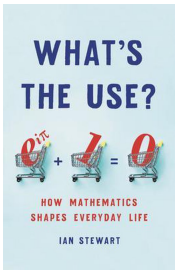
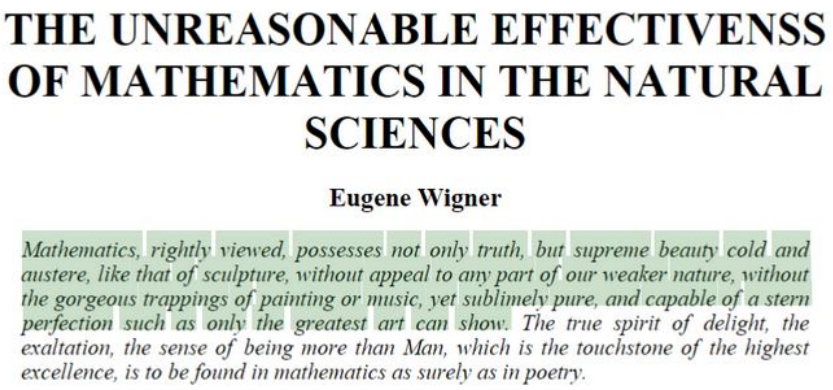
What's the use? How Mathematics Shapes Everyday Life by *Ian Stewart*, Basic Books, 2021 (263 p.), isbn: 978-1541699489.




Ian Stewart

This is not the first book written about the use of mathematics in everyday's life. There are *Figuring it out* (N. Crato, 2010), *Raising public awareness of mathematics* (Berends et al, 2012), *The grapes of math* (P. Bellos, 2014), *A numerate life* (J.A. Paulos, 2015), *Mathematics in everyday life* (J. Haig, 2016), and *The mathematics that power our world* (J. Khoury et al, 2016), to name just a few. Sooner or later, Ian Stewart was bound to add a simiar one on top of his pile of popularizing books on mathematics. In his usual witty entertaining style, he does again an excellent job in contradicting the sigh of resignation that is heard all too often: Why should I care about mathematics, I have no use for it in my life.

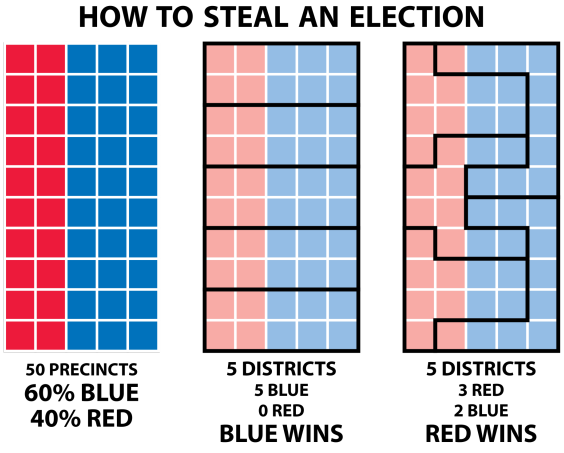

THE UNREASONABLE EFFECTIVENSS OF MATHEMATICS IN THE NATURAL SCIENCES

Eugene Wigner

Mathematics, rightly viewed, possesses not only truth, but supreme beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry.

The way Stewart tackles it is to start his book with an introductory chapter, which explains very well the meaning of Wigner's often cited quote about mathematics being so unreasonably effective. A typical example of this is the fact that the number π, which is the ratio of the circumference of the circle divided by its diameter, a purely geometric concept, shows up is so many formulas and applications that have not the slightest relation to geometry. But there are of course many other examples of this unreasonable effectivity, where some concepts in mathematics turn out to be applicable in totally different areas. Abstraction, so characteristic for mathematics, is of course an essential element in explaining why this is happening so often. But Stewart goes over six ingredients to answer *why* mathematics is so effective and he concludes his first chapter with the line: "Reality, beauty, generality, portability, unity, diversity. Which, together, lead to utility. It's as simple as that." This statement is amply illustrated in the thirteen subsequent chapters of the book.

