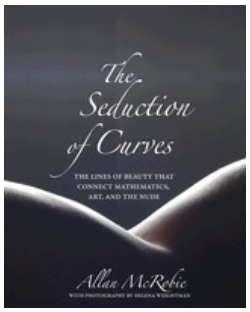
The seduction of curves: The lines of beauty that connect mathematics, art, and the nude by *Allan McRobie*, with photography by *Helen Weightman*, Princeton University Press, ISBN 978-0-691-17533-1 (hbk), 2017.

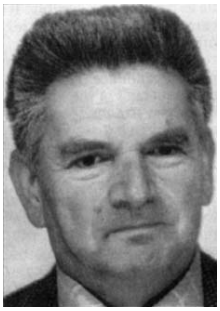




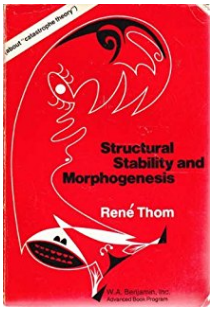
Allan McRobie

Many books do exist that link mathematics and art. Some of them were reviewed in this Newsletter or the Bulletin before [1–8]. The main geometric topics treated are symmetry and polyhedra with applications in architecture or it is placed in a cultural historical perspective as in [4]. Much of this involves straight lines, but McRobie has chosen for curves and more particularly he has used the catastrophe theory of René Thom as a guide and illustrates it by many pictures and graphs. That could have led to quite complicated mathematics, but he has left that wisely under the carpet and uses only graphical arguments.

Catastrophe theory is a special case of singularity theory in geometry, that has applications in the analysis of dynamical systems and bifurcation theory. The theory was started by René Thom in the 1960's, later made more polular by Christopher Zeeman. The equilibrium of the system corresponds to the minimum of a smooth potential manifold. The manifold has folds which may explain sudden changes in the behaviour of the system when there is a drop from a top fold to a lower fold. By projecting the manifold on a lower dimensional space one obtains the curves or the umbilics. Catastrophes are the singularities in the projections of manifolds. In the order of increasing complexity the elementary catastrophes considered by Thom are the fold, cusp, swallowtail, and the butterfly which are all one-dimensional curves. For two-dimensional surfaces one may have elliptic, hyperbolic, or parabolic umbilics. These are the 7 fundamental shapes for at most 2 active variables, and at most 4 active parameters. There are however several other variants, like the wigwam, the lips, the beaks, the gull and the goose. All this is beautifully illustrated by artistic (mostly black and white) pictures of nude bodies or details of hips, shoulders, neck, or whatever detail illustrates the idea. By projecting parallel lines on a body, the Moiré effect shows which parts are elliptic and which parts are hyperbolic.



R. Thom



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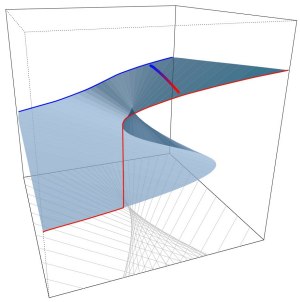
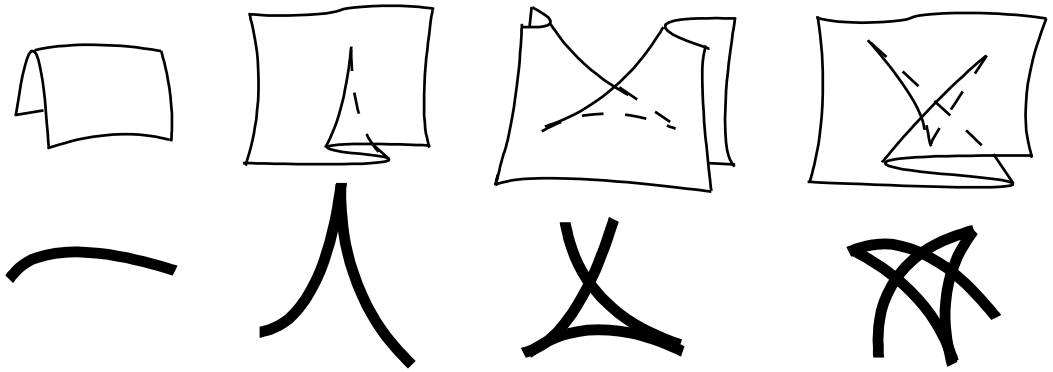
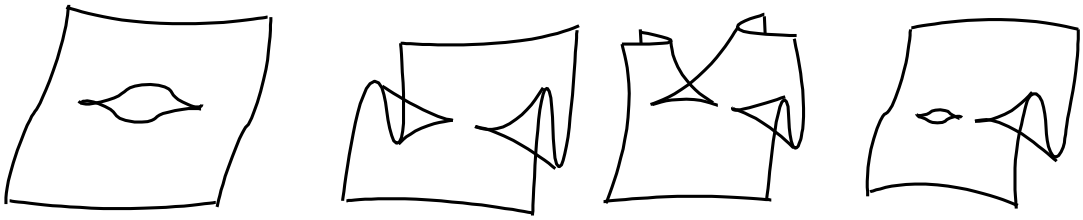


