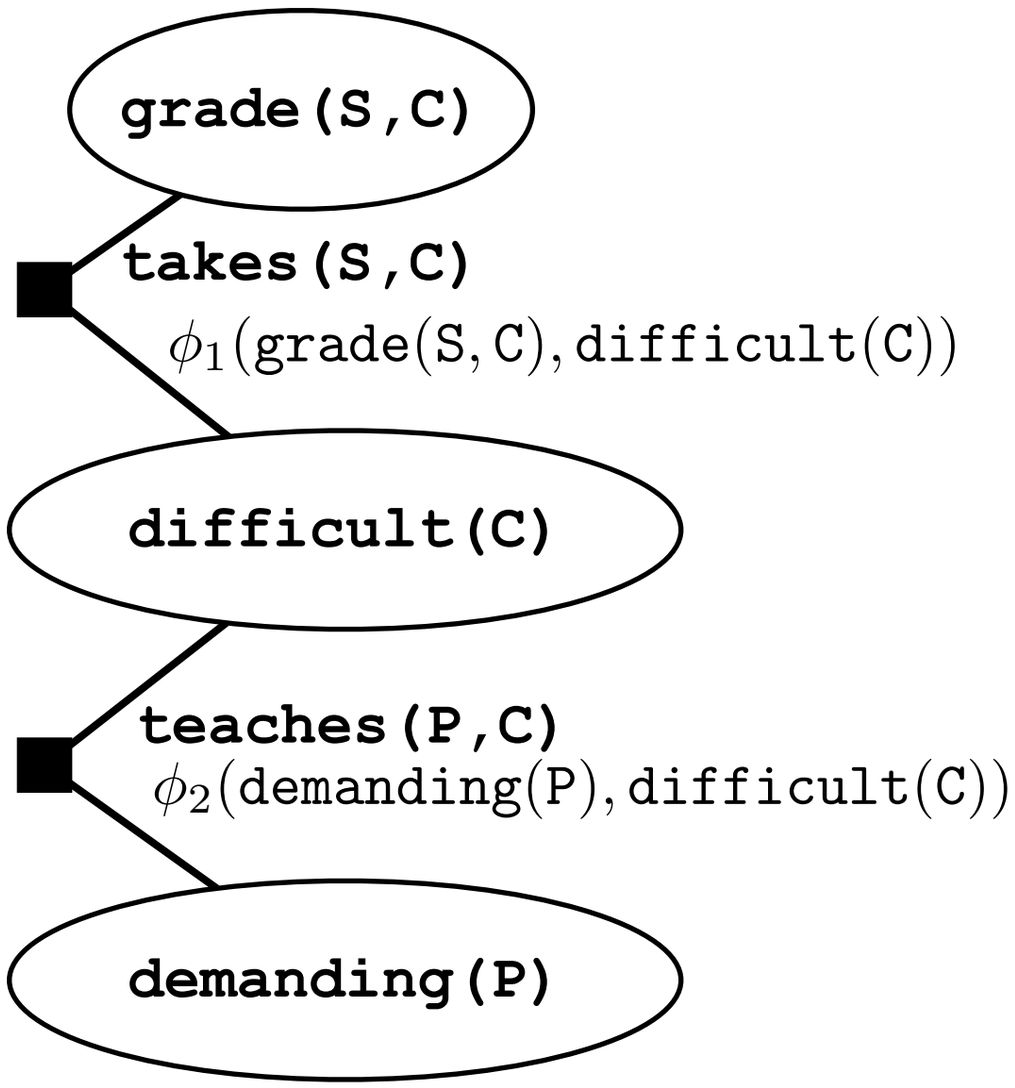
D=d(c1) with p(...,[])



CLP(BN) Summary

- Templating Bayesian networks via constraint logic programming
- Knowledge-based model construction (KBMC):
  - construct relevant ground BN by backward reasoning, adding constraints to constraint store
  - run any propositional inference technique

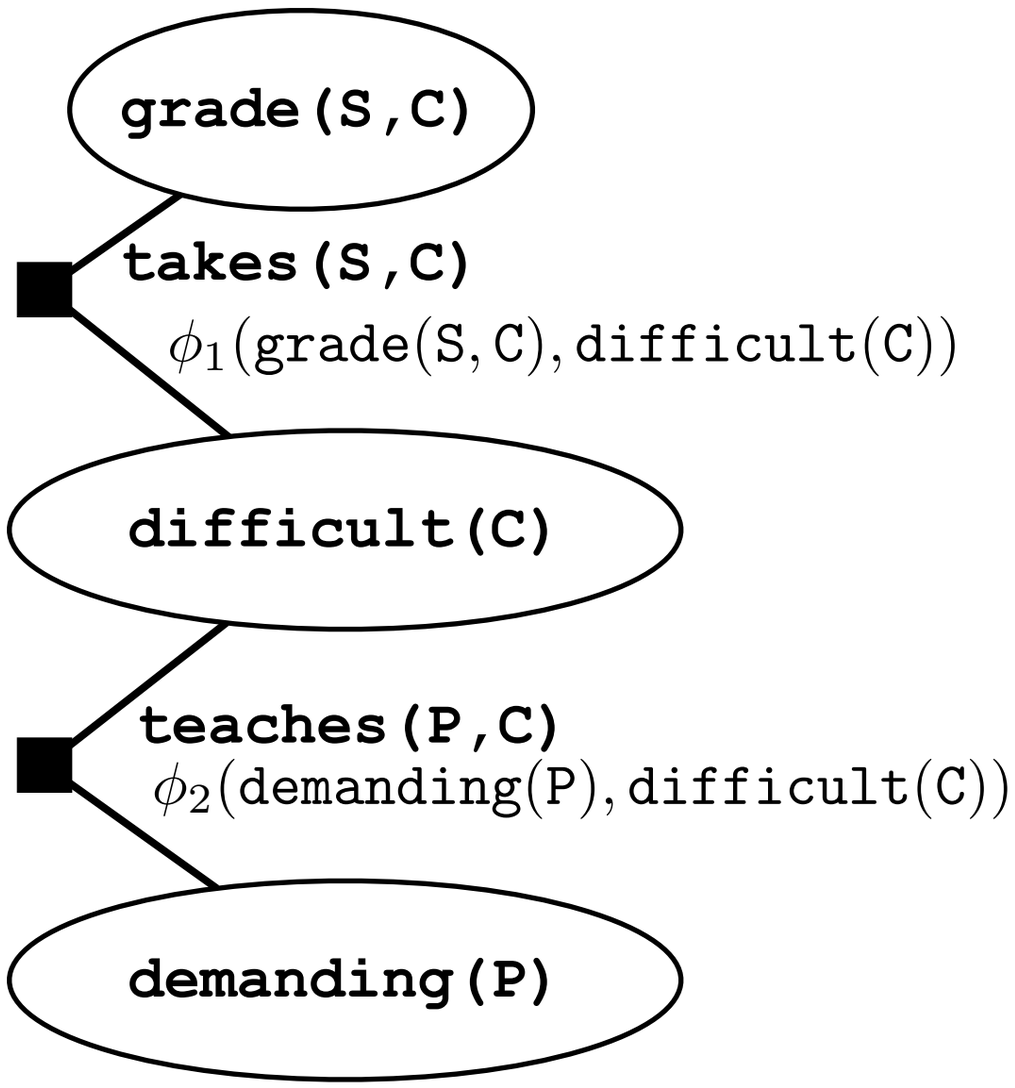
Inference by Grounding



demanding(tom)?

```
 takes(ann,ml).
 takes(bob,ml).
 takes(ann,db).
teaches(dan,db).
teaches(tom,ml).
```

Inference by Grounding




```
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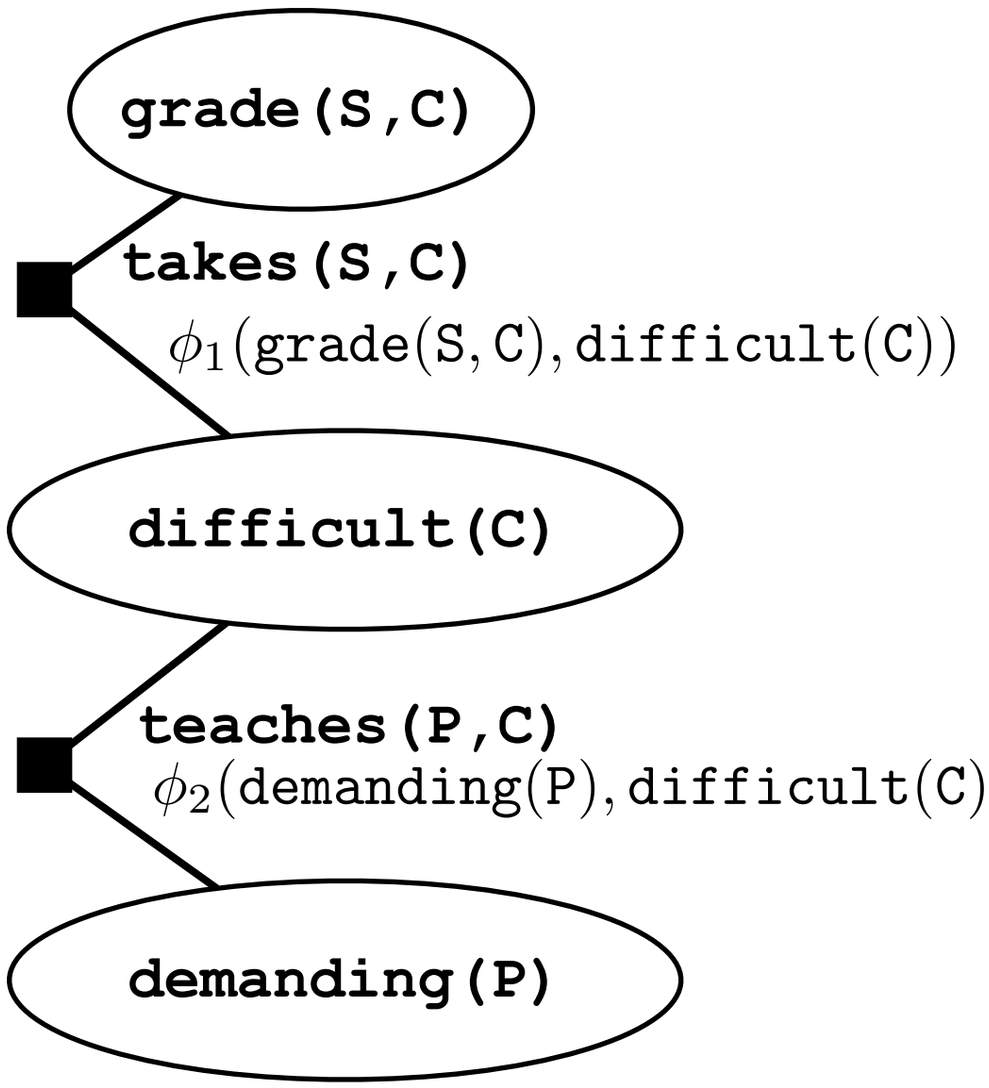
teaches(tom,ml).
```

demanding(tom)?

1. construct factor graph
2. run any propositional inference technique

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takes(ann,db).
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teaches(tom,ml).
```
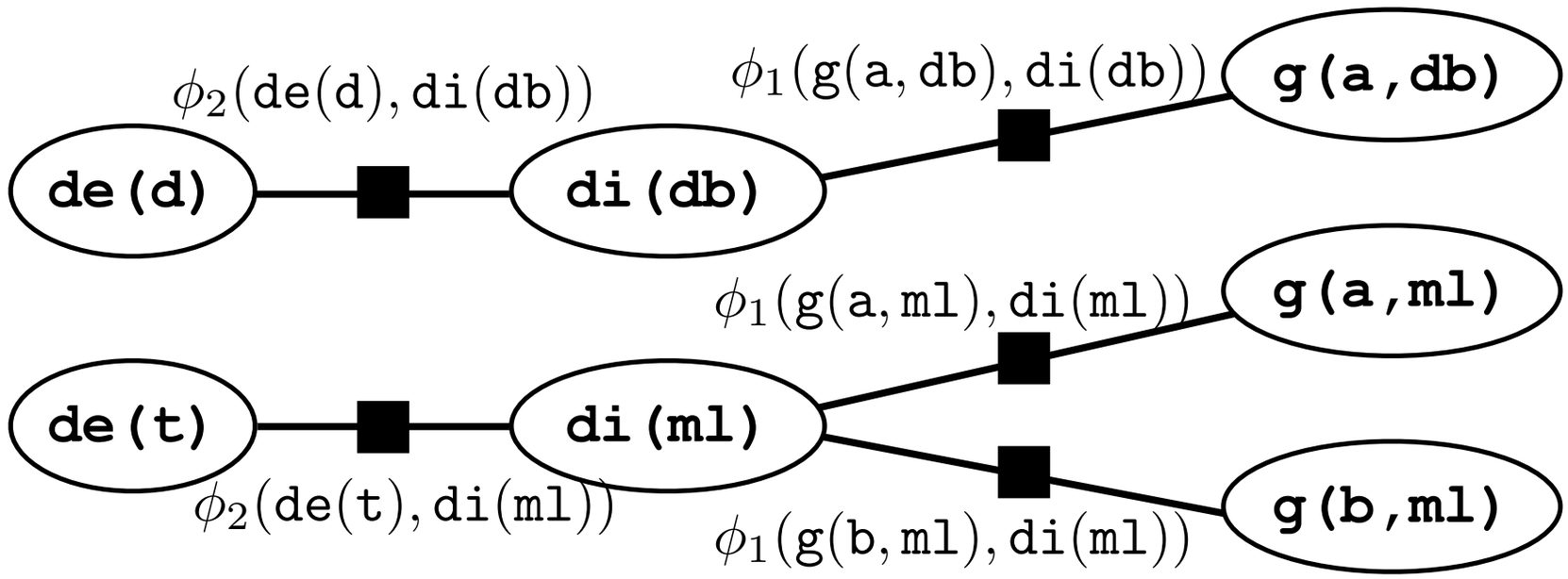
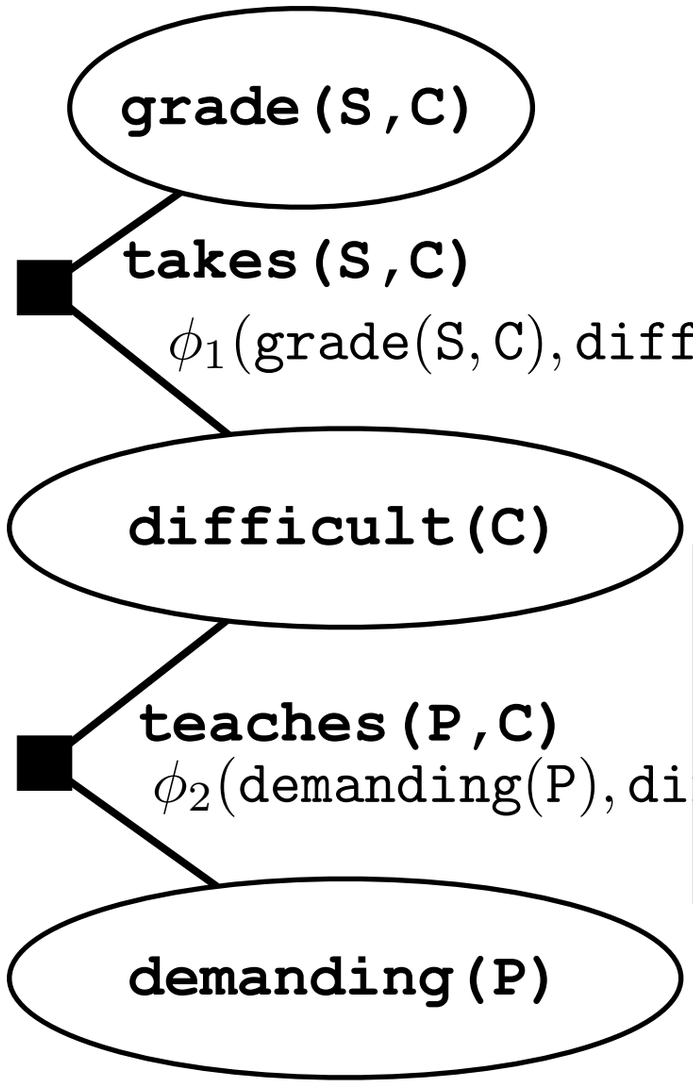

Inference by Grounding

demanding(tom)?

