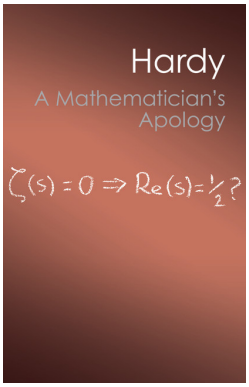
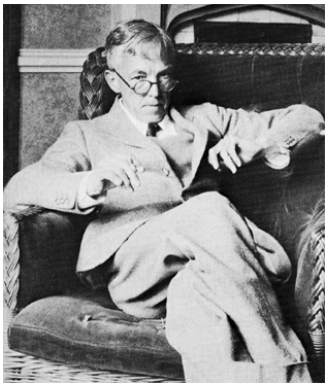
A Mathematician's Apology *G.H. Hardy* Cambridge University Press, Cambridge, 2012, (153 p.), ISBN 978-1-107-6046-36, pbk.





Godfrey Harold Hardy (7 February 1877 – 1 December 1947) was the brilliant English mathematician, and one the star players in the high days of mathematics as it flourished during the first half of the 20th century. His work with Littlewood and with Ramanujan, whom he lifted from an anonymous Indian to a phenomenon among the leading mathematicians who then flocked together in Cambridge[1].

Hardy worked mostly in Cambridge, but in 1919 he left for Oxford, where he stayed until 1931. His move to Oxford was a consequence of the fact he disagreed with several of his colleagues in Cambridge about WW I. On the one hand he was a convinced pacifist like Bertrand Russell, but on the other he was an admirer of German society, and he was convinced the England should not have started a war against Germany. His return to Cambridge was because it was indeed the mathematical spearhead of those days. It was around that time that he became acquainted with the young C.P. Snow with whom he shared a passion for cricket. They became good friends ever since. After Cayley, Forsyth, and Hobson, Hardy was the fourth Sadleirian Professor of Pure Mathematics from 1931-1942.

Hardy had a heart attack in 1939, and after WW II, his health deteriorated considerably. He even made a failed attempt to take his own life with an overdose of pills in the Summer of 1947, but he finally died on December 1 later that year at the age of 70.



Hardy and Littlewood

Hardy said: *One can be horrified or disgusted, but one can't be bored.*[2] To get him out of his depression in 1940, at the verge of WW II, Snow and other friends urged him to write this *A mathematician's apology*. In this treatise, very accessible for anyone, not being mathematician he asks himself the question: is the work that a man does during his life worth doing, and why does he do it? His answer is a remarkable attempt to explain what it is like to be a mathematician and why and how a mathematician can draw his pleasure in doing mathematics.

He starts by answering the second question: why would anyone choose the job that he does? There are two possibilities. A minority can do something particularly well: *... perhaps only five, or perhaps ten percent can do something rather well. It is a tiny minority that can do something really well, and the number of man that can do two things well is negligible.* On the other hand one may not be good at anything, and then the answer could be that it just so happened that one is doing what one is doing. The first reason applied to himself: *There is no doubt that I was right to be a mathematician, if the criterion is to be what is commonly called success.* Note the past tense. Hardy is convinced that mathematics, *more than any other art or science is a young man's game.* All great mathematicians achieved their main results at a young age. For example Galois, Abel, Ramanujan died very young. Newton lived longer but he obtained his best results at 24, was creative till 40, but did something else later in life (he became master of the Royal Mint). So

[1] See also the review of D. Leavitt's *The Indian Clerck* in this Newsletter issue 72, March 2009, which is more an account on Hardy than it is on Ramanujan.

[2] All italic text are quotations from the booklet.

this essay has some silent veil of sadness over it because Hardy is looking back on his life that he considers to be almost over, at least as far as mathematical creativity is concerned.



GODFREY HAROLD HARDY
MATHEMATICAE OXONII MOX CANTABRIGIAE
PROFESSOR DISCIPLINAE APVD VTROSQVE
EMENDATOR PRAECLARVS IN STVDIIS PROPRIIS PER
ORBEM TERRARVM INTER PRIMOS OMNIVM
CONSENSV EMINENS VIR OPINIONVM IMPATIENS
ALIENARVM SVARVM STRENVVS ET HILARIS
DEFENSOR HVIVS COLLEGII PER XIX PRIMVM
DEINDE POST REDITVM PER XVI ANNOS SOCIVS
AMICIS CARISSIMVS MORTEM OBIIT KAL.DEC.
A.S.MDCCCCXLVII AETATIS SVAE LXX

Godfrey Harold Hardy was Professor of Mathematics at Oxford and then at Cambridge, and was famous for reforming methods of teaching in both places. In his own field he was universally recognised as pre-eminent among the world's best mathematicians. He had little time for the views of others, and he defended his own with energy and humour. He was a Fellow of the College for nineteen years, and for a further sixteen years after his return to Cambridge. He was much loved by his friends. He died on 1st December 1947.

