KU LEUVEN

ARENBERG DOCTORAL SCHOOL
Faculty of Science

Dynamics of Branes and Fluxes

Brecht Truijen

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Dissertation presented in partial fulfilment of the requirements for the degree of Doctor of Science (PhD): Physics

September 2018

Dynamics of Branes and Fluxes

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September 2018

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Uitgegeven in eigen beheer, Brecht Truijen, Celestijnenlaan 200d - box 2415, B-3001 Leuven (Belgium)

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Acknowledgements

By now I have spent more than five years in Leuven in pursuit of my PhD. I would not have made it through those years without the help and support of good friends. Therefore a few words of gratitude are in order.

Firstly, let me thank the members of my examination committee, Enrico Carlon, Andres Collinucci, Thomas Hertog, Christian Maes, Niels Obers, Antoine Van Proeyen and Thomas Van Riet. In particular, I would like to thank my supervisor Thomas for giving me the opportunity to start a PhD in Leuven and my co-supervisor Thomas for his guidance, many useful discussions and especially for sharing his enthusiasm for physics.

I would also like to thank all the people of the Leuven Institute for Theoretical Physics (ITF) for making it such a wonderful work environment. Let me start with all the lovely people with whom I had the pleasure to share my office. Thank you Alessandra, Alice, Magnus, Marco and Ruben for all the fun times which made coming to the office a pleasure. In addition I am deeply grateful to Adam, Ellen, Fridrik, Jesse, Juan, Nikolay and Yannick for all the good times at the ITF and for thinking of me during the difficult times. Fridrik, if you ever need to get some old school camera equipment, let me know! I can probably find some greasy Belgian food along the way. The list does not end here, all members of the ITF are responsible for the great atmosphere: Alexander, Alexandre, Anneleen, Annemie, Ben, Bert, Bruno, Curtis, Edoardo, Federico, Fotis, Frederik, Gabriele, Gerben, Hans, Jesper, Kristof, Kwinten, Lihui, Manuel, Marco, Marina, Marius, Marjorie, Matt, Mattia, Michiel, Natascia, Pablo, Petar, Pierre, Pieter, Rob, Stefanos, Tarek, Tom, Thibaut, Vincent, Ward, Winny and Zehui. Let me also use this opportunity to single out Anneleen and Filip and thank them for their efforts in organising many of the social gatherings at the ITF. I definitely want to thank Filip and the ITF for throwing a bachelor party for Kristof and me and I am sorry for wasting a good opportunity to perform a small bungee jump. A wedding usually follows a bachelor party and I want to thank the ITF for being so numerously present. I am sure my family can still

remember all those physicists on the dancefloor!

I also want to thank my friends from the PhD school for sharing their passion for physics: Diego, Dries, Fotis, Laurens, Mario, Marcelo, Nava, Luca, Panagiotis, Watse and Yannick. Additionally, let me thank the fellow organisers of the Modave Summer School in Mathematical Physics for all the wonderful times: Amaury, Andrea, Blagoje, Ellen, Gabriele, Jonathan, Joris, Laura, Laure-Ann, Marco, Pujian, Ruben, Thomas, Yannick as well as everybody who attended the school. Needless to say I am also grateful to all my research collaborators and many other friends outside of Leuven that I have forgotten to mention here.

To end I would like to thank my family and friends for their support over the last years. In particular my parents for granting me the opportunity to study and supporting my choices. I also want to thank my sister Tine for correcting many typos, my friends from UHasselt with whom my physics adventure started almost ten years ago and my friends from Utrecht. In particular, I want to thank Jelle for sharing a flat with me in Utrecht and Leuven where I watched him slowly embrace the dark side and develop an interest in finance. To conclude, I thank all of my friends for trying to understand and (pretending) to be interested in my PhD and, most of all, for taking my mind off my PhD often enough!

Last but not least, I want to thank my wife Stéphanie and daughter Liss without whom none of this would have been possible or wouldn't have even made sense.

Thank you all!

The research in this thesis is supported by an FWO PhD fellowship.

Abstract

Our universe has a small, positive cosmological constant that is difficult to understand from a microscopic perspective. String theory could provide an understanding by describing the current state of our universe as a compactification. However, these compactifications are difficult and we lack an explicit calculable example. We can introduce additional symmetries to aid in the construction of a string model for our universe. Supersymmetry is a useful tool in this regard but it is necessarily broken in our universe. In this thesis we study the breaking of supersymmetry in string theory.

If we wish to benefit from the added structure of any symmetry then it better be broken spontaneously. In the context of supersymmetric gauge theories the breaking is expected to happen dynamically. This means that it happens due to nonperturbative effects. Moreover, the non-supersymmetric state is usually associated with a metastable state that can eventually decay to the supersymmetric ground state.

The AdS/CFT correspondence is a concrete realisation of the holographic principle in string theory. It relates a gravity theory with anti-de Sitter (AdS) boundary conditions to a conformal field theory. In string theory, certain branes are known to provide a holographic description of spontaneous supersymmetry breaking. These branes are called antibranes as they carry charge opposite to other background fields called fluxes. Moreover, antibranes are also used to break supersymmetry in string compactifications and create a metastable vacuum. Spontaneous supersymmetry breaking can thus be achieved using branes and fluxes of opposite charge. However, there is an ongoing debate on whether their interaction results in a perturbatively unstable situation rather than a metastable state.

The instability is due to the dynamics of branes and fluxes. Branes can materialise out of flux to annihilate with the antibranes. This is possibly prevented by a potential barrier so that the annihilation can only proceed

after tunnelling. In this thesis we study antibranes in the hopes of better understanding their interaction with background fluxes.

In the first part we introduce the necessary string theory framework in chapter 2 after motivating our research in chapter 1. In chapter 3 we review different string constructions that are relevant to this thesis. Afterwards, we introduce the necessary background involving antibranes and supersymmetry breaking in chapter 4.

In chapter 5 we study the dynamics involved in brane-flux annihilation for anti Dp with $p < 7$. This involves brane polarisation as was previously understood in the case of $p = 3$. These dynamics are subsequently used in chapter 6 to study antibranes with AdS worldvolume instead of the usual flat worldvolume. We find that perturbative brane-flux decay can be prevented when the worldvolume is AdS. In the case of flat worldvolume, that is of interest to our universe, we can not make the same statement. We show that supersymmetry of the solutions with AdS worldvolume depend on p nontrivially.

In chapter 7 we study non-supersymmetric deformations of fractional p-brane solutions. In the case of $p = 3$ this describes a deformation of the Klebanov-Strassler solution. Klebanov-Strassler is the gravity dual of an $\mathcal{N} = 1$ gauge theory with interesting features such as confinement and chiral symmetry breaking. Consequently, the solution presented in chapter 7 is a potentially interesting playground for the holography of non-supersymmetric gauge theories.

In chapter 8 we study supersymmetric antibranes in the context of AdS_7 solutions of massive type IIA string theory. These solutions are holographically dual to six-dimensional (1,0) superconformal field theories. We improve on this by more firmly establishing the holographic interpretation of the background as a near-horizon limit of an NS5-, D6-, D8-brane intersection. Using a consistent truncation to seven-dimensional gauged supergravity we construct a universal supersymmetric deformation of these AdS vacua. In the holographic dual this deformation describes a universal RG flow on the tensor branch of the vacuum moduli space triggered by a vacuum expectation value for a protected scalar operator of dimension four.

Beknopte samenvatting

Ons universum bezit een kleine, positieve kosmologische constante die we moeilijk microscopisch kunnen verklaren. Snaartheorie kan een verklaring geven door ons universum als compactificatie te beschrijven. Zulke compactificaties zijn echter moeilijk en we hebben geen expliciet model waarin berekeningen mogelijk zijn. Men kan extra symmetrie introduceren om te helpen bij een snaartheoretische constructie van ons universum. Vanuit dit perspectief is supersymmetrie een nuttig hulpmiddel maar in ons universum is die symmetrie noodzakelijk gebroken. In deze thesis bestuderen we de breking van supersymmetrie in snaartheorie.

De structuur afkomstig van supersymmetrie is enkel nuttig wanneer de symmetrie spontaan gebroken is. Men verwacht dat deze breking dynamisch gebeurt in een supersymmetrische ijktheorie. Dit betekent dat de breking een gevolg is van niet-perturbatieve effecten. Daarenboven is de niet-supersymmetrische toestand meestal een metastabiele toestand die uiteindelijk kan vervallen naar de supersymmetrische grondtoestand.

De AdS/CFT correspondentie is een concrete realisatie van het holografische principe in snaartheorie. De correspondentie relateert een gravitationele theorie met anti-de Sitter (AdS) randvoorwaarden aan een conforme veldentheorie. Snaartheorie bezit branen die een holografische beschrijving van spontane supersymmetrie breking geven. Zulke branen hebben een lading die tegengesteld is aan die van andere achtergrondvelden die we fluxen noemen. Vandaar dat we spreken van antibranen. Daarenboven worden zulke antibranen ook gebruikt voor de breking van supersymmetrie in snaarcompactificaties om een metastabiel vacuum te creëren. Men kan spontane supersymmetrie-breking dus beschrijven met behulp van branen en fluxen van tegengestelde lading. Het is echter onduidelijk of de interactie tussen branen en fluxen resulteert in een metastabiele toestand of eerder in een perturbatief onstabiele situatie.

De instabiliteit is een gevolg van de dynamica van branen en fluxen. Branen kunnen gecreëerd worden uit flux en annihileren met antibranen. De annihilatie

kan voorkomen worden door een potentiaal barrière zodat deze enkel nog via tunneling kan gebeuren. In deze thesis bestuderen we antibranen in de hoop de interactie met achtergrond fluxen beter te begrijpen.

In het eerste deel van deze thesis introduceren we de nodige snaartheorie in hoofdstuk 2 nadat we ons onderzoek motiveren in hoofdstuk 1. We gebruiken hoofdstuk 3 om bepaalde snaarconstructies te herhalen die van belang zijn in het vervolg van de thesis. De nodige achtergrond over antibranen en supersymmetrie breking kan men terugvinden in hoofdstuk 4.

In hoofdstuk 5 bestuderen we de dynamica van braan-flux annihilatie voor anti Dp met $p < 7$. Dit process gebeurt door middel van braan polarisatie zoals reeds begrepen was voor $p = 3$. Vervolgens gebruiken we deze kennis in hoofdstuk 6 om antibranen met AdS wereldvolume in de plaats van het meer gebruikelijke vlakke wereldvolume te bestuderen. We laten zien hoe perturbatief braan-flux verval voorkomen kan worden wanneer het wereldvolume AdS is. Het geval van vlak wereldvolume is belangrijk voor het modelleren van ons universum maar hier kunnen we niet dezelfde conclusie trekken. We tonen tevens aan dat de supersymmetry van de AdS oplossingen niet triviaal afhangt van p.

In hoofdstuk 7 beschrijven we niet-supersymmetrische deformaties van fractionele p-braan oplossingen. Wanneer $p = 3$ dan zijn dit deformaties van de Klebanov-Strassler oplossing. Klebanov-Strassler is de graviationele duale van een $\mathcal{N} = 1$ ijktheorie met interessante eigenschappen waaronder confinement en chirale symmetrie breking. Bijgevolg zijn de oplossingen uit hoofdstuk 7 potentieel interessant voor het holografisch bestuderen van niet-supersymmetrische ijktheorieën

In hoofdstuk 8 bekijken we supersymmetrische antibranen in de context van AdS_7 oplossing in massieve IIA snaartheorie. Deze oplossingen zijn holografisch duaal aan zes-dimensionele (1,0) superconforme veldentheorieën. We versterken de holografische interpretatie van deze achtergronden als deel van een NS5-, D6-, D8-braan intersectie. Door middel van een consistente truncatie van zeven-dimensionele geijkte supergravitatie construeren we een universele supersymmetrische deformatie van deze AdS vacua. Holografisch komt deze deformatie overeen met een universele RG flow van de tensor tak van de vacuum moduli ruimte door een vacuum verwachtingswaarde te geven aan een beschermde scalaire operator van dimensie vier.

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Chapter 1

Introduction

1.1 Length scales

It is difficult not to overemphasise the importance of length scales in physics. We do not describe colliding biljart balls with the same equations as we would to understand the microscopics of the hydrogen atom. Most of the time physics at a certain length scale is rather insensitive to both smaller and larger length scales. Exceptions do exist of course, most well known are phase transitions which have very long correlation lengths whereas details depend on the microphysics. A few length scales and the associated energy scales can be found in table 1. To one extreme we find the largest of length scales, namely the size of the observable universe, roughly 10^{26} m. In natural units ($\hbar = 1$ and $c = 1$) we may associate 10^{-42} GeV with the size of the universe using:

$$1 \frac{\hbar c}{\text{eV}} = 1,973 \cdot 10^{-7} \,\text{m}\,. \tag{1.1}$$

There is another observable that has roughly the same associated length scale as the size of the universe, it is called the cosmological constant (cc) . This constant can be thought of as a constant energy density that provides the simplest model of the accelerated expansion that is observed in our universe:[1]

$$\Lambda = 1,11 \cdot 10^{-52} \,\text{m}^{-2}\,. \tag{1.2}$$

The size of the universe is indeed of the same order as $\sqrt{\Lambda^{-1}}$.

[1] Value estimated using data from the 2015 Planck collaboration [3].

On the other end of the length spectrum we find the Planck length at which quantum gravity effects are expected to become important. The Planck length is formed by combining the gravitational coupling constant G with $\hbar$ and c:

$$l_p = \sqrt{\frac{\hbar G}{c^3}} = 1,616 \cdot 10^{-35}\,\mathrm{m}\,. \tag{1.3}$$

This is the smallest distance known to physics and we do not have a working description of physics at these length scales but expect that gravity becomes equally important as the other forces in nature. The smallest length scales at which we still understand physics is the Large Hadron Collider's TeV scale (10^{-20} m), slightly smaller than the length scale associated with the electroweak symmetry breaking: 10^{-18} m (10^2 GeV). This is still many order of magnitude below the energies associated with the Planck scale:

$$M_p = \sqrt{\frac{\hbar c}{8\pi G}} = 2,435 \cdot 10^{18}\,\frac{\mathrm{GeV}}{c^2}\,, \tag{1.4}$$

where an explicit factor of 8π was included because this will simplify equations later on. Looking at our current understanding of the physics at small length scales there is the expectation that the electroweak energy scale should be derivable from a higher energy scale. This would naively correspond to the Planck scale, 16 orders of magnitude bigger.

This is not inconsistent but it is unexpected and therefore puzzling why the Planck and electroweak scale are so far apart[2]. The above is often formulated as the electroweak hierarchy problem. Now, if we include cosmological observations then we can formulate a second hierarchy puzzle known as the cosmological constant problem. We will argue that the cosmological constant should also be derivable from the Planck scale, but in this case we have a mismatch of 60 orders in magnitude.

1.2 Dark energy

1.2.1 The cosmological constant

Einstein's theory of general relativity describes gravity as the curving of spacetime. In the absence of matter the Einstein equations read:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0\,, \tag{1.5}$$

[2]Our expectation is based on reasons, partly historical, that will be explained in the following pages.

Planck	LHC	Electron	Milky Way	Universe
$1,6 \cdot 10^{-35}$ m	10^{-20} m	10^{-12} m	10^{21} m	$8,8 \cdot 10^{26}$ m
$2,4 \cdot 10^{27}$ eV	10^{13} eV	10^{5} eV	10^{-28} eV	$2,2 \cdot 10^{-34}$ eV

Table 1.1: The observable universe provides us with the largest length scale discussed in physics. It differs by some 60 orders of magnitude from the smallest length scale known to physics, the Planck length. To grasp this enormous difference a few intermediate scales are given, complemented by the associated energies in electron volt.

where $g_{\mu\nu}$ is the spacetime metric and $R_{\mu\nu}$ its curvature or Ricci tensor. For historical reasons Einstein found this equation unsatisfactory and noticed that one could add an extra term consistent with all symmetries:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R - 2\Lambda\right) = 0 \,. \tag{1.6}$$

The term Λ was later called the cosmological constant and there are multiple equivalent descriptions that we will briefly outline.

When we include matter the Einstein equations change and the right hand side gets a source term called the energy-momentum (EM) tensor $T_{\mu\nu}$:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R - 2\Lambda\right) = M_p^{-2} \, T_{\mu\nu} \,. \tag{1.7}$$

The cosmological constant can be seen as a constant energy density, $\rho_\Lambda = M_p^2 \Lambda$ that contributes to the EM tensor as $T_{\mu\nu} = -\rho_\Lambda \, g_{\mu\nu}$. It is instructive to compare this to the EM tensor of an ideal fluid, moving with velocity U^μ:

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu} \,. \tag{1.8}$$

We see that the cosmological constant is similar to an ideal fluid with constant negative pressure equal to its energy density: $p = -\rho$. Moreover, if we couple scalar fields to gravity the minimum of the potential (V_0) acts as a constant energy density:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[(\partial \phi)^2 + V(\phi) \right] \,. \tag{1.9}$$

Hence, we may also identify V_0 with ρ_Λ. The cosmological constant can thus manifest itself in a multitude of equivalent ways: as a constant energy density, a negative pressure state of an ideal fluid, the minimum of a scalar field potential or a simple parameter added to the Einstein equations. In this context it is important to notice that the Einstein equations are sensitive to the absolute

value of the energy, contrasting all other situations in physics where only energy differences have any significance. This is a consequence of physics being independent of a chosen coordinate system and is a feature of any theory invariant under general coordinate transformations.

The solution to (1.6) reduces to Minkowski space for zero cosmological constant but when we consider nonzero values it has an interesting effect on the spacetime geometry. In particular, its sign determines the global structure of spacetime. When Λ is negative the solution is given by anti-de Sitter space or AdS (see appendix A for details):

$$\mathrm{d}s^2_{\mathrm{AdS}} = L^2\left(-\cosh^2\chi \mathrm{d}\tau^2 + \mathrm{d}\chi^2 + \sinh^2\chi \mathrm{d}\Omega^2_{n-1}\right). \tag{1.10}$$

Anti-de Sitter is a solution to Einstein's equations with constant negative energy density. Initially parallel geodesics will eventually intersect in AdS. If we take a constant positive energy density, as is the case in our universe, we find de Sitter space (dS) as a solution (more details can be found in appendix A):

$$\mathrm{d}s^2_{dS} = -\mathrm{d}t^2 + L^2\cosh(t/L)^2\mathrm{d}\Omega^2_n\,. \tag{1.11}$$

This can be thought of as an exponentially expanding universe.

In Einstein's classical gravity theory the cosmological constant is associated with a finetuning puzzle. The previous section mentioned that Λ differs by 60 orders of magnitude compared to the natural length scale in general relativity, the Planck length. In the following pages we will argue that Λ and l_p are related such that a fine tuning of 60 significant numbers is required. It can be shown that there is little or no room to change the Einstein equations to solve this finetuning puzzle. The reason is due to Weinberg [202] who proved that any theory similar in structure to general relativity necessarily needs to be finetuned if Λ is not of the Planck scale.

We related the cosmological constant to a constant energy density. This is the lowest possible energy without including any other contributions from the dynamics of matter fields. One might wonder whether we can understand this using a fundamental theory describing the matter in our universe. The lowest possible energy is then the vacuum energy. To use the quantum description of matter fields in combination with classical gravity we may simply substitute the energy-momentum tensor by its quantum expectation value:[3]

$$T_{\mu\nu} \to \langle T_{\mu\nu}\rangle\,. \tag{1.12}$$

From this viewpoint we would like to understand how the vacuum energy interacts with gravity and move away from the classical finetuning issue. If we

[3]This can be seen as an extension of Ehrenfest's theorem in quantum mechanics.

expect classical gravity to remain a valid description until we reach length scales closer to l_p then this is the expected scale of the vacuum energy or cosmological constant for reasons explained section 1.3.1.

As a final remark note that in semiclassical gravity we combine general relativity with a quantum description of matter. The quantum description is valid and tested up to the electroweak scale but gravity is too weak to be tested at such short distances. General relativity has been tested up to 10^{-6} m [4] with the state of the art experiments trying to go down to 10^{-8} m in strong contrast with quantum electrodynamics being tested at 10^{-15} m, even 10^{-18} m. This shows that quantum gravity is not only theoretically challenging.

1.2.2 Vacuum energy

One of the remarkable things in quantum mechanics is the appearance of a zero point energy. The quantisation of the harmonic oscillator shows that the minimum energy is not what one would expect from the classical theory. If the potential for the oscillator is normalised such that it is zero in the minimum, one can show that the ground state energy is $\frac{1}{2}\hbar\omega$ where ω is the natural frequency of the oscillator. This value can then be associated with the vacuum energy. If we move towards quantum field theory, a quantised field is similar to having a harmonic oscillator at every spacetime point. Without a proper understanding of renormalisation the vacuum energy would be infinite. We already mentioned that absolute energies are only important when gravity is involved. Hence, without gravity the vacuum energy is only indirectly observable. Indeed, one can observe the physical effects of quantum fluctuations but only by measuring (vacuum) energy differences.

Perhaps the most well known example is the Casimir effect: consider two parallel conducting plates, due to the difference in vacuum fluctuations in between the two plates and outside of the plates there is a net force pushing the plates together. This force has been measured in the lab, directly confirming quantum vacuum fluctuations. If the plates have area A and are separated a distance L the force is:

$$F = \frac{\pi^2 \hbar c A}{240 L^4} \,. \tag{1.13}$$

Similar observations of vacuum fluctuations include the Lamb shift which is a small splitting of atomic energy levels due to electron-proton interactions involving the quantum fluctuations of virtual particles (photon polarisation). The splitting can be predicted using quantum electrodynamics and for the hydrogen atom it is of the order:

$$\Delta E = \alpha^5 m_e c^2 = 1,06 \cdot 10^{-5}\,\mathrm{eV} \,. \tag{1.14}$$

The general theory of relativity was build with the equivalence principle in mind, stating that inertial and gravitational mass are equal. Actually, a stronger form of the equivalence principle holds: any form of energy interacts gravitationally. It then becomes puzzling why the vacuum energy contributes to the Casimir effect and Lamb shift but does not couple to gravity, in disagreement with the expectations from general relativity [174]. The Lamb shift is for instance much larger than the vacuum energy measured in our universe.

From here on we will no longer make the distinction between the cosmological constant Λ, constant energy density, minimum of a scalar field potential or vacuum energy. The puzzle at hand is about the equivalence principle at small distances and the vacuum energy which affects our late time cosmology. We will define the cosmological constant problem as the puzzle why the vacuum energy does not gravitate as would naively be expected and we forget about the classical finetuning issues. To discuss these matters in a bit more depth we will formulate the puzzle in the context of effective field theory. This is the framework in which we understand the standard model of particle physics, our quantum theory of all matter and forces with the exception of gravity.

1.3 Effective field theory

1.3.1 Wilson

From this point on we will describe all matter in the language of quantum field theory. This is how the standard model is formulated. We will also think of general relativity as a field theory, even as the low-energy description of a quantum theory. The modern perspective and understanding of quantum field theory as developed by Wilson is as a low-energy description called an effective field theory. Let us briefly sketch Wilson's perspective as can be found in many good field theory textbooks (e.g. [184]).

With any quantum field theory we should associate a length or energy scale. The usual ultraviolet (UV) divergencies which cancel in renormalised quantities are then understood as coming from smaller length scales that are irrelevant to the physics of interest. We think of the quantum field theory at lengths L as having coarse grained over smaller lengths $\ll L$. In the context of particle physics it is more useful to discuss energy scales and we associate a mass (energy) scale M with our effective field theory such that we have integrated over all momenta above the scale M. In this way one can construct the effective theory starting from a more fundamental theory valid at higher energies. However, it is often more useful to construct the effective theory from the bottom up. This can be

the case when we don't know the more fundamental theory or when it becomes too complicated to perform the integrations over high momenta.

At a more technical level, think of a quantum field theory with energy scale M to be build out of fundamental fields ϕ and operators $\mathcal{O}_i(\phi)$ with mass dimensions Δ_i. Bottom up, we have to include all operators that are not explicitly forbidden by symmetries. If we have an action principle for our theory it takes the following form:

$$S[\phi] = S_0[\phi] + \sum_i c_i(g) \int \mathrm{d}^4x \, \frac{\mathcal{O}_i(\phi)}{M^{\Delta_i - 4}} \,. \tag{1.15}$$

The action S_0 is the standard free action associated with the ϕ and the c_i are dimensionless numbers associated with the usual couplings g_i of the operators $\mathcal{O}_i$. We may think of the Wilson coefficients c_i as:

$$c_i(g) \approx \frac{g_i}{M^{4-\Delta_i}} \,. \tag{1.16}$$

If we are interested in physics at low energies E (the infrared or IR) we may estimate the importance of each operator in the effective theory in the following way. The fluctuations in the operator $\mathcal{O}_i$ cannot exceed energies of the order E and so the contribution to the effective action is:

$$\int \mathrm{d}^4x \, c_i \frac{\mathcal{O}_i}{M^{\Delta_i - 4}} \approx c_i \left(\frac{E}{M}\right)^{\Delta_i - 4} \,. \tag{1.17}$$

In the infrared $E \ll M$ and so the contributions for operators with $\Delta_i > 4$ become smaller, these operators are called irrelevant. However, for $\Delta_i < 4$ the contribution to the action grows as we move to smaller and smaller energies, these are called relevant operators and they are sensitive to the UV physics. There also exist marginal operators which have $\Delta_i = 4$. The above naive estimates are inconclusive for marginal operators and one has to study the theory in more detail (in particular take quantum corrections into account) to understand their importance in the infrared.

It is instructive to understand the relation with power counting renormalisability. A theory is renormalisable if all couplings have nonnegative mass dimensions. In the notation used above this means $4-\Delta_i \geq 0$ and so the corresponding operators are relevant and marginal. Thus, for relevant and marginal operators one has to impose a renormalisation condition but we are guaranteed that the results are independent of any short distance cutoff. This means we have to determine the c_i from experimental data for relevant and marginal operators and loose

predictive power to some degree. Irrelevant operators are nonrenormalisable but from the Wilsonian perspective this is not an issue. They only depend on short distances through a power series in E/M which becomes small in the IR. They can be calculated in perturbation theory but keep a dependence on the physical cutoff.

We now argue that the smallest of length scales are important for the value of the cosmological constant. This means that the cosmological constant is UV dependent. In the language of effective field theory the vacuum energy is a relevant operator ($\mathcal{O}_\Lambda = 1$) having mass dimension 0, it naively depends quartically on the large mass scale (M^4) and should be supplemented with a renormalisation condition. Let us for illustrative purposes consider a fundamental theory valid up to the Planck scale and try to calculate the vacuum energy $\rho_\Lambda = M_p^2\Lambda$ in effective field theory[4]. Each term in perturbation theory depends on the cutoff as M^4. When we include gravity, the natural energy scale is the Planck mass and we schematically obtain:

$$\rho_\Lambda \approx M_p^4 \left(\epsilon_0 + \epsilon_1 + \cdots\right) . \tag{1.18}$$

The ϵ_i are dimensionless numbers in perturbation theory: they are "small order one" numbers[5]. If we now take ρ_Λ as the observed energy density of our universe then we would be summing a series of small order one numbers to obtain something that is of the order 10^{-120}, an unlikely endeavour[6]. We would have to sum every term in perturbation theory, not to mention nonperturbative corrections, to calculate the value of the vacuum energy. We seem incapable of predicting its value in effective field theory. This is not a surprise from Wilson's perspective, effective field theory cannot predict observables associated with relevant operators and these need to be renormalised using experimental data as input. In the standard model, we have only two relevant operators: the vacuum energy and Higgs mass. In this way we may also understand the Higgs mass or electroweak hierarchy problem.