1. construct factor graph
2. run any propositional inference technique



Inference by Grounding



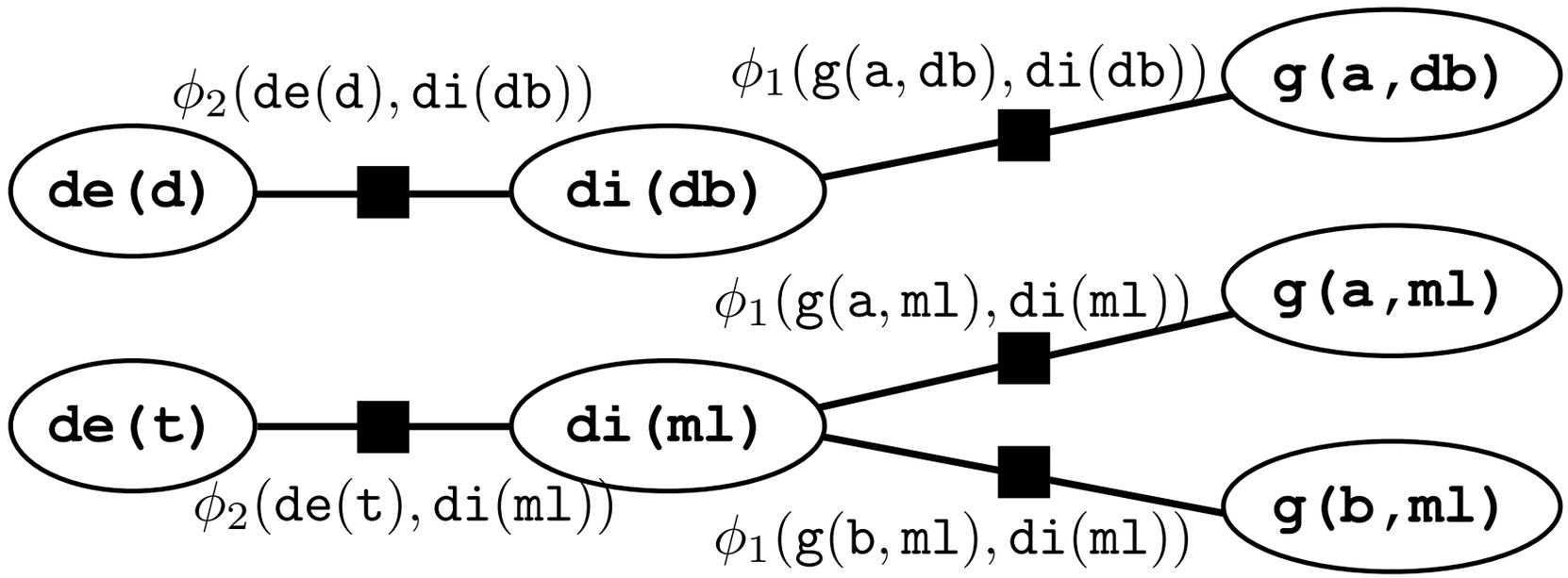
Problem: often huge factor graphs with many repetitions or symmetries (even if restricted to relevant parts)

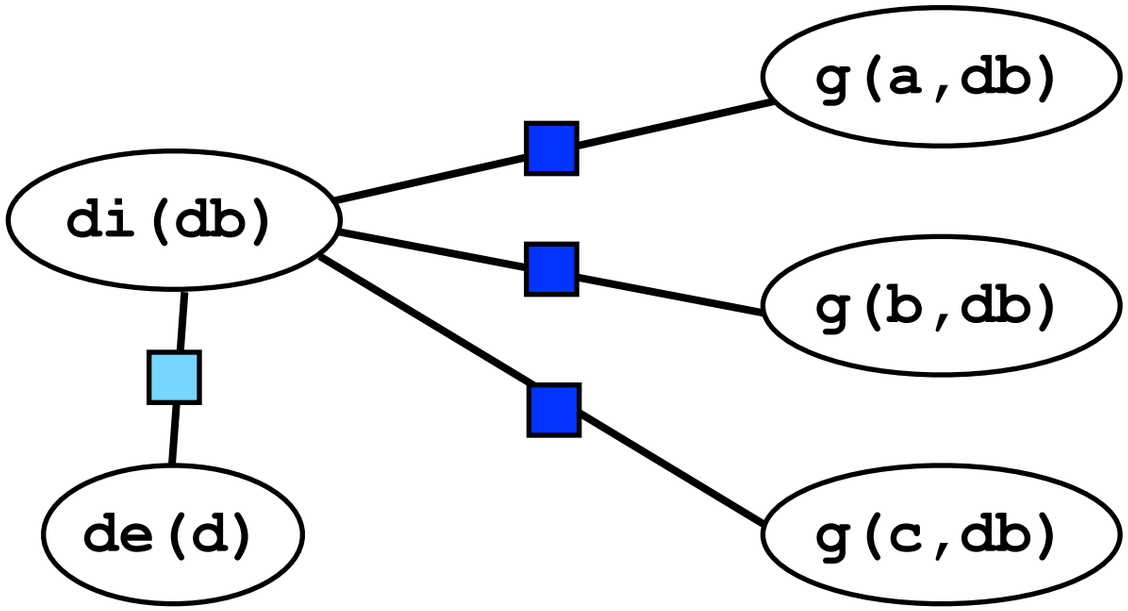
demanding(tom)?

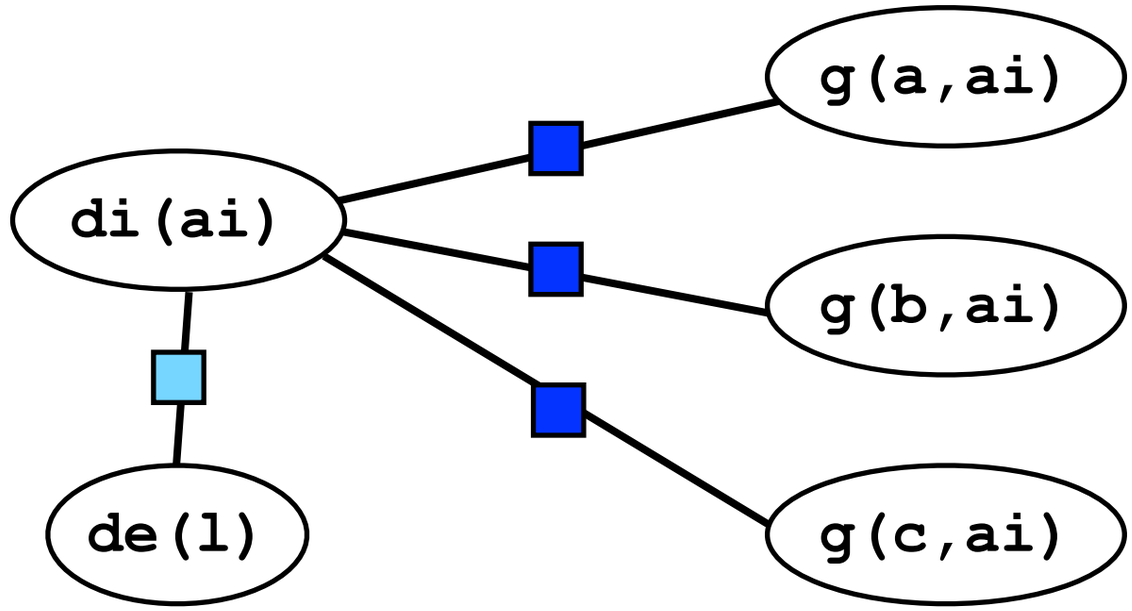
ctor graph

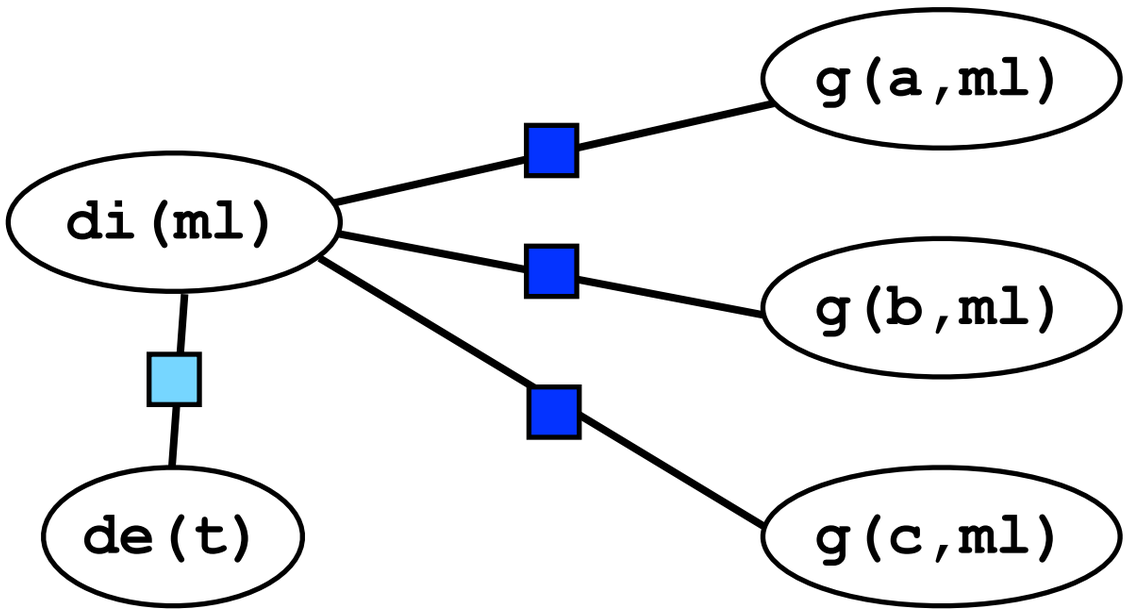
2. Run any propositional inference technique

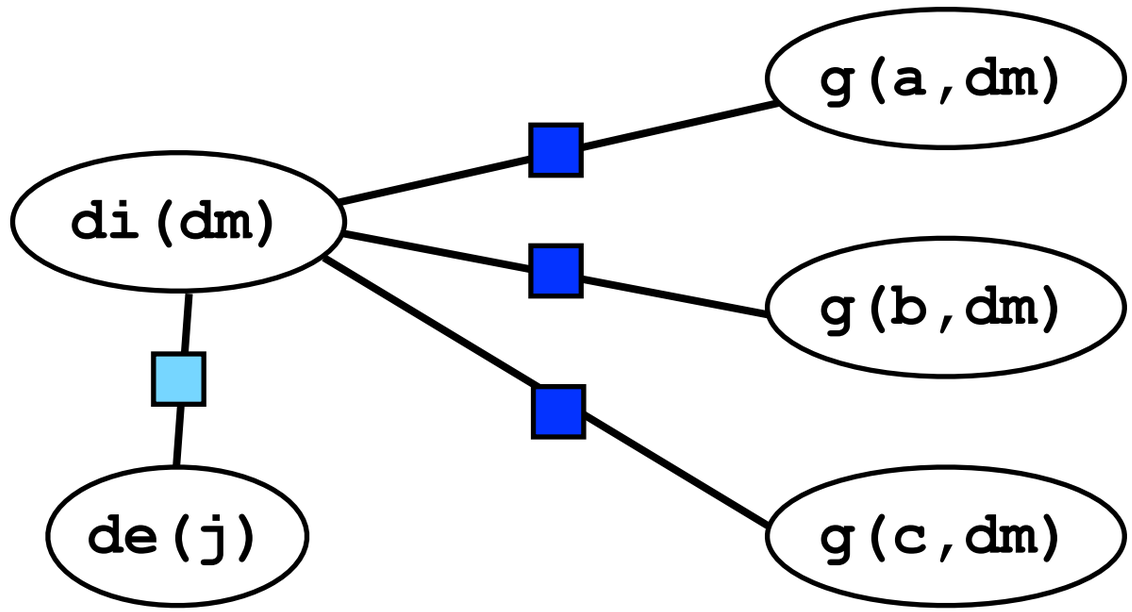
```
takes(ann,ml).
takes(bob,ml).
takes(ann,db).
