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Solving differential equations and inverse problems with AI. With problems and solutions – Part I. (English) Zbl 08218058

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This is an introduction to the use of physics informed neural nets (PINNs) to solve differential equations. It starts by explaining in Chapter 1, how processes can be described by ordinary and partial differential equations, with their initial and boundary value conditions (IBC), and how analytical solutions can be obtained. In Chapter 2, some classical numerical methods are recalled and in Chapter 3 an introduction to (artificial) neural nets is given. A network consists of layers of neurons. Each neuron receives the weighted outputs of a previous layer and transforms them with some activation function in an output that serves as an input for the next layer. In particular, it is explained how these PINNs can be trained to solve a differential equation.

The principle is simple: Suppose $u(x)$ is the solution of a differential equation $\mathcal{D}u = f$ satisfying some IBC, where $\mathcal{D}$ is some differential operator. If u^{NN} is a (proposed) solution then during the learning phase the weights w for the PINN are modified to minimize the cost function which collects the mismatch in the IBC and the differential equations. These mismatches are computed by mean squared errors for a set of sample points.

The next two chapters explain how to choose these sampling points. Sampling methods are discussed for the IBC (Chapter 4) and for the domain of the differential equation (Chapter 5) and how to incorporate them in the optimization methods used to minimize the cost function.

The remaining two chapters explain the transfer learning processes. This means, if a PINN has been trained, for a set of problems, but then a somewhat different problem has to be solved, how can the trained parameters be modified to solve the new deviating problem without starting the training process all over.

In Chapter 6 the transfer for the IBC is explained. One may freeze most layers and retrain only a few, like for example the first or the last layer. Another strategy is to modify the architecture of the network and separate the dynamics part into a basic network separated from the IBC part, for which different possibilities can be collected in a multi-head network. MIC-PINNs are Multi-Input Conditional networks that differ from these because they have the different possible networks trained cyclically, which are combined in an overall network. A particular situation will then need minimal adaptation.

Chapter 7 explains on-shot transfer learning. It is explained how for linear ODE, a minimal adaptation will suffice to adapt to the new problem.

The discussion is rather elementary and minimal, meaning that the text is highly structured like an executive summary with bullet points explained the solution with a minimum of sentences, just enumerating the essential components and enumerating the advantages and disadvantages of all the methods. Each chapter starts with an introduction explaining the objective and concludes with a summary of what has been achieved.

There are some typos that will hopefully be corrected in a future edition.

Reviewer: Adhemar Bultheel (Leuven)

MSC:

65-02 Research exposition (monographs, survey articles) pertaining to numerical analysis
34-02 Research exposition (monographs, survey articles) pertaining to ordinary differential equations
68T05 Learning and adaptive systems in artificial intelligence
68T07 Artificial neural networks and deep learning

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