There is no reason in effective field theory why ϵ_0 in (1.18) should provide a good estimate for ρ_Λ/M_p^4. However, if for some reason the UV completion conspires to make the sum $\epsilon_1 + \epsilon_2 + \cdots \ll \epsilon_0$ then ϵ_0 can be used as a good estimate. We will argue how such a situation can arise in string theory.

[4]With this definition the vacuum energy differs from the Planck density by 120 orders because of the explicit M_p^2.

[5]Anything between 0.001 and 0.1 sound like acceptable numbers to classify as "small order one".

[6]Note that this is unlikely but not impossible, this is a subjective statement that we will try to motivate a bit more in the next section.

1.3.2 Naturalness

In the above vacuum energy calculation (1.18) we expected the ϵ to be "small order one". This is related to the concept of naturalness, which has been a useful guiding principle in constructing the standard model. We will consider what is called "technical naturalness" in the context of effective field theory. *Dimensionless parameters in a theory are technically natural if they are of order one w.r.t. the natural energy scale. Moreover, they are allowed to be extremely small if the theory becomes more symmetric in the limit where the parameter vanishes.* This is in general used to construct bottom up effective field theories (see for instance [21]). The dimensionless parameters are the Wilson coefficient $c_i(g)$ introduced in (1.15) where M is the natural scale associated with the theory.

With this in mind the parameter $\rho_\Lambda/M_p^4 \sim 10^{-120}$ is called unnaturally small, the same holds for the Higgs mass parameter $m_H^2/M_p^2 \sim 10^{-32}$. Note that the smallness is associated with the parameters of the theory, not the physical value that is experimentally measured. Naturalness is a guideline and there is no strict mathematical reason for why it should hold, let us therefore motivate the concept with a historic example.

Let us go back to the pre QED era, Lorentz modeled the electron as a continuous charge distribution of a certain radius. The radius was defined as the point where this description breaks down, more precisely where the electromagnetic self energy becomes of the same order as the rest energy:

$$E = m_e c^2 + \delta E = m_e c^2 + \frac{\hbar c \alpha}{r_e} \approx 511\,\text{keV} + \frac{\hbar c}{137 r_e}\,. \tag{1.19}$$

Using (1.1) we can easily convert the radius to electron volt. This results in an electron radius of the order $10^{-14}\,\text{m}$, 5 orders of magnitude too large compared to modern experimental upperbounds ($10^{-19}\,\text{m}$). The mismatch was also puzzling to Lorentz, even though we formulate it in a slightly more modern way. In an even more modern language we may think of the self energy as a correction to the restmass which makes the electron mass an unnaturaly small parameter. Weisskopf later explained the mismatch using Dirac's sea of positive charge, including the screening of the electron charge due to the presence of its antiparticle, the positron. This cancels the $1/r_e$ term leaving:

$$E = m_e c^2 \left(1 + \alpha \log\left(m_e r_e\right)\right). \tag{1.20}$$

The second term should be familiar as the self energy in QED. Using the upperbound for the electron radius produces a correction to the restmass of

only 10%, in other words the two terms are only a single order of magnitude apart. This restores naturalness for the electron mass, even though it is still a small paramater compared to the typical energy scale in QED. The smallness is explained in QED due to the restored chiral symmetry for massless fermions. From this viewpoint we see that naturalness points towards the existence of new physics, the positron and an approximate symmetry.

Similar arguments involving naturalness have been very successful in the past, for example the existence of the ρ meson was predicted before it was experimentally confirmed. More examples and information can be found in [94, 21, 117].

Another way to formulate the two hierarchy problems in the standard model would thus be to call both the Higgs mass and vacuum energy unnaturally small[7]. Given the historical succes of naturalness in particle physics it is not unreasonable to expect a more complete theory to explain these puzzles. Once again, the absolute value of the vacuum energy does not have a meaning in the standard model so the cosmological constant problem only appears when we introduce gravity.

1.4 Large hierarchies

In this section we briefly discuss some proposed solutions to the standard model hierarchy puzzles. The most popular solution for the electroweak hierarchy problem involves adding an extra symmetry to the standard model, called supersymmetry (SUSY). This is the only possible extension of the spacetime symmetries without reducing the standard model effective theory to a trivial theory[8]. Supersymmetry relates every bosonic degree of freedom (d.o.f.) to a fermionic d.o.f.: it acts by transforming fermions into bosons and vice versa. One implication of this is that fermions and bosons that transform into each other have the same mass, these particles are then called each others superpartner.

Supersymmetry is not observed in nature and for this reason we think of it as a broken symmetry. To benefit from the added structure the symmetry has to be broken spontaneously[9]. There is an associated energy scale to the spontaneous breaking of a symmetry. For supersymmetry breaking this would

[7] The θ angle also suffers from a naturalness problem but it is less severe and we will ignore it here.

[8] This is a famous theorem by Haag, Łopuszański and Sohnius in which they show that supersymmetry is the only extension to the spacetime Poincaré group if the theory has a nontrivial scattering matrix. The conformal group is another possible extension if we give up the concept of scattering matrix.

[9] A symmetry is spontaneously broken if the ground state of the theory is not invariant under the symmetry.

agree with the difference in masses between the superpartners. An interesting concept is dynamical SUSY breaking as it can provide a natural solution to the Higgs hierarchy puzzle. Dynamical breaking is understood as the breaking by nonperturbative effects such that the associated scale, M_{SUSY}, is exponentially small compared to a higher scale (e.g. the Planck scale). Take g to be some relevant coupling then the symmetry-breaking scale has the following form:

$$M_{SUSY} \sim e^{-\frac{1}{g^2}} M_p \,. \tag{1.21}$$

More details on this can be found in [203]. This provides a way to obtain a small Higgs mass from a more fundamental scale.

Regardless of any phenomenological applications, supersymmetry can be thought of as a calculational tool. For instance, SUSY quantum mechanics can be used to obtain the spectrum of the hydrogen atom in an easier way than the more conventional methods. At low energies it is also possible to construct cold atom systems with dynamics that are well approximated by supersymmetric Hamiltonians [209].

In summary, supersymmetry is heavily used in high energy physics and this makes studying its breaking an important subject. Even in the absence of gravity this is poorly understood. There are other mechanisms and extensions to solve the Higgs mass problem. The majority involve new symmetries, new particles and interactions to address the hierarchy issue in effective field theory.

In case of the cosmological hierarchy problem any new particle and interaction will contribute to the vacuum energy but cannot render its magnitude sufficiently small without introducing finetuning. The same holds for internal symmetries but one can extend the spacetime symmetries. The possible extensions are well studied and only supersymmetry guarantees the vanishing of the vacuum energy, if unbroken. When broken, the vacuum energy becomes of the order of the SUSY breaking scale, which is much larger than the observed vacuum energy. For these reasons it seems unlikely that we can address the smallness of the vacuum energy (perturbatively) in effective field theory and we may have to understand quantum gravity in a new framework. We will argue how string theory can provide such a new framework to go around the issues in effective field theory.

Most solutions proposed to solve the large hierarchies involve new physics to restore naturalness. There is another proposed solution worth mentioning that does not involve naturalness. It goes under the name of the multiverse and an associated anthropical reasoning [20, 202]. The name "multiverse" suggests that our universe is not unique but there are many others, we can think of them as different vacua in a large landscape. Such a landscape could justify

an anthropic reasoning to account for the small observed vacuum energy. The reasoning is best motivated with a historic example. At some time in ancient Greece, Aristarchus was able to determine the earth-sun distance. From that point on one started wondering what was special about this distance, could it be explained in a "natural" way? Nowadays we understand the distance as being an important requirement for the existence of life on earth. Combining this with the 10^{12} other solar systems (the landscape) makes for a strong statistical argument. From a scientific perspective, the landscape of other solar systems is crucial in a succesful application of the antropical principle. With this in mind, we can anthropically reason that the observed vacuum energy is small because our universe would not exist if it would be much larger [20, 202][10]. The reasoning makes sense when there is a multiverse and we will comment on its existence in string theory in later chapters.

1.5 Strings

The apparent issue with relevant operators in effective field theory is associated to the short distance behaviour of the theory. Interactions are modeled as happening at a specific point in spacetime, associated with the pointlike nature of particle excitations. Of course there is no reason to expect such a model to hold beyond the current experimental limits (think of the upper bound for the electron radius). We will entertain the possibility that at smaller distances these local excitations actually obtain a finite extend in the form of a one-dimensional object called a string. Strings were historically introduced to explain data from hadron scattering experiments. Only later was it realised that strings have the necessary ingredients to describe all particles and forces of the standard model with the addition of the graviton, the quantum of the gravitational field.

Low-energy physics can be obtained from strings by taking a pointlike limit in which we recover familiar concepts such as a metric for gravity. It should be stressed that these are not the fundamental objects of the quantised string and this is exactly the reason why strings can provide a way around the hierarchy puzzles in effective field theory. Moreover, we may derive an effective theory from string theory in a regime where we can argue for small corrections. It is then unnecessary to calculate corrections in the effective theory. Consequently, the calculation in equation (1.18) may be well approximated by only keeping ϵ_0.

We already mentioned supersymmetry as a solution to the Higgs hierarchy puzzle, which is compatible with string theory. Regarding the large hierarchy

[10] In such a situation the expansion would be too rapid for the formation of large scale structures so this would approximately correspond to an empty universe.

between the vacuum energy and Planck mass there are recent developments in understanding the vacuum structure of string theory. They have started a debate on whether there is evidence for a multiverse in string theory or not. The multiverse was previously mentioned as a possibility to explain the small cc. Another possibility to generate large hierarchies comes from brane world scenarios that can be constructed in string theory. In brane world scenarios the standard model is confined to live on a four dimensional brane embedded in a larger dimensional space where gravity propagates. The scenarios then provide a way to observe effectively four dimensional gravity on the standard model brane, where we would presumably live. We will discuss similar situations in chapters 3 and 4.

1.6 Motivation

The cosmological constant problem does not pose a mathematical problem. It should be viewed as a puzzle about gravity at small distances that seemingly effects our late time cosmology and therefore the largest distances observed in physics. As is often the case it is difficult to ask the correct question. Perhaps we should view the cosmological hierarchy problem as the motivation in search of the correct question. More details about the cc problem can be found in [174, 57, 65, 169].

We sketched the importance of quantum gravity, string theory from a perspective where our current particle physics standard model provides us with some intriguing puzzles. This makes the study of low-energy physics from string theory of academic interest, especially because it provides a clear opportunity to address the naturalness puzzles associated with effective field theory. Supersymmetry plays an important role in both the naturalness puzzles and in string theory. For that reason, understanding supersymmetry breaking goes hand in hand with the hierarchy puzzles of the standard model. These subjects form the motivation for the research described in this thesis.

In chapters 2 and 3 we introduce string theory and the elements needed to understand string compactifications, holography and supersymmetry breaking. These elements include branes and fluxes and their role in supersymmetry breaking is explained in chapter 4. We further develop our understanding of the dynamics between branes and fluxes in chapters 5 and 6. A model for explicit supersymmetry breaking is discussed in chapter 7 and chapter 8 discusses branes, fluxes and their dynamics in a supersymmetric context.

Chapter 2

Strings

2.1 History

We already gave a short introduction to string theory in the previous chapter. Let us go a little deeper in the historical introduction. The original reason to introduce a quantum theory of strings had nothing to do with gravity. In the late 1960s quantum chromodynamics (QCD) was not yet established as the fundamental theory underlying the strong interaction and there were interesting experimental observations that begged for a theoretical explanation. These experiments involved hadrons (bound states of quarks) and it proved particularly difficult at the time to explain a certain relation between the mass and spin of these hadrons (Regge trajectories). String theory provided a model for these Regge trajectories and by doing so already made contact with experimental data in the sixties. However, in the early 1970s the research concerning the strong interactions shifted towards QCD as it provided a better explanation of reality.

String theory was not completely abandoned at this point as it was soon realised that the theory contained a state that had all the necessary properties to describe gravitons, the quanta associated with the gravitational field. Since not only gravity but also other interactions and matter were automatically included, the potential of string theory as a unifying theory was understood. It was then already known that perturbative string theory consisting of purely bosonic degrees of freedom (d.o.f.) had a tachyon in its spectrum but this could be remedied by including fermionic degrees of freedom. This insight was further developed and lead to the first superstring revolution, a period of booming

string theory research between 1984 and 1994.

The name superstring is due to the supersymmetry involved to cure the tachyon problem found in the bosonic string. During the first revolution, five consistent, critical superstring theories were developed. They are called type I, type II A and B, heterotic SO(32) and heterotic $E_8 \times E_8$. We will not discuss the differences here but direct the interested reader to one of the following textbooks [118, 119, 175, 176, 54, 22]. They are called critical because they are associated with the critical string dimension, ten dimensions in case of the superstring. String theory does not necessarily live in ten dimensions but for technical reasons it is the critical string that we understand best.

At this point it was unclear how to understand these five seemingly different versions of the superstring and how to relate this to the hope of having a single unifying theory. Moreover, the relation between the massless spectrum of the superstring and "low energy" supergravity theories in ten dimensions was known but there was a unique maximal eleven dimensional supergravity that could not be obtained from a superstring. This badly understood theory associated with eleven dimensional supergravity was called M theory [1]. During 1994 and 1995 it was understood that these five theories were actually all related and are all related to a unique 11 dimensional framework: M theory [204]. This is nowadays seen as the start of the second superstring revolution. Two other importent landmarks in this second revolution are the understanding of D-branes in 1995 [173] and the anti-de Sitter/conformal field theory correspondence (AdS/CFT) [157, 124, 207]. These modern developments can be characterised as describing nonperturbative string theory whereas before the second revolution string theory was mostly studied in the perturbative regime. In what follows we will describe elements from the type II superstring that are relevant for the remainder of this thesis.

2.2 Perturbative type II strings

We will not attempt to introduce string theory using a perturbative worldsheet[2] description. The interested reader will find plenty of material in [118, 119, 175, 176, 54, 22]. Instead we will focus on the so-called low-energy limit of the type II superstring. It should not be thought of as low energy in the sense of condensed matter systems as it could very well be used to describe physics at the large hadron collider. A point-particle limit is probably a better way to describe it

[1] Depending on your liking the M stands for magic, mystery, matrix or membrane. It could even be the letter W upside down.

[2] The two-dimensional surface carved out by a string moving through spacetime.

as the low-energy theory is obtained by taking a limit in which all length scales are much larger than the string length l_s. Large lengths are associated to low energies but since we think of the string length as fundamental and associated to the Planck length, we may take physics at the LHC as being low energy.

The type II superstring has oriented strings and maximal supersymmetry in the 10-dimensional target space. It is useful to use Majorana-Weyl spinors in 10 dimensions, they have 16 real components and a definite chirality (hence called Weyl spinors). In this language the type II string has $\mathcal{N} = 2$, i.e. 32 supercharges described using two Majorana-Weyl spinors. If the two supercharges have the same chirality we obtain a consistent superstring called IIB, if they have opposite chirality the superstring theory is called IIA.

Roughly speaking, strings come in two varieties, open and closed. Open strings are like a piece of rope, they have two endpoints that can either move or be held fixed. The behaviour of the string at its endpoints will have important physical consequences. It turns out that open strings can be used to describe gauge bosons and matter. Closed strings, on the other hand are more like a rubber band being periodic. If we move along the length of the closed string we end up at the same point again, this is also important for the physics and turns out to describe the gravitational sector of string theory.

Before we continue, let us introduce some notation for the length and energy scales associated with strings. We already introduced the fundamental string length l_s, we will often use its tension T_s to describe the string energy scale per length unit (so T_s has mass dimension 2). There is no agreement in the literature on the definition of the string length l_s compared to its tension T_s, various different conventions involving different factors of 2 and π exist. However, literature does seem to agree on another parameter, α', that was historically favoured instead of the string length. It is the slope of Regge trajectories which played a crucial role in the historic setting for the birth of string theory. By definition we have $2\pi\alpha' = 1/T_s$. When we discuss string lengths we will always take $l_s = \sqrt{\alpha'}$.

The low-energy type II closed string theory turns out to be a 10-dimensional supergravity that carries the same name (sugra IIA and IIB). We will discuss the open string sector in the subsequent section.

2.2.1 Closed strings

In the point-particle limit only massless string excitations survive and since we are describing the superstring, we have both fermionic and bosonic d.o.f.. The

massless type II closed string excitations are well described by 10-d supergravity (IIA and IIB).

This thesis will mainly focus on describing the vacuum structure of string theory in which we usually have no nonzero values for the fermions[3]. For this reason we will only discuss the bosonic massless excitations, they are commonly subdivided into an NSNS and RR sector[4]. The NSNS sector is common to all superstring theories but the type I theory and consists of the graviton (symmetric traceless tensor), the Kalb-Ramond field (antisymmetric tensor) and the dilaton (scalar field). The long wavelength description involves the Ricci scalar $\mathcal{R}$ for the ten dimensional metric G_{MN}, a field strength 3-form that is locally the exterior derivative of the Kalb-Ramond 2-form field $H = \mathrm{d}B$ and the dilaton Φ:

$$S_{NS} = \frac{1}{2\tilde{\kappa}_{10}^2} \int e^{-2\Phi} \left(\star\mathcal{R} + 4 \star \mathrm{d}\Phi \wedge \mathrm{d}\Phi - \frac{1}{2} \star H \wedge H \right) . \tag{2.1}$$

Definitions and conventions for differential geometry can be found in appendix A.1. The RR sector is different for the two type II theories and involves additional n-form field potentials C_n. In case of type IIA we have $n = 1$ and 3, the case of IIB needs $n = 0$, 2 and 4. The action involves their field strengths F_{n+1}.

When dealing with form potentials and field strength we often use a language first introduced when studying Einstein-Maxwell theory[5]. The vector potential corresponding to the photon field A is the 1-form potential (also commonly called gauge potential) and $F = \mathrm{d}A$ is its 2-form field strength. The action is an integral of $\star F \wedge F$ from which the equation of motion (eom) $\mathrm{d} \star F = \cdots$ can be derived. The action alone is insufficient to describe the correct d.o.f., since F is locally given by $\mathrm{d}A$ we need to supliment it with the Bianchi identity $\mathrm{d}F = 0$ [6]. We will employ the same language to describe the relevant fields in IIA/IIB, starting with the NSNS action (2.1) we see that it should be supplemented with $\mathrm{d}H = 0$. For the RR action we need to relate the potentials with the field strength. In IIA we have:

$$F_2 = \mathrm{d}C_1 \,, \quad F_4 = \mathrm{d}C_3 - \mathrm{d}B \wedge C_1 \,, \tag{2.2}$$

[3] If we for example insist on a Lorentz symmetric vacuum the fermionic vacuum expectation values are zero with some exceptions that involve fermion bilinears.

[4] The abbreviations come from names of main contributors to the field of string theory, namely Neveu, Schwarz and Ramond.

[5] Electromagnetism coupled to general relativity.

[6] The right hand side is zero in the absence of any magnetic monopole sources.

and for IIB we have:

$$\begin{aligned} F_1 &= \mathrm{d}C_0\,, \quad F_3 = \mathrm{d}C_2 - C_0\mathrm{d}B\,, \\ F_5 &= \mathrm{d}C_4 - \frac{1}{2}C_2\wedge\mathrm{d}B + \frac{1}{2}B\wedge\mathrm{d}C_2\,. \end{aligned} \tag{2.3}$$

Contrary to the H Bianchi, the other Bianchi identities are slightly altered but can trivially be obtained from the above relations by acting with the exterior derivative, e.g. $\mathrm{d}F_4 = H\wedge F_2$. There is one extra constraint that has to be imposed separately on top of the Bianchis (or equivalently (2.2), (2.3)), namely the self duality of the 5-form $F_5 = \star F_5$. With this information, we only need to give the action that produces the correct equations of motion:

$$S_{R,A} = \frac{1}{2\tilde{\kappa}_{10}^2}\int -\frac{1}{2}\sum_{n=2,4}\star F_n\wedge F_n - \frac{1}{2}B\wedge\mathrm{d}C_3\wedge\mathrm{d}C_3\,, \tag{2.4}$$

$$S_{R,B} = \frac{1}{2\tilde{\kappa}_{10}^2}\int -\frac{1}{2}\sum_{n=1,3}\star F_n\wedge F_n - \frac{1}{4}\star F_5\wedge F_5 - \frac{1}{2}C_4\wedge H\wedge F_3\,. \tag{2.5}$$

There is an associated gauge transformation for every form potential, as is familiar from electrodynamics. The B field transforms as $\delta B = \mathrm{d}\xi_1$, the other potentials in IIA have the following gauge transformation:

$$\delta C_1 = d\lambda_0\,, \quad \delta C_3 = \mathrm{d}\lambda_2 - \lambda_0\mathrm{d}B \tag{2.6}$$

and for IIB the potentials transform as:

$$\begin{aligned} \delta C_0 = 0\,, \quad \delta C_2 &= \mathrm{d}\lambda_1\,, \\ \delta C_4 = \mathrm{d}\lambda_3 - \frac{1}{2}\mathrm{d}B\wedge\lambda_1 + \frac{1}{2}\mathrm{d}C_2\wedge\xi_1\,. \end{aligned} \tag{2.7}$$

Note that the action (2.1) does not have the standard Einstein-Hilbert term that you might have expected. This is due to the dilaton coupling in string theory. The above actions are often said to be in string frame, since this is the pointlike limit as you obtain it from doing worldsheet perturbation theory. We can make a simple field redefinition to get the standard Einstein-Hilbert term resulting in the action in Einstein frame (g_{MN}):

$$g_{MN} = e^{-\frac{(\Phi-\Phi_0)}{2}}G_{MN}\,. \tag{2.8}$$

Here we use the physical, fluctuating part of the dilaton to make the transformation[7] , the dilaton vacuum expectation value (vev) $\langle\Phi\rangle = \Phi_0$ actually

[7]The following identity in d-dimensions is useful: if $\tilde{G}_{MN} = e^{2A}G_{MN}$ then

$$\tilde{\mathcal{R}}(\tilde{G}) = e^{-2A}\left[\mathcal{R}(G) - 2(d-1)\nabla^2 A - (d-1)(d-2)\partial_M A\partial^M A\right]\,.$$

contributes to the gravitational coupling. Einstein frame is useful to discuss four-dimensional physics since we know that the low-energy limit should look like Einstein gravity (i.e. the action should be normalised to have the correct Einstein-Hilbert term).

There is one more remark regarding the closed string effective actions (2.1), (2.4) and (2.5). We need to explain the coupling $\tilde{\kappa}_{10}$ which should have come as a surprise since the worldsheet theory has only a single free parameter, the string length l_s. Matching string scattering amplitudes with scattering in the effective supergravity theory we find[8]:

$$2\tilde{\kappa}_{10}^2 = (2\pi)^7 \alpha'^4 \,. \tag{2.9}$$

We will use the tilde ($\tilde{}$) to indicate that this is not the physical coupling. The physical fields fluctuate around their vev, for the dilaton we see that its vev contributes to the ten dimensional gravitational coupling:

$$2\kappa_{10}^2 = (2\pi)^7 \alpha'^4 e^{2\Phi_0} \,. \tag{2.10}$$

It may be worth to pause for a second and explain how this is obtained from worldsheet perturbation theory. We already mentioned the point particle limit such that we are actually considering lengths $\gg l_s \sim \sqrt{\alpha'}$ but there is an additional small parameter expansion used in string theory. The dilaton couples to the worldsheet Ricci scalar. In two dimension, the integral of the Ricci scalar is a topological quantity related to the Euler number, χ. We can thus understand the contributions of the dilaton vev as coupling to χ. When calculating string scattering amplitudes with a fixed worldsheet topology it contributes as:

$$e^{-\Phi_0 \chi} = g_s^{-\chi} \,, \tag{2.11}$$

where we have define the string coupling $g_s = e^{\Phi_0}$. More details can be found in any string theory textbook [118, 119, 175, 176, 54, 22], for us it suffices to mention that χ grows more negative with more complicated worldsheet topologies such that we can focus on only the more simple topologies provided g_s is a small number. This is also called the string loop expansion and it is used to obtain the low-energy supergravity description.

If we now imagine living in 10 dimensions for a moment, we can compare the gravitational coupling (Planck mass) to the string scale M_s, or string tension ($T_s = M_s^2 = \frac{1}{2\pi\alpha'}$):

$$M_{p,10}^8 = \frac{1}{\kappa_{10}^2} = \frac{2}{g_s^2 (2\pi)^7 \alpha'^4} = \frac{2\, T_s^4}{g_s^2 (2\pi)^3} \,. \tag{2.12}$$

[8] Actually, this depends on the definition of Φ_0 which is fixed by requiring that the string tension and D1-brane tension are equal, we will introduce D-branes in the next section.

If we are in the perturbative regime where we can trust the effective actions we need $g_s \ll 1$, which immediately tells us that:

$$\left(\frac{T_s}{M_{p,10}^2}\right)^4 \ll 1\,, \tag{2.13}$$

meaning that the string scale is actually below the 10-dimensional Planck scale. We automatically get some form of hierarchy between the string and Planck scale in type II. The hope is of course to obtain a similar hierarchy involving the four-dimensional effective Planck mass (we will come back to this in chapter 3) but at least this already gives a hint as to how string theory could solve the problem of quantum gravity. At no point is it necessary (from the 10-d perspective) to quantise gravity or the metric field. Before we reach the Planck scale, massive string modes become important and the effective description breaks down.

Polyforms and the democratic formulation

We will often make use of polyforms and the democratic formulation of the type II theory [40]. Formally we double the number of gauge potentials and field strength such that we can derive both the equations of motion and Bianchi identities from the action if we remember that we need to impose the correct duality relation in the end to describe only the physical d.o.f., e.g. we relate F_7 to its Hodge dual F_3. The details can be found in appendix A.2.

Using this formalism we may define the IIA and IIB theory simultaneously (just remember to stick to even and odd polyform field strengths). We define the gauge potentials as $\mathbf{F} = \mathrm{d}\mathbf{C} - H \wedge \mathbf{C}$ and the action is:

$$S = \frac{1}{2\tilde{\kappa}_{10}^2}\int e^{-2\Phi}\left(\star\mathcal{R} + 4\star\mathrm{d}\Phi\wedge\mathrm{d}\Phi - \frac{1}{2}\star H\wedge H - \frac{1}{4}e^{2\Phi}\star\mathbf{F}\wedge\mathbf{F}\right)\,. \tag{2.14}$$

In the last term we implicitly take the integration over all of spacetime to project out only the 10-form part. Now we need to impose the H Bianchi $\mathrm{d}H = 0$ and duality $\star\sigma\mathbf{F} = \mathbf{F}$ on top of the eoms following from (2.14). The duality involves the reversal operator σ which introduces a sign depending on the degree of the form (see appendix A.2).

As a final remark, it is possible to add a 0-form field strength F_0 in the IIA theory. When F_0 is nonzero we call the corresponding supergravity massive IIA (mIIA) and have to change the definition of the gauge potentials (see appendix A.2).

2.2.2 Open strings

As we just saw, we only know the leading behaviour for the effective description of the closed massless spectrum (some corrections are also known). The situation is different for the open massless spectrum, we know pieces of the low-energy effective action to all orders in α'.

Open strings are not periodic, they can have a variety of different boundary conditions (bc) and they need not be the same in every direction. Common choices include Neumann bc for which the spatial derivative of the string coordinate in a specific direction vanishes but it can otherwise move freely. Another popular choice is the Dirichlet bc for which the string coordinate itself is fixed at a certain position. We will discuss the open string sector in the type II theory with Neumann and Dirichlet bc[9]. Imagine an open string moving in $9+1$ dimensional flat space with $p+1$ Neumann bc and $9-p$ Dirichlet bc. This means the string endpoints are fixed on a $(p+1)$-dimensional hypersurface called a Dp-brane. It was realised at the beginning of the second superstring revolution by Polchinski [173] that these D-branes are dynamic objects of equal importance in string theory as the string itself. Note that the situation where $p=9$ corresponds to a spacetime filling D9-brane so the absence of Dirichlet bc is not related to the absence of a D-brane. Moreover, when $p=0$ the string ends on a particle-like object and for $p=1$ it ends on a D1-string. The latter was mentioned previously in a footnote to define the 10-dimensional gravitational coupling. It turns out that D-branes can be both stable and unstable and so in both IIA/B all D-branes can be present but they are not all stable. In the IIA theory D-branes with p even are stable while the IIB theory only has stable D-branes for odd p.