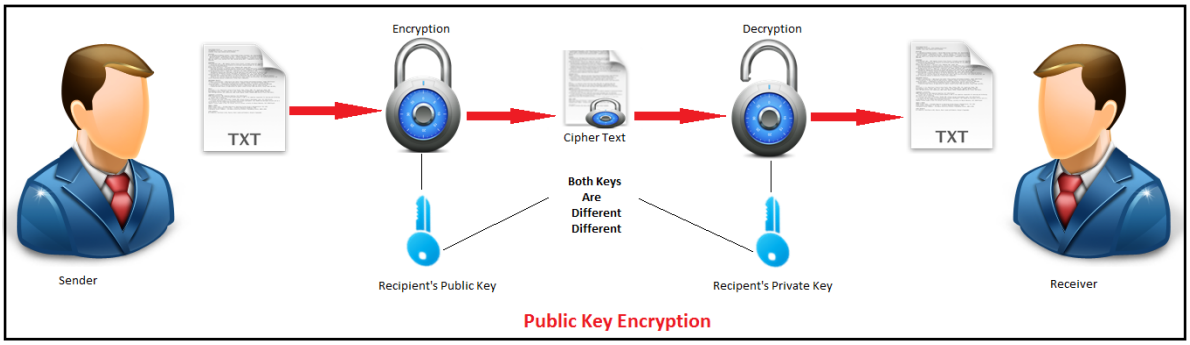
Much of our current mathematics finds its origin in Greece, which is also the cradle of a democratic society. So chapter two starts with an appropriate application of mathematics for voting systems, in particular a discussion of how gerrymandering (a phenomenon where the voting district is designed in a particular way so that a political party optimizes the number of its elected representatives) can be recognized by using mathematical analysis about the shape of the district, the bias that it causes, etc. The discussion is a mixture of simple examples that illustrate the ideas and real cases from USA and UK to show that this is a realistic problem.



simple gerrymandering examples

The next chapter deals with the traveling salesman problem (TSP) and related issues, including a discussion of complexity (e.g. the $P \neq NP$ millennial problem) and heuristic methods to find approximate solutions for TSP. But we are also

challenged to think about seemingly simple concepts such as the length of a curve (which is not trivial if it involves fractals), space filling curves (with the curious effect that every point in a square can be identified with one coordinate along the curve, so what is the dimension then?), Euclidean geometry (which is only valid in a flat space, and does not hold when relativity theory comes into play), the infinity concepts bringing along Peano, Cantor, Dedekind, and Hilbert (there are as many points on an interval as there are points in a square!), the influence of artificial intelligence (self-driving cars needing computer vision and image recognition, and error correcting codes to correctly understand road signs), and it's all mathematics!



Public key encryption

This idea of showing how many branches of mathematics are involved starting from a simple everyday problem is illustrated in all the other chapters too. The next one starts with a problem of kidney transplantation. Several patients may need a kidney and there may be several potential donors, but there is a problem of medical compatibility and a problem that some donors prefer to donate to a particular patient, for example because it is a relative. This matching problem can be cast into a graph theoretical problem, just like Euler solved the problem of the walking tour crossing all the bridges of Königsberg exactly once. Other chapters link number theory to cryptography and explain the RSA code, elliptic curves and quantum computing. More numbers are explaining the historical evolution of number systems to arrive at complex numbers and their unreasonable effectiveness in quantum mechanics, electrical engineering, optics, etc. Quaternions get their own chapter with applications in computer graphics.

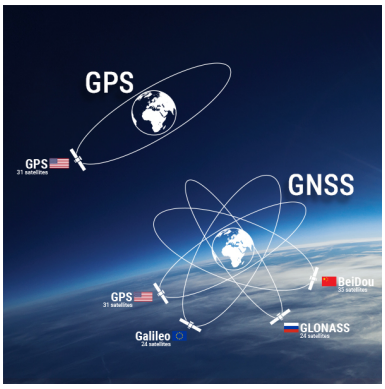


AI does not always distinguish between a puppy and a muffin.