teaches(dan,db).
teaches(tom,ml).
```

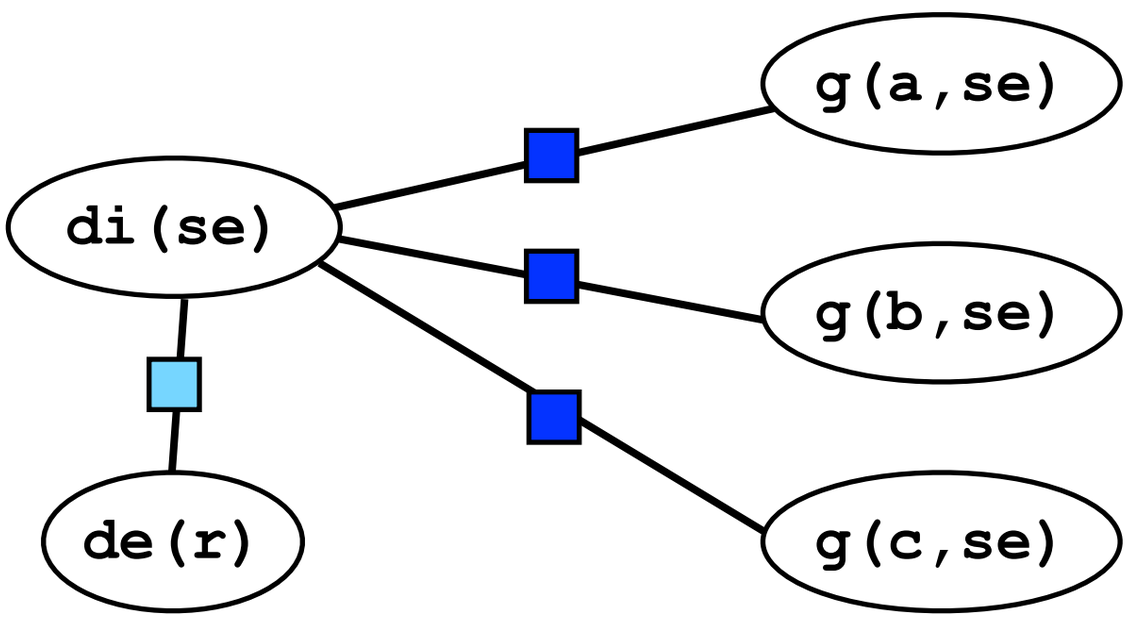


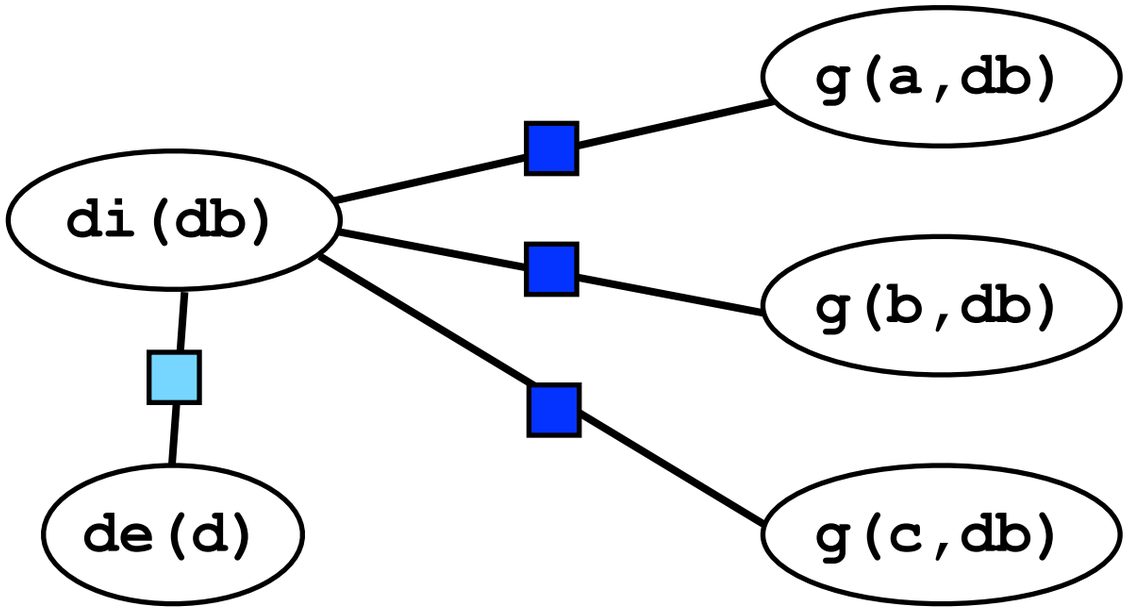

g(a,db)
di(db)
g(b,db)
de(d)
g(c,db)

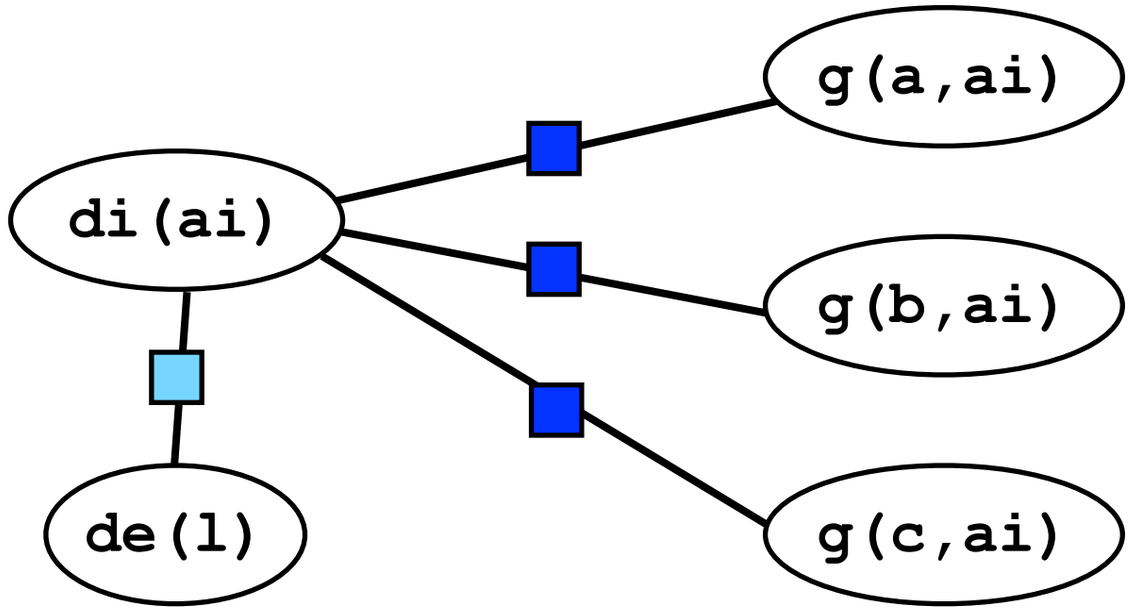

g(a,ai)
di(ai)
g(b,ai)
de(l)
g(c,ai)

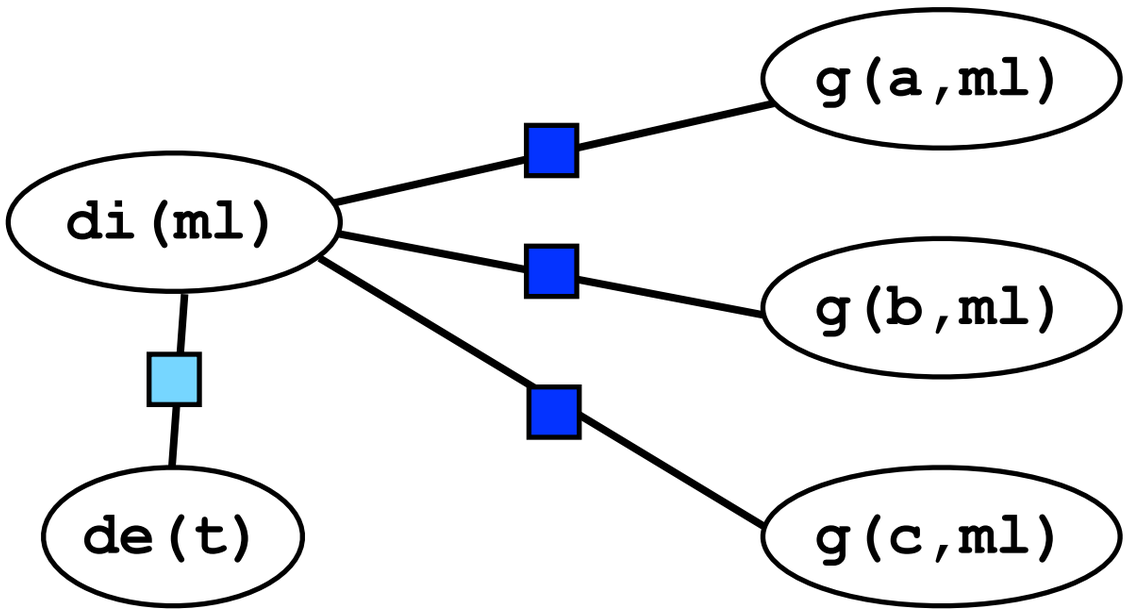

g(a,ml)
di(ml)
g(b,ml)
de(t)
g(c,ml)

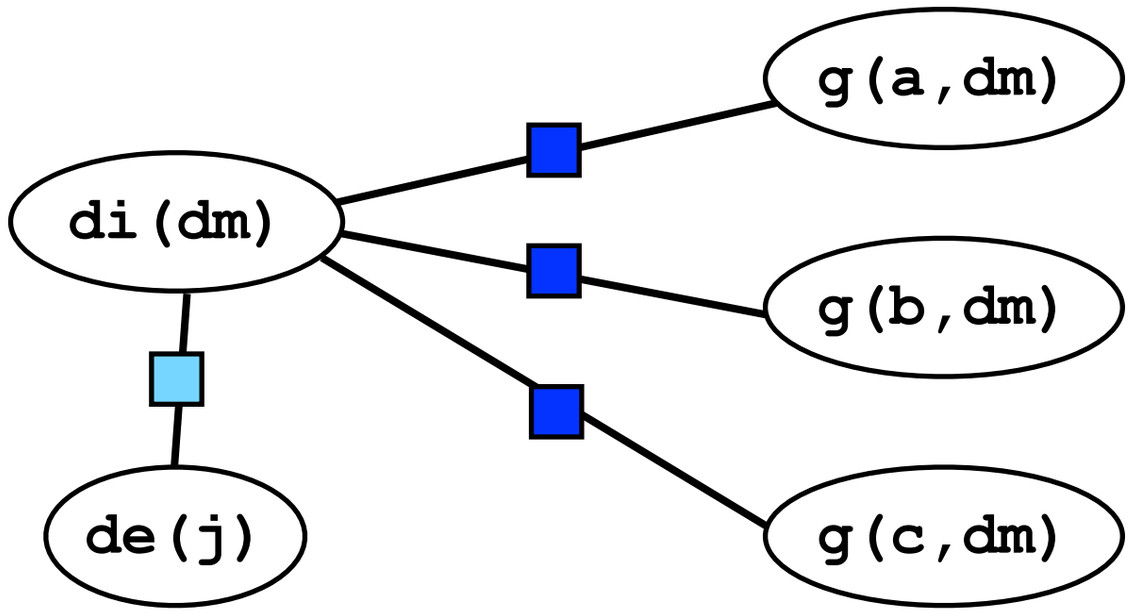

g(a,dm)
di(dm)
g(b,dm)
de(j)
g(c,dm)

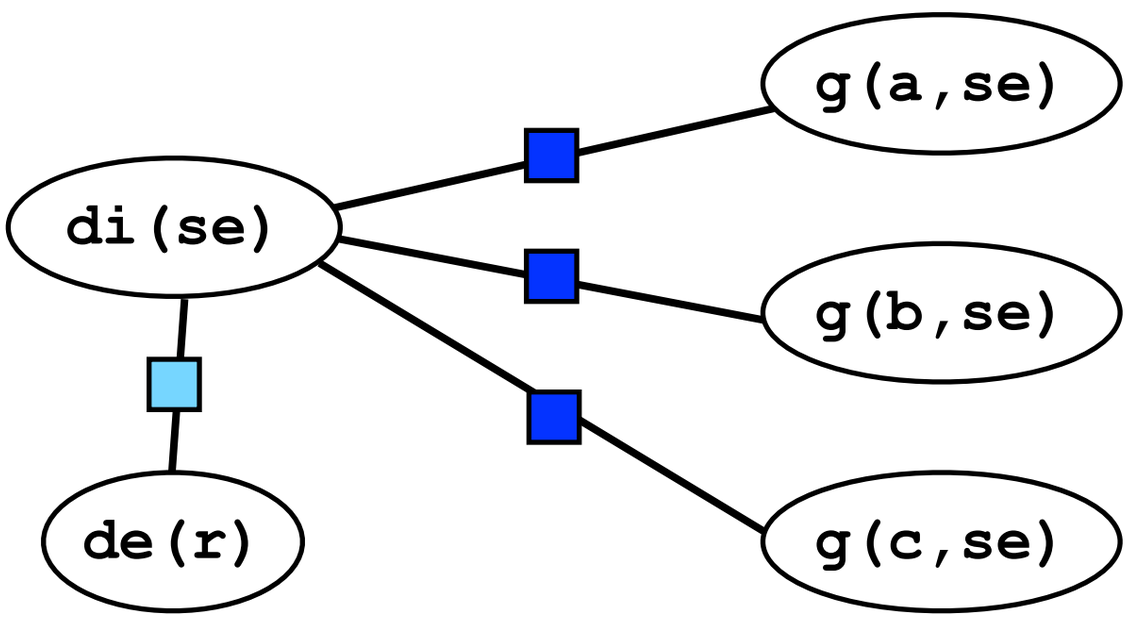

g(a,se)
di(se)
g(b,se)
de(r)
g(c,se)



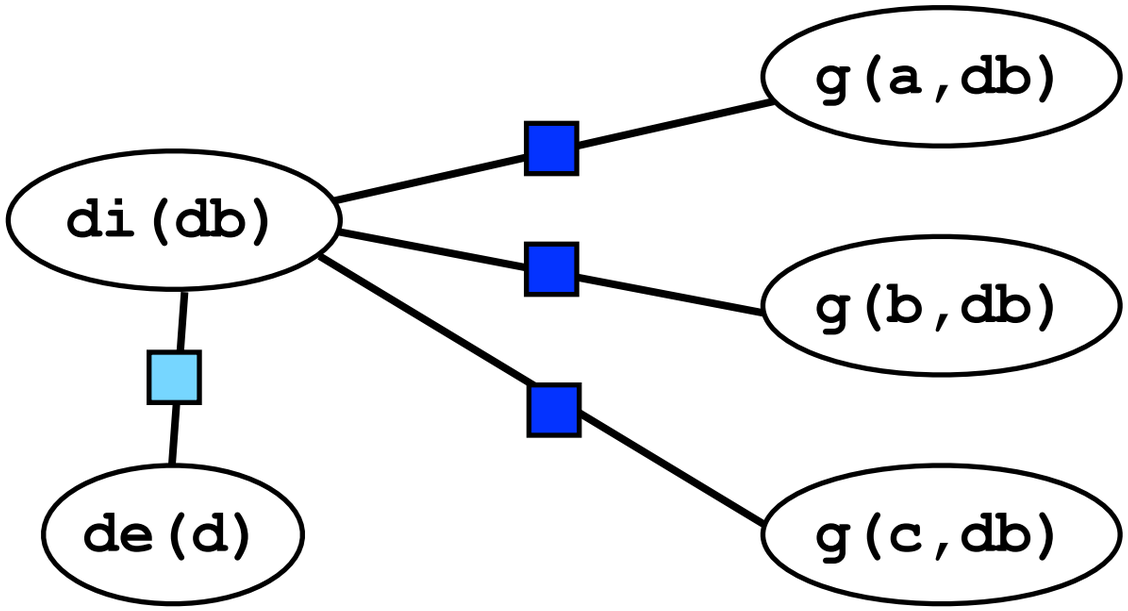


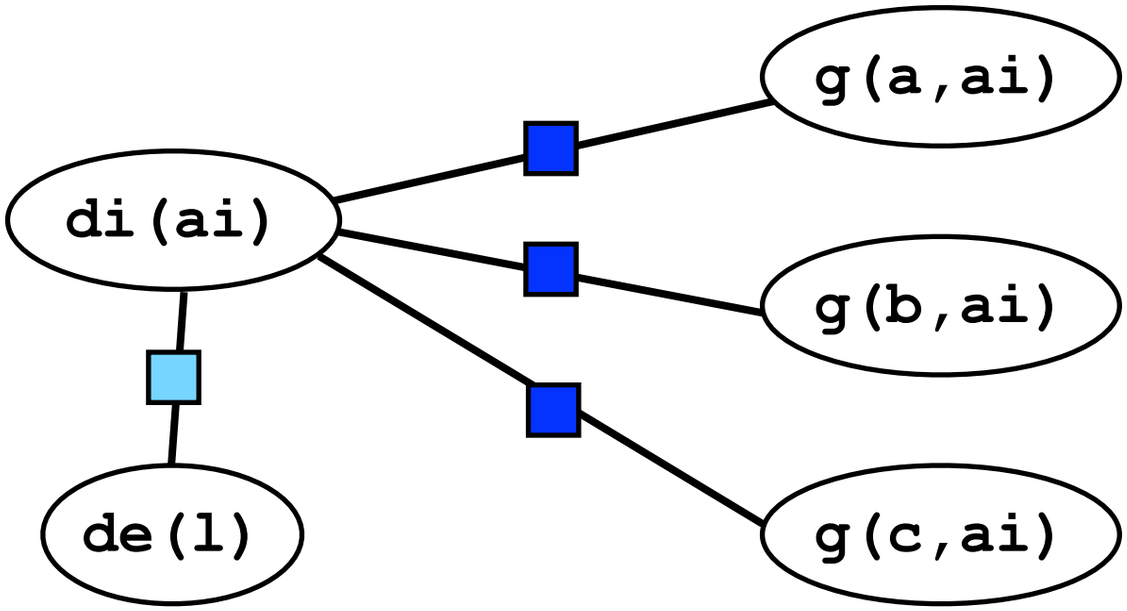


