Numerical quadrature on an arc of the unit circle

Adhemar Bultheel

KU Leuven, Department of Computer Science, Leuven (Heverlee), Belgium

We consider the construction of quadrature formulas for an integral with respect to a positive Borel measure that is supported on an arc of the unit circle in the complex plane.
We illustrate why Gauss quadrature formulas are not a solution for this problem but that appropriate Szegő-type formulas should be used.
Similar to the Gauss formulas, the nodes and weights of a proper formula can be computed by solving an eigenvalue problem for a matrix representing the recurrence relation. Where this is a 3-diagonal Jacobi matrix on the real line, it is a unitary Hessenberg matrix that depends on the Verblunsky coefficients in the case of the circle.
We propose a solution when these Verblunsky coefficients are not known, and we investigate how we can guarantee that all the nodes fall inside the support of the measure.
This is joint work with Leyla Daruis and Carlos Díaz Mendoza from the University of La Laguna (Spain).