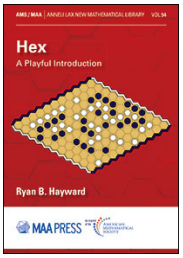
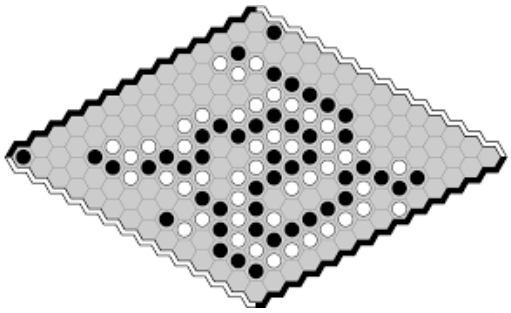
Hex: a playful introduction. Ryan B. Hayward. AMS, 2022 (124 p.), isbn: 978-1470464929(pbk).





Ryan B. Hayward

Hex is a relatively old game invented by Piet Hein in 1942 and yet it is little known, although it is simple to explain and fun to play. It is played on an $n \times n$ board with hexagonal cells. This implies however that the board is not square, which may explain its reduced popularity. Two opposite sides of the board are white and the other two are black. There are 2 players, lets call them Black and White. They can, each in turn, claim a cell by putting a marker or a pawn on it of their own color. A player wins if she is the first to connect the two borders of her own color with an uninterrupted path of cells that she constructed. The advantage of a hexagonal over a square grid is that there can be no deadlock and there will always be a winner.



This booklet contains everything you need to know to develop strategies to play the game. For small n, the strategy will be simple. Like for the trivial case $n = 2$ (actually this holds up to $n = 5$) there exists a winning strategy for the one who starts. For larger grids ($n = 11$ seems to be an interesting size) there are strategies that can be developed that have a complexity somewhat comparable to checkers or go. They are challenging enough to interest even the chess player.

The reader also learns about the origin of the game. The Danish engineer, designer, and inventer Piet Hein introduced the game with $n = 11$ under the name *Polygon*. He had a puzzle corner with that name in a Danish newspaper. After the war he moved to the USA and Polygon disappeared, until John Nash picked it up (or rediscovered it) and Martin Gardner made it popular again with his Scientific American columns.

The booklet also offers several variants and generalizations. The winner could be the last, and not the first, to conned the two borders of his color. One could play on an $n \times m$ grid, or on a triangular grid. In the latter case the winner is the one who is the first to have a connected path that touches the three edges of the triangle. Such a path may have the shape of a Y, so that this variant is called a Y board. Geodesic versions leave the idea of regular hexagonal cells and generates Y boards introducing pentagons near the center of the triangle. As long as at least 3 cells meet at every internal vertex, there is no deadlock possible.

Dark Hex is a battleship-like variant. Black and White have their own copy of the board and they cannot see each other's board. Then Black can ask White to claim a certain cell. If it is free on White's board he gets the cell, otherwise he knows that White has claimed the cell already and gets another try, and so on until he has found a free cell. Then it is White's turn to guess etc.

Of course there are theorems and properties to prove about this game and computers can be learned to play Hex and a complexity analysis has been made (but there are no details in this book because it is addressing anyone who is just interesting in learning to play the game). When theorems are too difficult to prove, a lot can be learned by letting a computer play random games and collect statistics from this Monte Carlo-like approach.

There are also a lot of puzzles (and solutions) since besides playing the game, this is how to learn the fine strategies. These puzzles (like for chess puzzles) give a certain constellation of the board, says that B(lack) is the next to move and asks what would be her winning strategy at this point. The booklet offers everything to get you started or improve you skills if you know hex already. There is a reference list for further reading if one is interested in more theoretical aspects, but this is not essential for enjoying the game.

Adhemar Bultheel