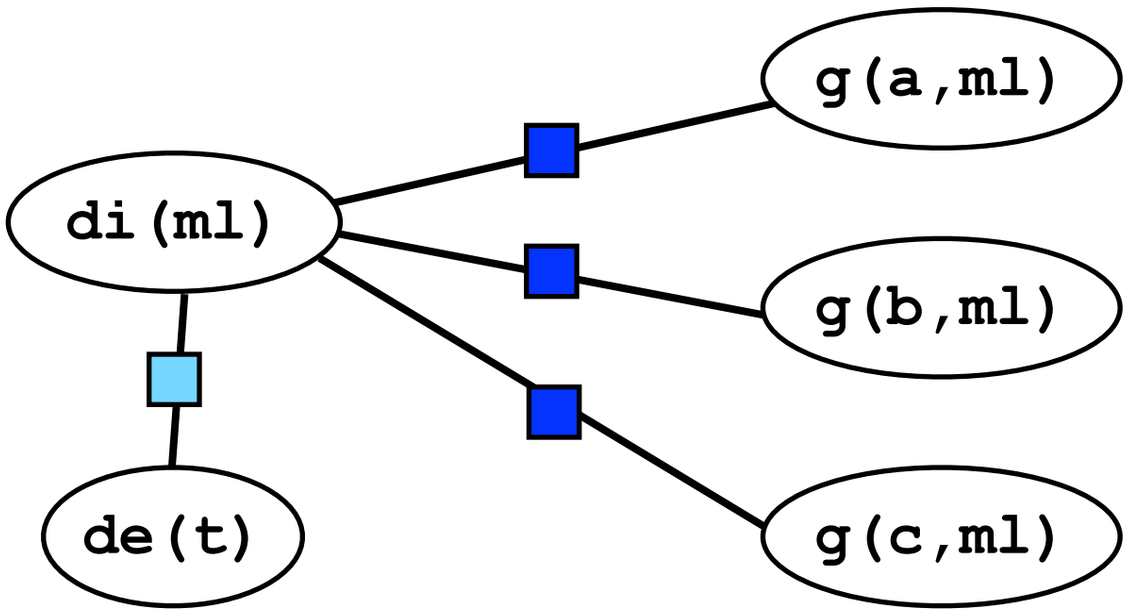


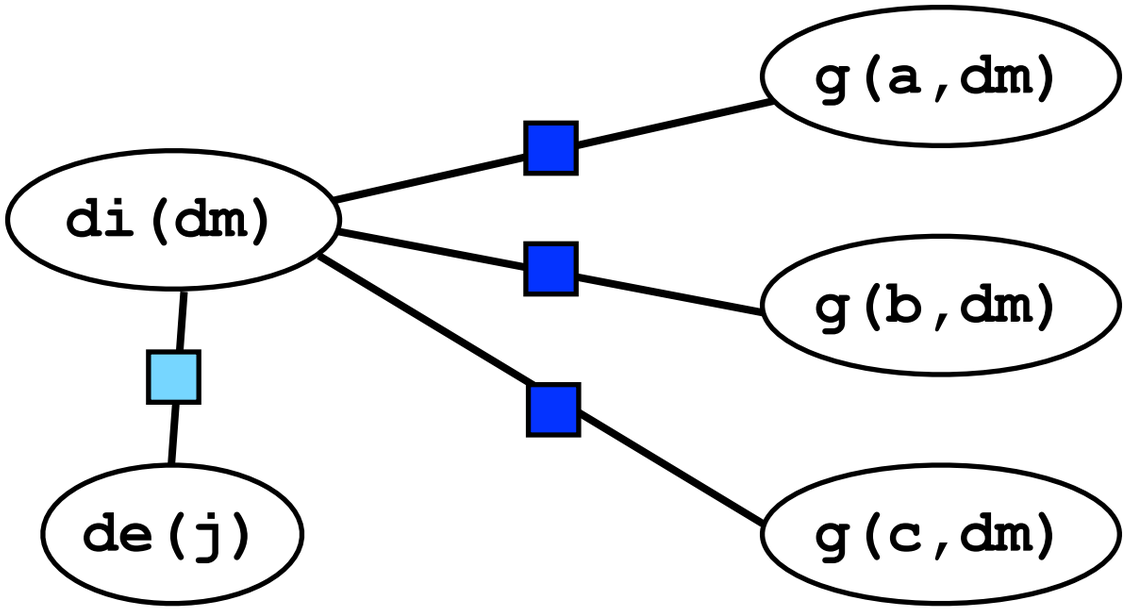


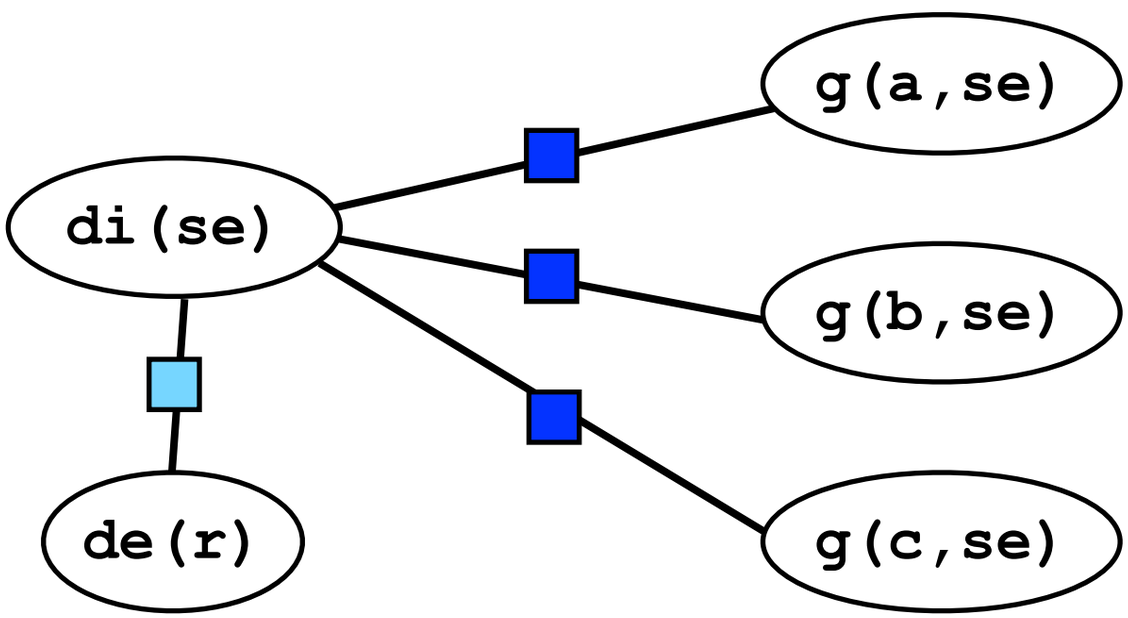
compute probability distribution over grades for every student-course pair





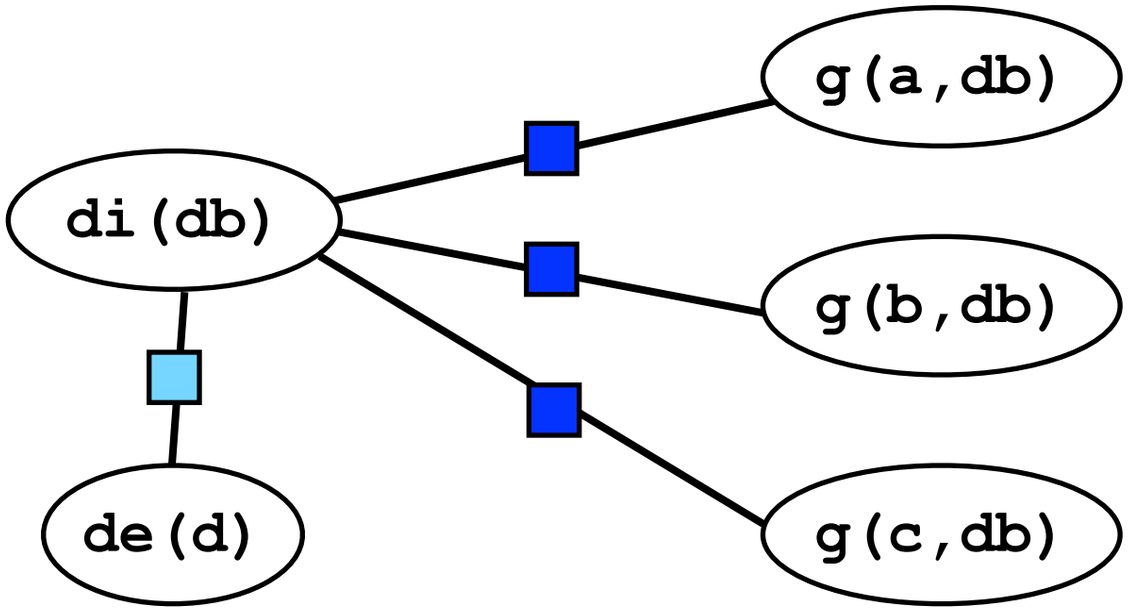


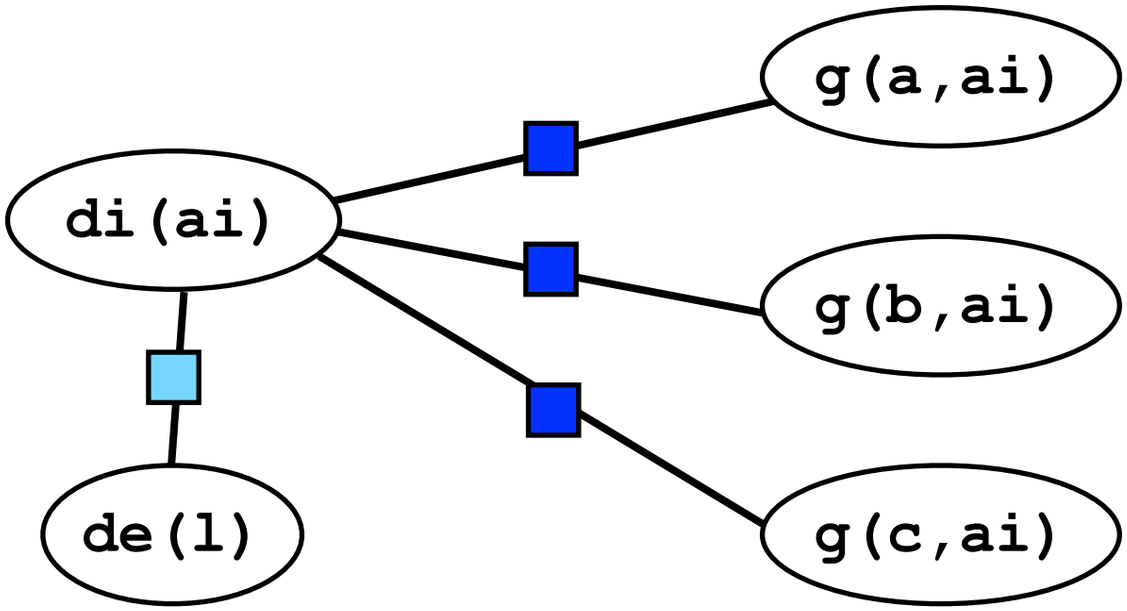


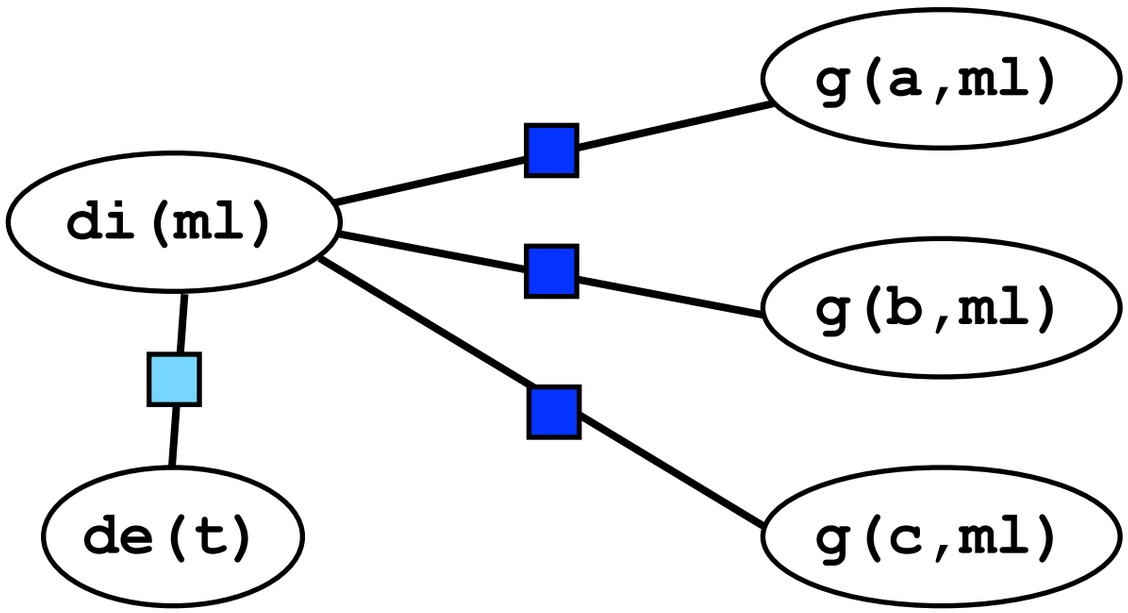


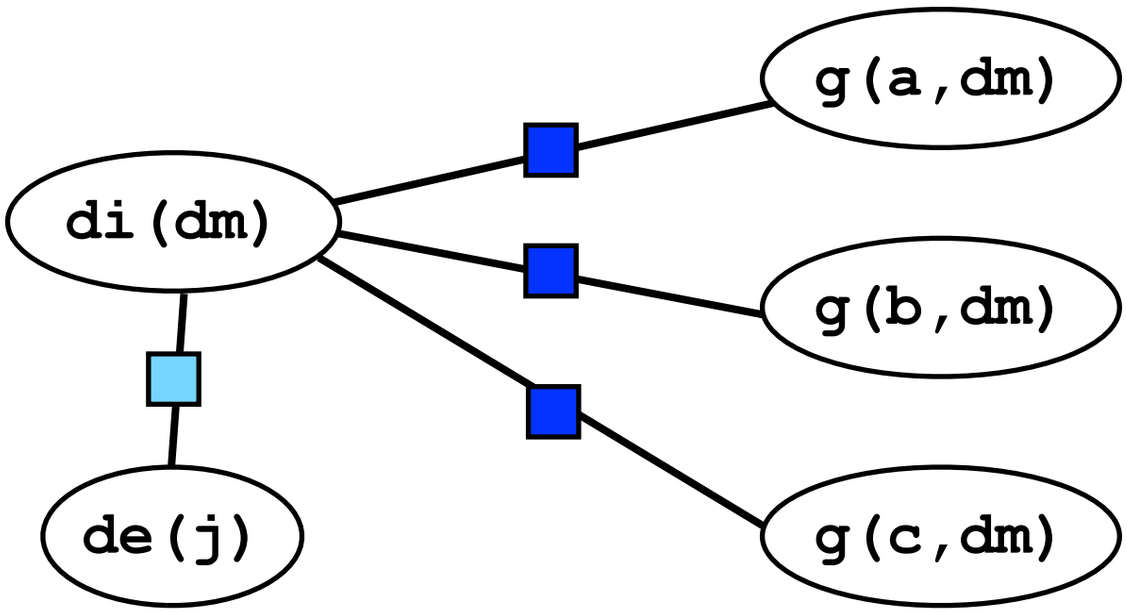
compute probability distribution over grades for every student-course pair

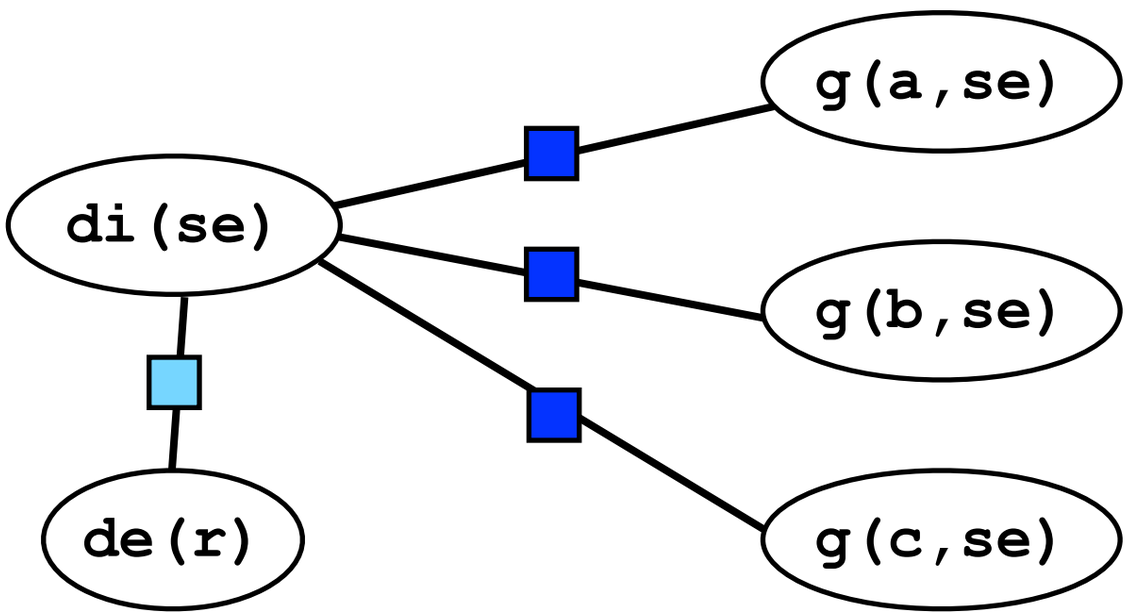
same computation for each pair!