Fold & Cusp



Fold Cusp Swallowtail Butterfy

There are however also more technical applications like for example the butterfly that defines the boundaries of the stability regions for the centre of gravity of a ship that will heel and even capsize when not kept inside the stable region. Or the cusp that is typically connected with the buckling of a steel pillar. The author is a structural engineer who used to design bridges and towers, but who is now teaching at the University of Cambridge. So that explains these examples that he can take from his own experience.

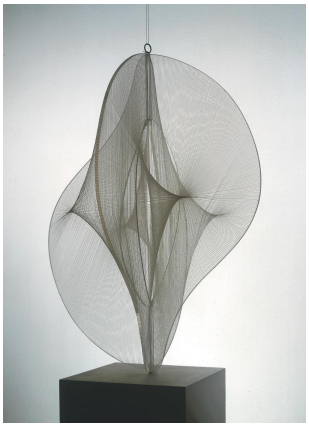


Lips Beaks Gull Goose





David Hockney



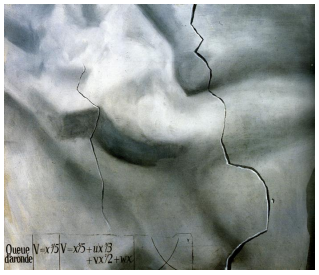
Naum Gabo
Linear Construction No. 2

The umbilics are 2D often self-intersecting objects in 4D space which is more difficult to illustrate. The results of the foldings and the corresponding singularities are illustrated by reflections on a calm sea surface, caustic effects on the floor of a swimming pool, or by a glass of water on the table, and even the formation of a rainbow. For example the multiple images of the sun reflected by a small wavelet in the sea is the result of optical folding that multiplies the images. Similarly gravitational lensing will show multiple images of a light source hidden behind a massive object. The observations allow to compute the mass distribution in the obscuring object, or it allows to get information about a massive planet orbiting a star. McRobie also discusses the morphogenesis part of Thom's book. Although biologists do not seem to pick up these ideas, it was the intention of Thom to explain the catastrophe that allows the asymmetry between male and female genitals while keeping left-right symmetry. McRobie suggests that perhaps it is a higher symmetry in the chemical processes that govern the development of the embryo that results in this kind of evolution.

The trailing part of the book is devoted to catastrophe theory in art. Simple line drawings have only folds (the outlines) and cusps. But as painters, sculptors, and architects developed their skills, more complex swallowtails started showing in their work, long before Thom developed his catastrophe theory. For example folds in the fabric of the clothes were shown in paintings. That was new. In sculptures, it was previously the responsibility of the observer to look at the object from the right angle to detect them. In paintings, it is the artist who is the observer and represents them in his painting. But it is not only the human body that shows these curves, other illustrations of Thom's theory are represented. For example David Hockney painted the flickering of the sun in swimming pools. However, the first one to really use Thom's form language was Naum Gabo, a contemporary of Thom, although they never met. In his *Linear Construction No. 2* (1970-71) made of strings, so that the surface can intersect itself so that the singularities are clearly seen. Organic forms have also entered architecture. Gaudi is an obvious example but others have followed: Frank Gehry, Santiago Salatrava, etc.

