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THE PRODUCTION SWITCHING HEURISTIC IN RETROSPECT

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ABSTRACT

The Production Switching Heuristic (PSH) has been proposed as a practical procedure to solve the important aggregate planning problem. In this paper the PSH is examined in a non-stationary demand environment. Our analysis shows that the PSH may be too inflexible to handle dynamic demand situations and as a consequence may result in far from optimal solutions. This statement is documented by means of a number of case studies.

THE PRODUCTION SWITCHING HEURISTIC IN RETROSPECT

I. INTRODUCTION

The aggregate production planning problem has been recognized as an important problem in the operations management area. The smoothing of fluctuations in production and inventory is the crux of the whole matter. Management may follow several strategies in order to respond to fluctuations in demand. Management may change the production level by adjusting the work force or by using overtime or idletime. Variations in demand may also be absorbed through changes in the inventory level and in some cases through subcontracting.

The importance of the topic has led to a stream of publications concentrating on a wide range of optimal and heuristic solution techniques. The most recent approach is the Production Switching Heuristic (PSH). The random walk inventory policy, suggested by Orr [6] , gives the theoretical background of the approach. Elmaleh and Eilon [1] and Mellichamp and Love [5] applied the PSH methodology to a limited set of production problems.

The basic idea of the PSH can be summarized as follows : an important goal of management in production planning is to avoid rescheduling too frequently, therefore, it makes sense to limit the production quantities to a small number (two or three) of discrete levels (a low, normal and high production level). Elmaleh and Eilon [1] justify this by pointing out that in various industries machines are operating in parallel, where one or more machines may be shut down, but their rates of production are fixed. In other situations production may be expanded by introducing extra-shifts. The PSH comes to meet this operating practice. Moreover, the PSH can easily handle different

types of cost functions, which is not the case for most other analytic models such as linear programming, quadratic programming and goal programming.

In this paper we'll examine the behavior of the PSH in a non-stationary demand environment and show that the inflexibility inherent to the PSH may be very costly. In most practical situations demand may be highly fluctuating through time whereas the PSH is from a theoretical point of view, based on a stationary demand distribution.

In section II a review is given of previous work. Section III is concentrated on the PSH in a non-stationary environment and in section IV an extension of the PSH is proposed.

II. REVIEW OF PREVIOUS RESEARCH

The random walk production-inventory policy by Orr [6] is based on the following observation : the main problem in production smoothing is not to choose an optimal production level at each period of time but to control the frequency of production changes. Orr assumed in his analysis that the demand distribution is stationary, time independent and random. The objective is to minimize the total expected cost. The formulation of the problem is as follows :

$$\text{minimize } E[L(t)] = E[\lambda_1(I_t | I_t > 0) + \lambda_2(I_t | I_t \leq 0) + \lambda_3(p_t) + \lambda_4(z)]$$

$$\text{subject to } I_{t+\Delta t} = I_t + \Delta t(p_t - s_t)$$

with the demand probability rule

$$\text{Prob.}\ (s_t < s) = \Phi(s)$$

Where

p_t = production rate at time t
s_t = demand rate at time t
I_t = inventory on hand
z = number of production rate changes in a time interval $(t, t+\Delta t)$
λ_1 = inventory holding costs per unit, per period
λ_2 = shortage cost per unit, per period
λ_3 = production cost per unit
λ_4 = production change cost.

The resulting decision rule may be summarized as follows : identify three inventory levels $a > c > b$, and three production rates $H > N > L$ with the following operating instructions :

$$p_t = \begin{cases} H & \text{if } I_t \text{ passes b from above} \\ N & \text{if } I_t \text{ passes c} \\ L & \text{if } I_t \text{ passes a from below} \end{cases}$$

This policy applies the same rationale to production problems as underlies the (s,S) policies for dealing with ordering problems (with fixed ordering costs).

Elmaleh and Eilon [1] applied Orr's random walk approach on a number of simple examples. They argue that this policy is valid for any demand pattern, although in the testproblems they assume that the demand pattern is stationary and taken from a normal distribution.

Mellichamp and Love [5] extended Orr's decision rule by incorporating the demand forecast. They suggest the following rule for P_t, the production rate in period t

$$P_t = \begin{cases} L & \text{if } F_t - I_{t-1} < L - C \\ H & \text{if } F_t - I_{t-1} > H - A \\ N & \text{otherwise} \end{cases}$$

Where L = the low level of production
H = the high level of production
N = the normal level of production
A = the minimum acceptable target inventory
C = the maximum acceptable target inventory
F_t = the demand forecast for period t
I_{t-1} = the ending inventory for period t-1.

$F_t - I_{t-1}$ is the net requirement in period t, the right hand side of the decision rule (L-C or H-A) gives the available production taking into account the planned inventory. This heuristic forces the planner to produce as much as possible at the normal production level. A switch to a high or low level occurs only if it is absolutely necessary. Given next period's expected net requirement, the low production level is choosen only if this results in an inventory position which is higher than the maximum acceptable target inventory, and the high production level is choosen only if the resulting inventory position is lower than the minimum acceptable target inventory.