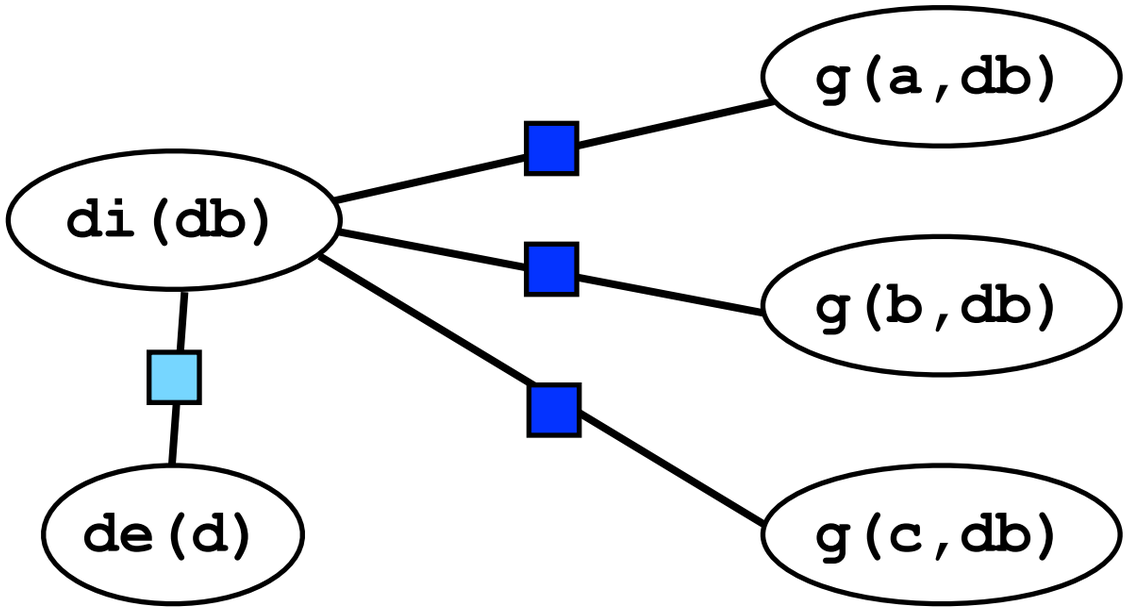


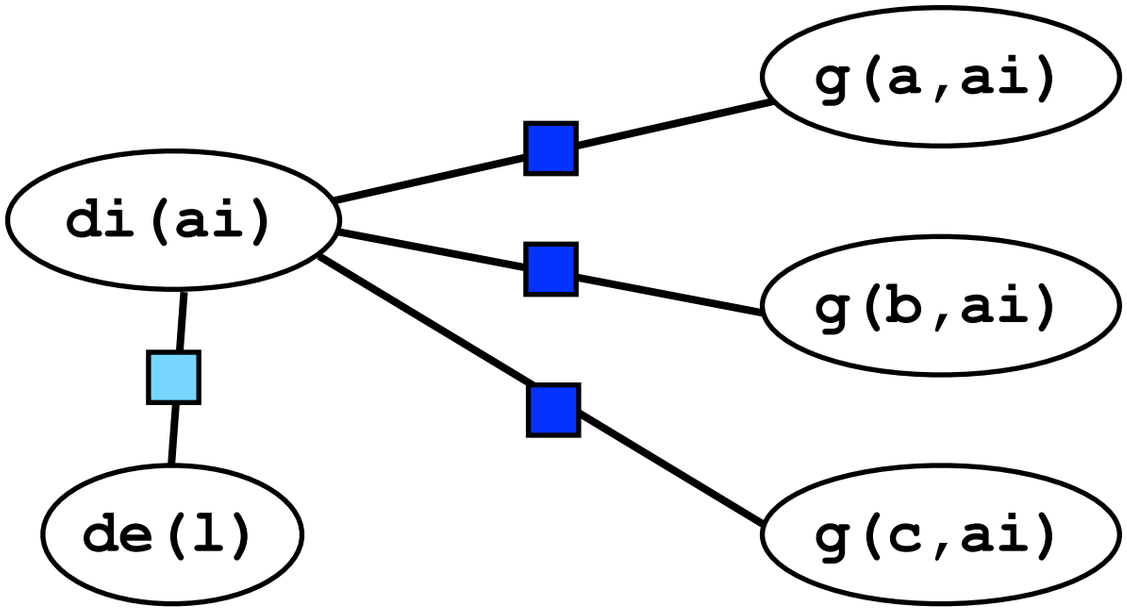


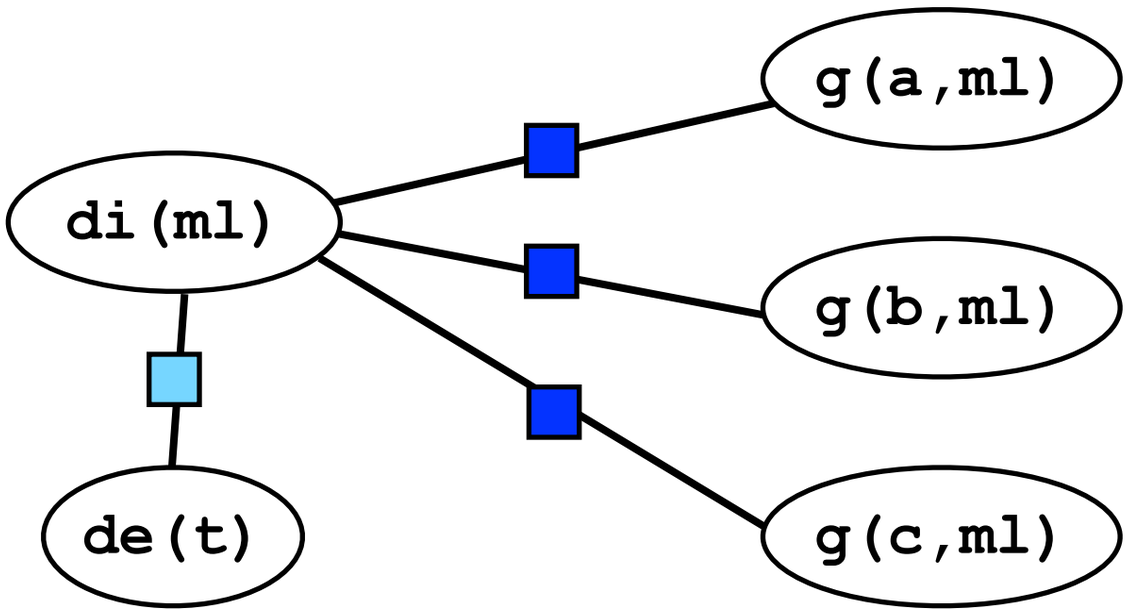


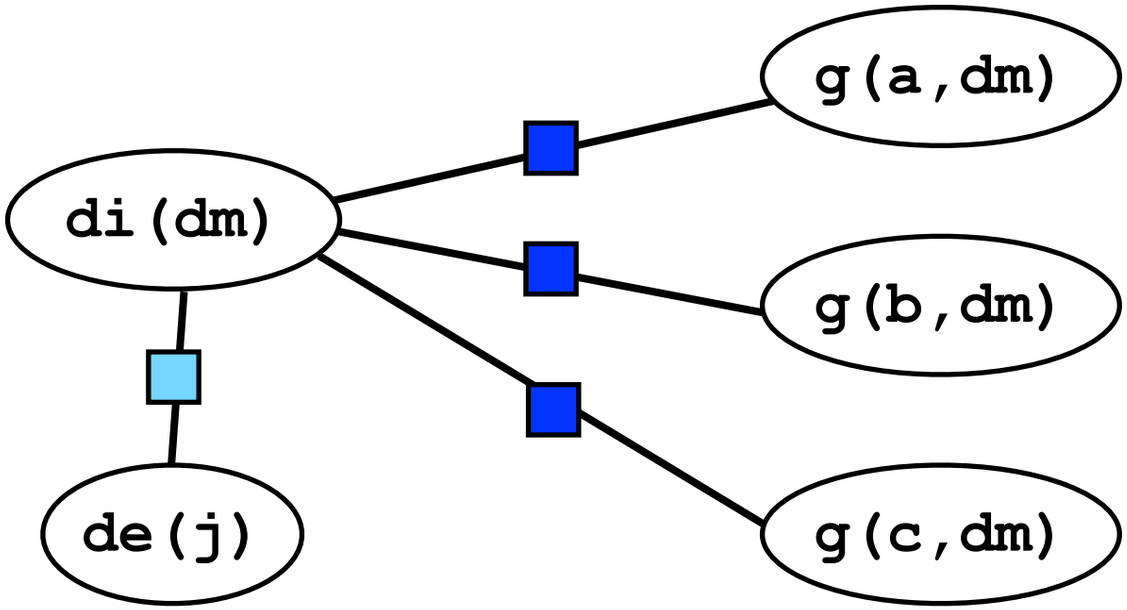


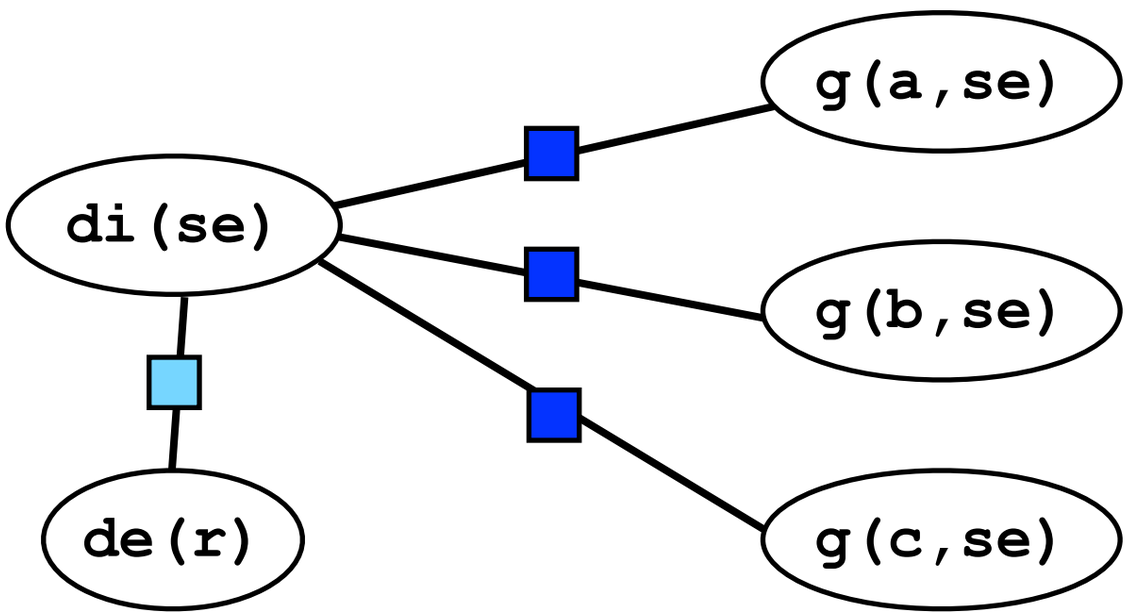
what is the probability that some professor is demanding?











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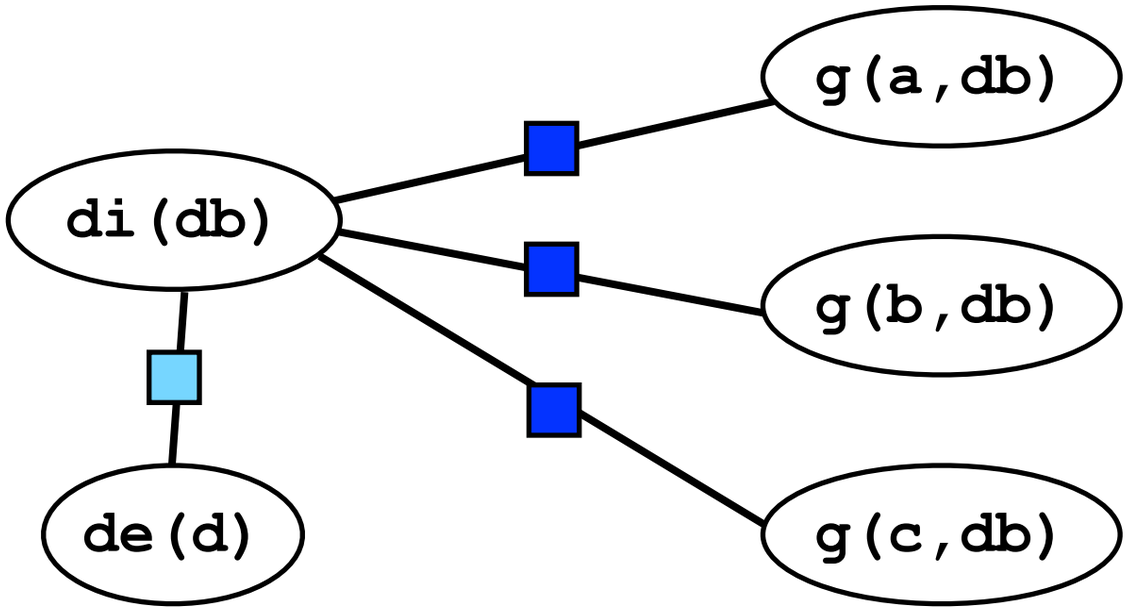
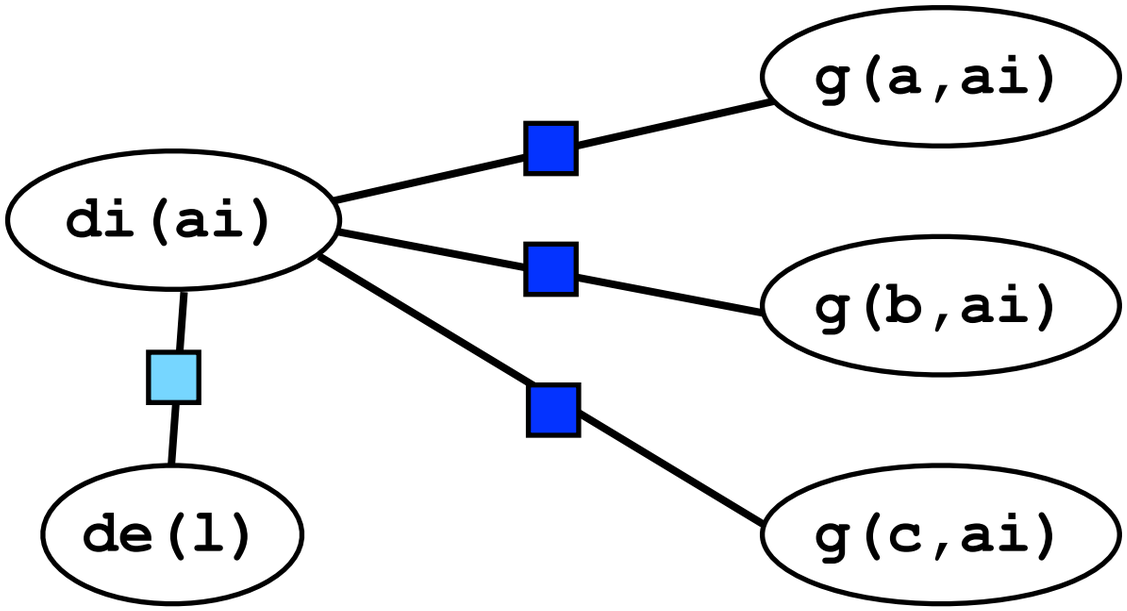
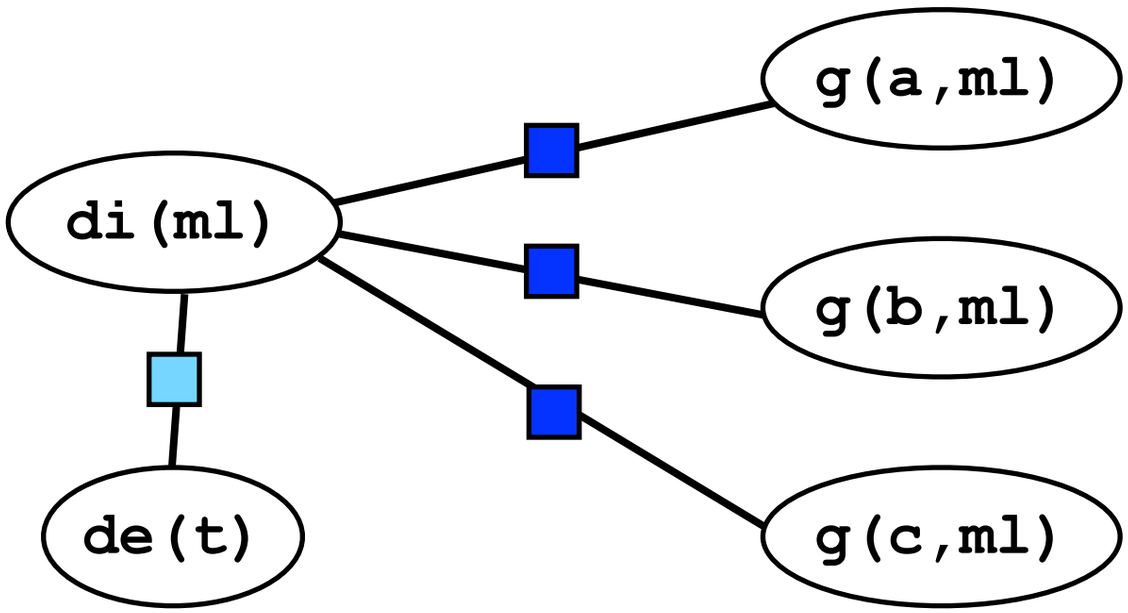
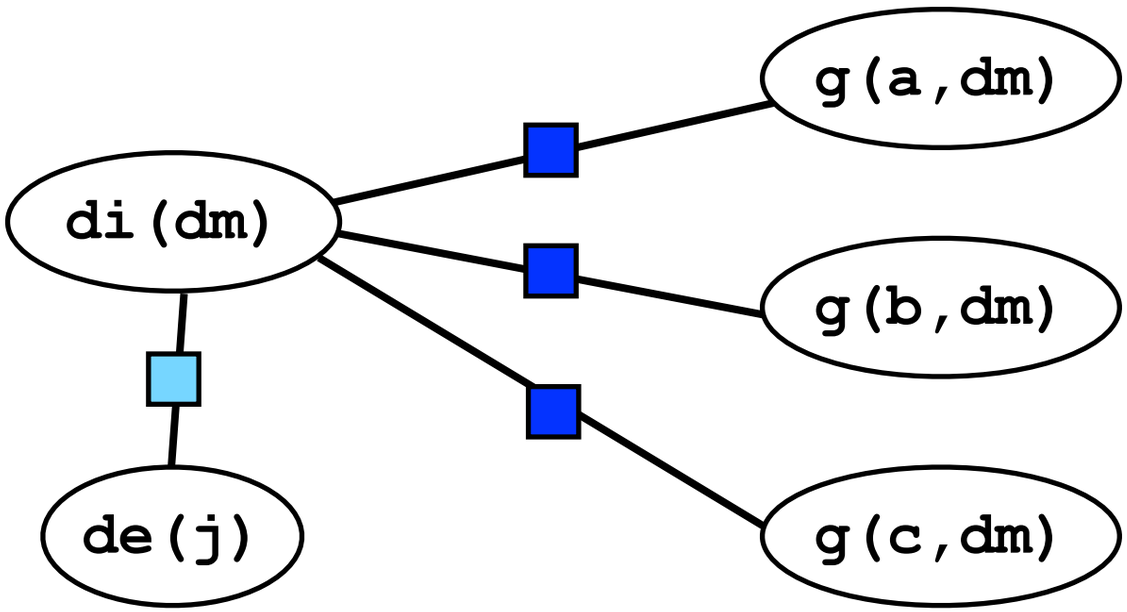
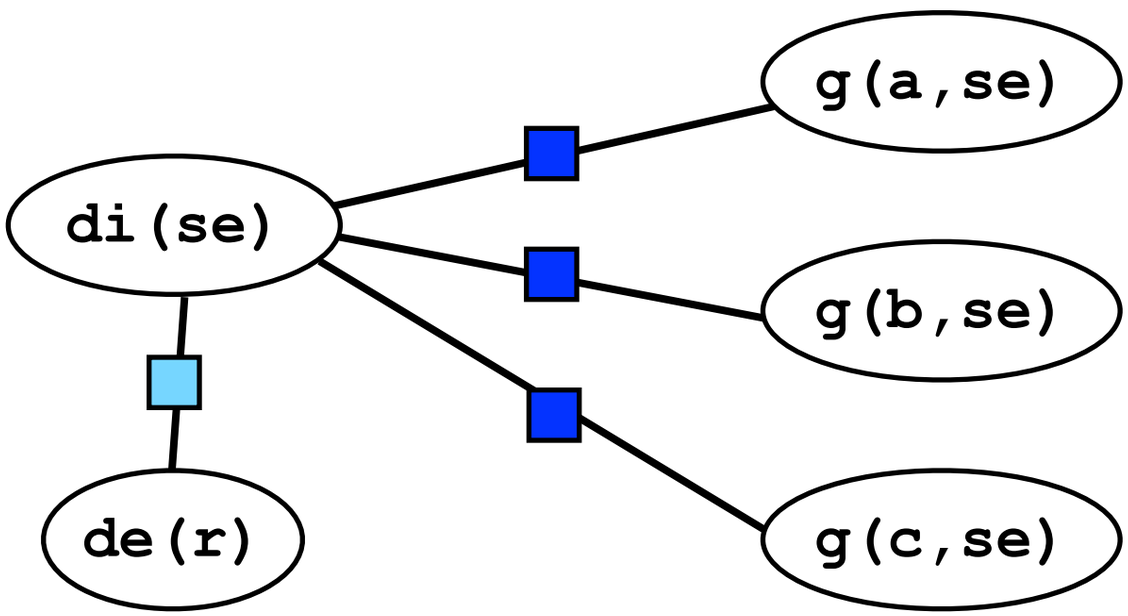
same computation for each professor!







