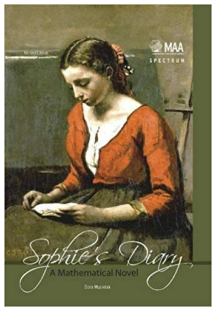
Sophie's Diary. A Mathematical Novel (2nd ed.) by *Dora Musielak*. MAA, 2012 (xii+279 p.) isbn: 978-0-88385-577-5.





Dora Musielak

Sophie Germain (1 April 1776 – 27 June 1831) is one of those women in mathematics who had to fight for being accepted as a mathematician. Her father was a wealthy merchant in Paris. He represented the bourgeoisie in the États-Généraux before the French Revolution that was sparked in 1789 by the fall of the Bastille (Sophie was then 13). In the subsequent social agitation radical sans-culottes, roamed the streets of Paris during a Reign of Terror, were it became common practice to imprison and/or kill anyone with the slightest suspicion of being anti-revolutionary. Especially clergy and nobility were targetted, but many of the bourgeoisie were terrorised as well. The last king and queen of France, Louis XVI and Marie-Antoinette, were guillotined in 1793. Sophie and her two sisters grew up in Paris in this turbulent and violent period.

According to her biographers, Sophie studied mathematics in that period on her own from books that she found in her father's library, which was originally not to the liking of her mother. It was in those days "not done" for a young girl to study mathematics that was still a "men only" privilege. So she had to study during the night in an unheated bedroom with candle light. She had to master also Latin and Greek to read the books by Newton and Euler.







When she was 18 in 1794, the École Polytechnique opened, still women-not-allowed, but the lecture notes were public. Under the alias of M. LeBlanc (a former student), she sent her work to Lagrange, who was impressed and when he requested a meeting, learned the real identity of the author. Later she worked on Fermat's Last Theorem (FLT) and corresponded with Legendre and Gauss. She also attempted several times to win a prize from the Académie about the vibration of Chladni plates, where she was in competition with Poisson. Eventually she won the prize in 1816 with her third attempt. Her work on FLT in which she proposed a general method for proof was however more relevant than her work on elasticity, but it was neglected for a long time since she did not publish it herself, but it was used and published by others. She died from breast cancer in 1831 at the age of 55.

Musielak in this book includes at the end a biographical sketch of Sophie Germain with more details than what I gave above. There is also the list of Sophie's publications and a list of references for further reading. The larger part of the book is however about a fictional Sophie in which Musielak imagines how a young girl like Sophie would have experienced the turmoil in Paris and learned mathematcs between 1 April 1789 (her 13th birthday) and 29 December 1794 (when she was 18). She does this in the form of diary entries written by Sophie. There are many entries dealing with the politics, the riots and the anarchy that ruled the streets of Paris and how she had to go against the dissuading measures from her mother (her father was more supportive as was also the philosopher and mathematician Marquis de Condorcet, who was a friend of the family). Those entries are genuinely credible as written by an emotional young girl. However many (if not most) of the entries are about mathematics. There I have more a problem to see this as realistic diary entries of a teenager. Here it is more Musielak who is instructing the reader about the mathematics that Sophie is learning, including proofs and historical remarks, and using mathematical notation

and ideas that we know from current mathematical schoolbooks. Most of them are about number theory and towards the end also about elasticity when Sophie detects calculus in the books by Newton, Euler and Kepler.



Many of the mathematical topics that this fictional Sophie writes about are the same as those found in books written for the popularization of mathematics, but of course the historical aspect is respected because only reference can be made to topics that are older than Sophie. So Sophie (i.e. Musielak) is writing about perfect numbers, Pythagorean triples, prime numbers and (approximations of) π, irrational numbers, Diophantine equations, the solution of linear, quadratic and cubic equations (including the Fior-Tartaglia dispute), the square root of -1, exponential growth, compound interest and the logarithm, Euler's constant, e^{ix} and the formula of de Moivre, infinite series and their convergence, Pascal's triangle, Fibonacci numbers, Goldbach's conjecture, and we see how Sophie is making her first steps in formulating her program she later develops in her attempt to prove FLT. She also tells about Hypatia of Alexandria (c. 360-415), Émilie de Châtelet (1706-1749, who translated Newton's work into French) and Maria Gaetana Agnesi (1718-1799) three other female mathematicians. With *Mécanique Analytique*, Lagrange's lecture notes that she obtains via a student Antoine LeBlanc, she learns about differential equations and equilibrium problems. She learns about the calculus of variations and finds the equation of tautochrone and brachistochrone curves.

