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Here is another volume of the *What's happening in the mathematical sciences* series highlighting some trending subjects in mathematics in which some progress has been made recently. The idea is that the mathematics are explained to the layman, interviewing the mathematicians that worked on the subject, very much like the articles of the online *Quanta Magazine* do for a broader scientific spectrum which includes besides mathematics also physics, biology, and computer science. On the other hand, this is definitely different from the annual anthology *The best writings on mathematics* edited by M. Pitici (see e.g. [*M. Pitici*, The best writing on mathematics 2021. Princeton, NJ: Princeton University Press (2022; Zbl 1506.00100)] for the latest issue), that is reproducing published papers that are about mathematics and mathematicians, and their impact and importance, but most often these do not treat the mathematics itself.

Like Pitici's survey of 2021, also this issue has a strong signature of the 2020 pandemic where mathematics had a rather important contribution. Hence, it should be no surprise that three chapters in this issue are devoted to this global catastrophe. One deals with the epidemic modeling of how it was spreading and how these models had to be adapted as more information became available. A second is about how models tried to catch the influence of vaccination and lock-downs which were essential for politicians to make decisions. The third one is a study of the virus itself on a RNA scale: how it operates and spreads in a human body which is essential if one wants to develop a vaccine.

With Covid-19 more or less under control, the other lurking global disaster of climate change that was temporarily pushed to the background, comes back to the foreground in full force. So there is understandably also a long chapter dealing with this threat. It explains the shift in appearance of extreme events like heat domes or water bombs, the relation between weather and climate, and how to identify the change in global measurements. Also here, like in so many other applications, machine learning is of increasing importance.

The four remaining chapters are somewhat lighter. One is about Apollonian tiling. Three touching circles enclose a curvilinear triangle in which a circle can be fitted touching the 3 bounding circles, which generates 3 other triangular holes, that can be filled etc. If the radii of the 3 circles are of the form $1/n_i$, where n_i, $i = 1, 2, 3$ are integer, then the circle in the triangle will have a radius $1/n_4$ with n_4 integer. This can be generalized in various directions, using Gaussian integers or more generally using numbers in $\mathbb{Z}[\sqrt{-p}]$ with p prime, or one may consider the 3D analogue by considering Apollonian sphere packing. There is an obvious connection with fractals and hyperbolic geometry. It is worth checking the Internet looking for the beautiful Schmidt arrangements obtained by Katherine E. Stange by applying Moebius transforms. These are generated, with the open source software SageMath, so you can experiment yourself. The problem was originally proposed in 1643 by Descartes as a homework problem for Princess Elizabeth of Bohemia, until he realized that the problem was way to hard for her to solve.

Another chapter is a discussion of what is known as the four-peg problem. If one draws any (smooth) non-intersecting Jordan curve, then one can always find four points on this curve that are the vertices of a perfect square. A simple problem, but difficult to prove, and once it is solved, many other problems come to the surface. What if squares are replaced with polygons with vertices on a circle, what with non-smooth or with self-intersecting curves?

Fermat's last theorem is a problem formulated in terms of a Diophantine equation, i.e., an algebraic equation with integer coefficients. This type of equations has a very long history and they are related to several aspects of number theory. Here the history of the problem of finding the algebraic solutions of a diophantine equation are discussed.

The remaining chapter of this volume starts from a simple graph problem which corresponds to computing the eigenvectors corresponding to the eigenvalue zero of a tridiagonal weighted adjacency matrix. Such problems are related to defining spins of a qubit network, but also to the stabilisation of buildings. The most famous example of the latter is the Taipei 101 skyscraper (once the world tallest building) that has

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a 660 ton pendulum hanging in its core to stabilise the building to survive an earthquake. The tridiagonal structure of the matrix corresponds to a simple serial connection of the nodes in a graph. What can be said about more general graphs? The original problem was to find out for a general graph where the value of a node defines the values of its neighbors: what is the minimal set of nodes that one has to fix to fix the whole network. This corresponds to finding in the corresponding class of (adjacency) matrices, the minimal set of entries that will maximize the dimension of the null space. Finding these 'zero forcing sets', is computationally a very hard problem and that is where recently some progress has been made. Solving the problem is important when the graphs represent social or traffic networks, or power grids, or certain pursuit problems.

So this is again a very nice collection of items where mathematics plays an essential role and that can be read by anyone with only a very elementary knowledge about mathematics. Several of the subjects discussed are not only interesting from a mathematical point of view, but they are also important from a social point of view. Dana Mackenzie once more succeeds in selecting these juicy topics and bring these in a pleasant and entertaining way.

See the reviews of Vols. 1, 2, 3, 4, 6, 8 and 11 in [Zbl 0858.00003; Zbl 0858.00004; Zbl 0858.00004; Zbl 1028.00002; Zbl 1244.00007; Zbl 1266.00007; Zbl 1477.00003]. For Vols. 5, 7, 9 and 10 see [Zbl 1242.00011; Zbl 1326.00011; Zbl 1326.00016; Zbl 1326.00017].

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MSC:

00A09 Popularization of mathematics
00A05 Mathematics in general
00A06 Mathematics for nonmathematicians (engineering, social sciences, etc.)
00B15 Collections of articles of miscellaneous specific interest

Keywords:

pandemic; RNA; climate change; Apollonian circles; plane tiling; square peg conjecture; Diophantine equations; zero forcing set; quantum spin; networks; graphs

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