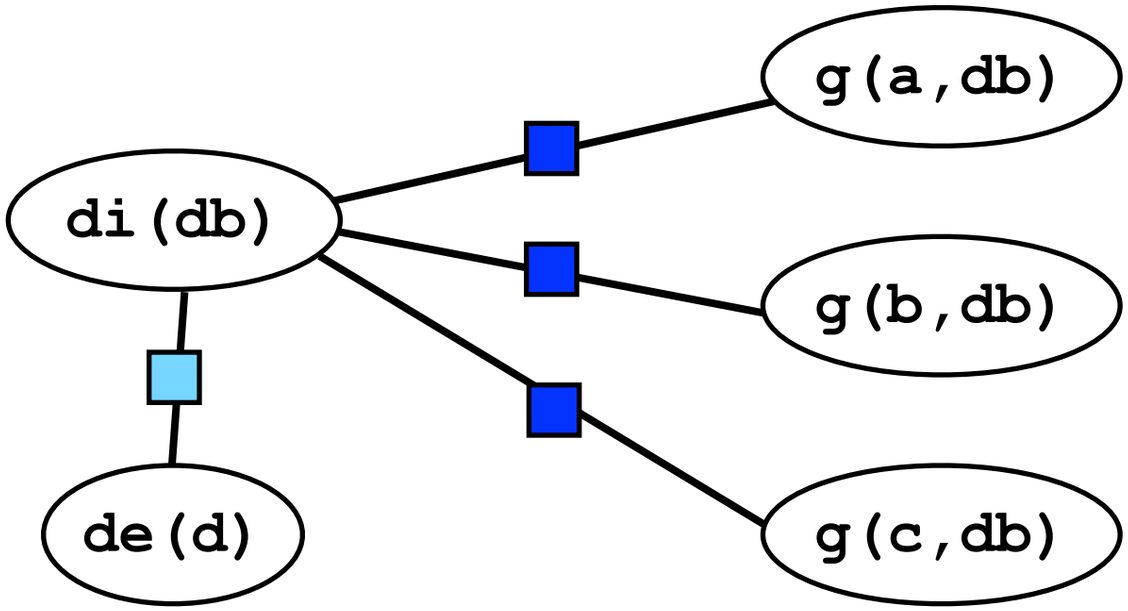


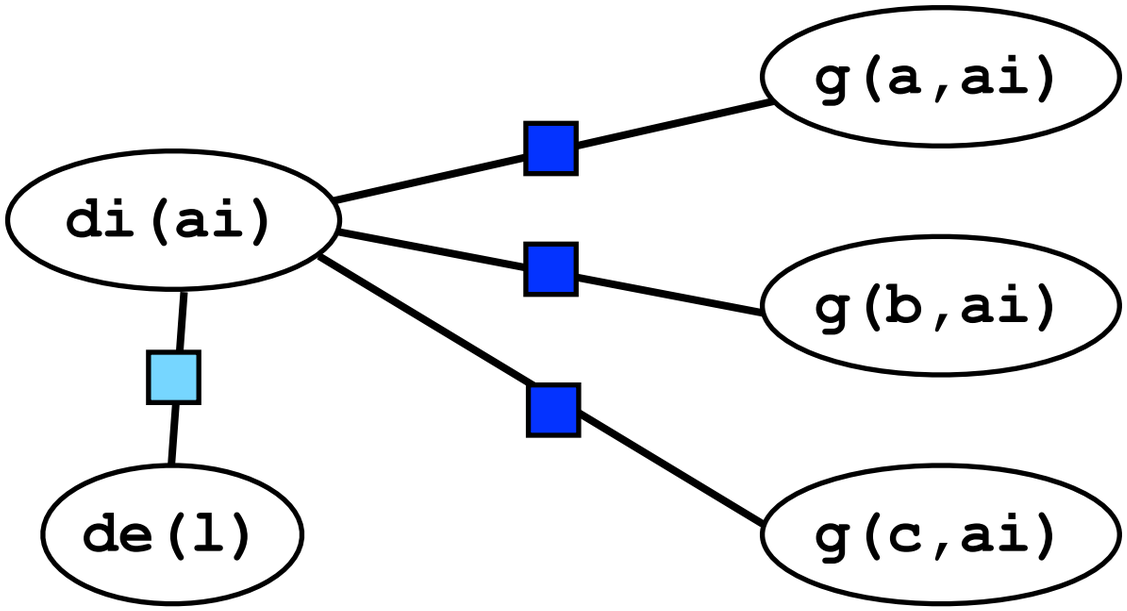


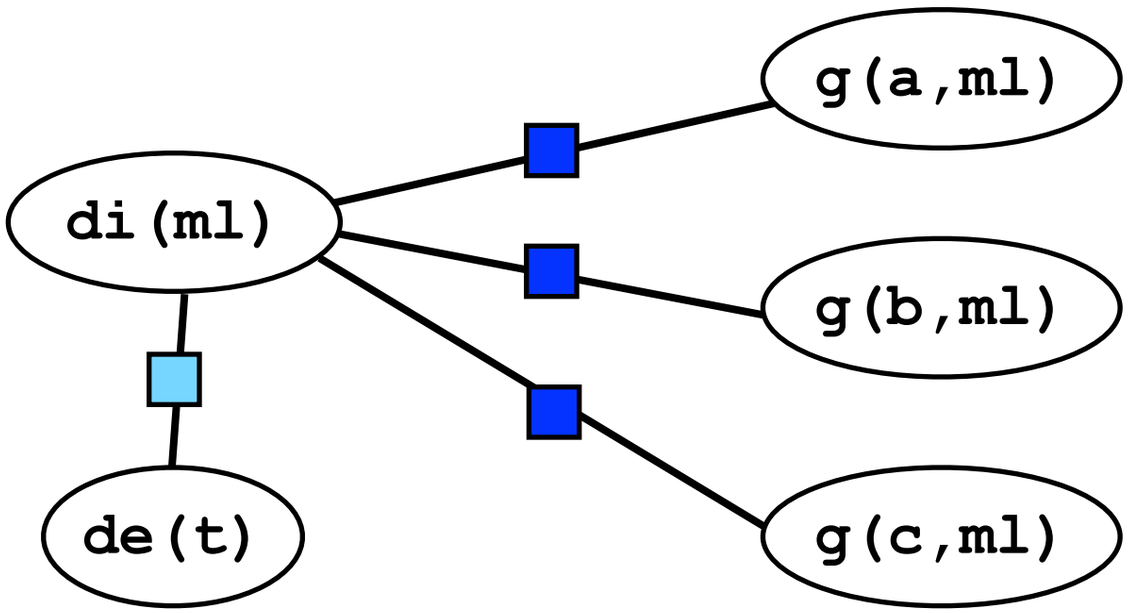
what is the probability that some professor is demanding?

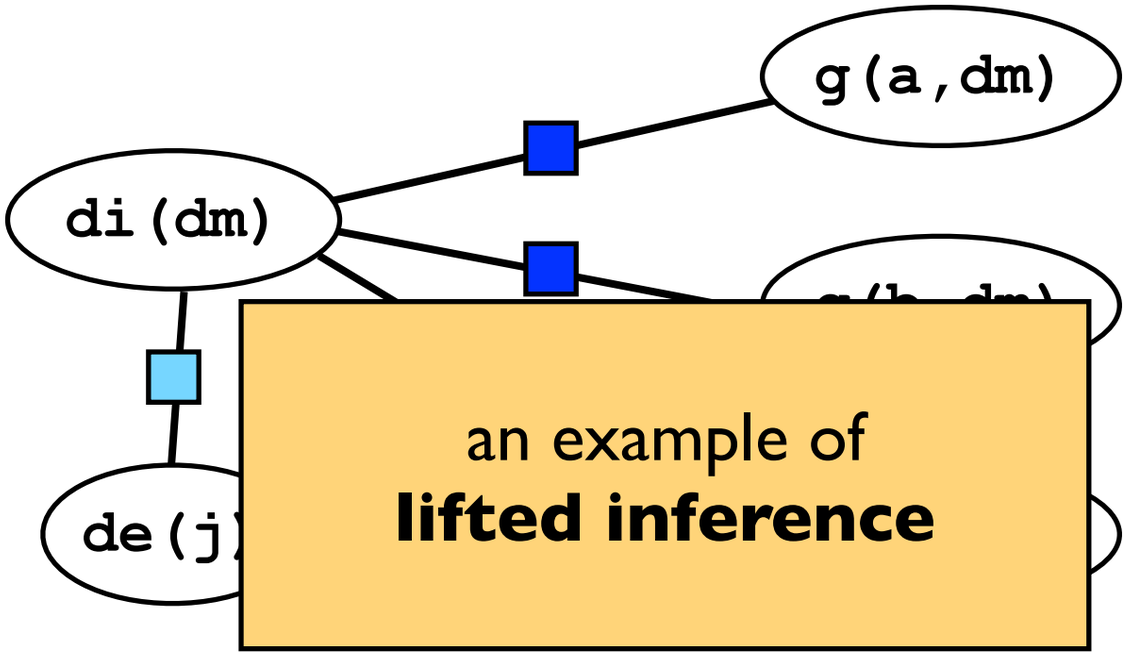
$$1 - \prod_{\mathtt{P}} (1 - P(\mathtt{de(P)})) = 1 - (1 - P(\mathtt{de(someP)}))^{\#\mathtt{P}}$$

$$= 1 - (1 - P(\mathtt{de(r)}))^5$$

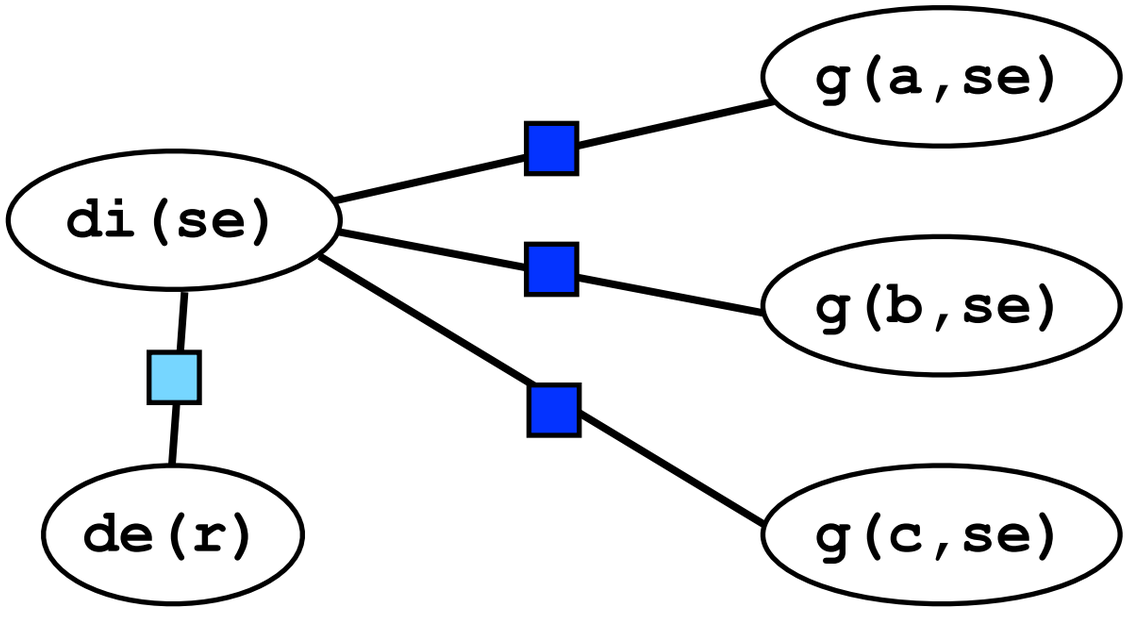








an example of
lifted inference



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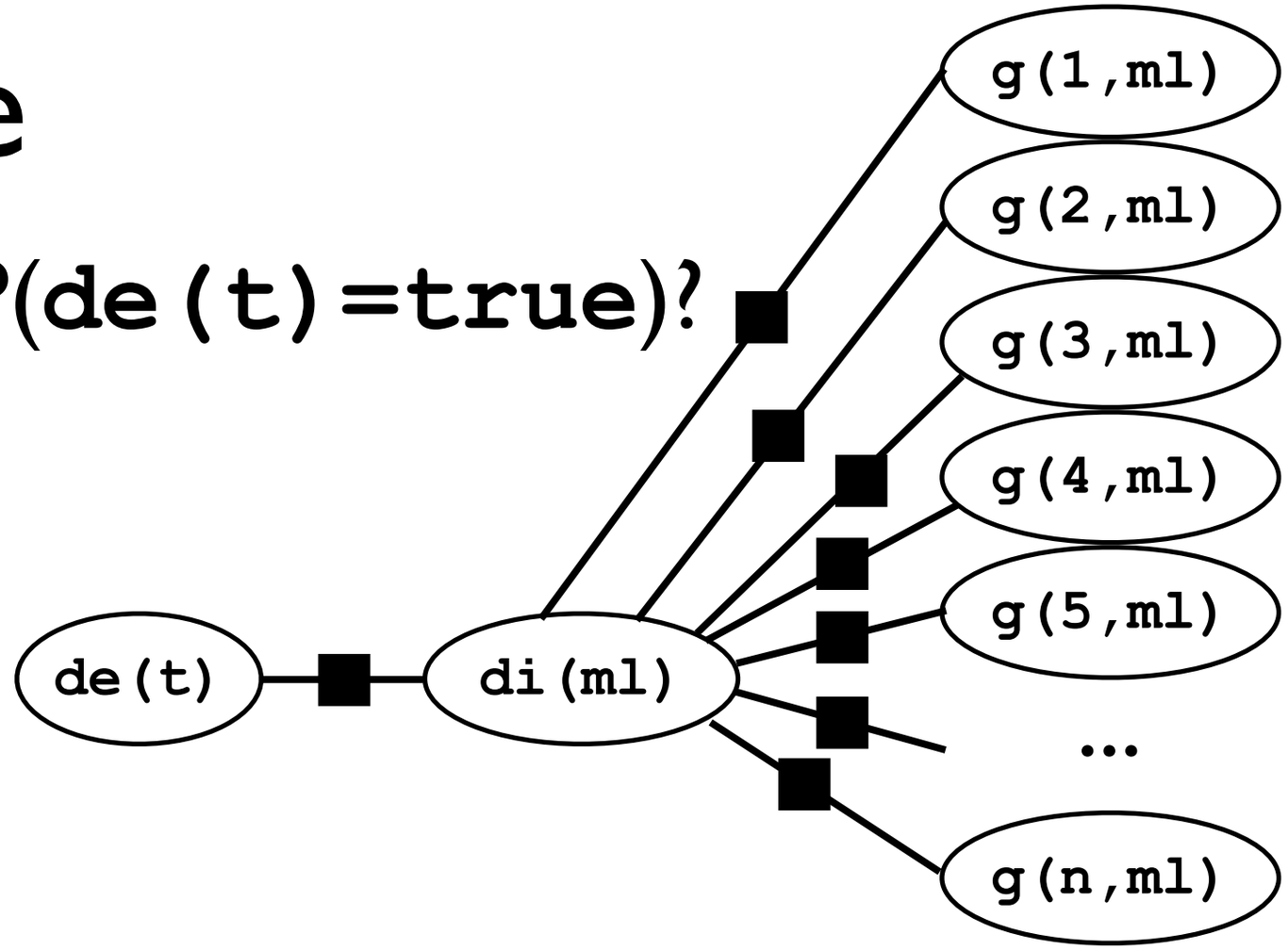
$$= 1 - (1 - P(\mathtt{de(r)}))^5$$

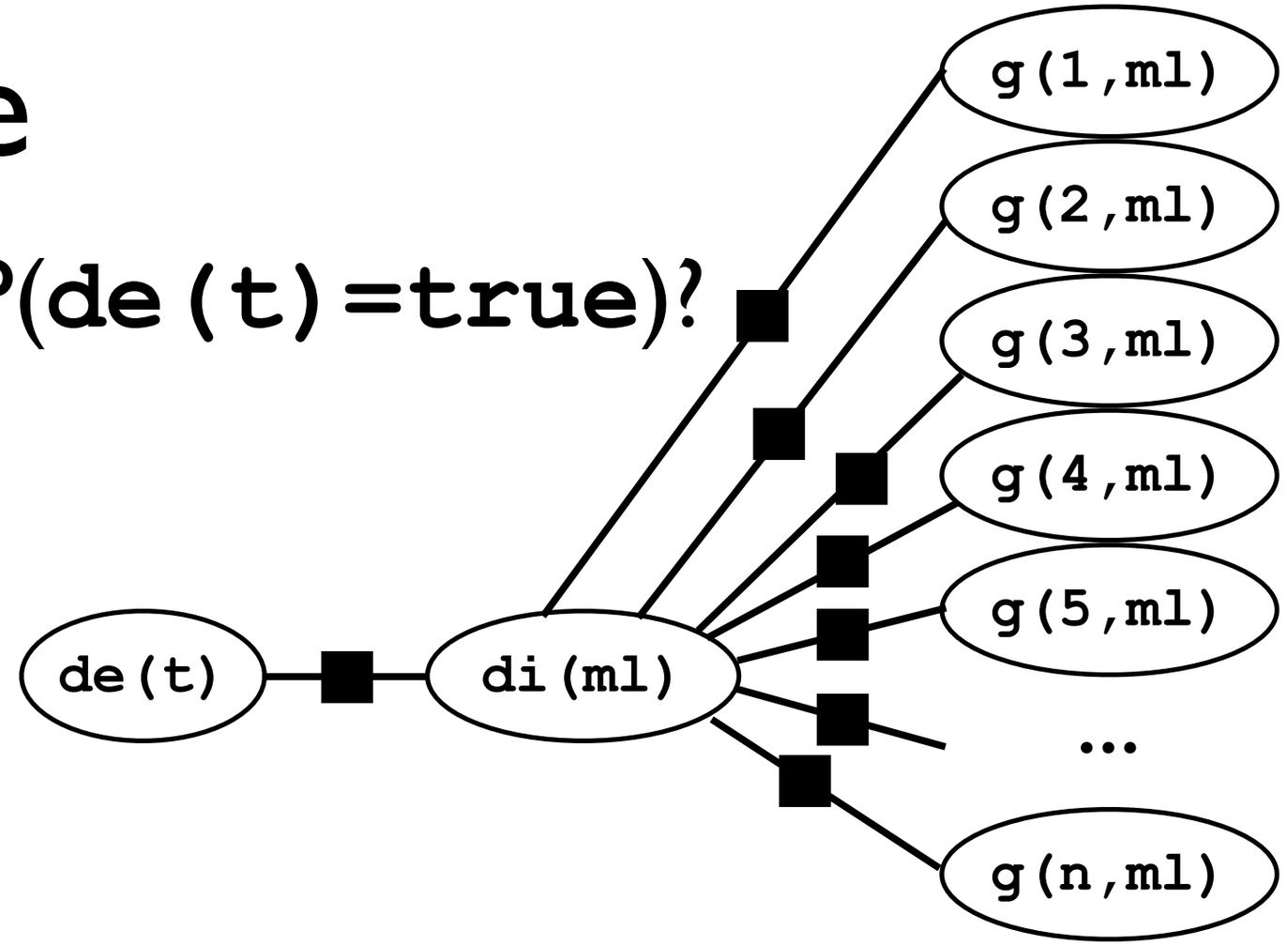
Another example



