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Integral and integro-differential equations. Wavelet-based numerical methods. (English)

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The first two chapters are short and serve to introduce all the necessary definitions and notation.

Chapter 1 introduces the different functional equations that are being discussed: Integral equations of Fredholm (FIE) and Volterra (VIE) type, integro-differential equations (IDE), partial IDE, and pantograph (i.e. with proportional delays) Volterra delay IDE. It also discusses the different kind of kernels used in their definitions (symmetric, separable, degenerate, iterated, or weakly singular). Some of their application fields are mentioned and it ends with a short survey of fractional differentiation and integration (both the Riemann-Liouville and Caputo definitions are introduced).

Chapter 2 serves to introduce the (continuous) wavelets that will be used. Most of these are generated by dilation and translation of several types of (classic) polynomials (Legendre, Bernoulli, Gegenbauer, Jacobi, Euler, Taylor and Laguerre) but also Müntz-Legendre (i.e., with non-integer powers), and Cosine and Sine (CAS) functions combining these two trigonometric functions.

The remaining chapters are all solving different kinds of the previously introduced equations with one of the wavelet based methods. The chapters start by giving a survey of the literature (each chapter has its own list of references), and then a method is proposed where the functions involved are expanded into a linear combination of wavelets. Using collocation or otherwise, (sometimes involving the derivation of operational matrices), the initial expansion coefficients of the solution can be computed by solving a set of algebraic equations giving a (wavelet based approximation of the) solution. An error (and convergence) analysis is given and this is all tested on a set of numerical examples resulting in plots and tables of the numerical values of solutions and the corresponding errors.

A brief survey of the different case studies in the subsequent chapters:

Chapter 3: Two-dimensional FIE of the second kind with Bernoulli wavelets using Karhunnen-Loève expansion or with a logarithmic weakly singular kernel.

Chapter 4: Two-dimensional nonlinear VIE or a system of 2D linear VIE with several kinds of wavelets.

Chapter 5: Linear and nonlinear VIDE with Bernoulli and Legendre wavelets, using collocation.

Chapter 6: Two-dimensional linear and nonlinear Volterra weakly singular partial IDE using Bernoulli wavelets.

Chapter 7: Müntz-Legendre wavelets are used to solve pantograph Volterra delay IDE of integer or fractional-order.

Chapter 8: Fractional order linear Volterra-Fredholm IDE with singular kernel using Euler wavelets and linear and nonlinear fractional Volterra IDE with weakly singular kernel using Taylor wavelets. The latter is continued in

Chapter 9: using Taylor wavelets to solve linear and nonlinear fractional order Volterra-Fredholm IDE, but here using an operational matrix approach instead of the functional approach in the previous chapter.

The book illustrates that there are many possible variants of the functional equations and there are many (wavelet based) approaches to solve them. The authors consider a set of possible problems and propose a solution. It can serve as a survey of the current literature and as a guide to find an appropriate method among the many possible approaches available to match with your own problem at hand.

Reviewer: Adhemar Bultheel (Leuven)

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MSC:

65-02 Research exposition (monographs, survey articles) pertaining to numerical analysis
65T60 Numerical methods for wavelets
65R20 Numerical methods for integral equations

Keywords:

functional equation; integral equation; differential equation; integro-differential equation; fractional differential equation; delay differential equation; Volterra integral equation; Fredholm integral equation; wavelet transform; collocation; Karhunnen-Loève; operational matrices; Muntz polynomial; singular kernel

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