The low-energy spectrum for the closed string was given by massless representation of the 10-dimensional Lorentz group. This is expected and agrees with our understanding of particle spectra in field theory. We just noticed that the open string involves a hyperplane so we should not expect to have 10-d Lorentz symmetry. In fact a Dp-brane breaks this to $SO(p,1)\times SO(9-p)$. We can thus expect the low-energy spectrum to fit into massless representations of the Lorentz group on the D-brane worldvolume ($(p+1)$-dimensional).

The open string spectrum is also analysed in perturbation theory. However, we have dynamical D-branes so we could be worried that they change the target space background. This happens and for that reason the perturbative spectrum is only understood in the limit of a small number of branes in flat space such

[9] We will take the boundary conditions in a specific direction to be identical on both endpoints of the string. The situation where the two endpoints have different boundary conditions also exist.

that spacetime is only slightly perturbed[10].

The massless spectrum has the same number of d.o.f. independent of the specific spatial dimensions of the D-brane. The spectrum exists out of a massless vector boson living on the $(p+1)$-dimensional brane ($p-1$ on shell d.o.f.) and $9-p$ scalar fields that correspond to the brane's position in the background spacetime such that the total number of on-shell d.o.f. is always equal to 8. This is exactly the amount of bosonic d.o.f. for the gauge multiplet with 16 supercharges, half of the 32 supercharges of the type II theory. This is no coincidence, the presence of the brane breaks half of the supersymmetries and the open string low-energy effective theory is a SUSY gauge theory. In 4-d this would correspond to $\mathcal{N}=4$ super-Yang-Mills theory (SYM).

For a single D-brane the low-energy action is best understood and describes an abelian $U(1)$ theory. We can also place N D-branes on top of each other and label the endpoints of the open string depending on the brane at which they end, this results in a non-abelian $U(N)$ theory.

Dirichlet-branes have tension T_p and interact gravitationally with the closed string. Let ξ be coordinates along the worldvolume of the brane and take X to denote the embedding of the brane (ξ) in spacetime ($X(\xi)$). The effective action involving the couplings to the NSNS closed string sector is described by the Dirac-Born-Infeld (DBI) action:

$$S_{DBI} = -\sum_p T_p \int \mathrm{d}^{p+1}\xi\, e^{-\Phi(X)} \sqrt{-\det P\left(G - 2\pi\alpha' \mathcal{F}\right)_{\alpha\beta}}\,. \tag{2.15}$$

We have also introduced $2\pi\alpha'\mathcal{F}_{\alpha\beta} = B_{\alpha\beta} - 2\pi\alpha' F_{\alpha\beta}$, where B is the Kalb-Ramond field and F is the field strength associated with the worldvolume gauge boson A. This is the first time we encounter a hint of nonperturbative effects since the physical tension of the brane is $T_p g_s^{-1}$ this should be contrasted with other string masses which are independent of g_s in perturbation theory.

The symbol P is used to denote the pull-back of, for instance, the spacetime metric to a metric on the worldvolume of the D-brane:

$$P\, G_{\alpha\beta} = \frac{\partial X^M}{\partial \xi^\alpha} \frac{\partial X^N}{\partial \xi^\beta} G_{MN}\,. \tag{2.16}$$

If we have a D-brane at positions x^a we may take $X^a = x^a + 2\pi\alpha'\sigma^a$ where σ^a describe the fluctuations of the D-brane, then the pull-back of the metric gives:

$$P\, G_{\alpha\beta} = G_{\alpha\beta} + 2\pi\alpha' G_{\alpha a}\partial_\beta \sigma^a + 2\pi\alpha' G_{\beta a}\partial_\alpha \sigma^a + 4\pi\alpha'^2 G_{ab}\partial_\alpha\sigma^a\partial_\beta\sigma^b\,. \tag{2.17}$$

[10] In fact, a more thorough analysis shows that the string coupling g_s times the number of branes should be small in order to ignore the backreaction on the target spacetime.

From here on we will stop explicitly writing the pull-back P but it should be understood to always be present. The reader that wishes to make the connection to the standard abelian action as known from electromagnetism can use:

$$\det(1+W) = 1 + \mathrm{tr}(W) - \frac{1}{2}\left(\mathrm{tr}W^2 - (\mathrm{tr}W)^2\right) + \cdots , \tag{2.18}$$

to find (momentarily take $B = 0$ and only consider terms quadratic in derivatives):

$$S_{DBI} = -\sum_p T_p \int \sqrt{-G} e^{-\Phi} \left(1 + \frac{(2\pi\alpha')^2}{4}(F^2 + 2(\partial\sigma)^2) + \cdots\right) . \tag{2.19}$$

The first term is just the Casimir energy, the second gives the kinetic term for the gauge field and scalars. All matter fields in the gauge multiplet transform in the bifundamental, for strings starting and ending on the same brane this becomes the adjoint representation of the gauge group. For a single D-brane this means that the scalars in the gauge multiplet are not charged under the $U(1)$. The DBI action is understood for a single D-brane or multiple branes sufficiently far apart but when we place branes on top of each other it is badly understood. In particular, it is only known how to incorporate the nonabelian nature to leading order by changing (2.19) to the corresponding nonabelian Yang-Mills action[11].

The above also identifies the gauge coupling g_p:

$$g_p^2 = \frac{g_s}{T_p(2\pi\alpha')^2} = g_s(2\pi)^{p-2}{\alpha'}^{\frac{p-3}{2}} . \tag{2.20}$$

The coupling to the RR sector is described in the Wess-Zumino action, using polyforms and the democratic formulation (see appendix A.2):

$$S_{WZ} = -\sum_p (-)^p \mu_p \int_{p+1} P\sigma\left(e^{-2\pi\alpha'\mathcal{F}} \wedge \mathbf{C}\right) , \tag{2.21}$$

where the integration is over the Dp worldvolume such that it only picks up the $(p+1)$-form part from the polyform expression. As before P denotes the pull-back and σ reverses the order of the form, more details are found in appendix A.2[12].

For Dp-branes: $T_p = |\mu_p|$ and by calculating scattering amplitudes from the perturbative worldsheet description we find:

$$T_p = 2\pi(2\pi\sqrt{\alpha'})^{-(p+1)} . \tag{2.22}$$

[11] Some results are known beyond leading order, most of them rely on T duality and the D9-brane action.

[12] The action of σ merely introduces a sign $(-)^{p(p+1)/2}$ that in combination with the $(-)^{p+1}$ simplifies formulas later on.

The physical tension is $\tau_p = T_p g_s^{-1}$. If we momentarily forget about $\mathcal{F}$ we may think of the Wess-Zumino part as:

$$S_{WZ} = \pm\mu_p \int C_{p+1}\,, \tag{2.23}$$

which resembles the coupling of a particle ($p = 0$) to the electromagnetic gauge potential: $e\int A$. In this case we have extended objects (D-branes) that are charged under RR form potentials C_{p+1} and we can thus identify μ_p with the charge of the Dp-brane. We will have more to say about these charges in the next section.

2.3 Nonperturbative strings

2.3.1 Ramond charges, Dirichlet-branes and flux

Let us have a closer look at the effective D-brane actions. We mentioned μ_p being the charge of the D-brane under the F_{p+2} (or F_{8-p}) Ramond form. To see this in more detail, we momentarily ignore background values of B and the worldvolume gauge potential A so we may consider the following part of the action:

$$S = -\frac{1}{2\tilde{\kappa}_{10}^2}\int \frac{1}{2}\star F_{p+2}\wedge F_{p+2} - (-)^p\mu_p\int \sigma(C_{p+1})\wedge\delta_{9-p}\,, \tag{2.24}$$

where δ_{9-p} is a delta-function form with support on the worldvolume of the D-brane (it is the Poincaré dual[13] of the worldvolume). This gives the corresponding eom:

$$\frac{1}{2\tilde{\kappa}_{10}^2}\mathrm{d}\star F_{p+2} = -\mu_p\sigma\delta_{9-p}\,, \tag{2.25}$$

or equivalently the Bianchi identity:

$$\frac{1}{2\tilde{\kappa}_{10}^2}\mathrm{d}F_{8-p} = \mu_p\delta_{9-p}\,. \tag{2.26}$$

We may define the Ramond charge by integrating F_{8-p} over an appropriate sphere enclosing the D-brane:

$$Q_p = \int_{S^{8-p}} F_{8-p}\,, \quad Q_p = 2\tilde{\kappa}_{10}^2\mu_p = \left(2\pi\sqrt{\alpha'}\right)^{7-p}\,. \tag{2.27}$$

[13]By definition the Poincaré dual γ of a cycle Γ is such that for any form ω we have:

$$\int_\Gamma \omega = \int \omega\wedge\gamma\,,$$

where the last integration is over the entire space.

This is the unit charge of a single D-brane, we will encounter situations with a number, N, of D-branes on top of each other with charge $Q_p = N 2\tilde{\kappa}_{10}^2 \mu_p$.

The above charge is electric w.r.t. F_{p+2} by analogy with the electric charge ($Q_e \sim \int \star F_2$), it can also be considered a magnetic charge for F_{8-p}. In electromagnetism one encounters the Dirac quantisation condition when dealing with both electric and magnetic charges. A similar condition holds for Ramond charges. Define $\hat{\mu}_p$ to be the charge of a $p+1$ dimensional object, not necessarily the charge of a single D-brane. We then need $\hat{\mu}_p \hat{\mu}_{6-p} 2\tilde{\kappa}_{10}^2 \in 2\pi\mathbb{Z}$. Note that the μ_p for D-branes discussed in the previous section satisfy this relation for the smallest quantum, $\mu_p \mu_{6-p} 2\tilde{\kappa}_{10}^2 = 2\pi$. This can be seen as evidence that D-branes are fundamental objects in string theory.

Fluxes can be thought of as vevs of the RR forms (or of the NSNS H field). To simplify the situation we will consider the case of vanishing H field and there are no local sources for the RR forms such that their equations of motion (or Bianchi) simply tell us they are closed and co-closed:

$$\mathrm{d}F = 0 \qquad \mathrm{d} \star F = 0 \,. \tag{2.28}$$

Giving it a nonzero vev is thus not consistent with these equations unless there is nontrivial geometry involved. Fluxes provide background values for the Ramond forms without the existence of a local source. The flux values are quantised in string theory, meaning they take integer values. Moreover, they are quantised in exactly the same way as the Ramond charge of D-branes:

$$\int F_{8-p} \in (2\pi\sqrt{\alpha'})^{7-p}\mathbb{Z} \,. \tag{2.29}$$

The same goes for H-flux:

$$\int H \in 4\pi^2 \alpha' \mathbb{Z} \,. \tag{2.30}$$

We already saw D-branes being charged under the corresponding RR field strengths F, there is also a corresponding 5-brane for the NSNS H field. This brane is called the NS5 and it is charged (magnetically) under H:

$$\mathrm{d}H = \mu_5 \delta_{NS5} \,. \tag{2.31}$$

The fundamental string itself (denoted $F1$) couples electrically to H. We have a perturbative understanding of D-branes but cannot say the same of the NS5-brane. The only insights we have are through strong-weak dualities of string theory. We will discuss this briefly in section 4.3. We mentioned D-branes and NS5-branes but string theory also contains more exotic branes, see [85]. For a review of brane effective actions see [189].

In a later chapter we will discuss compact geometries. This results in global constraints on the charges of form fields that lie inside the compact space. The easiest way to see this is to integrate e.g. the eom (2.25) over the compact directions and use Stokes' theorem. This shows that the total net Dp-brane charge Q_p has to vanish:

$$0 = \int \mathrm{d}F_{8-p} = Q_p \,. \tag{2.32}$$

The above equation is called the tadpole condition and it will play an important role later on.

2.3.2 p-branes

Almost everything discussed so far is knowledge obtained by studying the perturbative string. It assumes g_s small such that we can expand results in a power series in g_s and only keep the lowest terms. The connection with the low-energy effective actions then comes from an expansion in α'. Only when we discussed the DBI action for D-branes did we see a hint of nonperturbative strings.

Unfortunately we do not have a good understanding of nonperturbative string theory and can at best hope to find insights from the low-energy effective actions. Doing so, we can find classical solutions to the type II supergravity equations of motion that correspond to nonperturbative objects in the full string theory. The supergravity solutions are known as p-branes and they are in one-to-one correspondence with the D-branes [173]. These p-branes involve a nontrivial metric, dilaton[14] and F_{p+2} background. The relevant part of the action is:

$$S = \frac{1}{2\kappa_{10}^2} \int e^{-2\phi} \left(\star\mathcal{R} + 4 \star \mathrm{d}\phi \wedge \mathrm{d}\phi - \frac{1}{2} e^{2\phi} \star F_{p+2} \wedge F_{p+2} \right) . \tag{2.33}$$

In what follows we will restrict to $p < 7$. We rescaled the Ramond field strength ($F \to g_s^{-1} F$) such that the entire gravitational sector has the same physical coupling, $2\kappa_{10}^2$. The corresponding eoms are solved by:

$$\begin{aligned} \mathrm{d}s_p^2 &= W(r)^{-\frac{1}{2}} \mathrm{d}x_{p+1}^2 + W(r)^{\frac{1}{2}} \mathrm{d}x_{9-p}^2 \,, \\ F_{p+2} &= \mathrm{d}(W^{-1} \star_{p+1} 1) \,, \quad e^{\phi} = W^{\frac{3-p}{4}} \,, \\ W(r) &= 1 + \left(\frac{R}{r} \right)^{7-p} \,, \quad p < 7 \,, \end{aligned} \tag{2.34}$$

[14] From this point on we will always discuss the physical, fluctuating part of the dilaton $\phi = \Phi - \Phi_0$.

where $R^{7-p} = 2^{5-p}\pi^{\frac{5-p}{2}}\Gamma(\frac{7-p}{2})g_sN\alpha'^{\frac{7-p}{2}}$ is a constant in the sugra solution that we match to the corresponding D-brane charge, r is a radial coordinate in the space perpendicular to the brane and $\star_{p+1}$ is used to denote the Hodge star on the external space ($\mathrm{d}x^2_{p+1}$). The solution is characterised by a single function $W(r)$, commonly called the warp factor. Both the unwarped worldvolume of the brane ($\mathrm{d}x^2_{p+1}$) and the unwarped transverse space ($\mathrm{d}x^2_{9-p}$) are taken to be flat.

Let's verify that these brane backgrounds indeed correspond to D-branes and calculate the Ramond charge (notice the factor of g_s because of the rescaling):

$$Q_p = g_s^{-1}\int_{S^{8-p}} F_{8-p} = \epsilon_0 g_s^{-1}\int_{S^{8-p}} \tilde{\star}_{9-p}\mathrm{d}W = \epsilon_0 g_s^{-1}\int_{9-p} \mathrm{d}\tilde{\star}_{9-p}\mathrm{d}W\,, \quad (2.35)$$

where $\epsilon_0 = (-)^{(p+1)(p+2)/2}$ is a result of using the Hodge duality $\star\sigma F_{p+2} = F_{8-p}$ and $\tilde{\star}_{9-p}$ is the Hodge star of $\mathrm{d}x^2_{9-p}$. The last expression in (2.35) is actually the laplacian on the unwarped transverse space, a direct calculation using (2.34) gives $(2\pi\sqrt{\alpha'})^{7-p}N$, exactly the Ramond charge of N Dp-branes.

One can also calculate the mass (M_p) or tension (τ_p) of the p-brane solutions which equals their charge in agreement with the D-brane results: $M_p = |Q_p|$ or $\tau_p = |\mu_p|/g_s$. The p-branes can be thought of as extremal black brane solutions. They appear to be singular at $r = 0$ which is interpreted as the position of the D-brane. However, $r = 0$ is only a singularity for $p \neq 3$. The case of a 3-brane is special and we will investigate it in more detail in section 2.4. For $p \neq 3$ the singularity coincides with the horizon and when we move away from extremality by adding a little bit of mass (such that $M > |Q|$)[15] we can cloak the singularity with an event horizon[16]. For this reason they are acceptable supergravity solutions but we can do more. The $r \to 0$ limit can be expressed as a delta function corresponding to the position of the brane, similar to the situation with a point charge in electromagnetism. Formally, we do this by including the D-brane effective action that is also minimised by the p-brane background:

$$S = -N\tau_p\int e^{-\phi}\sqrt{-G}\wedge\delta_{9-p} - (-)^p N\tau_p\int \sigma\left(C_{p+1}\right)\wedge\delta_{9-p}\,. \quad (2.36)$$

This changes the F_{8-p} Bianchi identity (or F_{p+2} eom) to:

$$\mathrm{d}F_{8-p} = g_sQ_p\delta_{9-p}\,. \quad (2.37)$$

[15]The non-extremal p-branes or black branes with $M > |Q|$ do not have the simple form (2.34). They have an extra blackening factor in front of the $\mathrm{d}t^2$ term in the metric which should be compared to the black hole metric creating an event horizon.

[16]The situation is similar to (non-)extremal black holes where there are 2 horizons which only coincide when the black hole is extremal. As is familiar from non-extremal black holes the event horizon introduces a nonzero temperature.

If we include the brane position as a delta source at $r = 0$ we can actually make contact with the corresponding Green's function as should be familiar from electromagnetism. For $p < 7$ we have:

$$\begin{aligned} &\mathrm{d}\tilde{\star}_{9-p}\mathrm{d}\mathcal{G} = (-)^p\tilde{\star}_{9-p}\Delta\mathcal{G} = (-)^p\sqrt{\tilde{g}}^{-1}\partial_i\sqrt{\tilde{g}}\tilde{g}^{ij}\partial_j\mathcal{G}\tilde{\star}_{9-p}1 = \delta_{9-p}\,, \\ &\delta_{9-p} = \delta(r)\mathrm{d}r\wedge\hat{\mathrm{vol}}_{8-p}\,, \quad \mathcal{G} = (-)^{p+1}\frac{\Gamma\left(\frac{7-p}{2}\right)}{4\pi^{\frac{9-p}{2}}}r^{p-7}\,. \end{aligned} \tag{2.38}$$

We introduced the notation $\hat{\mathrm{vol}}$ in the delta function form to denote the normalised volume form which integrates to 1. This is exactly the structure in the warp function W and allows us to identify Q_p with $(2\pi\sqrt{\alpha'})^{7-p}N$, exactly the Ramond charge of N Dp-branes.

More evidence can be found from supersymmetry [173], the p-brane solution preserve half of the SUSY charges in the supergravity theory (16 supercharges). Recall that the low-energy description of D-branes is a super-Yang-Mills theory with the maximal amount of SUSY (16 supercharges). Thus the amount of supersymmetry agrees and it is concluded that p-branes and D-branes are two different descriptions of the same nonperturbative object. Notice that we originally described D-branes in perturbation theory and g_sN small such that the backreaction on the string background can be neglected. The p-brane background, on the other hand, actually requires g_sN to be large since only in this limit is the curvature of the supergravity background sufficiently small.

2.3.3 T duality

A remarkable duality in string theory is found when one of the coordinates is circular, e.g. when $X \sim X + 2\pi R$. In this setting, the closed string can wind n times along the circular direction: $X(\sigma + 2\pi) = X(\sigma) + 2\pi Rn$. We furthermore note that the momenta along this direction are quantised, single valuedness of e^{ipX} demands $p = m/R$, $m \in \mathbb{Z}$. The spectrum that is obtained by quantising the string in this setting is now observed to be invariant under:

$$R \overset{T}{\longleftrightarrow} \frac{\alpha'}{R} \quad , \qquad n \overset{T}{\longleftrightarrow} m\,. \tag{2.39}$$

A closed string wrapping a circle of radius R is equivalent to a string wrapping a circle of radius $\frac{\alpha'}{R}$ if momenta and winding numbers are interchanged. This was named T duality and it is one of many interesting string dualities, relating a perturbative situation where R is large with a possibly very nontrivial situation in which the circle is of stringy length α'/R. The spectrum of the open string involves more details considering the bc. It can be shown that an open string with

Neumann-Neumann bc in the circular direction with radius R and momentum m/R is T dual to the situation with Dirichlet-Dirichlet bc, radius α'/R and winding number m:

$$\begin{array}{ccc} \text{Neumann bc} & & \text{Dirichlet bc} \\ p = \frac{m}{R} & \overset{T}{\longleftrightarrow} & \text{winding } m \\ R & & \frac{\alpha'}{R} \end{array} \quad .$$

From the above it should be clear that T duality along a direction parallel to a Dp-brane results in a D$(p-1)$-brane because the NN bc turns into a DD bc. Similarly, T duality along a transverse direction results in a D$(p+1)$-brane. This also shows that T duality relates IIA with the IIB theory:

$$\text{IIA on } S^1 \simeq \mathbb{R}/2\pi R\mathbb{Z} \quad \overset{T}{\longleftrightarrow} \quad \text{IIB on } S^1 \simeq \mathbb{R}/2\pi\tfrac{\alpha'}{R}\mathbb{Z} \quad .$$

Let us briefly describe how T duality manifests itself in the low-energy effective description. For closed strings, the duality results in supergravity IIA and IIB being equivalent after dimensional reduction along the circular direction (with radius R vs α'/R). The reason being that dimensional reduction gives maximal supergravity in 9 dimensions, which is unique. Practically, T duality relates the metric, dilaton, B field and RR forms from IIA and IIB. These relations are known as the Buscher rules [66] and can be found in appendix A.4. The Buscher rules are constructed such that the reduction of IIA and IIB along the circular direction give the same 9-dimensional action. Similarly, the effective D-brane actions are mapped onto each other under T duality. The p-brane solutions discussed in section 2.3.2 can also be mapped onto each other using the Buscher rules if the brane source is smeared over the circular direction. Smearing can be though of as an approximation where the localised delta form distribution is replaced by a smooth form that integrates to the same value. By smearing the brane over the circle it becomes a smooth charge distribution instead of a point-like source. This is necessary for technical reasons, the circular direction under T duality has to be an isometry of spacetime. Note that the T dual to a perturbative string background need not have a good supergravity description, it can be both strongly coupled and have small curvature such that g_s and α' corrections become important.

2.4 AdS/CFT

In section 2.3.2 we discussed the p-brane supergravity solutions that were actually D-branes in the appropriate limit. This insight was crucial to the

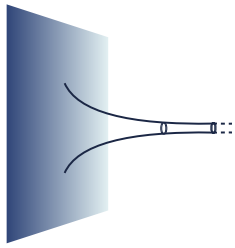


Figure 2.1: A schematic representation of the supergravity p-brane background (2.40) corresponding to D3-branes. It interpolates between flat space for large r and has an infinitely long throat region in the near-horizon limit where $r \to 0$.

development of the AdS/CFT correspondence, which is a duality between a quantum gravity theory and a gauge theory. AdS/CFT is the string theory realisation of the holographic principle introduced by 't Hooft, Susskind and Thorn [192, 193, 194, 197]. The idea is that, when gravity is included, the information in our (3+1)-dimensional world can be encoded on a (2+1)-dimensional surface, similar to a hologram. Prime motivation for such a principle came from black hole thermodynamics[17] where the entropy of the black hole scales with the area of its horizon, suggesting the information can be encoded on the surface of the horizon. We will discuss the canonical example in string theory pioneered in [157, 124, 207], namely type IIB string theory on $\mathrm{AdS}_5 \times S^5$ that is related to the four-dimensional conformal field theory $\mathcal{N} = 4$ $SU(N)$ super-Yang-Mills.

To motivate the correspondence we will take a heuristic approach [157] by comparing two different viewpoints[18]. We will look at a stack of N coinciding D3-branes in type IIB string theory and compare with the 3-brane background in IIB supergravity.

The D3 point of view has open string d.o.f. living on its worldvolume giving rise to 4-d $\mathcal{N} = 4$ SYM with gauge group $SU(N)$[19] at low energies. At the same time there are closed string degrees of freedom not confined to the D3 worldvolume which give type IIB supergravity for energies sufficiently small. Interactions between these open and closed strings are gravitational by nature so they should be proportional to $\kappa_{10} \sim g_s \alpha'^2$. We now consider the so-called decoupling limit and take $\alpha' \to 0$ such that the interaction between closed and open strings vanishes. We thus obtain a string theory picture where there are

[17] See [23, 24, 128] for the original papers on black hole thermodynamics.

[18] Different heuristic derivations can be found in modern textbooks such as [22, 54, 167].

[19] The full gauge group includes a $U(1)$ but this is a free, decoupled theory since the fields in the supermultiplet transform in the adjoint representation which is trivial for $U(1)$.

free gravitons in the 10-d bulk and $\mathcal{N} = 4$ SYM on the D3 worldvolume. In this picture the backreaction of the D3-branes on the geometry is not considered which is strictly speaking only valid for $g_s N$ small.

We now discuss the dual viewpoint using the extremal three-brane supergravity background (2.34):

$$\begin{aligned} \mathrm{d}s^2 &= W^{-1/2}\mathrm{d}s^2_{3+1} + W^{1/2}\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_5^2\right) , \\ F_5 &= (1+\star)\mathrm{d}W^{-1}\wedge \mathrm{vol}_4 \ , \quad W = 1 + \left(\frac{R}{r}\right)^4 , \end{aligned} \tag{2.40}$$

where $R^4 = 4\pi g_s N \alpha'^2$. To stay in the supergravity limit we need the curvature to be small everywhere which requires $g_s N$ to be large. We want to find the analogue of the decoupling limit in this background to understand the D3 worldvolume gauge degrees of freedom. Let us first consider the background in more detail. We can take r large and find that (2.40) reduces to 10-d Minkowski space. On the other hand, for r small we find that the radial coordinate combines with the worldvolume coordinates to form AdS_5 and the metric reduces to $\mathrm{AdS}_5 \times S^5$:

$$\mathrm{d}s^2 = \frac{r^2}{R^2}\mathrm{d}s^2_{3+1} + \frac{R^2}{r^2}\mathrm{d}r^2 + R^2\mathrm{d}\Omega_5^2 \, . \tag{2.41}$$

In section 2.3.2 we interpreted $r = 0$ as the position of the D3-branes and so (2.41) is called the near-horizon or near-brane geometry. It should now also be clear that $r \to 0$ does not correspond to a singularity since the near-horizon geometry is completely regular. This is a special feature of D3-branes and is crucial for the AdS/CFT correspondence. The supergravity background develops a macroscopic horizon for $g_s N$ large which corresponds to the geometry close to D3-branes. The metric (2.41) resembles an infinitely long throat since the S^5 keeps a fixed radius even as we move r closer to 0. In constrast, for large r the radius grows linearly with r. This is schematically depicted in figure 2.1.

Now consider low-energy excitations (gravitons for example) in this background. They have very long wavelength and cannot probe the throat region close to the D3-branes. They only see the large r behaviour of (2.40) and so they effectively probe IIB supergravity in 10-d Minkowski space. Previously we had two decoupled low-energy systems, in the supergravity background we should measure energy as observed at infinity (E_∞). Let E_0 be the energy as observed by a comoving observer at radius r_0, by the gravitational redshift we have $E_\infty = W(r_0)^{-1/4} E_0$. For $r_0 \to 0$ we find:

$$E_\infty \sim \frac{r_0}{\sqrt{\alpha'}} E_0 \, . \tag{2.42}$$

We conclude that there is another low-energy system in the near-brane region. For fixed E_0 the energy measured at infinity is small due to the gravitational redshift. One can indeed show that there are low-energy excitations confined to the throat region close to the D3-branes. Thus, we find long wavelength excitations corresponding to IIB supergravity in 10-d Minkowski space and low-energy excitations that cannot leave the near-horizon throat region. This is the supergravity analogue of the decoupling limit and we heuristically identify type IIB in the near-horizon geometry (2.41) with the gauge theory living on the worldvolume of D3-branes.