Another example

sum $\phi_2(\text{true}, d) \prod_{i=1..n} \phi_1(g_i, d)$

for all combinations of difficulty d and students' grades (g_1,...,g_n)



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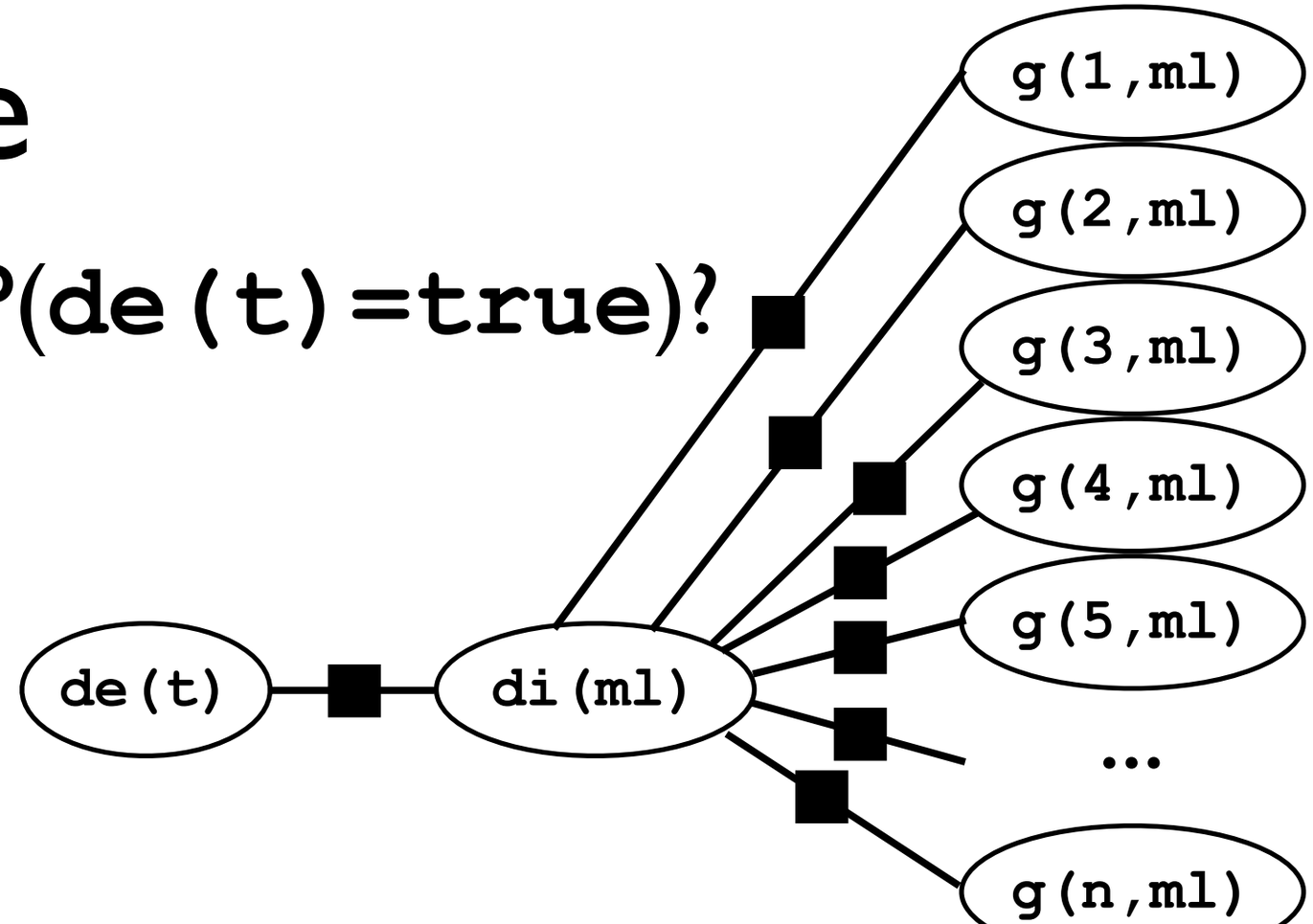
O(#grades$^{\#\text{students}}$) assignments to sum!



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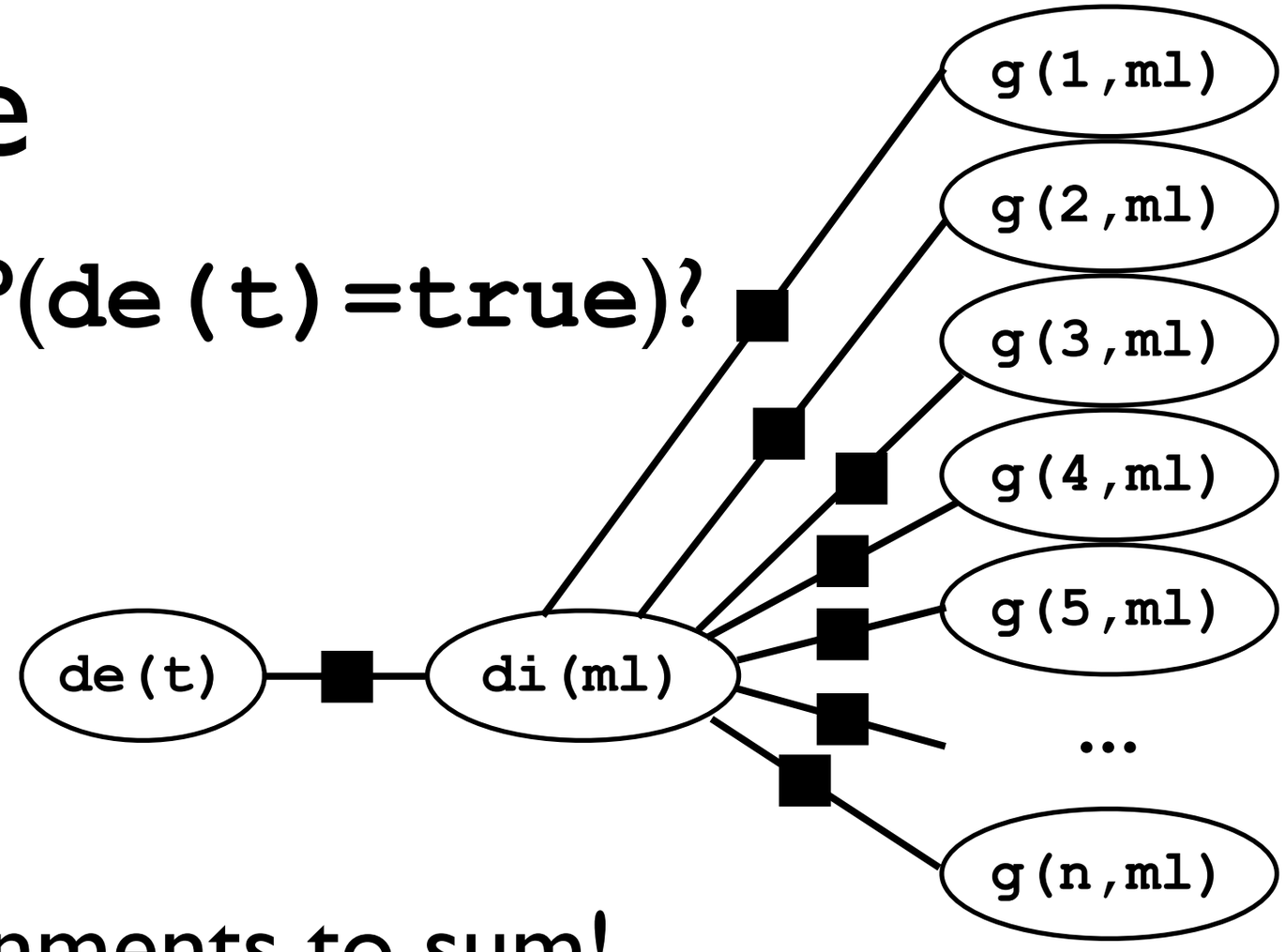

but: identity of students doesn't matter!

sufficient to count how often each grade m_1,...,m_k appears

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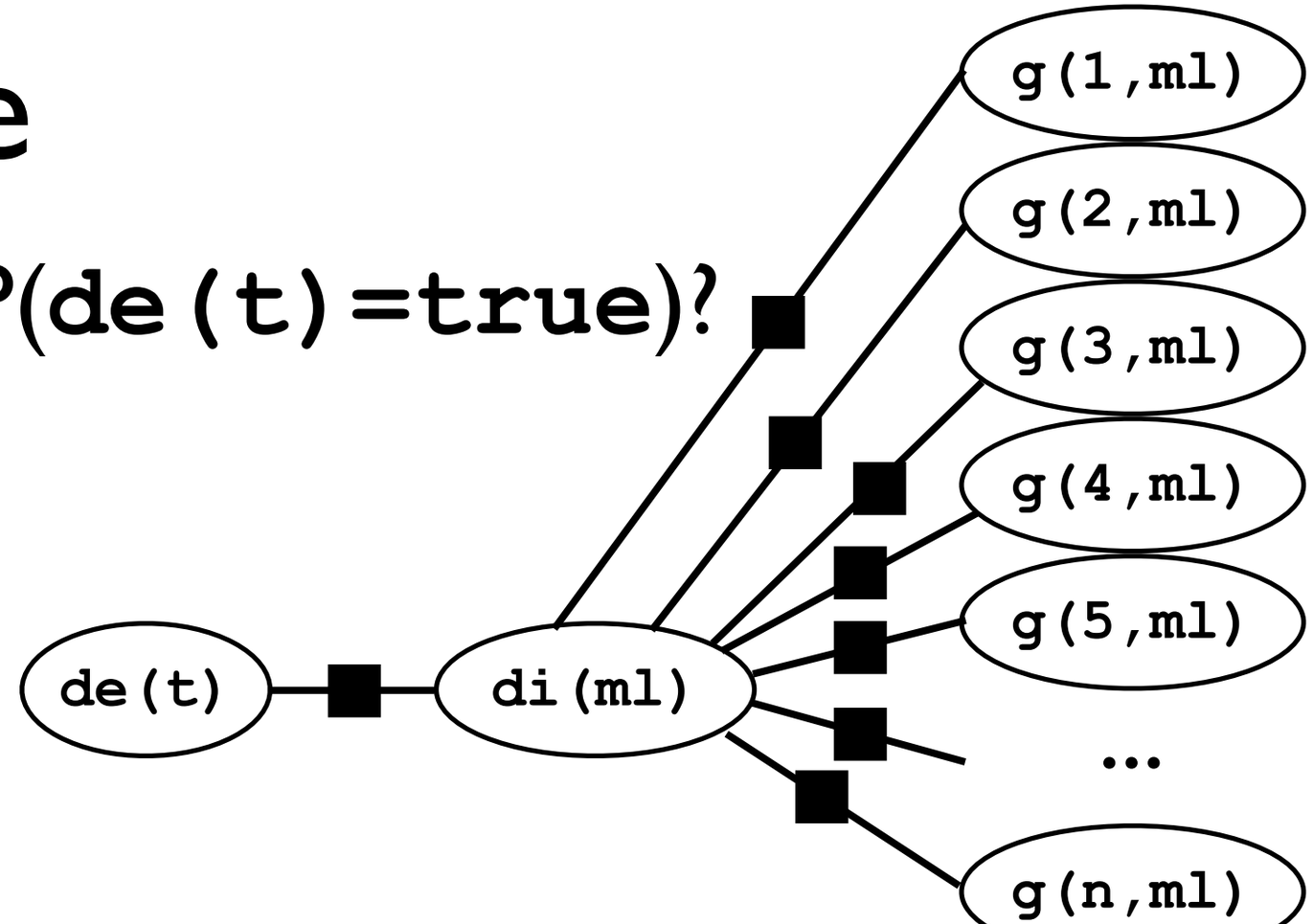

but: identity of students doesn't matter!