The control parameters of production and inventory are found by using a search routine. All combinations of the control parameters must be evaluated (various grid and gradient procedures may be used) and those values are retained for which total cost is minimum.

Our results in this paper are based on the Mellichamp and Love decision rules. The optimization rule used is based on the grid search method where each lattice was centered on the lowest cost point in the preceding lattice.

In a recent article Vergin [7] pointed out that cost comparisons between different heuristics must be based on the controllable variable costs resulting from the aggregate scheduling decisions. He very properly observed that the total costs of an aggregate production plan contain a very large element of fixed costs. If these fixed costs are eliminated, we can then compare different solution methods solely based on the controllable costs. Mellichamp and Love e.g. compare the PSH and the Linear Decision Rule (LDR) [3] over a horizon of 12 periods. The 1.13 % deviation quoted by Mellichamp and Love is in fact equivalent to a 19.5 percent deviation based on the controllable costs only. The cost deviations reported in this paper are also based on controllable costs.

III. THE PSH IN A NON-STATIONARY ENVIRONMENT

The PSH certainly comply more closely with managers current practice than do other suggested methods. But the PSH is from a theoretical point of view only justified in a stationary environment. The question now is if the PSH is still applicable in a world where demands are fluctuating through time, e.g. in case of a seasonal demand pattern. In such a case production flexibility is of crucial importance. The PSH limits the production rate to three discrete levels and this may create stockouts and high inventory levels. Our results indicate that the PSH is too inflexible to handle the non-stationary demand case. This conclusion will be documented by means of two examples.

A. The Linear Decision Rule (L.D.R.)

Mellichamp and Love [5] applied the P.S.H. over the first twelve periods of demand of the well known historical Paint Factory case. We extended the analysis to 20, 30, 40 and 60 periods. The results are summarized in Table 1. Total costs over a 60 period horizon deviate from the optimal solution

TABLE 1 : Paint Company Results with perfect forecasts : PSH vs. LDR

Time Horizon	20 periods	30 periods	40 periods	60 periods
1.Computation of the uncontrollable variable costs				
.Production = P_t = average demand	452.8	528	528.775	539.71
.Number of employees W_t :				
.$W_t = P_{t/5.67} - 4.59$ [1]	75.27	88.53	88.67	90.598
.Regular Labor costs per period :				
340 x W_t	25,591.8	30,100.2	30,147.8	30,802.96
.Costs of overtime/undertime :				
$0.2x(P_t-5.67W_t)^2+51.2P_t-231W_t$	2,167.89	2,292.23	2,292.37	2,310.4
.Uncontrollable costs per period	27,758.69	32,392.43	32,440.17	33,113.36
.Total uncontrollable costs	555,174	971,772	1,297.607	1,986,801.6
2.Cost performance based on total cost				
.Total costs L.D.R.	574,133	997,471	1,351,664	2,052,887
.Total costs P.S.H.	587,247	1,131,941	1,497,160	2,268,105
.Deviation	+2.28%	+13.48%	+10.76%	+10.48%
3.Cost performance based on controllable costs.				
.Contr. Costs L.D.R.	18,959	25,699	54,057	66,085.4
.Contr. Costs P.S.H.	32,073	160,169	199,533	281,303.4
.Deviation	+69.17%	+523.25%	+269.15%	+325.66%

1. This formula is known as the Yance transformation [8]. Vergin [7] computed W_t be means of the following formula : $P_t/5.67$. We believe that the Yance formula offers a better approximation to compute the controllable costs

by 10.48 % and in terms of controllable costs this is equivalent to an increase of 325.6 %. The main reason for this disappointing result (which holds for shorter horizons as well) can be found in the significant change in the average demand for different subintervals of the planning horizon. The average demand over the first 10 periods equals 390.5 whereas from periods 20 to 30 for example the average demand equals 678.11. As a result of this changing demand pattern, the PSH results in much higher inventory holding and shortage costs compared to the L.D.R. solution.

B. A Linear Programming Problem

As a second illustration we use the UNISIM case [2]. The data for this case are given in Table 2.

In Table 3 the costs of the PSH and the optimal solution (linear programming) are given

The PSH results in a cost increase of 43,2 %. Note that in this case all costs are controllable. From table 3 it can be seen that the LP solution follows the demand pattern much more closely than the PSH heuristic. As a result the inventory holding and backorder costs are much higher in the PSH solution than in the optimal one. Again, the non-stationary demand structure is the reason for this phenomenon.

IV. THE ADAPTIVE PRODUCTION SWITCHING HEURISTIC

As explained in section II, Orr's random walk production-inventory policy applies the same rationale as underlies the (s,S) policy for inventory problems. It is well known that in a non-stationary or dynamic inventory model the optimal policy at each stage or time period n is characterized by two critical numbers which however may vary in successive periods. In other words, in a dynamic environment the optimal policy is of the (s_n, S_n) type.

TABLE 2 : The UNISIM case.