Why do we find curves beautiful and pleasing? McRobie devotes a chapter pondering this question and ascribes this to an inherent result of evolution, something that is hidden in our genes. He even refers to the Berlin School of *Gestalt* psychology. He discusses *The Analysis of Beauty* (1753), a book by William

Hoghart, in which he praises the beauty of the S-shaped curve, which is also the basic shape in catastrophe theory. Alan Hollinghurst refers to this in his Booker Prize novel *The Line of Beauty* (2004).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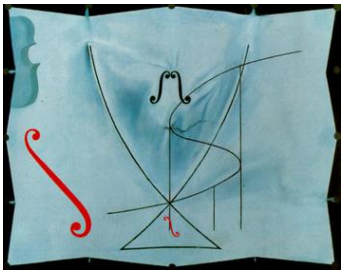
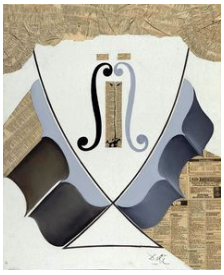

The Swallow Tail...




Other examples

In the penultimate chapter McRobie discusses the person of René Thom (1923-2002) and his work. Thom received the Fields Medal (1958) and the Brouwer Medal (1970). When he drifted away from topology and started applying his catastrophe theory to biology, morphogenesis, linguistics and epistomology, he got critisized by Smale who proposed chaos theory as a much more sound and general alternative. Also Vladimir Arnol'd did not approve Thom's philosophical writing style. His semi-mystical writings however were attractive amongst the postmodernist community. When in a 1980 publication he rejected indeterminism in sciene he earned Ilya Prigogine as a new opponent. Salvador Dalí was an admirer of Thom's work and organized in his Museum in Figueres a debate in 1985.



Dalí is the subject of the last chapter. He called Thom's theory "the most beautiful aesthetic theory in the world". When Thom told him he was using his theory on tectonic plate movements, Dalí made his penultimate painting *Topological Abduction of Europe: Homage to René Thom* (1983). It shows the text *Que d'aronde* (swallowtail) and Thom's formula $V = x^5/5 + (ux^3)/3 + (vx^2)/2 + wx$ and the swallowtail graph. The crack symbolises the catastrophe. His last painting in May of the same year was *The Swallow's Tail – Series of Catastrophes*.

This is a marvelous book, printed in two columns on glossy paper, with ample illustrations. The technical discussion is only minor and the formulas are left out. McRobie does hint at the applications and explains some at a rather high level, but this is nothing to be afraid of if you are not a mathematician. Thom has raised a lot of controversy in the later part of his life, but he has several quite reknowned contributions to mathematics too, but these are left aside in this discusson. A coffee-table book? Yes but there is plenty of interesting stuff to read too.

Adhemar Bultheel

References

[1] Filipe Cucker *Manifold Mirrors, The Crossing Paths of the Arts and Mathematics.* Cambridge U. Press, 2013. (BMS-NCM News issue 96 (2014))

[2] Michele Emmer *Mathematics and Culture I-VI.* Springer, 2004-2009.
A series of books. (BMS-NCM News issues 50 (2004) & 56 (2006) EMS Review here)

[3] Michele Emmer *Imagine Math 1-3.* Springer Verlag, 2012-2015.
Book series. (BMS-NCM News issue 93 (2013), EMS Reviews here & here)

[4] Lynn Gamwell *Mathematics and Art: A Cultural History.* Princeton U. Press, 2015.
A marvelous art book, relating mathematics, art and philosophy (EMS Review here)

[5] Georg Glaeser *Geometry and its Applications in Arts, Nature and Technology.* Springer, 2012.
Many pictures but with emphasis on the applications. (EMS Review here)

[6] Alexander J. Hahn *Mathematical Excursions to the World's Great Buildings.* Princeton U. Press, 2012. Here the shapes are more defined by conic sections. (EMS Review here)

[7] D. Schattschneider, M. Emmer *M.C. Escher's Legacy, a Centennial Celebration.* Springer, 2003. (BMS-NCM News issue 60 (2006))

[8] Marjorie Senechal *Shaping Space. Exploring Polyhedra in Nature, Art, and the Geometrical Imagination.* Springer, 2013. (EMS Review here)