The proper near-horizon limit was first formulated by Maldacena [157]: take $\alpha' \to 0$ and $r \to 0$ but keep $U = \frac{r}{\alpha'}$ fixed. The last requirement allows stringy excitations in the near-horizon to survive the decoupling limit. Indeed, take an excitation of string scale: $E_0 = 1/\sqrt{\alpha'}$ this corresponds to $E_\infty \sim r/\alpha' = U$. We can thus identify U with an energy scale in the gauge theory. The near-brane geometry becomes:

$$\mathrm{d}s^2 = \alpha' \left(\frac{U^2}{\sqrt{4\pi g_s N}} \mathrm{d}x_{3+1}^2 + \sqrt{4\pi g_s N} \left(\frac{\mathrm{d}U^2}{U^2} + \mathrm{d}\Omega_5^2 \right) \right) . \tag{2.43}$$

To summarise, we understand the gauge theory from N coincident D3-branes in the perturbative string (g_s, α' and $g_s N$ small), the dual picture gives us IIB supergravity in $\mathrm{AdS}_5 \times S^5$. We already took α' and g_s to be small, the supergravity approximation is valid when curvature is large, $R = (4\pi g_s N \alpha'^2)^{1/4} \gg \sqrt{\alpha'}$ meaning $g_s N$ should be large. This is the opposite regime as the validity for the D3-brane picture which makes this a strong-weak duality. Since we need small g_s we are forced to take N large in the supergravity approximation, while keeping $g_s N$ fixed and large.

The large N limit of a gauge theory was first studied by 't Hooft [133]. When $N \to \infty$ and $g_{YM} \to 0$[20] while keeping the 't Hooft coupling $\lambda = g_{YM}^2 N$ fixed. The 't Hooft coupling is the effective coupling constant for large N. In this situation only planar diagrams contribute and other diagrams contribute as N^{-1} corrections.

From this heuristic reasoning we conclude that IIB supergravity in $\mathrm{AdS}_5 \times S^5$ describes the large N limit of a strongly coupled ($\lambda \gg 1$) 4-d $\mathcal{N} = 4$ SYM theory. There is more evidence for this duality and many of the statements in this section follow from genuine calculations. Since AdS/CFT is a duality it can never be proven by perturbative arguments. But it has passed many nontrivial tests by studying perturbation theory and it is believed that the duality holds beyond the limits described above. In its strongest form the duality should hold for all values of g_s and N (see table 2.1).

[20] Recall the gauge coupling from (2.20): $g_{YM}^2 \sim g_s$.

$AdS_5 \times S^5$ IIB string theory	Full equivalence	$\mathcal{N} = 4$ SYM $SU(N)$ in 4-d
Classical IIB string theory $g_s \sim g_{YM}^2 \to 0$ $\forall \alpha'$	Strong equivalence	$\mathcal{N} = 4$ SYM $SU(N)$ in 4-d $g_{YM}^2 \to 0$, $N \to \infty$ $\forall \lambda \sim R^4/\alpha'^2$
Classical IIB supergravity $g_s \to 0$, $\alpha' \to 0$	Weak equivalence	$\mathcal{N} = 4$ SYM $SU(N)$ in 4-d $N \to \infty$, $\lambda \to \infty$

Table 2.1: The AdS/CFT correspondence. There is ample evidence supporting the weak equivalence and some evidence for the strong equivalence. In the most extreme view the two sides of the duality are taken to be completely equivalent.

To end this section we mention how the symmetries on both sides of the duality are related. The isometry group of AdS_{n+1} is $SO(n, 2)$ which agrees with the conformal group on the field theory side. The S^5 isometry group is $SO(6)$ and matches the R symmetry group of 4-d $\mathcal{N} = 4$ SYM. Later in this thesis, we will be interested in holography for non-conformal field theories. From the above we see that we have to break the isometry group of AdS. We will describe such an example in section 3.2.1.

Chapter 3

String vacua

In understanding low-energy implications from string theory there are usually two different directions to consider. The direction of particle physics, i.e. improving on our understanding of gauge theories and the direction of quantum gravity which involves understanding our cosmological history and various black hole puzzles. Our late-time cosmology is dominated by dark energy and we argued in chapter 1 that this is UV sensitive. This means that particle physics and cosmology are related and any serious phenomenological model will need to understand both at the same time. All models start from a string vacuum that provides the arena for particle physics and quantum gravity.

We will use a perturbative viewpoint in which we start from a certain fixed background in which strings and branes can propagate as small perturbations. This is similar to perturbative quantum field theory where we start from a vacuum and understand particles as small excitations on top of this background. Different string backgrounds do not define different string theories, rather they should be thought of as different vacua of the same theory and for this reason we refer to all string backgrounds as string vacua. The purist might consider this an abuse of language as these backgrounds need not be stable, may only be defined perturbatively or can perhaps even describe more complicated states in the full nonperturbative string theory.

We will restrict the discussion to type II string vacua that are characterized by background values of the 10-dimensional metric G, dilaton Φ, B field and all RR forms. For phenomenological reasons we will focus on having an external

4-d space $\mathrm{d}s_4(x)^2$ that is maximally symmetric[1] and provides the arena for low-energy particle physics. The remaining six internal dimensions $\mathrm{d}s_6(y)^2$ can be compact or noncompact and the string propagates in the 10-dimensional warped product space:

$$\mathrm{d}s_{10}^2 = W(y)^2 \left(\mathrm{d}s_4(x)^2 + \mathrm{d}s_6(y)^2\right) , \tag{3.1}$$

the function $W(y)$ is called the warp factor. The corresponding expressions for the dilaton, B field and Ramond forms should all respect the symmetries of the background, e.g. branes should fill $\mathrm{d}s_4(x)^2$ to be compatible with the isometries of the maximally symmetric spacetime, similarly fluxes are internal up to hodge duality[2].

The string worldsheet sigma model has a gauged Weyl invariance. Gauge symmetries should remain anomaly free at the quantum level. For the Weyl invariance this means the vanishing of the beta functions which, to leading order in g_s and α', reproduces the type II supergravity equations of motion. A consistent quantum theory of strings thus restricts the possible backgrounds. The study of string vacua is difficult since we only have a perturbative definition. We hope to learn more by studying the vacuum structure in a regime where the corrections are small and we can use supergravity arguments. Perturbatively, string backgrounds can often be connected by a continuous parameter without changing the value of the supergravity action. Such parameters appear as massless scalars in the 4-d effective action and are called moduli. Massless scalars generate long-range interactions and modify gravity so they pose a phenomenological problem [87]. Quantum correction to the string will almost certainly provide masses for these moduli but this is out of computational control in the perturbative regime [96][3]. This is known as the moduli stabilisation problem which further complicates the construction of consistent string vacua.

Ten-dimensional flat space is a good background but we are more interested in understanding backgrounds of the form (3.1) with a compact internal space. When this compact space is small compared to the external space we speak of a genuine compactification and there is a corresponding low-energy effective description in four dimensions. These vacua are introduced in the next section. Noncompact internal spaces are also important, they play a crucial role in the AdS/CFT correspondence and brane world scenarios. Moreover, they can

[1] This restricts the external space to be AdS, dS or Minkowski. The reader can consult appendix A.1 for more details.

[2] Consider a Ramond form F, then spacetime symmetries require either F to be internal or its Hodge dual $\star F$. In the later situation F fills spacetime (and possibly some part of the internal space) which is still consistent with spacetime symmetries.

[3] In other words, we are forced into a region of parameter space where our approximations are no longer valid.

often provide a good local approximation of more complicated compact spaces. Noncompact vacua are discussed later in this chapter.

3.1 Compact vacua

We will focus in this section on the search for phenomenological vacua, so we are interested in four large dimensions and a small compact 6-d space X_6: $\mathcal{M}_{10} \to \mathcal{M}_4 \times X_6$. In doing so we need length scales in our four-dimensional effective description l_4, much bigger than the string length $l_4 \gg l_s$, this is the basis of the pointparticle limit and our understanding of low-energy physics from string theory. The six compact dimensions should be sufficiently small, $l_4 \gg l_K$, the internal length scale l_K is commonly referred to as the Kaluza-Klein scale. From a worldsheet perspective these small dimensions can be of string scale but the low-energy limit of such constructions is badly understood. However, if l_K is large in string units $l_4 \gg l_K \gg l_s$, then we can hope to understand the background solely from the low-energy supergravity perspective. In this case we may integrate over the 6 compact dimension and produce a 4-dimensional effective theory. Let us write the internal volume in string units as $l_K^6 = V\alpha'^3$, then the gravitational coupling in 4 dimensions is:

$$2\kappa_4^2 = \frac{2\kappa_{10}^2}{l_K^6} = (2\pi)^7 \frac{\alpha' g_s^2}{V}\,, \tag{3.2}$$

where we used (2.10). In section 2.2.1 we hinted at a possible string theory solution to the problem of quantum gravity. Using the above identity we can compare the string scale $M_s^2 \sim 1/\alpha'$ with the 4-d Planck mass $M_{p,4}^2 \sim 1/\kappa_4^2$:

$$\left(\frac{M_{p,4}}{M_s}\right)^2 \sim \frac{V}{g_s^2} \gg 1\,. \tag{3.3}$$

For large internal volume V and small string coupling g_s do we find that the 4-d Planck mass is larger than the string scale. This suggest that massive string modes have to be incorporated before quantum gravity corrections in the effective theory are important.

For the string compactifications in this chapter, the 4-d spacetime is fixed to be maximally symmetric but its length scale is given by the vacuum energy. This is typically of the same order as the string scale such that the length scales are very small and there is little difference between the external and internal space, not a true compactification. Only when we can decouple the cosmological constant from the string scale physics can we have a compactification with small internal space and large external space such that you can effectively observe a

lower-dimensional spacetime. We will focus on compactifications to Minkowski space $\mathcal{M}_4$ and in section 4.4 discuss how we can decouple the vacuum energy from the string scale. Historically, the first attempts did not feature a warp factor:

$$\mathrm{d}s_{10}^2 = \mathrm{d}s_4(x)^2 + \mathrm{d}s_6(y)^2 \,. \tag{3.4}$$

This product space, where $\mathrm{d}s_4(x)^2$ is Minkowski space, requires $\mathrm{d}s_6(y)^2$ to be a manifold allowing a Ricci flat metric. The Ricci flatness follows from the equations of motion and we put all other backround fields to zero. The type II superstring has the maximal amount of supersymmetry allowed, incompatible with our 4-d physics. The search for vacua that do not allow for any supersymmetry is intractable since it becomes very complex to parametrise the internal geometry in full generality. Thus, for technical reasons, we keep the minimal amount of supersymmetry which can be broken in a later step. For type II strings this means that we compactify on a Calabi-Yau three fold (CY_3), leaving $\mathcal{N} = 2$ supersymmetry. The 8 supercharges are still too restrictive to describe the chiral matter in the standard model so we are forced to consider orientifold CY_3 which gives $\mathcal{N} = 1$.

From the worldsheet perspective an orientifold is a specific boundary condition on the string. Instead of the periodic bc we can consider orbifolding[4] and at the same time introduce an orientation reversal for the string. The result changes the global CY_3 structure, its local description away from singular points is unaltered. We call the internal geometry orientifold CY_3 and it allows for $\mathcal{N} = 1$ SUSY. The internal geometry has a defect corresponding to the points invariant under the orientifold procedure. The defect is called an O-plane and it contributes to the low-energy description of string theory in ways similar to D-branes. In particular, an Op-plane has tension $\tilde{T}_p$ and charge $\tilde{\mu}_p$. We will identify the O-plane tension $\tilde{T}_p$ with $-2^{p-5}T_p$, where T is de D-brane tension and we keep the relation between T and μ. Now, the low-energy contribution of O-planes is given by the D-brane DBI and WZ action after replacing $\mathcal{F} \to 0$ (there is no such coupling for O-planes). Note that the tension of an O-plane is negative but this is not an issue since the O-plane is non dynamical.

At this point we are not quite done, there are still massless moduli present in these string vacua. A good example is the size of the internal space, which is not fixed and corresponds to a scalar field without a potential in 4 dimensions. The solution is to search for ways to generate a classical potential and thus generate masses for all moduli at energy scales well above our effective 4-d theory. This is not yet understood for all superstring theories. In type II fluxes can be used to generate this potential [178]. For example, F_6 flux filling the internal space

[4]Think of orbifolding as replacing part of spacetime by a coset space where we divide by a discrete isometry group. As a simple example, we can think of a 2-d cone as the orbifold $\mathbb{C}/\mathbb{Z}_2$ where the $\mathbb{Z}_2$ identifies $z \sim -z$.

in IIA provides a term in the effective potential that wants to expand the size of the internal space, combining this with a positively curved internal manifold that wants to shrink under its gravitational weight can yield a stable value for the volume of X_6. The backreaction of such systems involves vacua that are warped product spaces as in (3.1). We will describe this situation in the Giddings-Kachru-Polchinski model [178] in IIB.

3.1.1 Giddings-Kachru-Polchinski

It was first realised in [178] how fluxes could be used to stabilise moduli which produced a new way to understand string vacua. However, this insight was not sufficient to find vacua which support lower-dimensional phenomenology and it was still unclear how to create a large hierarchy of scales (i.e. $l_4 \gg l_K \gg l_s$). There are well-known no-go theorems [88, 160] excluding Minkowski and de Sitter vacua and it can be extended to explain the lack of a simple pheno flux vacuum [83, 208, 112]. In short, it is shown that we need to include "negative-tension" objects to find a compactification to de Sitter or Minkowski space at tree level (not including higher-order corrections). This sounds like bad news because negative tension would correspond to unstable dynamics. Fortunately, string theory has non-dynamical negative-tension objects, as was already mentioned in the previous section. The orientifold or O-planes have negative tension, are non dynamical and therefore do not suffer from any instability.

To continue with a pheno perspective, we want to improve on the string vacua that use CY_3 orientifold compacitifcations by using additional branes and fluxes. This will also produce warped product geometries (3.1) such that the internal space is no longer Ricci flat.

We may thus look for string compactification backgrounds in type II by including D-branes, O-planes and fluxes as additional ingredients to search for Minkowski vacua. For strings IIB this was done in GKP [115], let us sketch these flux backgrounds.

We consider type IIB with D3 and O3-sources, a CY_3 with F_3 and H flux. The relevant type II supergravity action in Einstein frame, including local actions is

(see appendix A.3 without rescaling the RR forms):

$$S = \frac{1}{2\tilde{\kappa}_{10}^2}\int\Bigg[g_s^{-2}\left(\mathcal{R}(g) - \frac{1}{2}\star \mathrm{d}\phi\wedge\mathrm{d}\phi - \frac{1}{2}e^{-\phi}\star H\wedge H\right) \\ - \frac{1}{4}e^{\phi}\star F_3\wedge F_3 - \frac{1}{4}\star F_5\wedge F_5 - \frac{1}{4}e^{-\phi}\star F_7\wedge F_7\Bigg] \\ - N_3\tau_3\int\star_4 1 + N_3\mu_3\int C_4\,. \tag{3.5}$$

Ignoring the backreaction for the moment, we have no warp factor, a constant dilaton and the 10-d Ricci scalar $\mathcal{R}(g) = \mathcal{R}(g_4(x)) + \mathcal{R}(g_6(y)) = \mathcal{R}_4$. We may have a look at the scalar potential by integrating over the 6 compact dimensions:

$$\begin{aligned} S &= \int \frac{\nu_6}{2\kappa_{10}^2}\star_4\mathcal{R}_4 + \cdots - \star_4 V\,, \\ V &= \frac{1}{2\tilde{\kappa}_{10}^2}\int\left(\frac{1}{2}\left|g_s^{-1}\star_6 H + F_3\right|^2 + g_s^{-1}H\wedge F_3\right) + N_3\tau_3\,. \end{aligned} \tag{3.6}$$

We included the fluxes and DBI parts in the scalar potential and introduced the notation $\nu_6 = \int\star_6 1$. From $\mathrm{d}F_5 = H\wedge F_3 + 2\tilde{\kappa}_{10}^2 N_3\mu_3\delta$ and $\mu_3 = g_s\tau_3$ we have the following tadpole condition:

$$\int H\wedge F_3 = -2\tilde{\kappa}_{10}^2 N_3\tau_3 g_s\,, \tag{3.7}$$

and we may write:

$$V = \frac{1}{2\tilde{\kappa}_{10}^2}\int\frac{1}{2}\left|g_s^{-1}\star_6 H + F_3\right|^2. \tag{3.8}$$

We see that it is thus possible to have a vanishing potential such that $\mathrm{d}s_4^2$ is Minkowski, by taking $g_s^{-1}\star_6 H = -F_3$. This is often called ISD (imaginary self dual) flux, related to the corresponding property of the complex three-form $G_3 = F_3 - ig_s^{-1}H$. Moreover, for ISD flux (3.7) shows that $\tau_3 < 0$ which should remind you of the need to include O3-planes as was mentioned at the beginning of this chapter. Without considering the backreaction we thus see how this type of compactification requires a sufficiently large amount of O3-planes[5]. The full

[5]The interpretation of O-planes is different from that of D-branes. As a consequence we cannot have any number of O3 that we like, in particular there is a maximum amount of O3-charge.

backreacted solution is [115]:

$$\begin{aligned} \mathrm{d}s^2 &= W(y)^{-1/2}\mathrm{d}s_4(x)^2 + W(y)^{1/2}\mathrm{d}s_6(y)^2\,, \\ F_5^{ex} &= g_s^{-1}\mathrm{d}\left(W(y)^{-1}\wedge\tilde{\star}_4 1\right)\,, \\ \star_6 F_3 &= g_s^{-1}H\,, \\ \tilde{\star}_6\mathrm{d}\tilde{\star}_6\mathrm{d}W &= |H|_3^2 + 2\tilde{\kappa}_{10}^2 N_3\mu_3 g_s\tilde{\star}_6\delta\,, \end{aligned} \tag{3.9}$$

where the external four dimensions are given by Minkowski space. The last line gives a differential equation for the warp factor, it is the laplacian of the unwarped internal space with fluxes and local sources on the right hand side. By including D7- and O7-sources the dilaton and F_1 can also become nontrivial. Supersymmetry requires $\mathrm{d}s_6^2$ to be Calabi-Yau such that the internal space is actually conformal to an orientifold CY_3. Furthermore, SUSY restricts the allowed form of F_3 and H (it requires G_3 to have only a primitive (2,1) component).

To summarise, GKP type compactifications to Minkowski space are an interesting way to deal with the moduli problem and provide a first step towards a phenomenological vacuum. In section 4.4 we will see how additional ingredients can be used on top of GKP to decouple and even produce a small, positive cc. Even after the introduction of fluxes there are still moduli present that one has to take care of in a subsequent step. Examples include the internal volume and the position of D3-branes. Let us therefore have a look at probe D3 in the GKP background.

The above vacua have an interesting "no force" property for D3-branes. They can be added without distorting the background or they feel no force from the background. This also means that the position of the D3 is a modulus. This is easy to see from the D3 action: the pull back of the metric on the D3 worldvolume (at $y = y_0$) gives $W(y_0)^{-1}\tilde{\star}_4 1$ and the external part of F_5 shows that $C_4^{ex} = g_s^{-1}W(y)^{-1}\tilde{\star}_4 1$ such that:

$$S = -\tau_3\int\sqrt{-G} + \mu_3\int C_4 = -\int\tilde{\star}_4 W(y_0)^{-1}\left(\tau_3 - g_s^{-1}\mu_3\right)\,. \tag{3.10}$$

Now, D3-branes with $\mu_3 = g_s\tau_3 = T_3$ have zero potential $V = 0$, whereas the probe branes with $\mu_3 = -T_3$ will dynamically search for the region of highest warping $V = 2\tau_3 W(y_0)^{-1}$. The latter branes are called *antibranes*, they play an important role in the spontaneous breaking of supersymmetry and in this thesis. The above result can also be linked to the underlying supersymmetry algebra in which the branes and background fluxes are said to be mutually BPS and preserve the same amount of supersymmetry. Antibranes on the other hand,

preserve the opposite supercharges and therefore feel a force pushing them into regions of high warping. Before we turn to the subject of antibranes, let us consider the previous setting when the internal space is noncompact.

3.2 Noncompact vacua

We already encountered a noncompact flux vacuum, $\mathrm{AdS}_5 \times S^5$, in the context of the AdS/CFT correspondence in section 2.4. The AdS_5 factor is a noncompact warped product of 4-d Minkowski space and the real line: $\mathrm{Mink}_4 \times \mathbb{R}$. The duality between IIB on $\mathrm{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ SYM in Mink_4 has the maximal amount of supersymmetry in addition to being invariant under the conformal group. In the search for more realistic models one is interested in reducing these symmetries. If we hope to describe a standard model-like gauge theory then we need to have chiral matter which is only allowed when we have no more than 4 supercharges ($\mathcal{N} \leq 1$ in 4-d). A subsequent step would be to understand the chiral symmetry breaking that is observed in the standard model or to prove quark confinement in QCD. This involves non-conformal gauge theories and for that reason one wants to explore holography away from conformal invariance. From the gravitational point of view this means we have to break the isometries of AdS.

We saw in section 2.4 how $\mathrm{AdS}_5 \times S^5$ entered the story as the near horizon of a stack of D3s. The corresponding p-brane solution is very similar in form to the GKP vacua described in the previous section. The chief differences being that the internal space is noncompact and there are no O3-planes needed. It is thus interesting to study the above setting with noncompact internal space and relate this to a collection of string theory branes. From (3.2) we may heuristically argue for the existence of a decoupling limit since the effective 4-d gravitational coupling on the brane vanishes ($V \to \infty$). The hope is then that the near horizon is dual to the low-energy gauge theory living on the branes. The five sphere enters the game when we took $\mathbb{R}^6$ to be a cone over S^5. The most straightforward way would be to generalise this and consider different cone-like geometries for the internal space:

$$\mathrm{d}s_6^2 = \mathrm{d}r^2 + r^2 \mathrm{d}\Sigma_5^2 \,. \tag{3.11}$$

The coordinate r parametrises the radial direction of the cone over the base space Σ. Many results from section 3.1.1 (GKP) can be carried over to the noncompact setting. For example we still require the internal space to be Ricci flat but we no longer have to satisfy the tadpole condition. The base space of a Ricci flat cone is a positively curved Einstein space (see appendix A.1). If we

keep the minimal amount of supersymmetry we furthermore want the cone to be CY_3 which translates into an Einstein-Sasaki base space.

Noncompact vacua are also of interest for studying genuine compactifications as they can often be seen as a singular limit of a compact manifold or a certain region thereof. For that reason, we will focus on the external spacetime being Minkowski as this is of interest to both holography and with compact vacua of the GKP type in mind.

We will now discuss a specific example of a CY_3 cone called the conifold and its holographic interpretation. The conifold will frequently appear in the remainder of this thesis.

3.2.1 Conifold holography

The simplest noncompact Calabi-Yau threefold is flat space or a cone over a five sphere but it is highly nontrivial to find more involved examples for which the metric is explicitly known. The conifold is one such example and because of its explicitness it is heavily studied in the context of both holography and string vacua. Many details concerning the conifold geometry can be found in [67].

The conifold ($\mathcal{C}$) is a 6-d CY cone that can be embedded in $\mathbb{C}^4$ as:

$$\sum_{a=1}^{4}(w^a)^2 = 0 \quad , \qquad w^a \in \mathbb{C}\,. \tag{3.12}$$

The angular base of the cone is an Einstein-Sasaki space called tee-one-one, $T^{1,1}$. Let $\mathrm{d}\Theta^2$ denote the metric on $T^{1,1}$ then we can write the metric for $\mathcal{C}$ as:

$$\begin{aligned} \mathrm{d}\Theta^2 &= \frac{1}{9}\Big(2\mathrm{d}\beta + \sum_{i=1}^{2}\cos\theta_i \mathrm{d}\phi_i\Big) + \frac{1}{6}\sum_{i=1}^{2}\Big(\mathrm{d}\theta_i^2 + \sin^2\theta_i \mathrm{d}\phi_i^2\Big)\,, \\ \mathrm{d}s^2 &= \mathrm{d}r^2 + r^2\mathrm{d}\Theta^2\,. \end{aligned} \tag{3.13}$$

The radial coordinate $r \geq 0$ and the conical or conifold singularity is located at $r = 0$. Furthermore, we have $0 \leq \beta\,,\ \phi_i < 2\pi$ and $0 \leq \theta_i < \pi$. $T^{1,1}$ has an $S^2 \times S^3$ topology and can be thought of as a $U(1)$ bundle over $S^2 \times S^2$. The isometries of $T^{1,1}$ are $SU(2) \times SU(2) \times U(1) \simeq SO(4) \times U(1)$ as can also be seen from the conifold equation (3.12). The $SO(4)$ rotates the w^a while the $U(1)$ can be seen as a rescaling of every w^a. The $SU(2)$ structure is more transparant in the quotient description. For this we rewrite (3.12)[6]:

$$\tilde{z}_1\tilde{z}_2 - \tilde{z}_3\tilde{z}_4 = 0\,, \quad \tilde{z}_i \in \mathbb{C}\,, \tag{3.14}$$

[6]Use the linear transformation $\tilde{z}_1 = w^1 + iw^2$, $\tilde{z}_2 = w^1 - iw^2$ and $\tilde{z}_3 = w^3 + iw^4$, $\tilde{z}_4 = -w^3 + iw^4$.

which is solved by setting:

$$\tilde{z}_1 = A_1 B_1 \,, \quad \tilde{z}_2 = A_2 B_2 \,, \quad \tilde{z}_3 = A_1 B_2 \,, \quad \tilde{z}_4 = A_2 B_1 \,. \tag{3.15}$$

Now one $SU(2)$ acts on the A_i whereas the other acts on the B_j. The above solution A_i, B_j remains unaffected if we rescale $A \to \lambda A$, $B \to \lambda^{-1} B$. Let $\lambda = \rho e^{i\theta}$ where ρ is positive and θ real, we can fix ρ by requiring $|A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2$, we still have to address the freedom in θ. To identify the angular manifold $T^{1,1}$ we may quotient by the rescaling $w^a \to \tilde{r} w^a$ ($\tilde{r}$ positive) and set:

$$|A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2 = 1 \,. \tag{3.16}$$

This identifies the angular manifolds with $S^3 \times S^3 = SU(2) \times SU(2)$ now we can address the freedom in θ by quotienting $U(1)$ and we may identify:

$$T^{1,1} = \Big(SU(2) \times SU(2) \Big) \Big/ U(1) \,. \tag{3.17}$$

It will prove convenient to define the following one-forms:

$$e^1 = -\sin\theta_1 \mathrm{d}\phi_1 \,, \qquad e^2 = \mathrm{d}\theta_1 \,, \tag{3.18}$$

$$e^3 = \cos 2\beta \sin\theta_2 \mathrm{d}\phi_2 - \sin 2\beta \mathrm{d}\theta_2 \,, \tag{3.19}$$

$$e^4 = \sin 2\beta \sin\theta_2 \mathrm{d}\phi_2 + \sin 2\beta \mathrm{d}\theta_2 \,, \tag{3.20}$$

$$e^5 = 2\mathrm{d}\beta + \cos\theta_1 \mathrm{d}\phi_1 + \cos\theta_2 \mathrm{d}\phi_2 \,. \tag{3.21}$$

Using these there is a nice way to write the metric on $T^{1,1}$ as:

$$\mathrm{d}\Theta^2 = \frac{1}{9} \left(g^5\right)^2 + \frac{1}{6} \sum_{i=1}^{4} \left(g^i\right)^2 \,, \tag{3.22}$$

where

$$\begin{aligned} g^1 &= \frac{1}{\sqrt{2}} \left(e^1 - e^3\right) \,, \quad g^2 = \frac{1}{\sqrt{2}} \left(e^2 - e^4\right) \,, \\ g^3 &= \frac{1}{\sqrt{2}} \left(e^1 + e^3\right) \,, \quad g^4 = \frac{1}{\sqrt{2}} \left(e^2 + e^4\right) \,, \\ g^5 &= e^5 \,. \end{aligned} \tag{3.23}$$

Important to remember for the following pages is the topological structure of the conifold. It will be useful to think of it as a cone over $S^2 \times S^3$. We will frequently mention the S^2 (or S^3) in the conifold or $T^{1,1}$, keep in mind that

in reality we are referring to the nontrivial 2-cycle (3-cycle) in $T^{1,1}$ that is topologically a 2-sphere (3-sphere).