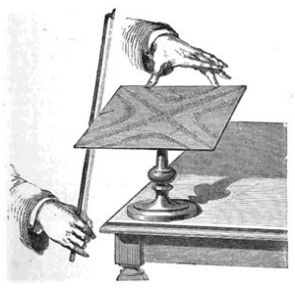




For her work on FLT Sophie Germain considers two cases for $x^p + y^p = z^p$. Case 1 when p does not divide x, y, or z, and case 2 when p does divide one of them. If p and $2p+1$ are both primes, then p is now called a Germain prime (of course she did not call it so). The first Germain primes are 2, 3, 5, 11, 23, 29, 41, 53, 83, 89, 113, and 131. She proved that in case 1, if p is an odd Germain prime then $x^p + y^p \neq z^p$. Legendre extended this soon to odd primes p such that $kp + 1$ is prime, $k = 4, 8, 10, 14$, and 16.

It is interesting to read about her interaction with Gauss (this is not in the diary but it is in the biographical notes). Gauss had published his *Disquisitiones Arithmeticae* in 1801 and some three years later when she was 25, Sophie Germain wrote her first letters to Gauss using the pseudonym of M. LeBlanc. After a period of silence she got worried and in 1806 she asked General Pernety to find out about the well-being of Gauss. Pernety was camped in Breslau (Wrocław) during the French occupation and he sends an officer to Gauss who was then in Brunswick to ask at the instance of Demoiselle Sophie Germain in Paris about Gauss and if needed offer protection. Gauss was perplexed, knowing only M. LeBlanc and he never heard of General Pernety or Demoiselle Germain. He only knew Madame Lalande in Paris, the wife of the famous astronomer.

Some of her work on Chladni's plates (plates sprinkled with sand showed particular patterns when the plate vibrates) that won the *Prix de Mathematiques* from the *Académie* was included in a memoir by Legendre in 1823. She never went to collect her prize (one kilogram of gold). Poisson, her prime competitor for the prize, was member of the committee and wrote to her that her work was lacking mathematical rigour. Poisson never sought further contact and ignored her in public. She continued working and published on the problem but never solved it completely. It was eventually successfully tackled by Kirchhoff (1850).



Chladni plate

Musielak writes in her justification that the diary is fictional and nothing justifies that Sophie has learned the mathematics in the way she describes. She also writes that "any errors in the mathematics are her own" but then adds that she never wanted to give it the rigour of a textbook and that some errors are intentional to reflect Sophie's learning steps. However, even though this is the second edition of the book there are several typos which I suspect not to be intentional. A superfluous power n in $\sum_{n=1}^{\infty}(\frac{1}{n})^n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ (p. 67); an extra factorial in $\frac{3n!}{(n+1)!} = \frac{3}{(n+1)!}$ (p. 147); an extra prime in $\dot{x} = \frac{dx}{dt'}$ (p. 163); a dx under the square root in $L[y] = \int_{x_0}^{x_1}\sqrt{1+y'(x)^2dx}$ (p. 220); spelling typos: *mémoirés* instead of *mémoires* (p. 189); *uvres* should be *Œuvres* in the footnote (p. 242); Chadlin instead of Chladni (p. 248). Also *étés* is misspelled when Sophie writes to Gauss: *Je vous aurez la plus grande obligation si vous étés assez bon pour prendre la peine de me dire ce que vous pensez de la marche que j'ai suivie.* (p. 260).



Père Lachaise

But all in all the whole book reflects very well how Sophie Germain fought her way into science, basically all on her own. It is probably a good thing that Musielak is a woman and hence may be better suited to get under the skin of a teenage girl. Also the description of the striking events of the French revolution are entries of the diary that stay with you after closing the book. The only illustrations in the book are historical gravures illustrating these events. The picture on the cover is a painting by Camille Corot from the mid 19th century, called *A girl reading*. It is not representing Sophie Germain. Although fiction, this book is an inspiring read about a remarkable woman in mathematics.


Sofia Kovalevskaya





Cecilia Payne-Gaposchkin


Marie Curie


Emma Noether

I have some afterthoughts about how women in mathematics are "mistreated" in pictures. The picture of the young girl in the beginning of this review is made by Auguste Leray and appeared in a book "l'Histoire du Socialisme" c. 1880 and it shows "Sophie GERMAIN à l'age de 14 ans". The statue is by Zacharie Astruc and is reconstructed from her death mask in the museum of the Louvre. It is located at a school in Paris named after her. The medal is a made to commemorate Sophie Germain and has an inscription to confirm that. The pictures above can be found on the web and are claimed to be Sophie Germain, but that has a lot of fake info. The two leftmost pictures are from Sofia Kovalevskaya. Since this is also a "Sophie" and given the ressenblance with the portrait of the young Sophie, the confusion is understandable. However the next pictures are less acceptable. We see on the first of them Cecilia Payne-Gaposchkin, an English Astonomer that is a popular Sophie on Spanish sites. Next are Marie Curie and Emmy Noether and on the right Mary Fairfax Greig Somerville, all pictures commonly mistaken for Sophie Germain. I think that the few women (and their pictures) that we have in mathematics should be treated a bit more carefully.


Mary Somerville

Adhemar Bultheel