Trinity College Cambridge. Brass located on the north wall of the Ante-Chapel.
Memorial text by Donald Struan Robertson.

The other question about why it is worthwhile doing mathematics takes the larger part of his discourse. According to Hardy *mathematics is an unprofitable, perfectly harmless and innocent occupation.* This needs some explanation. By 'unprofitable' he means 'useless' in the sense that there is no direct practical application like for example medicine or physiology is. If it can not be used, then it is not applicable for the welfare of society but it will also be harmless because neither can it be used in the opposite situation like e.g., in war. Again this dictum needs more clarification. With 'mathematics' he refers to what he calls *'real' mathematics* done by *'real' mathematicians.* We would call it 'pure mathematics'. This is quite different from *the repulsively ugly and intolerably dull* elementary school mathematics. Of course computation is used in daily life and most of the mathematics of the first years at the university can well be applied by engineers and scientists. Pure mathematics however are universal, deep and beautiful. One may call a chess problem mathematics, and in a sense it is, but it is a simple succession of if-then's, not requiring great creativity on a mathematical level. It is *unimportant* and *not serious.* 'Real' mathematics is general and of all times: *Babylonian and Assyrian civilisation have perished; Hammurabi, Sargon, and Nebuchadnezzar are empty names; yet Babylonian mathematics are still interesting.* Pythagoras, Newton and Einstein are real mathematicians. They have influenced science considerably. To make his point more concrete, he gives two generally understandable proofs: Euclid's proof of the infinity of prime numbers and the proof of Pythagoras on the irrationality of $\sqrt{2}$. Now Hardy can make his point about the aesthetics of a proof. A proof should not be dull and predictable, but there might be some non-straightforward twists, exposing the creativity of the mathematician. The two proofs he gave *have a very high degree of unexpectedness, combined with inevitability and economy.* He also explains what it means to say that a theorem is 'deep'. For example finding the number of primes smaller than an arbitrary integer N is a deeper result than just saying that there are infinitely many primes. Many deep results can be obtained in number theory, understandable for many, but requiring very complicated proofs. To illustrate the difference between pure and applied mathematics, he gives also the example of geometry. Applied or physical geometry is used to describe our surroundings. There is a model and there are rules that hold. However, these will

change when a massive gravitational object will be brought into the room. The theorems of pure geometry that one has proved in that same room do not change. In this sense the mathematician is in more direct contact with reality than e.g. a physicist. This he said to an audience that consisted mainly of physicists. For example a chair can be a collection of whirling electrons or anything else, depending on the model used to describe the physics, but a mathematical object is exactly what it is, no interpretation possible. There are many possible models for the physical world, yet *317 is a prime not because we think so, or because our minds are shaped in one way or another, but because it is so, because mathematical reality is built that way.*



Different editions of the 'Apology'

The foreword by C.P. Snow is not just a short introduction, but it takes about one third of the booklet. It has been added since the 1967 edition of the *Apology*. Snow gives a very human biography of Hardy. It is not about Hardy's mathematical achievements but rather about his opinions and his character as a human being. He was a rather shy person. It is surprising how few pictures are available for Hardy. Everybody addressed him as Hardy, and only very few called him by his first name. However, after a while, and surrounded by friends, he was most often the center of attention, and he didn't dislike it. He certainly was not a narcissist because did not tollerate mirrors. In an hotel, one of the first things he did was to covered up the mirror with a towel. Hardy was also obsessively heliotropic, and he played this game with God: when he wanted to watch a cricket game, he brought his 'anti-God battery', which consisted in some sweaters, an umbrella and some work because he said that then God would think that he was hoping that it would rain so that he could work, and thus God would counter that and arrange for a sunny afternoon, which is what Hardy was actually looking for: watching the cricket game in the sun. Strange enough he did not like the telephone. Snow once got an urgent call from Hardy who said with an angry voice *I shan't hear a word you say, so when I'm finished I shall immediately put the receiver down. It's important you should come between nine and ten tonight.* Clonk. These and other anecdotes give an idea of what kind of person Hardy was.

Hardy's text is now more than 70 years old, and it is clearly a plea for pure mathematics, but obviously Hardy is also justifying his own life. Some of his viewpoints were and still are controversial. Some of his arguments may no longer hold. E.g., his beloved number theory became an important element in public key encryption, although he may have considered that as dull and applied, and not of the pure and creative sort. Yet his ideas are so clearly formulated, that it is still highly recommended for mathematicians as well as those who wish to understand what a mathematician is like. So we are very lucky that the booklet is still available in print.

Note that the apology by Hardy is also publicly available courtesy of the University of Alberta Mathematical Science Society. The foreword by Snow is not. See *www.math.ualberta.ca/mss/misc/A Mathematician's Apology.pdf*.

Adhemar Bultheel