The birth and early development of orthogonal polynomials. A chronological history *Claude Brezinski and Michela Redivo-Zaglia.* SIAM, 2025 (xvi+593 p.) isbn: 978-1-61197-850-6, Other Titles in Applied Mathematics, vol 207

In this book, the authors search the period from approximately the second half of the 18th century until the start of WWII for traces of the first appearances of orthogonal polynomials (OP). The adjective "orthogonal" in relation to polynomials seems to have been used for the first time in the work of Sturm and Liouville around 1936-37, but the concept of functions being orthogonal became familiar only at the beginning of the 20th century. A bibliography on OP was collected and published by the National Research Council of the US in 1940, showing that by that time the topic had gained mainstream interest.

Orthogonal polynomials are related to many different topics in mathematics, and hence the first traces can be found among rather diverse papers. The book starts by explaining these relationships for readers not familiar with the subject so that they may appreciate the disguises in which OP appear within historical papers. The core of the theory is a positive weight on a finite or infinite interval of the real line. Approximating its power series expansion (its coefficients are called its moments) by rational functions (Padé approximants) one solves a truncated moment problem, and the denominator and numerator of the approximant can be seen as (formal) OP and polynomials of the second kind, respectively. These rational approximants can also be written as truncated continued fractions, and its backward evaluation scheme illustrates the three-term recurrence relation for OP of three successive degrees.

The first traces of OP appeared in the context of generating functions. To solve a second-order Sturm-Liouville differential equation, a power series expansion $g(z)$ of its solution is proposed, but if it also depends on a parameter x, then its coefficients depend on x. In certain cases these coefficients turn out to be OP because they satisfy the three-term recurrence relations for some weight (Favard's Theorem), and $g(z)$ is a generating function for the OP sequence. Several systems of OP have been well studied and their properties cataloged. Depending on the measure, they are linked to the names of mathematicians: Legendre, Jacobi, Chebyshev, Bessel, Hermite, Laguerre– even though these were not always the first to have used them in their work.

The zeros of OP are often used as the nodes of Gaussian quadrature formulas for the highly accurate numerical approximation of integrals.

The period covered was a rather turbulent one in European history: the French and Russian revolutions; the Napoleonic wars; the formation of Italy, Germany, Switzerland as unified nations; and the disaster of WWI. Since these events influenced the lives and work of the mathematicians of that period, this history is described in a second chapter. The major scientific projects were the encyclopedia of Diderot and d'Alembert and the expeditions to measure the world's meridian by triangulation (to prove that the Earth was an ellipsoid flattened at the poles).

Chapters 4 to 40 then discuss the lives and work of mathematicians starting with Lagrange in 1770 and ending with Shohat in 1938. Legendre polynomials were the first to appear, showing up in generating functions related to computations for the meridian expeditions. Other OP were studied by following similar ideas in connection with problems of gravitational attraction or electrical potentials for ellipsoid bodies. Gauss discovered OP from a very different perspective when he wanted to compute numerical approximations of integrals. The authors of this book have to explain the technical details because in many cases the mathematicians involved did not even know they were using OP since the concept did not exist yet. Because these ideas were circulated in letters or were presented before being published, priority disputes arose, which the authors of this book try to resolve.

Apart from the mathematical ideas, the book also devotes a lot of attention to the history and the biography of all the mathematicians involved, covering not only their education, employment, character, and physical appearance, but also their family history and what became of their siblings. Even the origin and history of their birthplaces or where they worked and sometimes the biography of their biographer are included.

An index of names (15 pages long) provides an indication for the large number of people discussed in this book. I particularly liked Chapter 2 on the historical background. Many events that happened in people's lives in the subsequent chapters are placed in a broader perspective. This book also clearly illustrates that new mathematical ideas (such as OP) often originally arise from a need to solve practical technical problems, and only in a later stage do they become formal mathematical concepts that develop further independently.

The careful dissection of mathematical formulas within historical papers to unveil the polynomials hidden underneath requires both the skills of a

mathematician familiar with the subject and that of an historian to bring them to light. The authors do this carefully and well. The many historical and biographical details are entertaining and interesting, but they may be a hindrance if the reader is only interested in the history of the mathematical ideas. The background information is detailed and factual with some repetition in the format. A chapter on mathematician M starts with something like: M was born on [date] in [place], followed by a history of [place] and the occupation of the parents of M. The chapter ends with: M died on [date] in [place] because of [cause]. Often a description of the mathematician's personality is provided, citing a colleague, a student, a biographer, or an obituary. Historians will appreciate that all the historical fragments of the texts are cited in the original language, while the lesser polyglot reader will appreciate that they are translated into English.

The history is restricted to orthogonality on the real line because orthogonal polynomials on the unit circle were only studied by Gábor Szegő around 1920 (although there are probably earlier traces to be found) and became popular after his book of 1939.

Most importantly, this is the first book on the history of orthogonal polynomials, a tremendously important tool in current mathematical analysis and applications. The amount of historical information and evidence presented in this book is overwhelming. It has the potential to become a reference book on the subject.

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