The convergence of Fourier-Takenaka-Malmquist series*

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Abstract

The Takenaka-Malmquist basis is defined as

$$\mathcal{B}_0 = 1, \quad \mathcal{B}_n(z) = \frac{\sqrt{1-|\alpha_n|^2}}{1-\overline{\alpha}_n z} \prod_{k=1}^{n-1} \frac{z-\alpha_k}{1-\overline{\alpha}_k z}, = \overline{\mathcal{B}_{-n}(1/\overline{z})} \;\; n = 1, 2, \ldots, \quad z \in \mathbb{C}$$

where $\{\alpha_k\}_1^\infty$ is a sequence of points in the open unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. These form an orthogonal system with respect to the Lebesgue measure on the unit circle $\mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$. See for example [2].

If $f(\omega)$ is a 2π-periodic function, we discuss the convergence of the truncated Fourier series

$$f_n(\omega) = \sum_{k<|n|} a_k \mathcal{B}_k(e^{i\omega})$$

to $f(\omega)$ as $n \to \infty$.
Some of this work is published in [1].

References

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[2] J.L. Walsh. *Interpolation and approximation*, volume 20 of *Amer. Math. Soc. Colloq. Publ.* Amer. Math. Soc., Providence, Rhode Island, 3rd edition, 1960. First edition 1935.

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