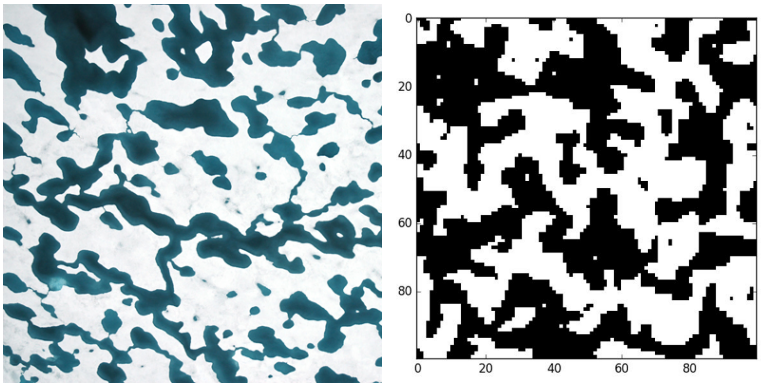
Chaotic systems and stochastic ones (a chaotic system is predictable over short periods, a random system isn't) is linked to quality control of the coiling process in spring production. Fourier and other transforms are essential to analyse the scans (X ray, CAT, PET, MRI, NMR...) that people are submitted to in the hospital. In this chapter we also meet functionals, calculus of variation, and computer tomography. And talking of images, digital pictures are nowadays only manageable because of compression techniques such as jpeg using the cosine transform and the associated ways of coding colour images. But there are other methods for compression like iterated function systems during the heydays of emerging fractal geometry, although that has now been largely abandoned for compression. Wavelets on the other hand were successfully incorporated in modern jpeg and they are now conquering several other applications a well.

All new cars have standard GPS and satnav to bring you from point A to point B on our globe. We take this all for granted, but the mathematics involved is enormous: Bringing up satellites above in a geostationary position, the production of the gadget in the car, that has to find the satellites, compute their positions, regulate the communication between the car and the satellites, taking into account relativity theory, and solving a TSP-type problem to compute your route, estimate the time of arrival, track the car as it moves,... it's an unreal mathemagical marvel.

Another excellent example of how mathematics connects completely different fields is the Ising model. Because of climate change the melting polar ice forms ponds that are irregularly connecting to form some random black-and-white patterns corresponding to melted-and-frozen ice. These patterns bear a remarkable similarity with patterns of electron spin orientation in a ferromagnetic material when simulated by the Ising model. In fact the Ising model is in a so called universality class, meaning that models at quite different scales all behave asymptotically the same near critical exponents where phase transitions take place. The formation of hese Arctic melt ponds is important for climate change since they influence the amount of sunlight reflected, and hence the heat absorbed by the earth.



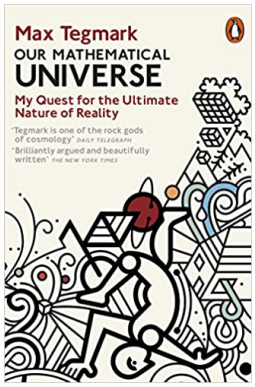
GPS is American
GNSS joins several systems



Left: melt ponds on the Arctic sea
Right: simulation of ferromagnetic Ising model.

In a penultimate chapter, the reader is introduced to topology. When it was used to study the working of our brains, unreal structures like Klein bottles have been observed. But topology is also applied to data mining to recognize clusters or 'big' holes using something called persistent homology, This connects isolated points at different scales and colours all the triangles that are formed. The holes remain as large blank spaces as one zooms out at larger scales.

In a final chapter Stewart reflects on the subjects covered, which, he admits are only a small selection, leaving many other subjects untouched. He feeds back on his original six reasons that may explain the reason why mathematics is so effective and he refers for it to the previous chapters for their justification. Some scientists, like Max Tegmark, claim that our universe just consists of mathematics. Stewart thinks this is nonsense, but still admits that if some mathivore aliens conquer Earth and just 'eat' all our known mathematics, then our world would certainly look quite different. But does not Stewart make a logical mistake here? If everything is mathematics and the mathivores consume it, then there would be no Earth left at all.



This is a very readable book on not always simple mathematical topics. Most of the time only the idea is explained, not going into the technicalities. It sometimes requires the reader to go-with-the-flow and accept some handwaving justification. But Stewart always convinces with a pinch of humour and a sauce of seriousness, gently pushing the reader to corners of mathematics where he or she probably never went before..

Adhemar Bultheel