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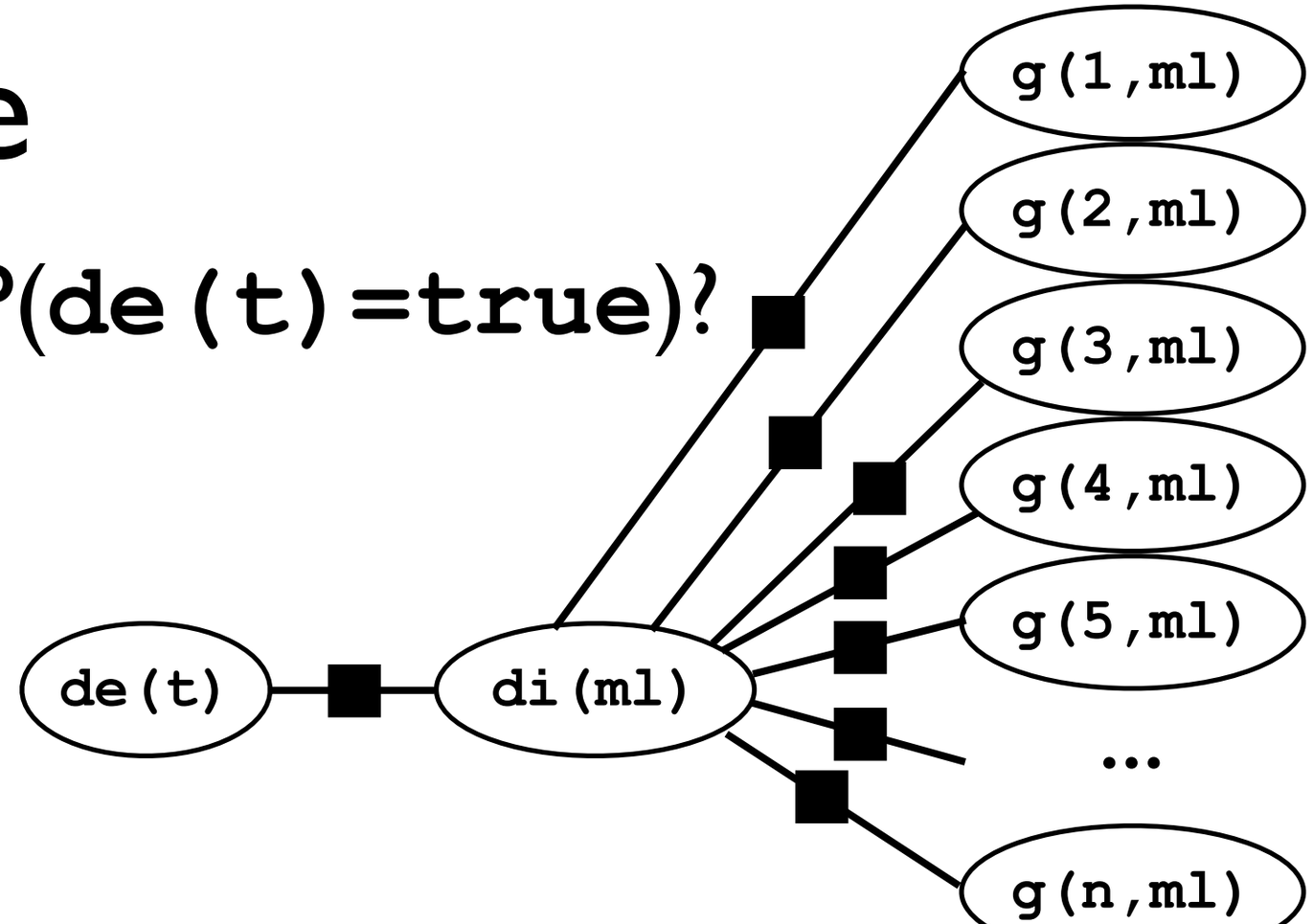
sum $\phi_2(\text{true}, d) \prod_{i=1..k} \phi_1(m_i, d)^{\#m_i}$ instead

Another example

sum $\phi_2(\text{true}, d) \prod_{i=1..n} \phi_1(g_i, d)$

for all combinations of difficulty d and students' grades (g_1,...,g_n)

O(#grades$^{\#\text{students}}$) assignments to sum!



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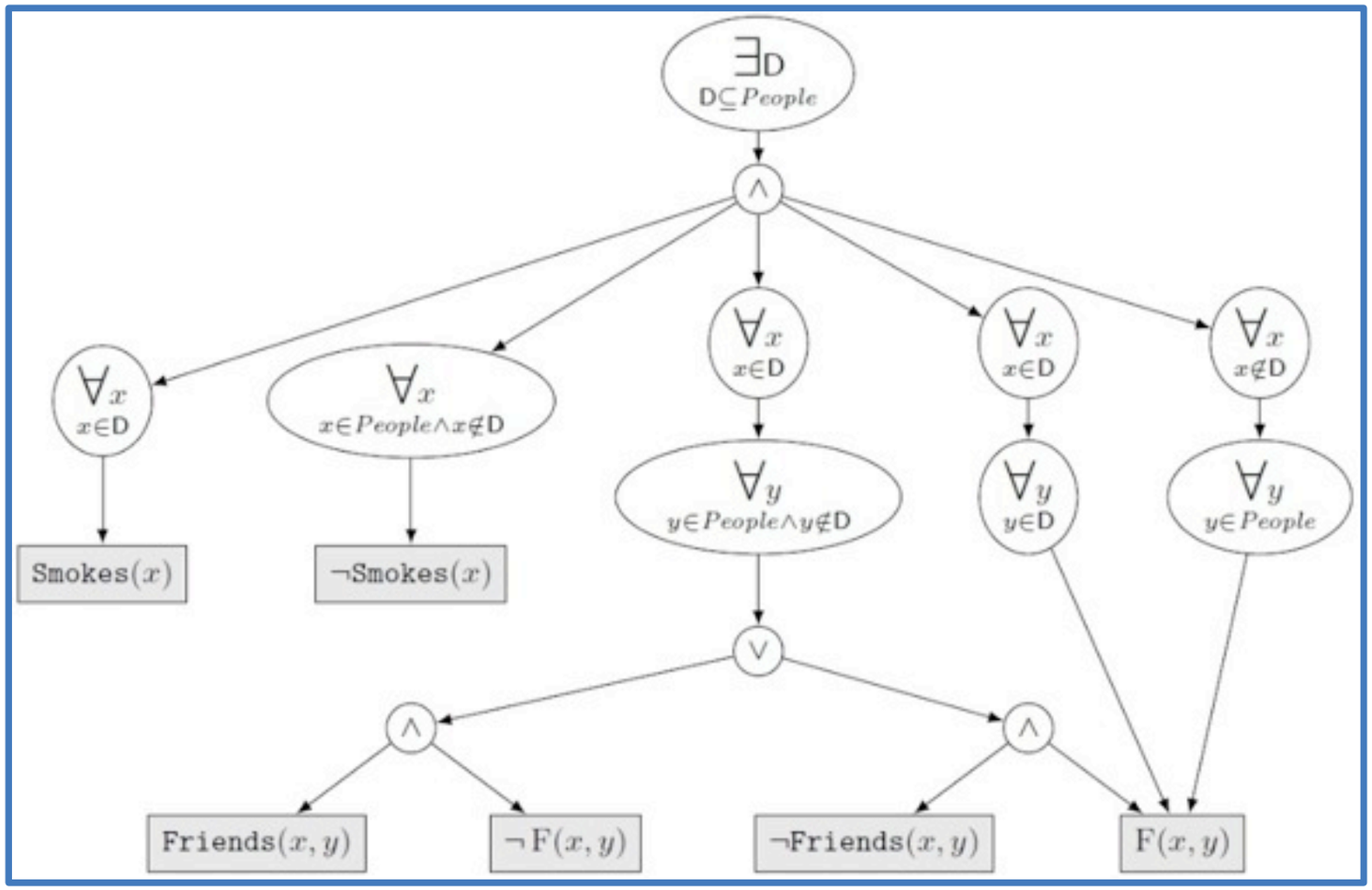
sum $\phi_2(\text{true}, d) \prod_{i=1..k} \phi_1(m_i, d)^{\#m_i}$ instead

e.g., k=3, n=15: >14M grade vectors vs 136 count vectors

Lifted Inference

- exploiting symmetries & repeated structure
- reasoning on first order level as much as possible
- aiming at independence from number of objects
- approximation: grouping similar computations
- very active research area

Example: Weighted First-Order Model Counting (WFOMC)



First-Order d-DNNF Circuit

Smokes → 1
¬Smokes → 1
Friends → 1
¬Friends → 1
F → exp(3.14)
¬F → 1

Weight Function

Alice
Bob
Charlie

Domain

Weighted First-Order Model Count is 1479.85

Roadmap

- Modeling
- Inference
- Learning & Dynamics
- Advanced Inference and KBMC

... with some detours on the way

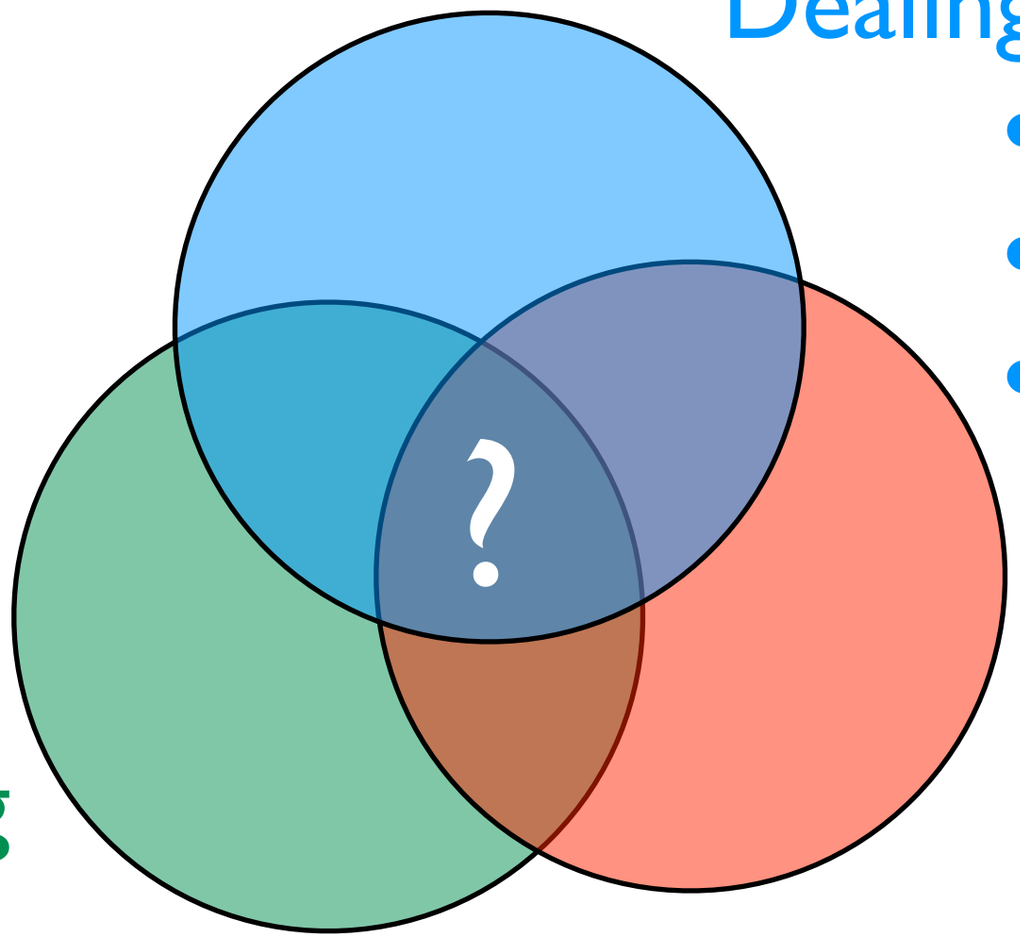
A key question in AI:

Dealing with uncertainty

- probability theory
- graphical models
- ...

Reasoning with relational data

- logic
- databases
- programming
- ...

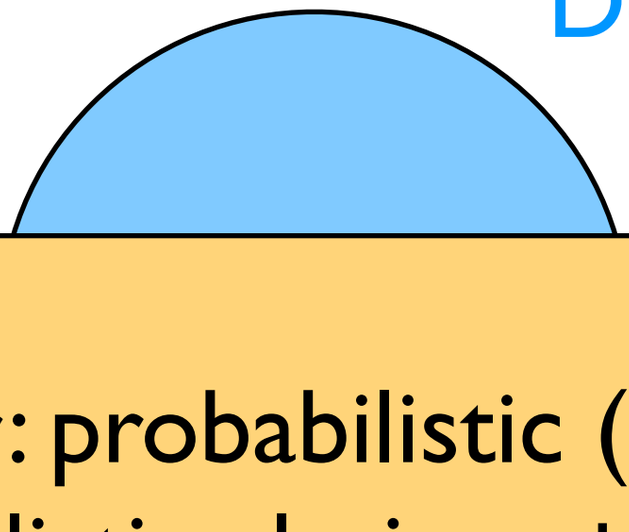


Learning

- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

A key question in AI:



Dealing with uncertainty
- probability theory

Reas
relat
- log
- da
- pr
- ...

- Our answer: probabilistic (logic) programming
  = probabilistic choices + (logic) program
- Many languages, systems, applications, ...
- ... and much more to do!

structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

Maurice Bruynooghe
Bart Demoen
Anton Dries
Daan Fierens
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Bernd Gutmann
Manfred Jaeger
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Theofrastos Mantadelis
Wannes Meert
Bogdan Moldovan
Siegfried Nijssen
Davide Nitti
Joris Renkens
Kate Revoredo
Ricardo Rocha
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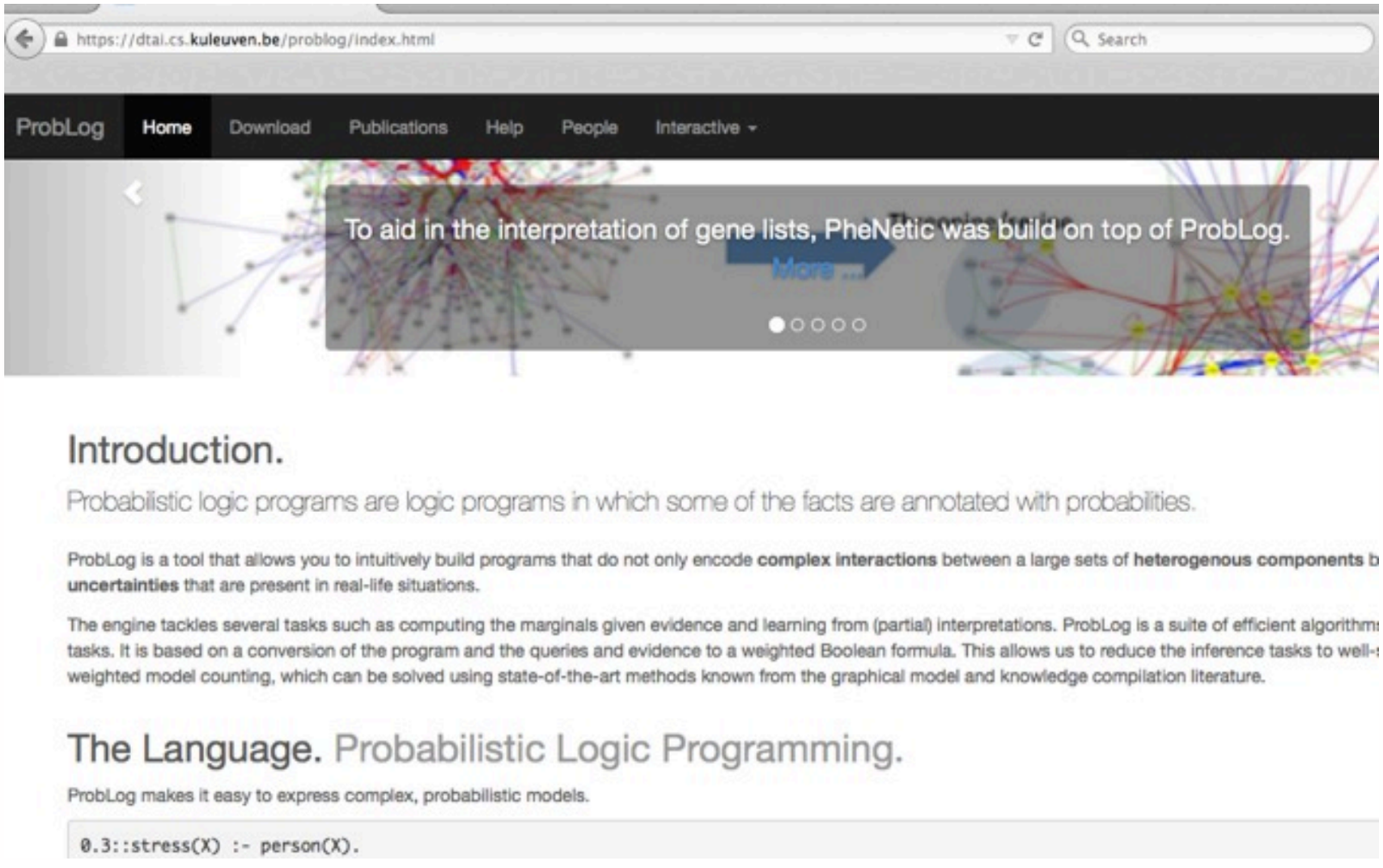
Dimitar Shterionov
Ingo Thon
Hannu Toivonen
Guy Van den Broeck
Mathias Verbeke
Jonas Vlasselaer

Thanks !

http://dtai.cs.kuleuven.be/problog



PLP Systems

- **PRISM** http://sato-www.cs.titech.ac.jp/prism/
- **ProbLog2** http://dtai.cs.kuleuven.be/problog/
- Yap Prolog http://www.dcc.fc.up.pt/~vsc/Yap/ includes
  - **ProbLog1**
  - **cplint** https://sites.google.com/a/unife.it/ml/cplint
  - **CLP(BN)**
  - **LP2**
- **PITA** in XSB Prolog http://xsb.sourceforge.net/
- **AILog2** http://artint.info/code/ailog/ailog2.html
- **SLPs** http://stoics.org.uk/~nicos/sware/pepl
- **contdist** http://www.cs.sunysb.edu/~cram/contdist/
- **DC** https://code.google.com/p/distributional-clauses
- **WFOMC** http://dtai.cs.kuleuven.be/ml/systems/wfomc

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