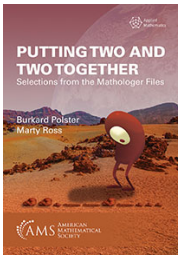
Putting two and two together, *Burkhard Polster, Marty Ross*, AMS, Providence, Rhode Island, 2021 (274 p.) isbn: 978-1-4704-6011-2.






B. Polster M. Ross

This is a second collection of *Math Masters* columns that Polster and Ross published weekly between 2007 and 2014 in the Melbourne's newspaper *Age*. Since 2009 their columns moved online which allowed more illustrations and longer columns. A first collection was called *A dingo ate my math book*[1] (AMS, 2017). These columns are treating mathematics related issues from everyday life, addressing a general readership. While the previous collection were discussing typical Australian topics, the current collection is in general more international. They are arranged in seven parts focussing on a common theme.

In the opening of their introduction they announce that this book contains 1.000.000 columns from the 11.111.111 that they wrote already. This is a binary mathematical wink: it contains indeed 64 of their columns, and they probably had written 255 columns at the time they wrote this introduction. This sets the tone of the whole book: they clearly enjoy the math that is all around them and they want to convey this joy to every occasional reader, the regular readers are already hooked. I'll give some examples of the seven parts.

The first part is about general calculus issues. An error often made is $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$. This is obviously wrong with the usual meaning of addition, but it makes sense if it indicates the mediant of the two items summed. Geometrically the vector $(b+d, a+c)$ is the sum of the vectors (b, a) and (d, c). But that is considered dull mathematics. Here it is explained as mixing a container with fluid A and B in ratio a/b and another one in the ratio c/d giving a mixture in the ratio $(a+b)/(c+d)$. Another example shows that with the parabola $y = x^2$, one can find geometrically the prime numbers. Indeed, the chord joining the points $(-a, a^2)$ and (b, b^2), $a, b \in \mathbb{N}$ intersects the y-axis in the integer point $y = ab$, which allows to multiply integers. Therefore the integers on the y-axis that are not on any chord are the prime numbers.



Many of the columns actually contain (rather informal) proofs. For example in part two, which is mostly about geometric topics, it is proved that in an equilateral triangle, the sum of the distances from an internal point to all the sides does not depend on the point chosen (Viviani's theorem). The tractrix is related to the turning of a long vehicle and there are classical cycling problems where one has to derive the direction of the bicycle from its wheel trails. But there are also problems of wrapping a Mozartkugel and the surface of a dodecagon is shown to be $3R^2$, so that for dodecagon "circles" π equals 3.

And these fun topics go on and on. In the part devoted to sports topics, we can find classic problems about the design of tournaments. How to arrange the matches, and what will be the chance that Djokovic and Nadal will play the finale of the Australian Open? But it is also shown that even Hawk-Eye is not infallible, because it depends on the viewpoint of the observer to see a ball in or out and it is always an approximation. Should e.g. the height of he grass be taken into account? But other sports are represented as well: soccer (how round is a soccer ball?), running (mechanics of walking), cycling (gear ratios), etc.

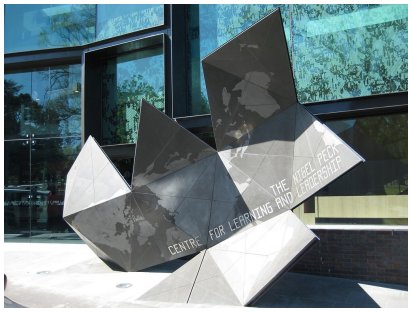
Like in their first collection, there is an interesting part related to architecture. A discussion of Voronoi diagrams is triggered by the facade of the *Victorian College of Arts' Center for Ideas* in Melbourne. The catenary is related to hanging cables in the city. A skyscraper called Eurika

[1] Reviewed in this Newsletter, issue 123, 2019

Tower is an incentive to discuss results obtained by Archimedes. A sculpture at the entrance of the *Nigel Peck Centre for Learning and Leadership* is a map of the world in the from of a icosahedron. The AAMI Park Stadion is consisting of bubble domes around a square field. The bubbles consist



VCA's Center for Ideas



NPC for Learning and Leadership



AAMI fooball stadium

of iterated triangulations of an elementary icosahedron.

The next part is about probability and statistics. In the Penny game, a penny is tossed and two players each predict a sequence of three outcomes like for example HTT or TTT. The player whose outcome comes up first wins. It can be computed which triplet one has to choose to have a better chance to occur. Another problem occurs in bkindlpurchasing a new set for your collection of different Playmobil figures (or Pokémon cards of football players, or whatever). The purchase is blind since it is packed up hiding the content. What is the chance that you will get one of the items that you are still missing? If you want to find the number of errors in a new course text you prepared, let it proofread by Bert (who finds 20 errors) and Ernie (who finds 30 errors) and 5 of these errors are the same, then it is explained that you might suspect some 120 errors in total.

Part six is about astronomical problems. For example the trajectory of our moon as it rotates around the earth which rotates around the sun or of other moons around other planets (assuming circular rotations). They explain also the position of the five Lagrange points of two bodies, and of course the strange computation of the Easter date, and the probability of getting a Friday the 13th. Exponential growth is illustrated by the mechanical sculpture *Machine with concrete* by Arthur Ganson. (You can find movies on YouTube.) It consists of a sequence of 24 gears connected with ratio 14/120. The first gear wheel is turned at 9.24 rpm, so the second takes about 56 seconds etcetera till the last one will take about 594 trillion years, which explains why it can be safely embedded in concrete.



Machine with concrete, Arthur Ganson

The last part is about some historical facts or persons. It is illustrated that many identities or theorems that are named after some mathematician, are very often not invented or proved for the first time by this mathematician, but they existed sometimes centuries earlier. Examples are Pythagoras theorem, Pascal's triangle, de l'Hôpital's rule, Möbius strip, Benford's law, and many more. Other items deal with the Babylonian number system, Julian and Gregorian calendars, Napier and his logarithm, the Goldbach conjecture, and Maria Agnesi and the curve that became known as the witch of Agnesi.

All of these relatively short chapters of only a few pages discuss the topic in a pleasant entertaining and thralling manner, and are certainly accessible for a general public. Except for one with knowledge aversion, anybody will enjoy these columns. For the more dedicated, there is a puzzle or problem added after each chapter, and these get an explanation and solution at the end of the book. If you likes *A dingo ate my math book*, you will love this book too.

Adhemar Bultheel