The heuristic AdS/CFT reasoning would now identify type IIB on $\mathrm{AdS}_5 \times T^{1,1}$ with the gauge theory on the worldvolume of D3-branes at the tip of the conifold, the gauge theory was first studied by Klebanov and Witten [150]. The AdS part reminds us that the dual gauge theory is conformally invariant but the $T^{1,1}$ breaks supersymmetry down to $\mathcal{N} = 1$.

To identify the gauge theory we note that $T^{1,1}$ can be obtained from the orbifold $S^5/\mathbb{Z}_2$ by blowing up the fixed circle oribifold singularity [150]. This viewpoint is helpful in constructing the worldvolume gauge theory, an orbifold construction requires one D-brane for each image. Thus, the $\mathbb{Z}_2$ orbifold sees 2 copies of the same D-brane and open strings ending on one copy can subsequently end on the second copy such that the gauge group for N coincident D-branes is enhanced to $U(N) \times U(N)$ with matter in the bifundamental representation $(N, \overline{N})$. In practice the $U(1)$ decouples[7] and we will identify the gauge theory on N D3-branes at the tip of the conifold with an $\mathcal{N} = 1$ SYM with gauge group $SU(N) \times SU(N)$. To determine the gauge theory further we specify its matter content to consist of four chiral superfields, A_i and B_j $(i, j = 1, 2)$. As notation suggests[8] the chirals A_i transform as a doublet of $SU(2)$, whereas the B_j transform as a doublet of the second $SU(2)$ and (A_i, B_j) have charge $(+, -)$ under the $U(1)$ that make up for the global $SU(2) \times SU(2) \times U(1)$ symmetry[9]. The superpotential is given by:

$$W = g\epsilon^{ik}\epsilon^{jl} \operatorname{tr}(A_i B_j A_k B_l) \ . \tag{3.24}$$

The A_i transform as $(N, \overline{N})$ and the B_i as $(\overline{N}, N)$ under the gauge group.

In the AdS/CFT correspondence, the vacuum moduli space of the gauge theory can be mapped to the degrees of freedom of the N D3-branes on the transverse space. The naive counting of d.o.f. in the chiral multiplets does not work because we have redundancy in our description. The potential in a 4-d $\mathcal{N} = 1$ supersymmetric gauge theory consists of two positive definite terms, one called the F term (part of a chiral multiplet), the other term is called the D term (part of a vector multiplet), (schematically $V \sim \sum F + \sum D$). To understand the moduli space for a SUSY vacuum we need the potential to vanish $V = 0$, which results in conditions for the vevs of the fields. Let us briefly discuss how this works on the Conifold. We can use a gauge transformation to put the chiral field vevs in diagonal form: $\langle A_i \rangle = \mathrm{diag}(a_i^1, \cdots, a_i^N)$ and $\langle B_i \rangle = \mathrm{diag}(b_i^1, \cdots, b_i^N)$. It

[7] The reason is identical to the situation of coincident D3s in flat space, see section 2.4.

[8] See equation (3.15) and the discussion following it.

[9] There are two additional $U(1)$s, one is the R symmetry group, the other is an axial symmetry. The axial $U(1)$ is anomalous and we will soon introduce the gauge group $SU(N + M) \times SU(N)$, in that setting the R symmetry is also anomalous as long as $M \neq 0$.

turns out that this automatically solves the F term equations, for details see [148]. The D term still needs to vanish, this requires:

$$|a_1^x|^2 + |a_2^x|^2 - |b_1^x|^2 - |b_2^x|^2 = 0\,,\ x = 1, \cdots, N\,. \tag{3.25}$$

Since we started with $4 \times 2N$ real d.o.f. (vevs for complex A_i and B_i) this equations further reduces this to $7N$ but we still have residual gauge invariance: the diagonal form of the vev is invariant under $U(1)^N$ which further reduces the d.o.f. to $6N$. This agrees with N D3-branes moving on the 6-d transverse space. Moreover, the D term equation can be mapped onto the defining equation for the conifold, matching the moduli space as is required.

The correspondence between the gauge theory described above and IIB on $\mathrm{AdS}_5 \times T^{1,1}$ has been checked to some extent. There is a detailed match between gauge invariant field theory operators and supergravity modes on the corresponding background [150, 121, 68]. Also, the following moduli are identified [150, 165]:

$$\begin{aligned} \frac{4\pi^2}{g_1^2} + \frac{4\pi^2}{g_2^2} &= \pi e^{-\Phi}\,, \\ \frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2} &= e^{-\Phi}\left(\frac{1}{2\pi\alpha'}\int_{S^2} B - \pi \pmod{2\pi}\right)\,, \end{aligned} \tag{3.26}$$

where g_1 and g_2 are the coupling of the first and second $SU(N)$. The dilaton Φ is constant in this background. The Kalb-Ramond B field is integrated over the S^2 at the tip of the conifold, the 2π periodicity of the relative coupling follows from the quantisation conditions of H.

Remembering the goal to describe more realistic gauge theories we want to break the isometries of AdS for non-conformal holography. This can be done by including fractional branes[10], for our purpose these are branes that wrap contracting cycles and are therefore called spherical branes. Moreover, they carry lower-dimensional brane charge e.g. a D5 carrying D3 charge by wrapping a collapsing 2 cycle (S^2) at the tip of the conifold. From the GKP point of view the story so far (Klebanov-Witten) did not involve three-form fluxes such that it could be interpreted as purely D3-branes with an AdS near horizon.

[10]Fractional branes were originally introduced when studying D-branes on orbifolds. As mentioned previously the orbifold normally sees one D-brane per image but at orbifold singularities (fixed points) this is not necessary and a single copy of the D-brane suffices. Such constructions can only exist at the fixed point (orbifold singularity) and are called fractional branes. For a $\mathbb{Z}_k$ orbifold we need k fractional branes to obtain a normal brane that can exist away from the singularity. As is usually the case, these conclusions follow from studying string boundary conditions. The low-energy supergravity description involves higher dimensional branes wrapping the contracting cycle at the singularity. Details can be found in [44].

The spherical D5 source F_3 and the near horizon is no longer AdS. By analogy with p-branes one expects this situation involving D3 and spherical D5 to be described by 5- and 3-form fluxes, the full supergravity dual was constructed by Klebanov and Tseytlin in [149].

We started with N D3-branes at the tip of the conifold and we now include M D5 wrapping the S^2 in $T^{1,1}$ (fractional D3). We can easily derive the solution and let's simplify the setting in which we replace $T^{1,1}$ by $S^2 \times S^3$ (this will only change some numerical factors in the end result). The fractional branes give $\int F_3 = 4\pi^2 \alpha' M$ and the ISD condition fixes $H = 2 g_s \alpha' \frac{M}{r} \mathrm{d}r \wedge \Omega_2$ (we used the 4π volume of S^2 and $2\pi^2$ volume of S^3). The equation for F_5 can now directly be obtained from (3.9). We may take the warp factor to resemble the D3 case with an effective, radially dependent number of branes and find a solution of the form:

$$W = c_0 + c_1 \frac{g_s \alpha'^2 N(r)}{r^4} \quad , \qquad N(r) = N + c_2 g_s M^2 \left[1 + 4 \log \left(\frac{r}{r_0} \right) \right] . \tag{3.27}$$

Where the c_1 and c_2 are order 1 numbers that depend on the size of the cycles corresponding to F_3 and F_5 flux[11]. Because of the warp factor (3.27) the geometry has a naked singularity at r_* where the effective number of branes goes to zero.

In [123] it is shown how the gauge group jumps from $SU(N) \times SU(N)$ to $SU(N+1) \times SU(N)$ when we add a fractional D3. This can be understood as the $\mathbb{Z}_2$ orbifold only seeing a single D3 (fractional) whereas the actual D3s all come in pairs (for the two images of the orbifold). From this we deduce the gauge group to be $SU(N+M) \times SU(N)$. We still have 4 chiral superfields: A_i in the representation $(N+M, \overline{N})$ and B_i in the representation $(\overline{N+M}, N)$.

Let us consider the supergravity background again for a moment, the dilaton remains a modulus and so should $g_1^{-2} + g_2^{-2}$ in the dual field theory. From the expression for H we do expect the following behaviour for the relative coupling:

$$\frac{1}{g_1^2} - \frac{1}{g_2^2} \sim g_s \alpha' M \log \left(\frac{r}{r_0} \right) + \cdots , \tag{3.28}$$

which agrees with the field theory β functions if we identify r_0 with the UV scale. Considering $r < r_0$ we see that g_1^{-2} must be decreasing whereas g_2^{-2} is increasing along the RG flow. At some point the description breaks down because the field theory becomes strongly coupled as g_1 diverges. From the

[11] These are different for $T^{1,1}$ then for the simplified $S^2 \times S^3$ setup. The reader can find the relevant details in [148], e.g. F_3 fills the cycle $g^5 \wedge \left(g^1 \wedge g^2 + g^3 \wedge g^4 \right)$ and the internal part of F_5 fills the entire $T^{1,1}$.

supergravity perspective we reach a point r_* where $N(r_*) = 0$ (3.27) and we hit a naked singularity in agreement with our physical intuition about negative N.

In [148] Klebanov and Strassler were able to smoothen the singularity by understanding the field theory side and they constructed a completely regular supergravity dual. The field theory mechanism to describe the theory beyond strong coupling is called a Seiberg duality. In [186] Seiberg showed how an $\mathcal{N} = 1$ SUSY gauge theory with F flavours and gauge group $SU(N)$ could equally well be described as an $SU(F-N)$ gauge theory with F flavours though with slightly different matter content and superpotential. The duality can be thought of as an extension of the usual electric-magnetic duality in electromagnetism, we will refrain from any details and direct the interested reader to the reviews [8, 191]. Important for us is that we can move past the point where g_1 diverges by using the (Seiberg) dual description. The remarkable thing for the conifold theory is that its Seiberg dual after integrating out heavier fields can be described using the exact same theory, same superpotential, only slightly different gauge group: $SU(N) \times SU(N-M)$. However, note that the gauge group that dualised was actually $SU(N+M) \to SU(N-M)$ since there are $2N$ flavours (A_i) such that $2N - (N+M) = N - M$. Now, in this dual theory, the gauge coupling of what was originally the second gauge group $(SU(N))$ diverges as we lower the energyscale. We can continue moving the renormalisation group scale further down (towards the IR) until we reach a point where the gauge group is $SU(M+p) \times SU(p)$ for $0 \leq p \leq M$. The cascade naturally stops at this point because the moduli space for the gauge theory with flavours $F \leq N$ is very different from the situation where there are more flavours than colours and there is strong dynamics involved. In fact, if the above analysis proves to be correct, there are nonperturbative corrections to the superpotential in the theory with less flavour than colour [5]. These nonperturbative effects give rise to chiral symmetry breaking and confinement. The supergravity background involves the deformed conifold which will now be explained.

There are two ways to smoothen the singularity at the tip of the conifold. One is to consider the deformed conifold:

$$\sum_{a=1}^{4} (w^a)^2 = \epsilon^2 \quad , \qquad w^a \in \mathbb{C}\,, \tag{3.29}$$

which results in a geometry similar to the conifold but only the S^2 shrinks to zero size whereas the volume of the S^3 remains finite at the tip. The other alteration is called the resolved conifold in which the S^2 remains of finite size at the tip and the S^3 shrinks to zero. The conifold and its smoothened versions are schematically depicted in figure 3.1.

One can redo the analysis that we sketched to match the moduli space of the

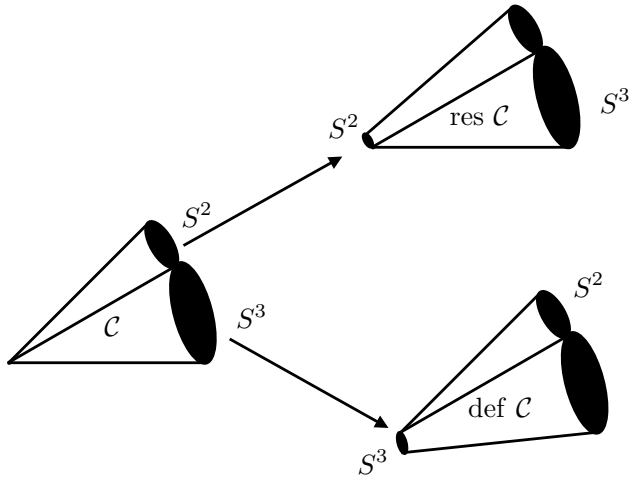


Figure 3.1: Schematic depiction of the conifold and its smoothened versions. The angular manifold of the conifold is topologically the product of an S^3 and an S^2, which both shrink to zero size at the singular tip. The resolved conifold is regular at its tip because the S^2 keeps a finite size whereas the deformed conifold is regular due to the finite-size S^3 at the tip.

field theory with N D3-branes moving on the conifold. We saw how the gauge group in the IR is $SU(M+p) \times SU(p)$ for $0 \leq p \leq M$. For this reason, we first introduce some common notation from $SU(M)$ gauge theories that has its origin in QCD. We have chiral matter fields A_i and anti-chiral matter B_i (i =1, 2). Similar to mesons in QCD (quark-antiquark pairs $q\overline{q}$) we introduce mesons:

$$N_{ij} = A_i B_j \,. \tag{3.30}$$

By including nonperturbative effects in the field theory [5] one can show that there are M distinct vacua in the theory, labeled by nonzero vevs of the mesons[12]. In [148] it is shown how the nonperturbative field theory analysis matches with p D3-branes probing the deformed conifold. In the limiting case where $p = 0$ the supergravity analysis is insufficient to capture the pure SYM field theory. When $p = M$ the moduli space is modified [185] and we have to introduce baryons. In QCD we have 3 flavours and 3 colours and we can make colour neutral combinations of 3 quarks (qqq), these bound states are called baryons. For $p = M$ we have $2M$ flavours from the $SU(2M)$ perspective (A_i with fixed

[12]The vacuum structure breaks the $U(1)$ R symmetry that was already broken to $\mathbb{Z}_{2M}$ further down to $\mathbb{Z}_2$.

$SU(M)$ index) and we introduce colour-neutral baryons $\mathcal{B}, \tilde{\mathcal{B}}$ (schematically):

$$\mathcal{B} = A_1^M A_2^M \,, \quad \tilde{\mathcal{B}} = B_1^M B_2^M \,. \tag{3.31}$$

Now the vacuum moduli space consists of two branches, a baryonic and a mesonic branch. In the baryonic branch, baryons obtain a nonzero vev whereas the mesons do not. The mesonic branch is similar to the moduli space discussed for D3-branes probing the deformed conifold, mesons obtain a nonzero vev and baryons do not.

For p small we may construct the branes as probing the $p = 0$ supergravity solution. This background is constructed in [148] and is no longer singular because of the finite S^3 at the tip, where it is well approximated by $(r \to 0)$[13]:

$$\mathrm{d}s_{10}^2 \approx \frac{\epsilon^{4/3}}{\alpha' g_s M} \mathrm{d}s_4^2 + \alpha' g_s M \left(\mathrm{d}r^2 + \frac{1}{2} r^2 \left((g^1)^2 + (g^2)^2 \right) + \mathrm{d}\Omega_3^2 \right) . \tag{3.32}$$

The Klebanov-Strassler background provides a gravity dual to more realistic gauge theories that feature chiral symmetry breaking, confinement and other interesting phenomena [148]. The deformed conifold provides a mechanism for the dynamical creation of a scale (see the factor in front of Mink_4 in (3.32)):

$$\epsilon^{\frac{2}{3}} = e^{-\frac{2\pi K}{3 M g_s}} \,, \tag{3.33}$$

where ϵ is the deformation parameter (3.29) and K corresponds to the units of H flux:

$$\int H = 4\pi^2 \alpha' K \,. \tag{3.34}$$

The scale (3.33) is in agreement with the expected scale generated by dimensional transmutation in $\mathcal{N} = 1$ SYM [115].

The chiral symmetry is associated with the anomalous U(1) mentioned in footnote 9. It is spontaneously broken to $\mathbb{Z}_{2M}$, this is a subleading effect in the supergravity picture for large M. However, the further symmetry breaking to $\mathbb{Z}_2$ is captured in supergravity and is a result of the deformed conifold [148]. Similarly, one can argue that the interquark potential is linear (quark confinement) due to the finite warp factor and smooth tip of the deformed conifold.

For large r the supergravity background is well approximated by the Klebanov-Tseytlin [149] background (see (3.27)). The sugra approximation is everywhere valid for $g_s M$ large as this agrees with small curvature throughout the solution. In chapter 7 we describe non-SUSY perturbations of these solutions.

[13] This is the Einstein frame metric.

3.3 Phenomenology and model building

To end this chapter, let us go back to the hierarchy problems introduced in chapter 1 and specifically focus on the cosmological constant. Since we are discussing type II string theory where there is a classical potential to stabilise the moduli and the regime is such that supergravity approximations are valid, we are necessarily fine tuning to a very specific corner of the vacuum space of string theory. Should this produce a single model with a small vacuum energy then this would not be an explanation to its smallness, merely a sign of what we already know, that it should be possible to have a small cosmological constant. However, we might find evidence for a large number of vacua with varying vacuum energy. In that situation we can take the multiverse and anthropic reasoning as a serious proposal to solve the cosmological constant problem. This finetuned search for string vacua is crucial to address the cosmological constant. We need to put ourselves in the situation where all approximations are valid such that we may argue for a good worldsheet description. If not, we know that there are corrections to consider and we have to resort to effective field theory methods where there is no perturbative way to address the value of the vacuum energy (or any other relevant operator).

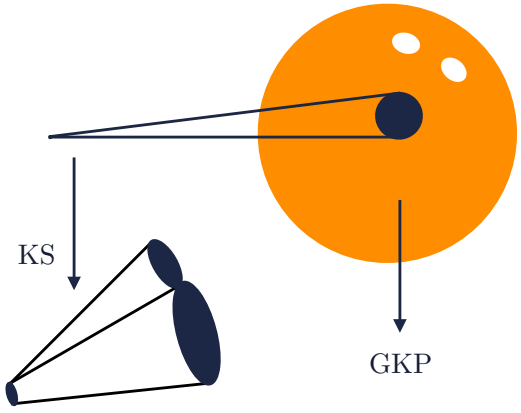


Figure 3.2: A GKP-type string compactification [115] with a region of large warping that is locally described by the Klebanov-Strassler background [148].

String model building could start with the Klebanov-Strassler [148] region of high warping in mind as part of a genuine compactification provided by Giddings-Kachru-Polchinski [115] (schematically depicted in figure 3.2). This should provide a Minkowski compactification with only a few massless moduli left, e.g.

the volume modulus (dilaton and axion can be stabilised with D7-branes). The warp factor sets the length scale for the external spacetime:

$$W(y)^{-1/2}\mathrm{d}s_4^2\,. \tag{3.35}$$

Hence, large warping can create a large hierarchy of scales, as was already used in brane world scenarios [179, 180]. The defining property of such a scenario is that standard model fields are confined to live on a brane (e.g. the gauge theory on the worldvolume of a D3) whereas gravity propagates in a larger number of dimensions. The internal space need not be compact and warping is used to produce an effective 4-d Planck mass on the Standard model brane and a large hierarchy. By combining the conifold [148] and GKP compactifications [115] such scenarios can be constructed in string theory. Moreover, the scale generated at the tip of KS (3.33) should be compared to the warp factor close to the tip of the undeformed conifold (3.27) or close to a stack of D3-branes (2.40). In both situations does the warp factor diverge whereas in KS it stays finite due to the deformed conifold, creating a stable hierarchy [115].

The subsequent step would now be to understand supersymmetry breaking in this setup and produce a phenomenologically small vacuum energy. Central to this is the study of antibranes, which is the subject of the next chapter.

Chapter 4

Antibranes and SUSY breaking

We came across antibranes ($\overline{\mathrm{D}p}$) in the previous chapter in section 3.1.1 when discussing the no-force property of string vacua with ISD three-form fluxes (3.10). Let us understand this better and consider a flux vacuum as discussed in the previous chapter. The F_5 Bianchi reads:

$$\mathrm{d}F_5 = H \wedge F_3 + 2\tilde{\kappa}_{10}^2 N_3 \mu_3 \delta \,, \tag{4.1}$$

δ indicates the positions of three-brane/plane sources. From this equation it is clear how the background three-form flux acts as a three-brane charge distribution. Moreover, if the background has ISD flux $H \wedge F_3$ is positive definite and we can define antibranes to carry the opposite charge w.r.t. the background fluxes. In this way they feel both an electromagnetic and gravitational attraction where the electromagnetic analogue comes from F_5. Actual branes feel no force because of the cancelation between gravitational pull and electromagnetic repulsion[1].

Antibranes play an important role in String model building and are most well known for their role in supersymmetry breaking. They preserve the opposite supercharges compared to the branes and fluxes in the supersymmetric vacuum. However, there are also special situation in which they do not break supersymmetry and we will see an example of this in chapter 8. In this chapter we will mainly be concerned with the supersymmetry breaking even though some aspects are of broader importance.

[1] However, when nonperturbative effects [142] are added the no-force property disappears.

The interest in breaking supersymmetry in string theory is twofold. Firstly, if our universe can be described using a string vacuum then we are obviously interested in breaking SUSY in this context. For this reason antibranes in GKP type compactification will be discussed and we will connect this with a possible solution to the cosmological constant problem. From the discussion around equation (3.10) we know antibranes to dynamically search for regions of high warping. Already in section 2.3.2 did we encounter a highly warped region when discussing the geometry close to D3-branes but also in the absence of branes can such highly warped conical regions exist. The geometry in the vicinity of regions with high three-form flux density also creates strong warping since the fluxes act as a charge density. It is difficult to find explicit expressions for highly warped compact manifolds. For this reason antibranes are studied in cone-like geometries with three-form fluxes such as the conifold setting described in section 3.2.1. Even though the conifold is noncompact, we may view it as though we are zooming in on a region of a compact manifold with high warping.

Secondly, we care about improving our understanding of gauge theories such as the standard model. We may exploit holography to understand the dynamical breaking of SUSY in strongly coupled gauge theories from a classical gravitational perspective. From this perspective it is natural to study antibranes in the conifold which provides a good holographic playground for non-conformal gauge theories. We see that both holographic SUSY breaking and non-SUSY compactifications are linked to the conifold.

4.1 Supersymmetry breaking

There are two perspectives on the breaking of supersymmetry in string theory. Since gravity is necessarily included we may look at its effective coupling (3.2):

$$2\kappa_4^2 = (2\pi)^7 \frac{\alpha' g_s^2}{V} \,. \tag{4.2}$$

Heuristically, we may argue that for noncompact string vacua there is no effective gravitational coupling localised on Mink_4. This can be made more precise in the AdS/CFT correspondence and gives rise to a string theory description of a gauge theory. Thus, we can study SUSY breaking in field theory, where gravity decouples. On the other hand, in the subject of string compactifications we have an effective 4-d gravitational coupling. This gives a second perspective: supersymmetry breaking when gravity cannot be decoupled. Let us start by discussing the SUSY breaking in field theory, a useful review is [95].

To benefit from the added structure of supersymmetry we want it to be spontaneously broken. Generically, if there is a dimensionless coupling g and

SUSY is unbroken at tree level then it is unbroken to all orders in g due to nonrenormalisation theorems. The energy scale associated with tree-level breaking is the typical energy scale of the theory, e.g. think of the Higgs mass and Electroweak symmetry breaking. Since we expect new physics at the grand unified scale (10^{16} GeV) or Planck scale this would be of little interest to current issues in particle physics phenomenology such as the Higgs hierarchy problem. Nonperturbative effects can also break SUSY, this is called dynamical breaking and it naturally creates the type of scale hierarchy that is desirable in discussing physics beyond the standard model [203]. Concretely, the SUSY breaking scale M_{SUSY} can be made much smaller than any UV cutoff scale M_{UV}:

$$M_{SUSY} = e^{-\frac{1}{g^2}} M_{UV} \,. \tag{4.3}$$

This can be seen as a first step towards creating a large hierarchy between the electroweak scale and M_p.

It turns out that dynamical SUSY breaking in gauge theories is complicated from a model building perspective. As a result, interest shifted to metastable (dynamical) SUSY breaking. The metastability indicates that the non-SUSY vacuum is not a true vacuum but a long lived state that eventually decays to a supersymmetric vacuum. Current model building suggests that metastable SUSY breaking is a more likely scenario compared to previous attempts but it requires the gauge theory to be nonrenormalizable or strongly coupled [95].

Understanding why model building points us in the direction of metastable SUSY breaking is beyond the scope of this text. However, we will look at holographic SUSY breaking in section 4.3 where antibranes are used to create a non-supersymmetric metastable state [143]. Furthermore, the state created by antibranes is dual to spontaneous SUSY breaking in the field theory [45].

From the perspective of string compactifications there has been some effort in finding a supergravity model to describe the small cosmological constant. Such a compactification can at best be metastable. To understand this, recall that sugra is valid in a regime where length scales are large compared to l_s. This means that there is a formal limit taking all length scales to infinity such that string corrections vanish. In this limit we have 10-dimensional Minkowski space as a well-understood, supersymmetric string vacuum. Moreover, this vacuum has zero vacuum energy so any state with positive vacuum energy can at best be metastable. I turns out that we can obtain such a state using antibranes in GKP-type compactifications [142], we will have more to say about this in section 4.4.

4.2 Dielectric branes

When we place an electron in a background electric field it feels a force. An electron can be thought of as a zero-brane charged under F_2, feeling a Lorentz force when a background F_2 is present. Similarly, a Dirichlet p-brane is charged under F_{p+2} (or by Hodge duality F_{8-p}) and feels a force reminiscent of electromagnetism when a background F_{p+2} is present. Since we are considering situations involving branes and background fluxes we should be careful about their interactions. One important feature of their interaction is called brane polarisation or the Myers effect [166] and it has some similarities with the dielectric effect in electromagnetism.

In the presence of a background electric field, the charges in a dielectric medium will align to form an extended object with a net polarisation. The reason can simply be understood from classical physics as the alignment provides a lower-energy configuration. When branes interact with background flux a similar phenomenon occurs where the brane extends itself over a certain distance and by doing so lowers its energy due to the interaction with the flux. A collection of N Dp-brane can extend itself over a certain distance by actually forming a bound state of N Dp and a D$(p+2)$-brane. The $(p+2)$-brane wraps a spherical 2-cycle and is often referred to as a spherical D$(p+2)$.

The effective actions of single D-branes were reviewed in section 2.2.2 and it was mentioned that the nonabelian generalisations are not completely understood[2]. The Wess-Zumino action is easiest to discuss as its nonabelian generalisation is often taken to include a trace over the gauge group indices and an implicit agreement to use gauge covariant derivatives $D = \partial - A$ instead of ∂ in the pullback P [134, 97]. This can be confirmed to leading order by string scattering amplitudes [110] but it is unclear whether this is the full story. In fact, it should not be the full story if the nonabelian generalisation is to be consistent with T duality. The situation is worse for the DBI part of the action as there is no consensus on how to modify it for coincident branes. Myers proposes a nonabelian generalisation relevant to studying polarised branes that is based on consistency with T duality[3].

