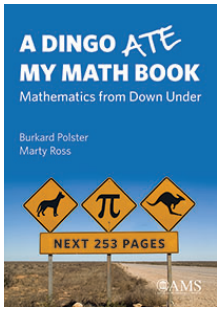
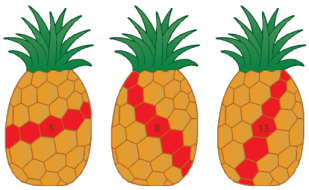
A dingo ate my math book by *Burkhard Polster and Marty Ross*. AMS, Providence, Rhode Island, 2017, isbn 978-1-4704-3521-9 (pbk) xv+253 p.





B. Polster (L) & M. Ross (R)

The two authors of this book had a weekly mathematical column *Math Masters* in the Australian newspaper *The Age* from 2009 till 2014. The present book is a selection of 64 of the most "Aussie" flavoured of their contributions but the mathematics can be appreciated also by non-Australians. Moreover, since these columns are written for a general public, the mathematics are very elementary. Some of the themes are familiar and appeared also in other similar collections written by mathematical popularizing authors.





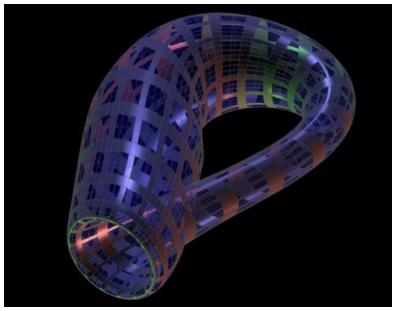
Big Pineapple

Each topic is just a few pages and usually ends with "a puzzle to ponder" (solutions can be found in an appendix at the end of the book). Enumerating all the topics here would be silly, but if you are familiar with popular mathematics books, you will recognize subjects such are organizing sports competitions, the trickiness of voting systems, Fibonacci numbers and the pineapple's hexagonally tiled surface, the traveling salesman problem, prime numbers, sphere packing, gambling, traffic problems (Braess's paradox: an extra road may be slowing traffic), etc.

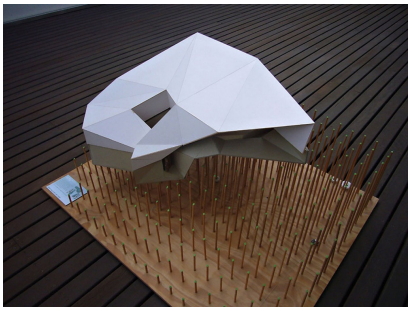
Interesting to read is their opinion about stereotype mathematical ideas (the golden ratio appearing everywhere in nature and that it is a key to beauty and perfection is a myth), and their critique on the Australian educational system that was implemented around 2009, almost banning mathematics completely (except from using a calculator).

Let me pick some of the topics that are related to remarkable Australian architecture (all the topics in the book are nicely illustrated). Left on this page you can admire the 16 m high *Big Pineapple* in Woombye, Queensland which is 'the biggest in the world'. Real pineapples have three families of spiraling sequences of hexagonal waffles with matching sides, and the number of these spirals is usually 5-8-13 or 8-13-21 (Fibonacci numbers, they grow with the pineapple), but the one in Woombye has 13 left and 13 right spirals and 26 vertical ones. Even if these are not Fibonacci numbers, they satisfy the theorem that says that on a cylindrical surface the largest number is always the sum of the two others.

Another remarkable building is the *Klein Bottle house* designed by Robert McBride and Debbie-Lyn Ryan, a beach holiday house on the Mornington Peninsula, 1.5 hrs drive from Melbourne. It is a playful design that resembles an origami version of a Klein bottle. The concept started as a spiral and while the tail of the spiral intrudes into the house, the idea of a Klein bottle-ish structure is realized.



Concept



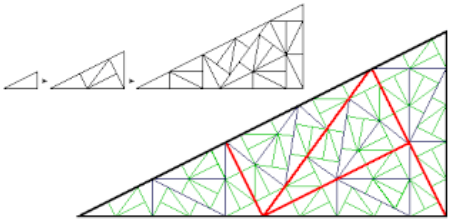
Scale model





Klein Bottle house

It cannot be a true Klein bottle because that surface is one sided. Thus there is no inside, and it would thus be impossible to live in.



pinwheel tiling



ACMI façade (middle)



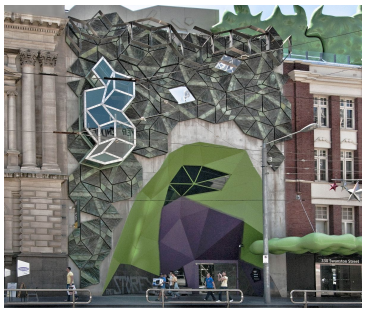
ACMI close-up

The Federation Square in Melbourne has several remarkable façades decorated with triangular structures. In the book, the Math Masters picked the front of the Australian Center for the Moving Image (ACMI, that is the film museum), that has a pinwheel tiling. At least that is what it is supposed to be. Pinwheel tiling is illustrated above. Five rectangular triangles are pieced together into a larger triangle of the same shape, and this can be iterated to result in an irregular tiling of the plane. This idea is used to cover the ACMI façade. At least it works for the larger part of it up to 3 iterations, but then the left part breaks down at the third iteration step because the triangles of iteration 2 are not arranged in the same way as in the other level 3 triangles.

