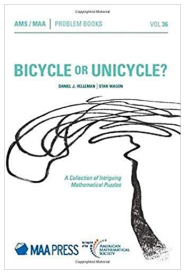
Bicycle or unicycle? by *Daniel J. Velleman and Stan Wagon*, MAA/AMS, 2020 (xvi+286 p.), isbn: 9781470447595. Problem Books vol. 36





D. Velleman



S. Wagon

Since 1968, Joe Konhauser at the Macalester College (a private liberal arts college in Saint Paul, Minnesota) started weekly posting a mathematical problem. Since 1993 Stan Wagon took over. After his retirement, he kept mailing problems to subscribers, although not on a weekly basis. A collection from the first 21 years of the archives was published with the title *Which way did the bicycle go?* (MAA, 1996). It refers to the now well known problem where Sherlock Homes has to find out from the tracks of the two wheels of a bicycle in which direction the bicycle went. This book is a new selection from the Macalester archives. The title refers again to a bicycle problem: can a bicycle track generate a single track like a unicycle does?

The book contains a collection of 105 diverse challenging mathematical puzzles that can be solved with rather elementary mathematics, but by pushing the problem a bit further, one may easily arrive at more demanding mathematics. Problems are somewhat comparable to some of the mathematical olympiads, or the Putnam competition problems.

The solution of a problem is often surprising so that guessing a solution does not work in general. The book starts by posing the problems, subdivided into 8 different chapters. The bicycle problem is the starter. Subsequent chapters refer to the mathematics involved: geometry, number theory, calculus,.... The problem settings are usually rather short (only the first 50 pages for the 105 problems). The larger second part of the book consists of a discussion of the solutions. Some solutions are simple and concise, but others (like the cycle problem) is extensively treated.

The solution of the cycling problem, when it is not a straight path, requires the solution of a differential equation and in particular to define the angle of the front wheel as a function of time. Some solutions are simple, but a general solution is quite surprising.



S. Wagon on a bike

Some of the problems are set in classical mathematical terms, others as a narrative puzzle of cutting pizzas, weighting coints, etc. An example of the first kind is asking the path of the focal point when the parabola $y = x^2$ rolls without sliding over the x-axis. This relates again to a cycling problem for a bike with square wheels. In another one, one has the prove that 3^{3^n} is a sum of two integer squares.

An example of a more puzzler's type problem is a baby version of public key encryption. Charlie deals seven cards 1,2,3,4,5,6,7: 3 to Alice and 3 to Bob, keeping one. Each can only see their own cards. What public statement can Alice make so that Bob knows her cards, while Charlie does not? The greater challenge is to solve the problem for n cards where n is odd and Charlie keeping one card while Alice and Bob each get $(n-1)/2$. There is also a revision of the Monty Hall problem, where there are four instead of three doors with a prize behind one of them. You choose a door and Monty shows you another one without a prize. You are allowed to choose another door but you do not. Then Monty shows you another door without the prize. Should you switch then or not?

There are also problems based on physics laws. For example, a heavy block of mass M is to the right of a small block of mass m. To the left of the small block is a wall. The mass M is pushed to the left, hitting m, making it bounce back and forth between the wall and M. Gliding is frictionless and all collisions are perfectly elastic. How many times collides the smaller block with the wall and M? The appearance of π in the solution is a surprise, don't you think?

Where possible, the origin of the problem is added. There are e.g. problems from national Math Olympic team selection competitions, others are taken from the literature. There is a list of 134 references for further reading about the problems and their generalizations.

Adhemar Bultheel