When discussing multiple D-branes, we have $9-p$ massless d.o.f. (scalars σ^i) that parametrise the position of each D-brane in the transverse space. However, when they are coincident additional fluctuations become massless s.t the d.o.f. can be arranged into $U(N)$ representations [205]. In fact, we obtain $9-p$ scalars transforming in the adjoint representation, easiest described using $N \times N$

[2] For a review see [189].

[3] However, it is unclear whether the proposal is correct and it is known not to be complete (there are other α' and g_s corrections), see [100] for further information and references.

matrices Σ^i (i runs from 1 to $9-p$, we suppress matrix indices)[4]. They have the following low-energy potential, directly related to 10-d super-Yang-Mills:

$$V = \frac{(2\pi\alpha')^2 T_p}{4} \operatorname{Tr}\left([\Sigma^i, \Sigma^j]^2\right) + \cdots . \tag{4.4}$$

The ground state then has all scalars commuting, such that they can be diagonalised simultaneously: $\langle \Sigma^i \rangle = \operatorname{diag}(\sigma_0^i)$, precisely agreeing with N copies of the $9-p$ positions in the transverse space. However, in the case of background fluxes or other fields, the potential may be altered and there may be a lower-energy configuration in which the scalars no longer commute. The interpretation as the positions of individual D-branes is then no longer clear and a more extended, non-commutative object is formed, called a polarised brane. In [166] Myers shows how this non-commutative object corresponds to a fuzzy S^2 or more general 2-cycle. The concept of N individual Dp-branes breaks down and the extended object can be understood as a bound state of a spherical D$(p+2)$ and N Dp-branes. The proces is schematically shown in figure 4.1.

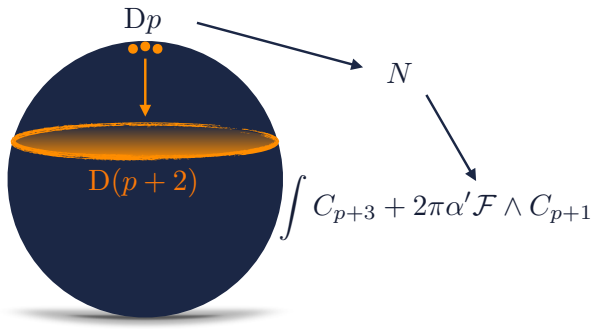


Figure 4.1: The polarisation of N Dp branes into a spherical D$(p+2)$.

The low-energy effective description of such a bound state goes through the usual D$(p+2)$ action with N units of worldvolume $U(1)$ flux for the same amount of Dp-branes [205]. The $U(1)$ flux corresponds to a monopole solution on the worldvolume of the $(p+2)$-brane:

$$F = -\frac{N}{2} \sin(\theta) \mathrm{d}\theta \wedge \mathrm{d}\phi \,, \tag{4.5}$$

[4]The Σ have complex entries but satisfy a reality condition.

here we took an S^2 for the 2-cycle. This worldvolume flux enters the Wess-Zumino action (2.21) through $2\pi\alpha'\mathcal{F} = B - 2\pi\alpha' F$:

$$S_{WZ} = -(-)^p \mu_{p+2} \int \sigma \left(C_{p+3} - 2\pi\alpha' \mathcal{F} \wedge C_{p+1} \right) + \cdots , \tag{4.6}$$

and induces N units of Dp charge:

$$\mu_{p+2} \int_{p+3} 2\pi\alpha' \mathcal{F} \wedge \sigma C_{p+1} = -\mu_{p+2} \int_{S^2} 2\pi\alpha' F \int_{p+1} \sigma C_{p+1} \,, \tag{4.7}$$

$$= \mu_{p+2} (2\pi)^2 N \alpha' \int_{p+1} \sigma C_{p+1} \,. \tag{4.8}$$

The reader will be pleased to learn that indeed $\mu_{p+2}(2\pi)^2\alpha' = \mu_p$ showing that the correct amount of Dp charge is carried on the spherical D$(p+2)$. Moreover, the $(p+2)$-brane wraps a trivial 2-cycle and the unpolarised state corresponds to the 2-cycle shrinking to zero size, we will see an example in the next section. The above will often be abbreviated to a Dp polarising into a (spherical) D$(p+2)$.

Polarisation highly depends on details and can happen in a multitude of ways in which different background fields are involved. It is pointless to discuss this in full detail, rather we will look at some examples throughout this thesis in section 4.3 and chapters 5 and 6. In the above setting, polarisation is found by studying the effective potential $V(\Sigma)$ of the non-abelian brane configuration and understanding the ground state. However, the non-abelian extension is an unsettled debate and for that reason we will take a different point of view.

Polarisation can also be understood by studying the final bound state, a D$(p+2)$-brane wrapping a spherical 2-cycle around the original position of the Dp-branes. In this case we associate a coordinate, e.g. ψ, to the position of the 2-cycle and since the cycle is trivial there is a limit, $\psi \to 0$, that corresponds to the cycle shrinking to zero size. When this happens, there is no $(p+2)$-brane and only the original N Dp-branes are left. From the effective action for the D$(p+2)$ we may now deduce an effective potential $V(\psi)$. If we find a minimum for some ψ_0 such that the 2-cycle has a stable nonzero size then we expect the D-branes to polarise. In other words, the probe action for the $(p+2)$-brane indicates that the system can lower its energy by forming a polarised bound state.

We end this section with an intuitive picture of polarisation in situations inspired by the flux backgrounds discussed in the previous chapter. Consider the following Bianchi identity again:

$$\mathrm{d}F_5 = H \wedge F_3 + 2\tilde{\kappa}_{10}^2 N_3 \mu_3 \delta \,. \tag{4.9}$$

This shows how three-form fluxes can behave in ways similar to local three-brane sources. This also means that a probe anti D3 will feel a force when there is background F_3 and H around. A result of this interaction is a high flux energy density surrounding the antibranes which are pointlike in the internal geometry. Now imagine the 3-branes forming an extended object involving a spherical D5. This is no longer pointlike and the energy density is lowered because it can be distributed over a larger surface. This agrees with the possibility to form a non-commutative lower-energy configuration in which we say the brane has polarised. We have primarily focused on D-branes but we will see in the next section the idea applies more generally. Let us now discuss a specific example with important applications for supersymmetry breaking: the case of antibranes polarising into spherical NS5.

4.3 Metastable antibranes

In [143] the authors consider placing a small number, p, of anti D3-branes[5] in the deformed conifold with M units of F_3 flux on the S_3 (called the A-cycle) and corresponding ISD H flux that will be denoted K (on the dual B-cycle):

$$\int_A F_3 = 4\pi^2\alpha' M\,, \quad \int_B H = 4\pi^2\alpha' K\,. \tag{4.10}$$

We discussed the deformed conifold in section 3.2.1 as part of the Klebanov-Strassler gravity dual of an $SU(N+M)\times SU(N)$ gauge theory. We may associate N with the total number of three-brane charge such that $N = KM - p$ in the absence of local D3 sources[6].

Antibranes were already known for their ability to break supersymmetry [159] but a concrete realisation was prohibited by brane-antibrane annihilation (the analogue of particle-antiparticle annihilation) which would render the system perturbatively unstable. When $N = KM$, the final Klebanov-Strassler geometry no longer involves any brane sources, only flux. It therefore provides the opportunity for antibranes to create a non-supersymmetric vacuum.

In section 3.2.1 we saw how the RG flow towards the infrared involved a series of Seiberg dualities. This eventually had to stop when the number of flavours became equal to or less than the number of colours in the gauge group. If we start with $N = KM + p$, where p is a small number of D3, the IR can be described using the gauge group $SU(M + p) \times SU(p)$ after K successive

[5] More precisely, p is small compared to three-form fluxes M and K such that the p antibranes can be considered as probe branes in the flux background.

[6] This follows from the F_5 Bianchi: $\mathrm{d}F_5 = H \wedge F_3 + 2\tilde{\kappa}_{10}^2\mu_3\delta$.

Seiberg dualities. Now, strictly speaking we cannot start with a small amount of antibranes ($p \to -p$), they would immediately annihilate but we can hope to get the correct quantitative picture in the IR by analogy. If we start with $N = KM - p$ we can only perform $K - 1$ Seiberg dualities such that we end up with $SU(2M - p) \times SU(M - p)$. The theory is supersymmetric and can be understood as $M - p$ D3-branes probing the deformed conifold with M units of F_3. This agrees with the supersymmetric moduli space sketched in section 3.2.1 where mesons obtain a nonzero vev. However, the probe antibranes break supersymmetry so there should exist a non-supersymmetric state, KPV state from here on.

We may consider p $\overline{\text{D3}}$ as probing the $SU(2M) \times SU(M)$ ($p = 0$) gauge theory for p small ($p/M \ll 1$). In [143] the authors argue that the KPV state is related to the baryonic branch of the $p = 0$ moduli space (baryons were introduced in (3.31)). However, for $p \neq 0$ we cannot construct colour-neutral baryons and so giving a vev to a colour-charged baryon costs energy. This then corresponds to the metatstable non-SUSY state dual to antibranes probing the deformed conifold. It is metastable since it has more energy than the zero-energy SUSY vacuum. These are mostly heuristic arguments. It was later verified that the KPV state indeed corresponds to spontaneous SUSY breaking using the conifold holography to compute field theory quantities [45].

We can also imagine a compact setting where the deformed conifold merely describes a local region of high warping. Now we also need to satisfy the tadpole condition. Since we expect p $\overline{\text{D3}}$-branes in this setting to eventually decay into $M - p$ D3, we conclude that M D3-branes seem to have materialised out of the flux to annihilate against the antibranes. This can only happen if there is a corresponding jump in H flux (K) as can be inferred from the tadpole condition:

$$KM - p = (K - 1)M + M - p\,. \tag{4.11}$$

This shows an interesting feature of the interaction between branes and fluxes. One unit of H flux has decayed such that M D3 could materialise to annihilate against the p anti D3.

The metastable SUSY breaking state is not simply p $\overline{\text{D3}}$ added as probe branes to the Klebanov-Strassler geometry. Because of the strong background fluxes the antibranes are believed to form a spherical NS5, i.e. they polarise. To show this we will consider a probe NS5 carrying the relevant antibrane charge as worldvolume flux. As was explained in section 4.2, we will look at the effective potential for the spherical NS5. In the absence of any background fluxes the spherical brane will collapse under its own tension leaving only the original antibranes. However, due to the background fluxes we may find a stable position for the spherical brane which corresponds to the position of the polarised configuration. The complete proof of polarisation would be to find

the backreacted supergravity solution corresponding to this spherical brane but this is often out of reach[7].

We can model the Klebanov-Strassler background close to the tip as (3.32):

$$\mathrm{d}s^2 \approx \frac{\epsilon^{4/3}}{\alpha' g_s M}\mathrm{d}s_4^2 + \alpha' g_s M \left(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2 + \mathrm{d}\Omega_3^2\right) , \tag{4.12}$$

where $r \to 0$. We take the NS5 to fill 4-d Minkowski space (as the $\overline{\mathrm{D3}}$ do the same) and wrap an S^2 inside the finite S^3 at some angle ψ:

$$\mathrm{d}\Omega_3^2 = \mathrm{d}\psi^2 + \sin^2\psi \mathrm{d}\tilde{\Omega}_2^2 . \tag{4.13}$$

In other words ψ is the position of the NS5 that is otherwise pinned at the tip $r = 0$. Note that the 2-cycle given by a constant value of ψ is trivial in homology (in other words δ_{NS5} is exact) such that the NS5 cannot carry any global NS5 charge. The 2-cycle collapses at $\psi = 0$ and π such that the NS5 reduces to the corresponding (anti) D3 at these points. We will place the antibranes at $r = 0$, $\psi = 0$. The metric on the NS5 reads:

$$\mathrm{d}s^2_{\mathrm{NS5}} = \frac{\epsilon^{4/3}}{\alpha' g_s M}\mathrm{d}s_4^2 + \alpha' g_s M \left(\dot{\psi}^2 \mathrm{d}t^2 + \sin^2\psi \mathrm{d}\tilde{\Omega}_2^2\right) \tag{4.14}$$

$$= \mathrm{d}s_4^2 + \mathrm{d}s_2^2 , \tag{4.15}$$

so g_4 is the induced metric along the Minkowski directions and g_2 along the trivial S^2 inside S^3.

The NS5-brane effective action for the IIB theory can be found by S duality of the D5 action. S duality is a string symmetry similar to T duality, it relates the IIB theory to itself but inverts the value of g_s amongst other, details can be found in [176, 54, 22]. We will only give the couplings in the action relevant for the KPV story[8]:

$$S_5 = -\frac{T_5}{g_s^2}\int \mathrm{d}^6\xi \sqrt{-\det g_4 \det\left(g_2 + 2\pi\alpha' g_s \tilde{\mathcal{F}}\right)} - T_5 \int \left(B_6 + \tilde{\mathcal{F}} \wedge C_4 + \cdots\right) , \tag{4.16}$$

where $2\pi\alpha'\tilde{\mathcal{F}} = 2\pi\alpha' F - C_2$ and F is the worldvolume field strength. This field strength is chosen such that it induces the correct amount of antibrane charge

[7] In a different setting such a backreacted solution has been found [177].

[8] Note that the D5 action is only valid in the small coupling regime $g_s \ll 1$. We actually need the NS5 at small coupling but the S dual D5 is then necessarily at strong coupling and far outside its regime of validity. However, the resulting intuition obtained by studying the probe NS5 action in this way has proven to survive this subtlety. As an example, consider [45] where the KPV state has been observed in a different analysis. In addition, the nonabelian analysis matches the NS5 calculation but it also involves S duality.

at $\psi = 0$, i.e. $2\pi \int F = 4\pi^2 p$. Moreover, B_6 is defined using $H_7 = \mathrm{d}B_6 + \cdots$ the form dual to H_3 such that the H_7 Bianchi identity agrees with the H_3 eom, the dots are not important in this paragraph. We may thus take (in Einstein frame):

$$\mathrm{d}B_6 = g_s^{-2} e^{-\phi} \star H_3 = -g_s^{-1} \star_4 1 \wedge F_3 \,, \tag{4.17}$$

where we used that we have ISD three-form flux and the dilaton is constant. The final ingredient needed to write down the potential $V(\psi)$ for the probe NS5-brane is the RR potential C_2. We started with M units of RR flux (4.10) and require that $\tilde{\mathcal{F}}$ induces p $\overline{\mathrm{D3}}$ charge at $\psi = 0$. This results in:

$$\int_{S^2} C_2 = 4\pi\alpha' M \left(\psi - \frac{1}{2}\sin(2\psi)\right) . \tag{4.18}$$

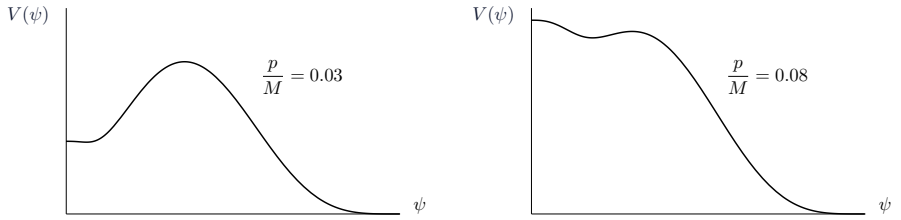


Figure 4.2: The potential (4.19) of a probe NS5 carrying $\overline{\mathrm{D3}}$ charge p wrapping an S^2 inside the finite S^3 at the tip of the deformed conifold with M units of F_3 flux. For $p/M = 0.03$ a metastable state exists that corresponds to a SUSY-breaking state in the dual field theory. When $p/M > 0.08$ the metastable state no longer exists.

We may now write $S_5 = \int T - V$ with T the kinetic term depending on $\dot{\psi}$ and $V(\psi)$ the potential:

$$V(\psi) = \frac{4\pi\alpha' T_5 M}{g_s} \left(\sqrt{\sin^4\psi + \left(\frac{\pi p}{M} - \psi + \frac{1}{2}\sin 2\psi\right)^2} - \left(\psi - \frac{1}{2}\sin 2\psi\right) \right) . \tag{4.19}$$

The potential is shown in figure 4.2. For $p/M = 8\%$ or larger the antibranes are perturbatively unstable and the system classically evolves towards the supersymmetric vacuum with $M - p$ D3. The instability is understood as brane-flux decay in which M D3-branes materialise out of the flux to annihilate against the p $\overline{\mathrm{D3}}$. On the other hand, when $p/M = 3\%$ we clearly see a metastable position for the NS5-brane. The supersymmetric vacuum seems to only be available after a tunnelling proces. The full proces is summarised in figure 4.3.

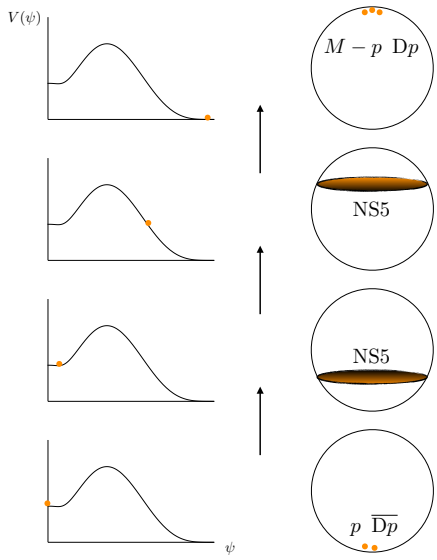


Figure 4.3: A summary of the processes involved in KPV [143]. The left hand side shows the potential (4.19) for $p/M = 3\%$ whereas the right hand side shows the finite S^3 at the tip of the deformed conifold. A small number p of anti D3 probing the deformed conifold can polarise into a spherical NS5. The NS5 wraps an S^2 close to the southpole ($\psi = 0$) of the S^3 at the tip of the deformed conifold. It can eventually tunnel close to the northpole $\psi = \pi$ where it induces $M - p$ D3-branes.

We will later (in chapter 5) show in more detail how the actual amount of D3 depending on the angle ψ is given by:

$$N_3(\psi) = -\frac{1}{2\pi}\int \tilde{\mathcal{F}} = \frac{M}{\pi}\left(\psi - \frac{1}{2}\sin(2\psi)\right) - p\,, \tag{4.20}$$

which nicely interpolates beween $-p$ and $M - p$ as ψ moves from 0 to π. We can also consider the difference in energy as the brane moves from $\psi = 0$ to π. This gives

$$\Delta V = \frac{2p}{g_s} 4\pi^2 \alpha' T_5 = \frac{2p}{g_s} T_3\,, \tag{4.21}$$

which is the expected contribution[9] associated with p $\overline{\text{D3}}$.

In summary, [143] provides a holographic model for dynamical, metastable supersymmetry breaking by studying antibranes in Klebanov-Strassler. Addi-

[9] The factor of 2 comes from the equal contribution of DBI and WZ.

tionally, the antibranes provide a positive contribution to the overal energy. If we imagine this setting as part of a compactification, this leaves the possibility to create a state of positive vacuum energy, though naively of string scale at this point. We will see how this idea is used in section 4.4.

4.3.1 Brown-Bunster and jumping flux

In the above section we saw how the process called brane-flux annihilation forced flux quanta to jump due to the tadpole constraint. In the internal space the process is understood as a form of brane polariation but the flux numbers seem to jump instantaneously from the perspective of the external spacetime. This is of course not true and there is a corresponding process describing the lowering of flux units in spacetime. This process goes through the formation of Brown-Bunster[10] bubbles [60].

Brown and Bunster investigated gravitational instantons in d-dimension where there was field strength F_d carrying flux around. In [60] they construct spherical domain walls or bubbles, called membranes at the time, that change the flux number by one unit as one moves from one side of the wall to the other. The jump in flux number can only be consistent with equations of motion if the bubbles are charged under the d-form such that the desired coupling in the eom or Bianchi is provided. The proces is pictorially represented in figure 4.4.

Intuitively, Brown-Bunster bubbles can be understood as the analogue to electromagnetism in $1+1$ dimensions where a background electric field (F_2 flux) can be lowered by the spontaneous creation of an electron-positron pair (the analogue of the membranes in $d > 2$). It is clear that, without any extra dynamics, the lowering proceeds through a tunnelling process.

In string theory, the F_d flux can correspond to the external spacetime description of any internal flux through Hodge duality. We may then understand the Brown-Bunster bubbles as the thin wall approximation of brane-flux annihilation. The bubble is then an NS5-brane in the case of H flux decay or a D-brane in the case of jumping RR flux. More details can be found in chapter 5 where we discuss brane-flux annihilation.

[10] Formerely known as Brown-Teitelboim.

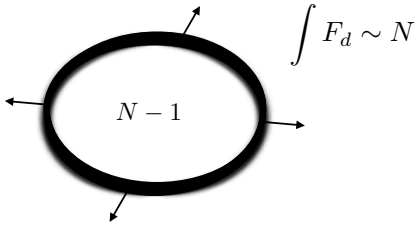


Figure 4.4: Brown-Bunster bubbles and the lowering of flux quanta. The spherical brane is charged under F_d, which has a background value corresponding to N flux units. If energetically favourable it may expand such that the N units of F_d flux change to $N-1$ inside the bubble.

4.4 Compact backgrounds and stringy dS

We now return to string model building and the hope to understand our universe using string theory vacua. The starting point is a GKP compactification to Minkowski space where only a few moduli remain and we will assume that only the volume modulus ρ is present[11]. To have a genuine string compactification we still need a way to deal with ρ and keep the internal length scales small compared to our 4-dimensional space.

In [142] (KKLT) the authors subsequently use 4-d quantum corrections (nonperturbative) to induce a mass for ρ. This is a tricky step and we mentioned before that the ideal situation is to induce a classical potential such that one has a good view of its regime of validity. However, there are good arguments for why the quantum corrections that can be used to deal with the remaining moduli are sufficiently well understood at low coupling g_s and large length scales such that they do not interfere with the supergravity regime of validity or any other approximation.

The authors argue that the quantum corrections are such that there exist a resulting AdS vacuum with small (negative) cosmological constant. Prior to any corrections ρ was a modulus and so the 4-d effective potential $V_4 = 0$, the

[11] In general, more moduli are present but they are more or less treated in the same way as the volume modulus.

nonperturbative corrections generate[12]:

$$V_4(\rho)\alpha'^2 = \frac{aAe^{-a\rho}}{2\rho^2}\left(Ae^{-a\rho}(1+\frac{a\rho}{3})+\omega_0\right). \tag{4.22}$$

More details on the explicit form of the effective potential can be found in [142]. The values of a, A and W_0 depend on details of the compactification and on the corrections that are included. For $\omega_0 = -10^{-4}$, $A = 1$ and $a = 0.1$ the AdS vacuum has a small minimum $V_0 \approx -2 \cdot 10^{-15}\alpha'^{-2}$. This can be thought of as the first genuine string compactification in the sense that the internal dimensions would be unobservable at low energies. However, it has so far proven too difficult to give an explicit construction of this step.

A second step involves adding a small number of probe $\overline{\text{D3}}$ to break SUSY and generate a positive vacuum energy. The resulting 4-d spacetime is then de Sitter and could correspond to the current state of our universe. We studied a similar effect of anti D3 in section 4.3 where the internal space was noncompact. Recall our discussion around equation (4.2), there is no 4-d gravity when the internal space is noncompact. In [142] the authors include antibranes in a compact setting where gravity cannot be decoupled and we have a finite 4-d effective Planck mass. We also saw in the previous section how p anti D3 gave a positive contribution to the potential ($2p\tau_3$). However, this was the result at the tip of the deformed conifold, in a more general GKP compactification one obtaines (see the discussion around (3.10)):

$$\Delta V = 2p\tau_3 W(y_0)^{-1}, \tag{4.23}$$

where the warpfactor is evaluated at the position of the anti D3 (y_0). This is a very attractive feature of the KKLT proposal and generates the type of large hierarchies desirable in understanding the cosmological constant. The string scale energy τ_3 can be made arbitrarily small since antibranes dynamically move into regions of high warping. In the 4-d effective potential, the antibrane contribution turns out to correspond to a power law ρ^{-3} [142]:

$$V_4(\rho)\alpha'^2 = \frac{aAe^{-a\rho}}{2\rho^2}\left(Ae^{-a\rho}(1+\frac{a\rho}{3})+\omega_0\right)+\frac{D}{\rho^3}. \tag{4.24}$$

The potential is plotted in figure 4.5 for $\omega_0 = -10^{-4}$, $A = 1$, $a = 0.1$ and $D = 3 \cdot 10^{-9}$. The antibrane contribution is nearly constant on the scale of the AdS minimum because it has a power law behaviour instead of the exponentials in (4.22). For this reason the procedure is often referred to as a "antibrane uplifting".

[12]The volume modulus is complex valued and the potential (4.22) is usually associated to the imaginary part of ρ so we have redefined ρ in this context for our convenience.

It seems safe to say this is the best string theory proposal to construct a background with a small positive vacuum energy. However, we still lack an explicit construction which shows how difficult it is to understand positive vacuum energy in string theory. If we forget about the subtleties and take this proposal as a sound construction then it is often seen as proof for a landscape of different string vacua with various values for the cosmological constant.

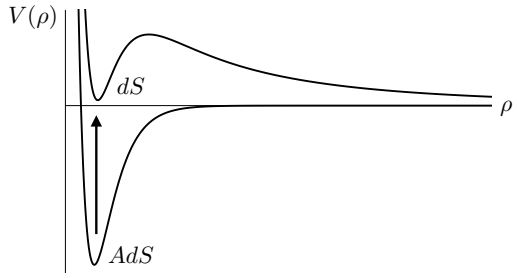


Figure 4.5: The potential (4.24) in string units for $\omega_0 = -10^{-4}$, $A = 1$, $a = 0.1$ and $D = 3 \cdot 10^{-9}$ (multiplied by 10^{15}). The antibranes produce a phenomenologically small positive vacuum energy corresponding to a dS universe.

A large part of this thesis will try to understand these antibranes beyond the probe regime by studying their backreaction. This is important to answer questions about the stability of the KKLT proposal. One can also wonder about the quantum corrections used to address the problem of massless moduli or whether there is no interference between these corrections and the addition of probe $\overline{\text{D3}}$. We will not address these issues but focus mainly on the problem of antibrane backreaction.

4.5 Antibrane backreaction

The backreaction of antibranes in flux backgrounds has received a lot of attention the past years. We explained the interest in SUSY-breaking anti D3 [143, 142] in sections 4.3 and 4.4. The backreaction of $\overline{\text{D3}}$ in the Klebanov-Strassler background [148] was first considered in [162, 31, 29]. In the same context $\overline{\text{D6}}$ in massive IIA were studied in [50, 51, 140]. They are related to anti D3

through a series of T dualities[13] and allow for a more explicit treatment of the backreaction.

By studying anti D3 in perturbation theory it was noticed that the three-form flux density close to the $\overline{\text{D3}}$ diverges (both in string and Einstein frame) [162, 31]:

$$|F_3|^2 \to \infty\,, \quad |H|^2 \to \infty\,. \tag{4.25}$$

Flux densities contribute to the energy-momentum tensor meaning that the backreaction on the geometry is very large, in disagreement with the probe regime. At first is was suggested that the diverging flux density is due to perturbation theory or because of approximations such as partial smearing of the branes [98, 21]. This was studied intensely [28, 29, 32, 51] and the singularity was eventually proven to survive beyond perturbation theory or other approximations [111, 48]. Moreover, in [48] a no-go theorem against finite temperature antibranes was found[14]. Physical singularities[15] are expected to be covered by a horizon at finite temperature [122]. This is similar to the cosmic censorship hypothesis, non-extremal (equivalently, nonzero temperature) black hole horizons are expected to shield singularities in our universe.