If we start from a triangle with orthogonal sides of length 1 and 2, then its surface is 1. Now we subdivide it into 5 smaller triangles of the same shape, and then subdivide the small triangle at the acute tip again and iterate this process, then we have a graphical proof that $\sum_{k=1}^{\infty} \frac{4}{5^k} = 1$. In fact, one could subdivide in $N > 5$ triangles and so prove that $\sum_{k=1}^{\infty} \frac{1}{N^k} = \frac{1}{N-1}$.



RMIT auditorium



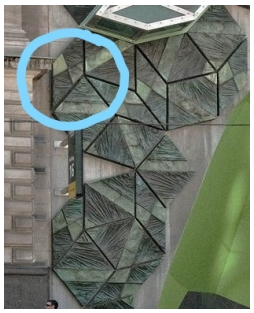
RMIT façade



RMIT close-up

The Storey Hall of the Royal Melbourne Institute of Technology (RMIT) front façade has the intention to honor Penrose for his irregular tiling. Irregular tiling means that it is not possible to rearrange the tiles in a regular pattern. In 1973 Penrose found an irregular tiling with only 6 different tiles and later only 2 tiles were needed. This is simulated here with 2 (regular) rhombi that are also used in the auditorium (see picture). The big pentagon consists of five rhombi  decorated by a green strip and their green tip form the pentagonal center spot. The 5 vertices of the pentagon are completed using more acute rhombi that have two green spots at the apposing obtuse vertices. Unfortunately only one green colour is used for the decoration. These tiles can be easily rearranged to form a regular tiling. One needs only the rhombi with the green strip to fill the plane in a regular fashion. Hence it is not a Penrose tiling. If the two green decorations on every tile would be in two different colour, then they could form a Penrose tiling, but using only one decorating colour destroys the original intention. The same idea is used to tile the façade.

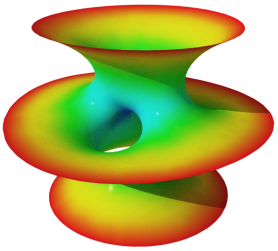
Note also that there are two blank spots in the front tiling. The reason is that the rhombi that fit into these holes do not have the proper decoration to blend with the surrounding tiles. This could be intentional, but on the left some tile is inserted that does not match the line pattern of the neighbouring tiles as can be seen on the encircled detail picture on the left below.



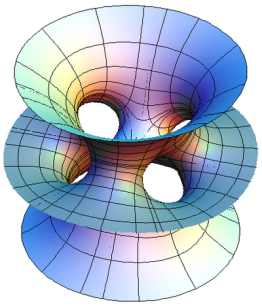
RMIT detail



Healesville Sanctuary



Costa surface

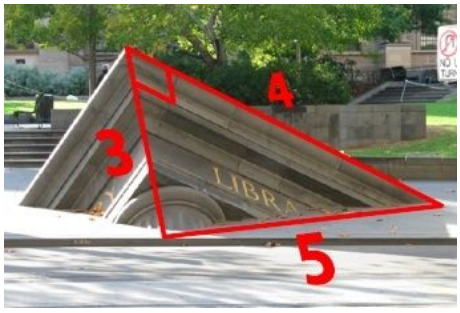


Meek-Hoffman-Costa

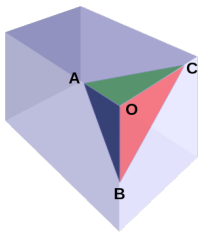
The Healesville Santuary is a zoo specializing in native Australian animals located in rural Victoria. The Health Center building has a remarkable mathematically constructed roof. It is a section of a minimal surface that is related to the (unbounded) Costa surface that was found by the Brazilian mathematician José da Costa in 1982. Before one only knew trivial minimal surfaces such as the plane, the helicoid and the catenoid. Later more general Meeks-Hoffmann-Costas surfaces were discovered that have n holes showing an n-fold symmetry when looked at from the top. A Costa surface has only two holes. The roof of the Health Center has three holes.



Architectural Fragment



in Swanston Street



Gua's theorem

On Swanston street in Melbourne, outside the State Library of Victoria you can see the *Architectural Fragment* (1992) by Petrus Spronk, a Dutch sculptor who emigrated to Australia in 1957. As indicated on the picture above, the pyramidal form is a cut corner of a cube and the larger rectangular triangle has sides 3-4-5 meters, which is a Pythagorean triple because $a = 3, b = 4, c = 5$ satisfies the Pythagorean theorem $a^2 + b^2 = c^2$.

The Pythagorean theorem is about the sides of a rectangular triangle, which can be seen as a corner snipped off a square by a line cutting the two incident edges. The abstract pyramid that is like the Archtectural Fragment is a part cut off a cube with a plane that intersects the three edges that coincide at the vertex. Like on the picture on the right, we obtain a pyramid with three right angles at the top vertex O and an arbitrary triangular base ABC. The generalization of Pythagoras is not about the length of the edges, but about the surfaces or the four faces and it is known as Gua's theorem which states that $\text{vol}_2(ABO)^2 + \text{vol}_2(BCO)^2 + \text{vol}_2(ACO)^2 = \text{vol}_2(ABC)^2$ and this can be generalized to n-dimensional volumes. The theorem is named after Jean Paul de Gua de Malves who published it in 1783 but it was known much earlier by for example René Descartes (1596-1650).

Adhemar Bultheel