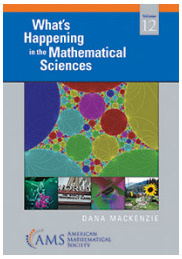
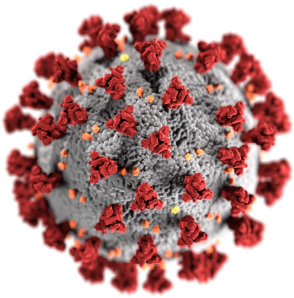
What's happening in the mathematical sciences, vol. 12. Dana MacKenzie. AMS, 2022 (126 p.), isbn: 978-1-4704-6498-1(pbk).





Dana MacKenzie

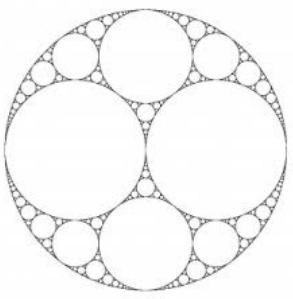
Here is another volume of the *What's happening* series highlighting some trending subjects in mathematics in which some progress has been made. The idea is that the mathematics are explained to the layman, interviewing the mathematicians that worked on the subject, very much like the articles in the *Quanta Magazine* does for a broader scientific spectrum. This is definitely different from the annual anthology *The best writings on mathematics* collected by M. Pitici that is reproducing published papers that are about mathematics and mathematicians, and their impact and importance, but most often do not treat the mathematics itself.



Like Pitici's survey of 2021, also this issue has a strong signature of the 2020 pandemic where mathematics had a rather important contribution. Bending the curve, reproduction factors, exponential growth, were mathematical concepts that were mentioned in about every news bulletin in the high-days of the pandemic by the news anchor with a spiky sphere of the corona virus as a background. Hence, it should be no surprise that not less than three chapters are devoted to this global catastrophe. One deals with the epidemic modeling of how it was spreading and how these models had to be adapted as more information became available. A second is about how models tried to catch the influence of vaccination and lock-downs which were essential for politicians to make decisions. The third one is a study of the virus itself on a RNA scale: how it operates and spreads in a human body which is essential if one wants to develop a vaccine.

With covid-19 more or less under control, the other lurking global disaster of climate change that was pushed to the background, comes back to the foreground. So there is understandably also a long chapter dealing with this threat. It explains the shift in appearance of extreme events like heat domes or water bombs, the relation between weather and climate, and how to identify the change in global measurements. Also here, like in so many other applications, machine learning is of increasing importance. At the moment of writing this review, we are in the middle of an energy crisis. This is obviously triggered by the Russia-Ukraine war, but the transition to sustainable energy consumption, is of course related to global warming and climate change as well. It will probably be another major issue in the following volumes of this series.

The four remaining chapters are somewhat lighter. One is about Apollonian tiling. Three touching circles enclose a curvilinear triangle in which a circle can be fitted touching the 3 bounding circles, which generates 3 other triangular holes, that can be filled etc. If the radii of the 3 circles are of the form $1/n_i$, with n_i, $i = 1, 2, 3$ integer, then the circle in the triangle will have a radius $1/n_4$ with n_4 integer. This can be generalized in various directions, using Gaussian integers or more generally using numbers in $\mathbb{Z}[\sqrt{-p}]$ with p prime, or considering Apollonian sphere packing. There is an obvious connection with fractals and hyperbolic geometry. It is worth checking the Internet look-

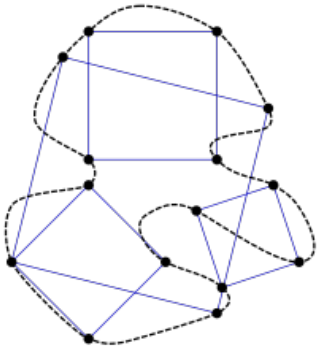


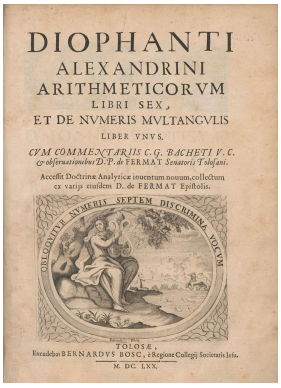
Apollonian tiling



Schmidt arrangement (K.E. Stange's homepage)

ing for the beautiful Schmidt arrangements obtained by Katherine E. Stange by applying Moebius transforms, generating results with the open source software SageMath. The problem was originally proposed in 1643 by Descartes as a homework problem for Princess Elizabeth of Bohemia, until he realized that the problem was way to hard for her to solve.





Another chapter is a discussion of what is known as the four-peg problem. If one draws any (smooth) non-intersecting Jordan curve, then one can always find four points on this curve that are the vertices of a perfect square. A simple problem, but difficult to prove, and once it is solved, many other problems come to the surface. What if squares are replaced with polygons with vertices on a circle, what with non-smooth or with self-intersecting curves?

Fermat's last theorem is a problem formulated in terms of a Diophantine equation, i.e., an algebraic equation with integer coefficients. This type of equations has a very long history and are related to several aspects of number theory. Here the history of the problem of finding the algebraic solutions of a diophantine equation are discussed.

The last chapter from this volume starts from a simple graph problem which corresponds to computing the eigenvectors corresponding to the eigenvalue zero of a tridiagonal weighted adjacency matrix. Such problems are related to defining spins of a qubit network, but also to the stabilisation of buildings. The most famous example of the latter is the Taipei 101 skyscraper (once the world tallest building) that has a 660 ton pendulum hanging in its core to stabilise the building to survive an earthquake. However the tridiagonal structure of the matrix corresponds to a simple serial connection of the nodes in a graph. What can be said about more general graphs? The original problem was to find out for a general graph where the value of a node defines the values of its neighbors: what is the minimal set of nodes that one has to fix to fix the whole network. This corresponds to finding in the corresponding class of (adjacency) matrices, the minimal set of entries that will maximize the dimension of the null space. Finding these sets, which are called 'zero forcing sets', is computationally a very hard problem and that is where recently some progress has been made. Solving the problem is important when the graphs represent social or traffic networks, or power grids, or certain pursuit problems.





The Taipei 101 skyscraper and the damping pendulum inside

So this is again a very nice collection of items where mathematics plays an essential role and that can be read by anyone with only a very elementary knowledge about mathematics. Several of the subjects discussed are not only interesting from a mathematical point of view, but they are also very important from a social point of view. Dana Mackenzie once more succeeds in selecting these juicy topics and bring these in a pleasant and entertaining way. For example the 3 pandemic chapters are entitled '50 ways to kill a virus' referring probably to Paul Simon's 50 ways to leave your lover and the last chapter has the Shakespearian title 'Much ado about zero'. Whenever you finish reading a volume in this series, you long for the next one to read. Adhemar Bultheel