With the singular behaviour of antibranes firmly established one can wonder whether this is any different from the expected singular behaviour that goes with any type of brane source. We already saw for example that the warp factor close to a stack of D3 is divergent. Unfortunately the diverging three-form flux density cannot be explained from local three-brane sources since these couple to F_5 and not to F_3 or H. Another possibility would be that the singularity is resolved by polarisation as is understood in [177]. It has proven difficult to find an interpretation that accepts the singularity as physical [33, 26, 25, 34, 48, 37]. However, the diverging flux density could also signal an instability of the antibranes. Such an interpretation was first given in [50] and further elucidated in [52, 84].

In the discussion surrounding equation (3.10) we saw how D3-branes do not backreact in a background with ISD three-form flux. The ISD thee-form flux behaves as a continuous 3-brane charge distribution (similar to smeared D3-branes) and the three-form flux and D3-branes do not feel any force from each other. Anti D3, on the other hand, are both electrically and gravitationally

[13] To be more precise, the branes have to be partially smeared along the T duality directions. This was also mentioned in section 2.3.3.

[14] There are toy models that show the possibility to evade the theorem [127]. However, one should be careful drawing conclusions since the analysis is performed in a simplified setting.

[15] A singularity is physical when it has a good microscopic description. The singularity in the electric field of an electron is physical as it corresponds to the position of the electron. Similarly, singularities in F_{p+2} forms can correspond to positions of Dp-branes (see section 2.3.2).

attracted to the ISD three-form fluxes. It is thus expected that the ISD condition does not survive after backreaction. The fluxes can only redistribute their density since they should integrate to:

$$\int F_3 = 4\pi\alpha'^2 M \,, \tag{4.26}$$

and similar for $\star_6 H = -g_s F_3$. This suggest that the diverging three-form flux is a consequence of the attractive force between the ISD flux cloud and $\overline{\text{D3}}$. Recall that the KPV state (see section 4.3) is unstable against brane-flux annihilation through tunnelling [143]. The interpretation was then given that this could destabilise the KPV probe analysis in such a way that there is no metastable state after backreaction, only perturbative brane-flux annihilation and the inevitable decay to the SUSY ground state [50, 52, 84].

There is still the possibility that the singularity is resolved by stringy corrections. Perhaps the flux cloud is sufficiently diluted for a small amount of antibranes (or even a single antibrane) [163]. In chapter 6 we investigate the antibrane backreaction and compare the situation of flat worldvolume (pheno compactification) with AdS worldvolume. We find that polarisation resolves the singularity for AdS worldvolume. In chapter 5 we explain how the brane-flux annihilation observed in KPV [143] proceeds in more general situations.

For completeness we mention that a similar setting exists in M theory where one studies anti M2-branes in the so-called CGLP (Cvetič, Gibbons, Lü and Pope) background [76]. Probe anti M2 were first studied in [147] where it was observed that a metastable state exists after polarising into an M5-brane. The backreaction was later considered in [30, 161, 74, 116, 47]. A similar singularity is found and an explicit construction to resolve the singularity is also not known.

Chapter 5

The many faces of brane-flux annihilation

In section 4.3.1 we explained how fluxes can decay via the nucleation of Brown-Bunster bubbles. When the decaying fluxes induce D-brane charges this process must be accompanied with an annihilation of D-branes. This occurs via dynamics inside the bubble wall as was well described for $\overline{\text{D3}}$-branes annihilating against 3-form fluxes in [143], see section 4.3. In this chapter we extend this to the other $\overline{\text{D}p}$-branes with p smaller than seven. Generically there are two decay channels: one for the RR flux and one for the NSNS flux. The RR channel is accompanied by brane annihilation that can be understood from the $\overline{\text{D}p}$-branes polarising into D$(p+2)$-branes, whereas the NSNS channel corresponds to $\overline{\text{D}p}$-branes polarising into NS5-branes or KK5-branes. We illustrate this with the decay of antibranes probing local toroidal throat geometries obtained from T-duality of the D6 solution in massive type IIA. We show that $\overline{\text{D}p}$-branes are metastable against annihilation in these backgrounds, at least at the probe level. This chapter is a reprint of [114].

5.1 Introduction

The search for metastable SUSY-breaking flux vacua in string theory is of particular interest to phenomenology and remains an active area of research. Verifying that a solution is metastable requires the knowledge of the masses of all moduli and the decay mechanisms for topological data such as brane charge and flux. In this chapter we focus on the latter and describe in general terms

how fluxes and branes can annihilate against each other. Our work can be thought of as a generalisation of the decay of $\overline{\text{D3}}$-branes against 3-form fluxes of [143] (see also [108, 58]) to the more general case where $\overline{\text{D}p}$-branes annihilate against either the NSNS 3-form flux H_3 or the RR $(6-p)$-form flux F_{6-p}. In this context *antibranes* refer to the D-branes carrying a charge that is opposite to the background fluxes. In other words, the two terms on the right hand side of the Bianchi identity:

$$\mathrm{d}F_{8-p} = H_3 \wedge F_{6-p} + 2\tilde{\kappa}_{10}^2 \mu_p \delta_{9-p}\,, \tag{5.1}$$

have opposite orientation. The δ-function form describes the brane sources.

Following the original work of Brown and Bunster (BB) [59] we anticipate the presence of nonperturbative processes that lower the flux quanta. This process is the generalisation of Schwinger pair creation to spherical domain walls (BB bubbles) in spacetime that nucleate and can grow to infinite size if the vacuum inside the bubble has lower energy.

For compact extra dimensions the integral of the LHS of equation (5.1) must always vanish which implies that fluxes cannot decay unless the brane charges change in the same process. This suggests that the BB bubble is the thin wall limit of a more complicated process where the finite thickness of the bubble has to account for the dynamics that change the brane charge [84]. We can understand these dynamics by considering the infinitely thick limit. This involves a (generalised) Myers effect where the polarised brane can decay leaving less flux and changing the brane charge accordingly. As suggested these two effects are two distinct limits of a single process called brane-flux annihilation. This can be thought of as a closed string versus open string description of the same phenomenon.

An equivalent picture of brane-flux annihilation is that Dp-branes can spontaneously materialise out of the flux. Those branes then annihilate with the antibranes and as a result there will be less antibranes and less flux. If the number of antibranes is smaller than the amount of nucleated Dp-branes then this process leads to the creation of Dp-branes. It is the aim of this chapter to explain how this phenomenon works for general branes and fluxes.

Our focus is on a pedagogical treatment of the physical process but an equivalent, mathematically oriented discussion invokes K-theory as in [195, 158, 103, 102]. We complement the latter references also by providing a concrete setup where these processes take place. Finding such backgrounds is a nontrivial matter since it requires supersymmetry to be broken in a controllable fashion.

This chapter is organised as follows. We first set notations and context in section 5.2 and then describe the brane-flux annihilation in the RR sector in section

5.3. After reviewing the NS5 Wess-Zumino (WZ) action in type II supergravity, we describe the brane-flux annihilation in the NSNS sector. We then apply this to local throat geometries of the form $\mathbb{T}^{6-p} \times \mathbb{R}^3$ filled with fluxes that carry Dp charge. These geometries are obtained from T-dualising a solution found in [137]. We demonstrate that, at the probe level, $\overline{\text{D}p}$-branes are metastable in these geometries. We end with conclusions in section 5.6. In appendix B we comment on different types of brane-flux decay that are not discussed in the main text.

5.2 Preliminaries and notation

Consider a compactification of type II supergravity which is supported by NSNS 3-form flux H and RR flux F_{6-p} on a compact space M to some lower-dimensional maximally symmetric vacuum. Such solutions typically involve Dp-branes and Op-planes to satisfy the RR tadpole condition:

$$\int_M H_3 \wedge F_{6-p} = -Q_p \ , \tag{5.2}$$

where Q_p denotes the total p-brane charge of the localized sources quantised according to (2.27):

$$Q_p = \left(2\pi\sqrt{\alpha'}\right)^{7-p} N_p \, , \tag{5.3}$$

with $N_p \in \mathbb{Z}$.

In order to have an explicit model in mind we assume that the compactification manifold is $(9-p)$-dimensional such that the sources are spacetime-filling and point-like in the compact dimensions. Our results are however valid for a general setup. Most widely known examples of this kind are IIB compactifications with 3-form fluxes to four dimensions [115], for which $p = 3$. These can be generalised to $p = 1, \ldots, 6$, see for instance [49]. To simplify the presentation even further we pretend as if only two cycles are threaded with fluxes. There is the A-cycle of dimension $6-p$ and the B-cycle of dimension 3 threaded by flux as follows:

$$\int_B H_3 = (2\pi)^2 \alpha' K \ , \qquad \int_A F_{6-p} = (2\pi)^{5-p} (\alpha')^{(5-p)/2} M \ , \tag{5.4}$$

with $K, M \in \mathbb{Z}$. Since the A and B cycle are each others dual we have from (5.2):

$$-N_p = KM \, . \tag{5.5}$$

In the rest of the chapter we work in string units for which $2\pi\alpha' = 1$.

Since we have both NSNS flux and RR flux there are two possible flux decay channels. Both of these channels proceed through the nucleation of a spacetime bubble where the inside of the bubble has less flux than the outside. This bubble will continue to grow if it is energetically favourable. The end state is either the true vacuum of the theory or another metastable state of lower energy. We recall the well-known Brown-Bunster picture of flux decay and its generalisations before describing the stringy resolution of this process which is the topic of this chapter.

The BB bubble that makes the NSNS flux decay is an NS5-brane wrapping the A-cycle and in the case of RR flux decay it is a $\mathrm{D}(p+2)$ wrapping the B-cycle. One readily checks that these wrapped branes indeed have a single codimension in the $(p+1)$-dimensional vacuum. Only for $p=6$ is the NS5 entirely inside the noncompact spacetime[1]. The intersection diagram for these BB bubbles can be found in table 5.1.

	Thin wall	$p+1$			A-cycle			B-cycle		
	Op/Dp	$\times$	...	$\times$	$-$	...	$-$	$-$	...	$-$
NSNS decay:	NS5	$\times$	...	$\uparrow$	$\times$	...	$\times$	$-$	...	$-$
RR decay:	D$(p+2)$	$\times$	...	$\uparrow$	$-$	...	$-$	$\times$	...	$\times$

Table 5.1: The intersection diagram for the microscopic description of the BB bubbles that describe the thin wall limit of brane-flux annihilation. The arrow $\uparrow$ reflects the fact that the brane is moving in the radial spacetime direction.

Because of the tadpole condition (6.8) a decay of the RR flux and NSNS flux must go together with a decay of brane charges as follows:

$$\text{NSNS decay} \quad : \quad K \to K-1 \quad , \quad N_p \to N_p - M\,, \tag{5.6}$$

$$\text{RR decay} \quad : \quad M \to M-1 \quad , \quad N_p \to N_p - K\,. \tag{5.7}$$

Hence the BB bubble describes the thin wall limit of a more involved decay mechanism that should also account for the correct change in brane charge. Clearly, there should be some dynamics inside the bubble wall that accounts for the change in brane charge such that the tadpole condition is satisfied on both sides of the wall. To understand how the brane charge changes we discuss the thick wall limit. For the RR channel this is understood as the Myers effect [166] where some $\overline{\mathrm{D}p}$-branes polarise into a $\mathrm{D}(p+2)$-brane filling spacetime

[1] One could also envisage a situation in which the spherical brane is inside a compact manifold and then a flux cascade can take place as conjectured in [146]. We leave an explicit string theory embedding of the cascade effect and its associated inflationary model [78] for future investigations.

and wrapping a trivial cycle inside the B-cycle. The NSNS channel is slightly more involved and requires a generalisation of the Myers effect where $\overline{\mathrm{D}p}$-branes polarise into an NS5 filling spacetime and wrapping a trivial cycle inside the A-cycle.[2] In both cases the polarised brane can move either perturbatively or by the means of tunnelling inside the A- or B-cycle to a point where the wrapped trivial cycle pinches off. At this point the flux has been lowered by one unit and the brane charge changes according to (5.6, 5.7). The intersection diagram for this process can be found in table 5.2.

	Thick wall	$p+1$			A-cycle			B-cycle		
	Op/Dp	×	...	×	−	...	−	−	...	−
NSNS decay:	NS5	×	...	×	×	...	↑	−	...	−
RR decay:	D$(p+2)$	×	...	×	−	...	−	×	...	↑

Table 5.2: The intersection diagram for brane-flux decay in the thick wall limit. The arrow ↑ reflects the fact that the brane is moving in the corresponding cycle.

In what remains we will discuss the dynamics in the thick wall approximation. We first analyse the decay of RR flux since it proceeds through the well-known Myers effect [166]. In the subsequent section we analyse the more involved NSNS channel.

5.3 The decay of RR flux

The BB bubble that decays a unit of RR flux corresponds to a D$(p+2)$-brane that wraps the three-dimensional B-cycle as explained above. The tension of this domain wall is roughly given by:

$$T = g_s^{-1}\mu_{p+2}\mathrm{Vol}(B)\,, \tag{5.8}$$

in string units. This process must go together with the annihilation of some brane charges and we now explain that this occurs through the polarisation of $\overline{\mathrm{D}p}$-branes into D$(p+2)$-branes wrapping a trivial S^2 inside B as depicted in figure 5.1. This D$(p+2)$-brane is not to be confused with the BB domain wall since the latter is a D$(p+2)$ that wraps the whole three-dimensional B-cycle. From here on we assume that B has spherical topology. In case the topology is different the process might involve tachyon condensation as we explain in section 5.4.

[2] When $p=6$ this description does not make sense and we will speculate in section 5.4.3 that this process involves a KK5-brane.

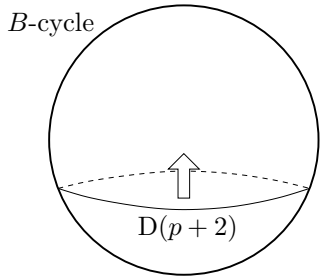


Figure 5.1: A D$(p+2)$-brane that wraps a trivial S^2 inside the spherical B-cycle. The D$(p+2)$ carries non-trivial worldvolume flux proportional to the volume form of the S^2 which ensures that the brane carries $\overline{\mathrm{D}p}$ charge.

Brane polarisation, a.k.a. the Myers effect [166], is enabled by the Wess-Zumino (WZ) coupling of the D$(p+2)$-brane (see section 4.2):

$$-(-1)^p \mu_{p+2} \int \sigma(C_{p+3} - \mathcal{F}_2 \wedge C_{p+1}) \,, \tag{5.9}$$

where the symbol σ is the operator that reverses all indices and induces only minus signs in such a way that the Bianchi identity always has the same sign for the localised source, and:

$$\mathcal{F}_2 = B_2 - F_2 \,, \tag{5.10}$$

with F_2 the worldvolume field strength living on the brane. The second term in the action can induce Dp charges.

We define the position of the spherical D$(p+2)$ by the polar angle Ψ on the 3-sphere. When $\Psi = 0$ the spherical D$(p+2)$ pinches off and corresponds to some number n of the original $\overline{\mathrm{D}p}$-branes. From the WZ action (5.9) we can read off that the charge (measured in units of antibrane charge) induced by the spherical brane is:

$$Q(\Psi) = -\frac{1}{2\pi} \int_{S^2(\Psi)} \mathcal{F}_2 \,, \tag{5.11}$$

where the factor of 2π comes from comparing μ_{p+2} with μ_p. We are mainly interested in the difference between the charge at the South Pole $Q(0)$ and the North Pole $Q(\pi)$. Without knowing the actual profile of B_2 we can already deduce that:

$$\int_{S^2(\Psi\to\pi)} B_2 - \int_{S^2(\Psi\to 0)} B_2 = \int_B H_3 = 2\pi K \,. \tag{5.12}$$

We can always consider a gauge of B_2 such that:

$$\int_{S^2(\Psi\to 0)} B_2 = 0\,. \tag{5.13}$$

In order to induce n antibrane charges at the South pole we must have that:

$$\int_{S^2(\Psi\to 0)} F_2 = 2\pi n\,. \tag{5.14}$$

This is achieved when the worldvolume gauge field has a monopole configuration:

$$F_2 = \frac{n}{2}\sin\theta \mathrm{d}\theta\wedge \mathrm{d}\varphi\,. \tag{5.15}$$

We assume this to remain unaltered throughout the motion along Ψ, see however [81]. Hence we deduce that:

$$Q(\pi) = n - K\,. \tag{5.16}$$

So if the spherical D$(p+2)$-brane moves from the South Pole towards the North Pole it induces first n $\overline{\mathrm{D}p}$ charges and in the end $(K-n)$ Dp charges such that effectively K Dp-branes nucleated from the flux cloud and annihilated with n $\overline{\mathrm{D}p}$-branes. The spherical D$(p+2)$ does not carry any monopole charge but it does source F_{6-p} locally which indeed causes the flux to decay $(M\to M-1)$ as is required by the tadpole condition.

The combination of the nucleation of the spherical bubble inside spacetime and the Myers effect inside the compact dimensions is visualised in picture 5.2. Consider some observer in noncompact spacetime that is about to be hit by an expanding BB bubble wall. In reality this wall is not infinitesimally thin, but has a finite thickness. When the outer wall passes the observer, the spherical D$(p+2)$ wrapping a 2-sphere inside the extra dimensions, starts rolling and reaches the North Pole the moment the inner wall passes the observer. The string theory realisation of the finite thickness wall is a pair of D$(p+2)$/$\overline{\mathrm{D}(p+2)}$-branes with M Dp-branes stretched between them. We have that p antibranes end on the outside of the wall while $M-p$ branes end on the inside of the wall. The M branes stretched between the pair are polarised in the internal space as we discussed.

Whether this process is energetically favourable, perturbatively or nonperturbatively, depends on the model under consideration. At leading order, in a probe approximation, this can be deduced from the energy given by the full D$(p+2)$ action (DBI+WZ). At first sight it seems puzzling that the brane annihilation process can be classical whereas its associated flux decay bubble is nonperturbative. However, as explained in [84] the bubble nucleation can become arbitrarily fast, such that it is effectively a classical process.

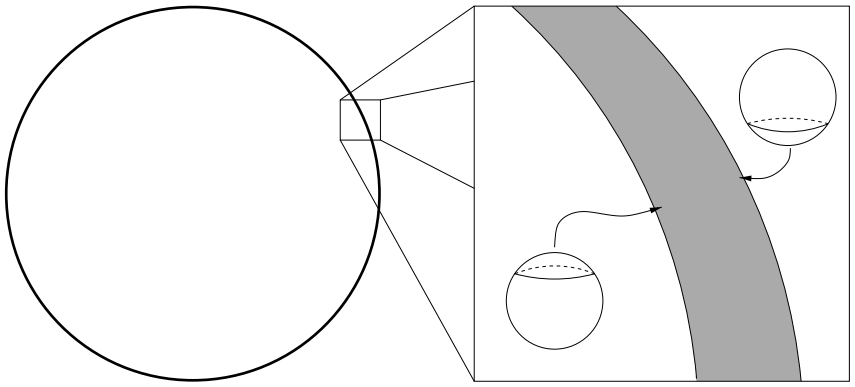


Figure 5.2: The S^2 moves across the S^3 when you go through the bubble wall.

Consider for instance the SUSY AdS_7 backgrounds of [14] built from D6-branes, Romans mass and H_3 flux. In this case there is no A-cycle (since $p = 6$) and the B-cycle is indeed topologically a 3-sphere. The D6-branes have been shown to polarise into D8-branes but the D8-branes do not move towards the South Pole, instead they are stuck at stable positions inside the internal manifold. This is consistent with the fact that the vacua are SUSY and hence there should not be any decay possible. However non-SUSY extensions of this model are possible, both compact models with AdS vacuum and noncompact Minkowski solutions [51, 140]. In the Minkowski case it has been shown that the D8 polarisation cannot occur [36] and instead these vacua are expected to decay via the NSNS channel which we turn to in the next section.

5.4 The decay of NSNS flux

To understand the decay of NSNS 3-form flux, it is useful to have a simple form of the NS5-brane action. Here we solely focus on the Wess-Zumino piece and in the next section we will discuss the details of the DBI action as well. NS5 worldvolume actions have been derived in various places in the literature, see for instance [104, 19, 43] (and see [189] for a useful review on general brane actions and their physics). We will review the derivation of the WZ action from first principles given in [43] since it gives the WZ action in a form slightly different from the existing literature, which is more useful for our purposes.

5.4.1 NS5 Wess-Zumino action

In what follows we use polyform notation (see appendix A.2) in which $\mathbf{C}$ is the sum of even RR-form potentials in type IIA and odd in type IIB.[3] We also have worldvolume gauge potentials combined into a polyform $\mathbf{a}$. For the IIA/IIB NS5-brane we have:

$$\begin{aligned} &\text{IIA:} \qquad \mathbf{a} = a_0 + a_2 + a_4 \\ &\text{IIB:} \qquad \mathbf{a} = a_1 + a_3 + a_5 \,, \end{aligned} \tag{5.17}$$

where the subscript denotes the rank of the form. Our notation is democratic and we naturally incorporate the dual gauge potentials as well. Starting from the gauge transformations of the RR potentials:

$$\delta\mathbf{C} = \mathrm{d}\boldsymbol{\lambda} - H_3 \wedge \boldsymbol{\lambda} \,, \tag{5.18}$$

where $\boldsymbol{\lambda}$ is a polyform of gauge transformation parameters. We see that the field strengths:

$$\mathbf{F} = \mathrm{d}\mathbf{C} - H_3 \wedge \mathbf{C} \,, \tag{5.19}$$

are gauge invariant. Similarly the gauge transformation of the B_2-field, $\delta B_2 = \mathrm{d}\Lambda_1$ shows that the field strength:

$$H_3 = \mathrm{d}B_2 \,, \tag{5.20}$$

is gauge invariant. We are interested in the magnetic field strength for B_2 under which the NS5 is charged. We then take a look at the equation of motion for B_2:

$$\mathrm{d}\left(\mathrm{e}^{-2\phi} \star H_3\right) - \frac{1}{2}\left\langle \mathbf{F} \wedge \sigma(\mathbf{F}) \right\rangle_8 = 0 \,, \tag{5.21}$$

The angle brackets denote the projection of polyform down to a specific form degree and σ is the reversal operator. We use the definition:

$$H_7 = \mathrm{e}^{-2\phi} \star H_3 \,, \tag{5.22}$$

and demand that it is gauge invariant. By integrating the equation of motion we find an expression for H_7 in terms of the 6-form gauge potential B_6:

$$H_7 = \mathrm{d}B_6 + \frac{1}{2}\left\langle \mathbf{C} \wedge \sigma(\mathbf{F}) \right\rangle_7 \,. \tag{5.23}$$

Gauge invariance then demands:

$$\delta B_6 = \mathrm{d}\Lambda_5 - \frac{1}{2}\left\langle \boldsymbol{\lambda} \wedge \sigma(\mathbf{F}) \right\rangle_6 \,. \tag{5.24}$$

[3] See [151] for a review.

We are now in position to write down a gauge invariant coupling to B_6, i.e. we look for a Lagrangian density $\mathcal{L}_{\rm NS5}$ that has the form:

$$\mathcal{L}_{\rm NS5} = B_6 + \cdots \tag{5.25}$$

and is gauge invariant up to a total derivative. To this end we define worldvolume gauge potentials $\mathbf{a}$ that transform under the RR gauge transformations:

$$\delta \mathbf{a} = \boldsymbol{\lambda} \ . \tag{5.26}$$

A gauge invariant field strength can be defined by:

$$\mathbf{G} = \mathrm{d}\mathbf{a} - H_3 \wedge \mathbf{a} - \mathbf{C} \ , \tag{5.27}$$

where we use the pullbacks of $\mathbf{C}$ and H_3 onto the worldvolume. With this information one can construct the full gauge invariant action:

$$S_{\rm NS5} = \mu_{\rm NS5} \int \left\{ B_6 + \frac{1}{2} \left\langle \mathbf{G} \wedge \sigma(\mathbf{C}) \right\rangle_6 \right\} \ . \tag{5.28}$$

The gauge transformation of the Lagrangian density is:

$$\delta \mathcal{L}_{\rm NS5} = \mathrm{d}\Lambda_5 - \frac{1}{2} \mathrm{d} \left\langle \mathbf{G} \wedge \sigma(\boldsymbol{\lambda}) \right\rangle_5 \ . \tag{5.29}$$

The democratic formulation of the NS5 action (5.28) is useful to understand how an NS5 can carry Dp-brane charge for $p < 6$, as discussed in the next section. However, it is unknown how to formulate the DBI part democratically and for that reason we will not make use of (5.28) for actual calculations but rather use the actions in [104, 19, 43].