Period	Sales Forecast	Optimal Solution L.P.	P.S.H.	Period	Sales Forecast	Optimal Solution L.P.	P.S.H.
1	116,000	100,000	106,826.8	19	104,000	104,000	106,826.8
2	127,900	106,700	106,826.8	20	94,000	104,000	106,826.8
3	133,000	106,700	106,826.8	21	91,000	104,000	106,826.8
4	129,900	106,700	106,826.8	22	96,000	104,000	106,826.8
5	120,000	106,700	106,826.8	23	108,000	104,000	106,826.8
6	106,000	106,000	106,826.8	24	124,000	138,000	106,826.8
7	92,000	92,000	106,826.8	25	151,000	140,000	141,228.7
8	82,000	85,250	106,826.8	26	157,000	140,000	141,228.7
9	79,000	85,250	106,826.8	27	150,000	140,000	141,228.7
10	84,000	85,250	106,826.8	28	143,000	140,000	141,228.7
11	96,000	85,250	106,826.8	29	140,000	140,000	141,228.7
12	112,000	133,200	106,826.8	30	130,000	130,000	141,228.7
13	128,000	133,200	106,826.8	31	110,000	110,000	106,826.8
14	140,000	133,200	106,826.8	32	106,000	106,000	106,826.8
15	145,000	133,200	106,826.8	33	100,000	100,000	106,826.8
16	132,000	133,200	141,228.7	34	90,000	100,000	106,826.8
17	118,000	132,000	141,228.7	35	80,000	100,000	106,826.8
18	104,000	118,000	106,826.8				

Production fluctuation cost = absolute value [Last period production - this period production] x 0,10

Inventory holding cost = ending inventory x 0,02

Backorder cost = 0,25 per unit per period

Beginning inventory = 100,000

TABLE 3 : Comparison of the PSH and L.P.

	L.P.(optimal)	P.S.H.
Inventory Holding Cost	10,262	25,287.43
Production fluctuation costs	18,130	14,443.43
Backorder costs	0	928.50
Total costs	28,392	40,659.36

Analogous with the observation just made, we may consider to extend the Production Switching Heuristic by allowing adaptive control limits in the original PSH. Technically this creates no difficulties but from a conceptual point of view the problem is that an adaptive switching heuristic is in contradiction with the "raison d'être" of the method itself namely to comply more closely with managers current practice of avoiding rescheduling too frequently.

With this important remark in mind, the following policy was examined. Assume the PSH is applied on a rolling schedule basis. First the PSH control parameters are computed based on the first T periods, with $T < N$ where N is the length of the total planning horizon. The objective is now to use these production levels as long as possible. Next using a rolling schedule the average demand over Y periods, $Y < T$ is computed. If this average demand is K percent above or below the average demand of the T period problem then the PSH control parameters (again over a T period horizon) are recomputed. The average demand of the new control period will now be compared with the coming Y period moving averages and no changes are made as long as these averages are within K % of the T-period average demand level.

The underlying rationale of this procedure is straighforward : by comparing the average demand levels we explicitely introduce the short run demand fluctuations.

It is possible to include the additional control parameters T, Y and K in the search routine, but at the cost of a drastic increase in computer time. In practice, however, T, Y and K are considered to be managerial parameters for which fixed values are choosen by the decision-maker depending on the particular characteristics of the business involved. For that reason, the parameters are not optimized in the examples treated below.

As a first illustration consider the linear programming problem introduced in section III.b. Let T=6, Y=6 and K=15 %, the resulting production program deviates by 7,7 % from the optimal solution. This solution is given in table 4.

As can be seen from Table 4, 10 different production levels are used, resulting in a 41 % decrease of inventory holding costs compared to the previous PSH solution (see section III.B).

A similar analysis was carried out for the Paint Factory case. The PSH solution can also be improved by introducing adaptive control limits as is illustrated in table 5..

TABLE 4 : Adaptive PSH, T=6, Y=6, K=15 %.

Period	Production Level	Period	Production Level
1	105,467.1	19	101,833.3
2	105,467.1	20	101,833.3
3	105,467.1	21	101,833.3
4	105,467.1	22	101,833.3
5	105,562.5	23	114,340.1
6	105,562.5	24	138,833.3
7	93,833.3	25	138,833.3
8	93,833.3	26	138,833.3
9	93,833.3	27	138,833.3
10	93,833.3	28	138,833.3
11	93,839.2	29	138,833.3
12	127,000.0	30	139,173.9
13	127,000	31	106,666.6
14	127,000	32	102,666.6
15	127,000	33	102,666.6
16	127,000	34	102,666.6
17	127,000	35	102,666.6
18	119,795.9		

TABLE 5 : Adaptive PSH for the Paint Factory Case (60 periods)

T	Y	K	Total cost deviation from LDR
8	5	25	8.75 %
12	6	25	7.07 %
10	6	25	6.12 %

V. CONCLUSION

The PSH has been proposed as a practical procedure to solve the important aggregate production planning problem. The approach of Mellichamp and Love therefore has great potential for acceptance and use by managers. In a non-stationary environment however this heuristic may be too inflexible and as a consequence result in far from optimal solutions. It is possible to introduce more flexibility by adapting the control parameters as a function of the demand variability. The question is now whether the alternative solution methods such as linear programming or the search decision rules are not more attractive after all.

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