5.4.2 Dp annihilation for $p < 6$

The BB bubble that decays the NSNS flux is an NS5-brane that wraps the A-cycle. As discussed above, the BB bubble does not account for the decay of brane charge. We now discuss the microscopic description of the brane charge which proceeds through an accompanying NS5-brane that wraps a trivial cycle inside the A-cycle. The relevant coupling in the NS5-brane action is:

$$-(-1)^p \mu_{\rm NS5} \int \sigma(C_{p+1}) \wedge (\mathrm{d}a_{4-p} - C_{5-p}) \ , \tag{5.30}$$

which is there for $p = 0, \ldots, 5$. Note that the factor of half is missing in equation (5.30) compared to eq. (5.28). This is due to the fact that the NS5 worldvolume gauge fields satisfy a self duality relation which halves the eight naive gauge field

degrees of freedom living on the brane to the correct four [43]. The NS5-brane must wrap a contractible cycle inside the A-cycle in order not to have any net NS5 charge (which it cannot have for the sake of charge conservation). This contractible cycle will have dimension $5-p$ such that the NS5 has one codimension inside A. Hence the homologically trivial cycle, $\Sigma_{5-p}(x)$, can be labeled by a single real number x denoting the position of the NS5 inside the A-cycle. From (5.30) we read off that the Dp charge induced by the contractible NS5-brane is:

$$Q(x) = (2\pi)^{\frac{p-5}{2}} \int_{\Sigma_{5-p}(x)} (\mathrm{d}a_{4-p} - C_{5-p}) \,, \tag{5.31}$$

where the factors of 2π are obtained by computing $\mu_{\rm NS5}/\mu_p$. We will use a gauge in which:

$$C_{5-p}(x \to 0) = 0 \ , \tag{5.32}$$

where $x \to 0$ denotes the limit where the contractible cycle shrinks to zero size. The NS5 induces $\overline{\mathrm{D}p}$ charge n at $x = 0$, by having a topological flux of $\mathrm{d}a_{4-p}$ on $\Sigma_{5-p}(x)$:

$$\int_{\Sigma_{5-p}(x)} \mathrm{d}a_{4-p} = (2\pi)^{\frac{5-p}{2}} n \,. \tag{5.33}$$

The way this is achieved is simply by equating $\mathrm{d}a_{4-p} = (2\pi)^{\frac{5-p}{2}} n \hat{\epsilon}_{5-p}(x)$, where $\hat{\epsilon}_{5-p}(x)$ is the *normalised* volume-form on $\Sigma_{5-p}(x)$ for any x. Since the flux is topological it will pick up a non-zero value even if $\Sigma_{5-p}(x)$ vanishes in size. The motion from one side of the chain of homological cycles, $x \to 0$, to the other side, $x \to 1$, corresponds to the decay of n units of $\overline{\mathrm{D}p}$ charge to $M - n$ units of Dp charge since:

$$Q(1) = n - M \,. \tag{5.34}$$

As before, we relied on Stokes theorem:

$$\int_{x\to 1} C_{5-p} - \int_{x\to 0} C_{5-p} = \int_A F_{6-p} = (2\pi)^{\frac{5-p}{2}} M \ . \tag{5.35}$$

5.4.3 D6 annihilation

An interesting case is the decay of $\overline{\mathrm{D6}}$-branes. From T-duality of the $\overline{\mathrm{D5}}$-brane decay we expect that the decay of NSNS flux in the thick wall limit does not proceed via an NS5-brane that carries $\overline{\mathrm{D6}}$ charge. Also, the NS5-brane does not possess the right couplings to carry $\overline{\mathrm{D6}}$-brane charge. T-duality suggests that the decay uses a KK5-brane carrying $\overline{\mathrm{D6}}$ charge. This decay is still under investigation, moreover, it poses a problem since a KK5-brane is an object with 6 worldvolume directions, and so, at first sight cannot carry $\overline{\mathrm{D6}}$ charges. However, the KK5 has one special transverse direction that possesses a circle

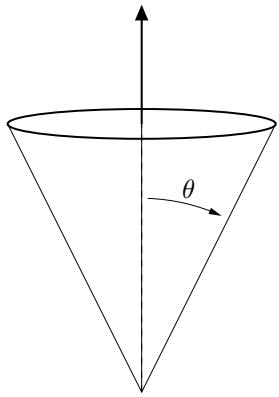


Figure 5.3: The infinitely thick limit of the domain wall KK5-brane that carries D6-brane charge.

isometry and in essence the spacetime solution looks like a seven-dimensional object. The KK5 worldvolume action also contains the coupling for it to carry $\overline{\text{D6}}$ charge:

$$S(\theta) = (n - \theta F_0) \int \iota_k C_7 \ , \tag{5.36}$$

where ι_k is the interior product in the direction of the isometry direction of the (Taub-NUT) circle describing the KK5-brane and θ is the angle along that S^1. This coupling can be obtained from massive T-duality of the NS5 coupling:

$$\int (n - C_0) \wedge C_6 \ . \tag{5.37}$$

The question is now what is the spacetime picture of the decay of H-flux involving the KK5-brane? The spacetime is seven dimensional Minkowski space which is the worldvolume of the decaying $\overline{\text{D6}}$-branes. The isometry direction of the KK5 must lie in spacetime and so we write the Minkowski metric as:

$$\mathrm{d}s^2_{\text{Mink}_7} = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \ \mathrm{d}\Omega_4^2) \ , \tag{5.38}$$

where $\mathrm{d}\Omega_4^2$ is the metric on the round 4-sphere. The KK5 extends through (t, r, Ω_4) and θ parametrises the contractible circle. The codimension of the KK5 in Mink_7 is the same as its isometry direction which is nothing but the polar angle in spherical coordinates. As time passes the θ position of the KK5 evolves from zero to π which means that the KK5 has a cone shape (see figure 5.3). To see that the KK5 carries the correct D6-charge we compare the coupling (5.36) at both endpoints of θ:

$$S(\pi) - S(0) = -\pi F_0 \int \iota_k C_7 = -F_0 \int C_7 \ , \tag{5.39}$$

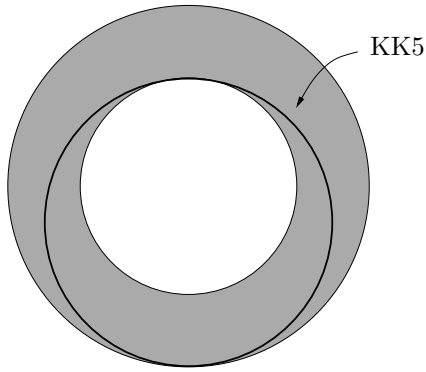


Figure 5.4: A domain wall NS5-brane with finite thickness that has the associated spherical KK5 in it.

where in the last step we used the fact that C_7 preserves the symmetry of Mink_7 and thus satisfies $\mathrm{d}\theta \wedge \iota_k C_7 = C_7$ and is independent of θ.

The resolved process where a BB bubble carries away H-flux *and* D6-charge, must be an NS5-brane of finite thickness with a KK5 flowing inside the wall. We have depicted such a process in figure 5.4. We do not provide more details here since we plan to present an in-depth study of this decay process in a separate work, which will also address the effect of the backreaction of the $\overline{\text{D6}}$-branes [82].

5.4.4 Tunneling rates

So far we have not discussed how to compute the actual tunnelling amplitudes for the decay processes. This requires knowledge of the energy it costs to expand a spherical brane or the energy it costs to separate brane/antibrane pairs. This is not captured by the WZ action alone but also involves the DBI action. The DBI actions for the NS5-brane in string frame are:

$$\text{IIA: } -\mu_{\text{NS5}} \int \mathrm{e}^{-2\phi} \sqrt{-\det\left(G + \mathrm{e}^{2\phi}\mathcal{G}_1 \otimes \mathcal{G}_1\right)} \left(1 + \frac{1}{4}\mathrm{e}^{2\phi}|\mathcal{G}_3|^2 + \cdots\right), \quad (5.40)$$

$$\text{IIB: } -\mu_{\text{NS5}} \int \mathrm{e}^{-2\phi} \sqrt{-\det\left(G - \frac{\mathrm{e}^{\phi}}{\sqrt{1+\mathrm{e}^{2\phi}\mathcal{G}_0^2}}\mathcal{G}_2\right)} \sqrt{1+\mathrm{e}^{2\phi}\mathcal{G}_0^2}\,, \quad (5.41)$$

where the gauge-invariant field strength $\mathcal{G}$ is defined in (5.27). Note, that these actions are not written in democratic form as we did for the WZ couplings in

section 5.4.1 and so one must sometimes dualise the field strengths in order to get a DBI action in a suitable form.

Once all interactions are known the potential energy for the spherical NS5-branes, or NS5/$\overline{\text{NS5}}$ pairs, can be computed and hence the tunnelling probability P through the standard WKB or instanton formula [71, 59]:

$$P \sim e^{-S_B/\hbar}\,. \tag{5.42}$$

S_B is the instanton action given by:

$$S_B = \frac{d^d 2\pi^{(d+1)/2}}{(d+1)\Gamma[(d+1)/2]}\frac{T^{d+1}}{\rho^d} \tag{5.43}$$

where d is the number of noncompact dimensions, T is the bubble tension and ρ is the drop in energy density through the bubble (the change in cosmological constant). The brane actions enter in the computation of the tension T. For that one constructs first the 1-dimensional effective action that governs the motion of the spherical NS5-brane or the NS5/$\overline{\text{NS5}}$ pair. This action allows classical *domain wall* solutions that interpolate between the two vacua and the on-shell action (tension) of that domain wall equals T (see for instance [143]). A classical motion, instead of tunnelling, would correspond to vanishing T and it has been argued that this is possible due to the backreaction of the antibrane probes [84].

Instead of computing T using the classical potential derived from the NS5-brane action wrapping some contractible cycle one could simply take the tension of the NS5 domain wall that wraps the *entire* A-cycle (the BB one). That tension equals the quantised NS5 tension $g_s^{-2}\mu_{NS5}$ times the volume of the A-cycle (see equation (5.8)). The latter way corresponds to the thin wall limit, whereas the first method is the thick wall limit. only in the limit $n/M \ll 1$ can one hope to match the thin and thick limits. But clearly, in the thin wall limit a classical decay can never be spotted.

5.5 Illustrations

5.5.1 $\overline{\text{D}p}$-probes in T-duals of the dissolved D6

Fractional brane solutions in the supergravity context [132, 53] are solutions for Dp-branes in which there is surrounding flux that carries Dp charge as well. This flux is simply the previously mentioned $H_3 \wedge F_{6-p}$ combination that contributes to the RR tadpole (6.8). The forces between the flux cloud and the

brane sources cancel out when the following condition is obeyed:

$$\star_{9-p}\, H_3 = g_s^{(p+1)/4} F_{6-p}\,, \tag{5.44}$$

where the Hodge star is taken with respect to the flat metric on the space transversal to the (parallel) Dp-branes. When this condition is met, simple solutions to the equations of motion can be found [132, 53, 154]. For instance for D6 solutions in massive IIA sugra one has, in Einstein frame [137]:

$$\begin{aligned}
\mathrm{d}s^2 &= S^{-1/8}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + S^{7/8}[\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2]\,,\\
F_2 &= -g_s^{-3/4} \star_3 \mathrm{d}S\,,\\
F_0 &= (2\pi)^{-1/2} M\,,\\
H_3 &= (2\pi)^{-1/2} g_s^{7/4} M r^2 \mathrm{d}r \wedge \mathrm{d}\Omega_2\,,\\
e^\phi &= g_s S^{-3/4}\,.
\end{aligned} \tag{5.45}$$

The Hodge star is taken with respect to the transversal metric with the warpfactor $S^{7/8}$ taken out. The function S is given by:

$$S = 1 + \frac{g_s T}{r} - \frac{g_s^{5/2} M^2 r^2}{12\pi}\,, \tag{5.46}$$

where T is the total D6 tension. At sufficiently large r the solution becomes singular, but at small enough distances, which is all we need in this chapter, this poses no problem. When there are no D6 sources present we can take $T = 0$ such that the solution is regular at small r. Such (locally) regular solutions can serve as toy models for the local geometries of resolved fractional branes near the tip [127].

T-duality of the above solution over the worldvolume directions generates the following solutions:

$$\begin{aligned}
\mathrm{d}s^2 &= S^{(p-7)/8}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + S^{(p+1)/8}[\mathrm{d}\mathbb{T}^2_{6-p} + \mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2]\,,\\
F_{8-p} &= -g_s^{(3-p)/4} \star_{9-p} \mathrm{d}S\,,\\
F_{6-p} &= (2\pi)^{(5-p)/2} M \hat{\epsilon}_{6-p}(\mathbb{T}_{6-p})\,,\\
H_3 &= (2\pi)^{(5-p)/2} g_s^{(p+1)/4} M r^2 \mathrm{d}r \wedge \mathrm{d}\Omega_2\,,\\
e^\phi &= g_s S^{(3-p)/4}\,, \qquad S = 1 + \frac{g_s T}{r} - \frac{g_s^{(p-1)/2} M^2 r^2}{6(2\pi)^{p-5}}\,,
\end{aligned} \tag{5.47}$$

where T is the total Dp tension and $\mathbb{T}_{6-p}$ is the torus generated by T-duality. We again consider the throat geometries which are locally smooth since $T = 0$ and we will focus on the geometry at the tip defined by $S = 1$ or $r = 0$.

Since it is useful for computing the WZ action for an NS5-brane we compute the dual NSNS gauge potential B_6, which is defined in (5.23). A gauge-fixed solution of this equation, at the tip ($S = 1$), is[4]:

$$B_6 = -g_s^{(p-3)/4} \mathrm{d}x^0 \wedge \ldots \wedge \mathrm{d}x^p \wedge C_{5-p} \ . \tag{5.48}$$

In this way the g_s dependence of the WZ action matches that of the DBI part in Einstein frame, as expected. In the same gauge we have that, at the tip:

$$C_{p+1}(r = 0) = 0 \, . \tag{5.49}$$

In what follows we consider perturbations of these local geometries by n $\overline{\mathrm{D}p}$ probes. The following conditions are necessary for having a sugra description and a probe limit at the same time:

$$n \ll M \, , \qquad g_s n \gg 1 \, , \qquad g_s \ll 1 \, . \tag{5.50}$$

The first condition makes sure that the probe approximation is naively justified, the second condition ensures that the *Schwarzschild radius* of the probe is large in string units in order to suppress higher derivative corrections and the last condition is the validity of perturbative string theory.

Below we verify how the probe branes can annihilate with one unit of H-flux to become $M - n$ Dp-branes. With regard to the previous sections the A-cycle in these backgrounds are the T-duality circles $\mathbb{T}^{6-p}$ and the B-cycle is noncompact.

5.5.2 Brane nucleation on tori

In the literature on antibranes polarising into larger objects over compact cycles (for instance [143, 147]) the compact cycles tend to have sphere topologies. The prime example comes from $\overline{\mathrm{D3}}$-branes inside the KS throat that polarise into NS5-branes wrapping a contractible S^2 inside an S^3 (the A-cycle). The S^2 pinches off at the poles and the motion between the two poles corresponds to the annihilation of branes and fluxes [143]. But when the compact cycles

[4]In the potential calculations below we will make use of a slightly different expression, which only makes sense for the pullback of B_6 on to the NS5 worldvolume. In stead of using (5.48) we will use:

$$B_6 = g_s^{(p-3)/4} \mathrm{d}x^0 \wedge \ldots \wedge \mathrm{d}x^p \wedge \mathcal{G}_{5-p} \ ,$$

where $\mathcal{G}_{5-p}$ is defined in (5.27). This shifts B_6 by a constant, and hence is just another gauge choice for it.

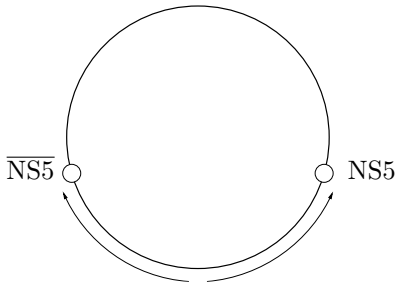


Figure 5.5: Nucleation of a NS5/$\overline{\text{NS5}}$ pair carrying D5 charges.

are toroidal, or have any other topology different from a sphere some extra complications arise.

Let us start with a compact A-cycle that is $S^1 = \mathbb{T}^1$, as is the case for $p = 5$. Then the NS5-brane carrying $\overline{\text{D5}}$ charge necessarily is an NS5/$\overline{\text{NS5}}$ pair as in picture 5.5. We take the circle of length 1 such that the circle-coordinate θ ranges from $(-1/2, 1/2)$. The profile for C_0 is then:

$$C_0 = M\theta\,. \tag{5.51}$$

Consider the nucleation of an NS5/$\overline{\text{NS5}}$ pair at $\theta = 0$. The pair separates and moves to opposite sides and meet again at $\theta = 1/2$. The NS5's carry worldvolume flux $n/2$ and $-n/2$. The total induced charge is:

$$Q(\theta) \sim \Big(\frac{n}{2} - M\theta\Big) - \Big(-\frac{n}{2} + M\theta\Big) = \Big(n - 2M\theta\Big)\,. \tag{5.52}$$

The two WZ actions are subtracted because one WZ term corresponds to an $\overline{\text{NS5}}$. The second WZ action also has a different sign for the $M\theta$ term because it moves in the opposite direction. So θ here is the positive θ that tracks the NS5 and not the $\overline{\text{NS5}}$ that is at the opposite side. So the motion ranges from $\theta = 0$ to $\theta = 1/2$ such that we indeed find that $Q(\theta)$ ranges from n to $(n - M)$.

Direct T-duality suggest that the other branes decay via NS5/$\overline{\text{NS5}}$ pairs wrapping $5 - p$ circles inside $\mathbb{T}^{6-p}$. This is correct but not unique. Similar to cycles with a spherical topology, we can have spherical NS5-branes nucleating inside the torus. Spherical NS5-branes are morally NS5/$\overline{\text{NS5}}$ pairs. We expect two possible things to occur for spherical NS5-branes carrying n $\overline{\text{D}p}$ charges:

1. The spherical NS5 grows and intersects itself in some, but not all directions of the torus to create a NS5/$\overline{\text{NS5}}$ pair that eventually decays by collision.

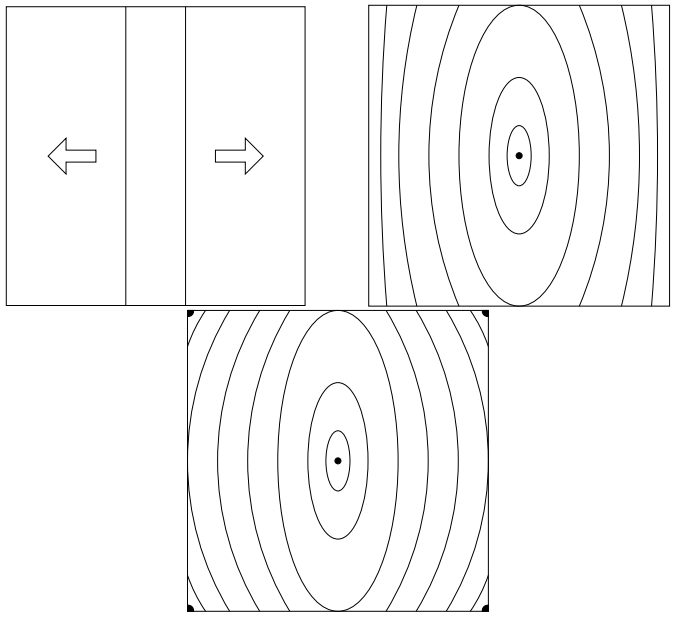


Figure 5.6: NS5 nucleation on tori. The tori are respresented as squares with the opposite sides identified. Left picture: Nucleation of an NS5/$\overline{\text{NS5}}$ pair that moves in opposite signs until they meet again. Middle picture: Nucleation of a spherical NS5 that transitions into an NS5/$\overline{\text{NS5}}$ pair or vice versa. Right picture: nucleation of a growing spherical NS5 that shrinks again as a spherical NS5 on a different part of the torus, by making a transition into an NS5/$\overline{\text{NS5}}$ pair.

2. The spherical NS5 grows and intersects itself to form an NS5/$\overline{\text{NS5}}$ pair that eventually reconnects again into a single spherical NS5-brane that shrinks to a single point at a different position in the torus than it started off.

We have depicted all the possible processes for a 2D torus in figure 5.6 .

The process in which a spherical NS5 breaks into an NS5/$\overline{\text{NS5}}$ pair wrapping non-contractible cycles, or the reversed process, is expected to happen through tachyon condensation. Some details of this can be found in [146, 79].

5.5.3 $\overline{D3}$ probes

For the sake of analogy with the work of Kachru, Pearson and Verlinde [143] we consider the situation in which a spherical NS5 nucleates and collapses as a

spherical NS5 in a different part of the torus. As explained above, this occurs via breaking the NS5 sphere into NS5/$\overline{\text{NS5}}$ pairs that later combine again into a single NS5 sphere.

Consider Cartesian coordinates on the straight torus with unit lengths. If we take the $\overline{\text{D3}}$ to polarise at the position $S = (0, 0, 0)$ into a round spherical NS5 then it will collapse again at position $N = (1/2, 1/2, 1/2)$. When the $\mathbb{T}^3$ is replaced by $\mathbb{R}^3$ the computation of the NS5 potential was performed in [127] and for a spherical NS5-brane that starts in the center of the $\mathbb{R}^3$ the polarisation potential reads:

$$V \sim \sqrt{\frac{r^4}{(2\pi)^2} + \left(n - \frac{M}{3} r^3\right)^2} + \left(n - \frac{M}{3} r^3\right) , \tag{5.53}$$

where r is the radial coordinate in the local patch with $S = (0, 0, 0)$ as the center of the radial coordinates. This potential has a local minimum at finite r and for n small enough we can make sure this minium occurs in a region of validity of the coordinate system. For large r it simply keeps on rising so for a noncompact A-cycle given by $\mathbb{R}^3$ decay via NS5 nucleation does not occur. To understand whether the potential goes down at some point on the torus it suffices to compute the potential at the point where one expects spherical collapse and verify whether the potential is indeed lower then at the starting point $S = (0, 0, 0)$. If this is the case, one expects the nonperturbative transition to occur, even though the full potential is not known. The point of spherical collapse in this example is engineered to be $N = (1/2, 1/2, 1/2)$, but the transition to the spherical collapse cannot be described in the same spherical coordinate system. We therefore consider the radial coordinates around the point N and denote the radial coordinate in that patch as $\tilde{r}$. To compute the potential at N we require the value of C_2 at that point which then fixes B_6 through (5.48). We already argued that in the gauge where $\int_S C_2 = 0$, then $\int_N C_2 = 2\pi M$. Straightforward calculation leads to the potential in the $\tilde{r}$-coordinates:

$$V \sim \sqrt{\frac{\tilde{r}^4}{(2\pi)^2} + \left(n - M + \frac{M}{3} \tilde{r}^3\right)^2} + \left(n - M + \frac{M}{3} \tilde{r}^3\right) . \tag{5.54}$$

We find that indeed the potential energy at N is lower than at S:

$$V(\tilde{r} = 0) = 0 \quad < \quad V(r = 0) = 2n \tag{5.55}$$

Hence we notice something very different from the noncompact A-cycle: the minimum of the brane polarisation at small r is only a local minimum since the potential at N is smaller then the potential at the minimum. At N the background is BPS again and the antibranes annihilated against the 3-form flux, in complete parallel with $\overline{\text{D3}}$-branes at the tip of the KS throat [143].

5.5.4 $\overline{\text{D4}}$ probes

Now we consider the T-dual process on a 2-torus. The blow up of D4-branes into NS5-branes has been considered earlier [27], but not in the background described here. As before there are multiple options. We need to compute C_1, which also fixes B_6 via (5.48), to understand this process. To describe the growth of a spherical NS5 we consider local radial coordinates on the torus such that, around N:

$$C_1 = \frac{\sqrt{2\pi}M}{2} r^2 \mathrm{d}\phi \,. \tag{5.56}$$

This is sufficient information to plug into the DBI and WZ action to get an expression very similar to what we found for the $\overline{\text{D3}}$ above. We hence find a local minimum for the spherical NS5 radius inside $\mathbb{T}^2$. Since the computation and expression for the spherical NS5 is in complete analogy with the $\overline{\text{D3}}$ example we do not bother the reader with the explicit expressions.

Alternatively one can consider an NS5/$\overline{\text{NS5}}$ pair wrapping an S^1 inside $\mathbb{T}^2$, move in opposite directions and annihilate to leave $M-n$ D4-branes at the other side. To find the potential for that process one uses Cartesian coordinates (x, y) and the following expression for C_1:

$$C_1 = \sqrt{2\pi} M x \mathrm{d}y \,, \tag{5.57}$$

The motion of the NS5-branes is from $x = 0$ to $x = 1/2$ where it meets the $\overline{\text{NS5}}$ that came from the other side. The potential for the process would therefore be a function of x. The way to compute the potential is identical to what we describe below for $\overline{\text{D5}}$-branes.

5.5.5 $\overline{\text{D5}}$ probes

anti D5 probes can only decay in the NSNS channel by transforming into an NS5/$\overline{\text{NS5}}$ pair since there is no option for spherical subcycles inside S^1. We already described above how the charge evolves from n to $n-M$ and in what follows we compute the difference in energies between the two configurations.

As before we take $C_0 = M\theta$ and the NS5/$\overline{\text{NS5}}$ pair at $\theta = 0$. The NS5's moves from 0 to opposite sides and meet at $\theta = 1/2$. The NS5's carry worldvolume flux $n/2$ and $-n/2$. The total energy can be found from summing the worldvolume actions and taking into account the mutual interaction term. This is technically different from spherical NS5-branes in which the energy that it costs to expand the NS5 is taken into account already by the DBI term. Instead the energy it costs to overcome the electric attraction between an NS5/$\overline{\text{NS5}}$ pair must be

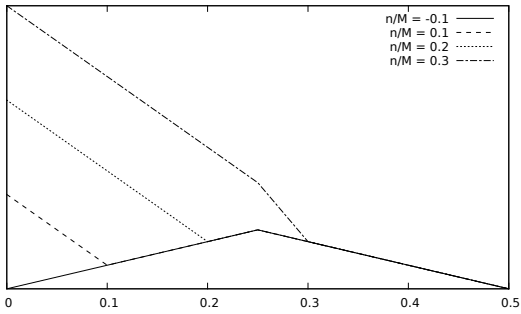


Figure 5.7: The brane-flux decay potential for $\overline{\text{D5}}$-brane plotted for various values of n/M. As expected for small enough n/M a metastable state appears.

added, just like in a Schwinger pair-creation computation. If we assume the length of the circle to be much larger than the length scale of the space threaded by H-flux then we can reasonably describe this interaction using a potential linear in the separation between the brane and antibrane:

$$V_{\text{interaction}} \sim -|\theta - \tfrac{1}{4}| \tag{5.58}$$

where we normalise the potential such that it has an unstable fixed point at $\theta = 1/4$.[5] The (normalised) energy at either $\theta = 0$ (N) or $\theta = 1/2$ (S) reads:

$$V \approx M g_s^{1/2} \left(\left| \frac{n}{M} - 2\theta \right| + \left(\frac{n}{M} - 2\theta \right) \right) , \tag{5.59}$$

where we threw away terms of order $(g_s M)^{-1}$. If we include the interaction term $-M g_s^{1/2}(|1/4 - \theta| - 1/4)$ we can plot the full potential:

$$V = M g_s^{1/2} \left(\left| \frac{n}{M} - 2\theta \right| + \left(\frac{n}{M} - 2\theta \right) - \left| \frac{1}{4} - \theta \right| + \frac{1}{4} \right) \tag{5.60}$$

over the entire circle and the result is shown in figure 6.1 for various values of n/M. As expected for small enough n/M a metastable state appears. At the endpoints of the circle the potential takes the value (for $p/M > 0$):

$$V(\theta = 0) = 2 g_s^{1/2} n \ , \qquad V(\theta = 1/2) = 0 \ . \tag{5.61}$$

We see that the potential at $\theta = 0$ gives twice the tension and zero at the other point which is expected since the fluxes and branes are mutually BPS.

[5] This linear behavior agrees with dimensional counting, but the worried reader might want to know that the form of this term does not affect the energy difference between the starting point and the end point since at both these points the binding energy of the NS5/$\overline{\text{NS5}}$ pair should be identical.

5.6 Discussion

We have explained how brane-flux annihilation is expected to proceed in flux compactifications. A useful lead is the RR tadpole condition (5.2) which implies that flux decay has to come together with brane decay. Perhaps confusingly, this implies that there are two contractible branes that mediate the decay: the Brown-Bunster bubble in spacetime and the polarised brane wrapping a contractible cycle in the internal manifold. We argue that this should be viewed as two distinct limits of the same process. In this way the polarised brane can be seen as the stringy resolution of the thin BB bubble as was also argued in [84].

Apart from explaining the general physical picture we have found a new family of local throat geometries in which antibranes can form metastable states in the probe approximation. These throat geometries have toroidal cycles, and to our knowledge, it was not yet explained how brane-flux decay proceeds in this case.

Since the discovery [31, 162, 111] that the backreaction of antibranes leads to singular H_3 and F_{6-p} fluxes (in addition to the normal F_{p+2} singularity) it has been questioned whether the backreaction invalidates the probe approximation and can cause perturbative instabilities [26, 52, 34, 48, 84, 81, 37], but see also [163] [127] for arguments against this interpretation. What obstructs a clear argument to settle the debate is a solution of a localised antibrane and an understanding of its decay channel in the sugra limit. For anti D6-branes however, the localised solutions are known [51] but the decay channel was not. In this chapter we speculate that the thick wall NSNS decay channel proceeds via KK5 nucleation. If correct, it should be possible to verify whether the backreaction indeed causes the background to decay perturbatively. This can be done consistently by considering a probe anti D6-brane in the background of $n-1$ backreacting anti D6, such that still $n \ll M$ required for metastability in the probe picture. This was later investigated in [82].

We can try to use the results of this chapter to explain the findings in [127], where it was claimed, contrary to [26, 48] that smooth finite temperature solutions can exist, which is an argument in favour of metastable states beyond the probe approximation. The crucial issue is that the A-cycle used in the smooth finite temperature solutions of [127] is noncompact. By increasing the size of the torus in the toroidal antibrane examples of section 5.5, one can make the classical barrier against brane-flux decay arbitrarily large (in the probe approximation). Hence in the limit $\mathbb{T}^{6-p} \to \mathbb{R}^{6-p}$ brane-flux decay becomes impossible. This is consistent with the findings of [127] that for noncompact A-cycles of the form $\mathbb{R}^{6-p}$ smooth finite temperature solutions exist. Since in that limit there is no brane-flux decay and the viewpoint of [52, 84] that

explains the absence of smooth finite temperature solutions does simply not apply (and in absence of a noncompact A-cycle neither does the nogo-theorem [48] for existence of smooth solutions apply).

Chapter 6

Smeared antibranes polarise in AdS

In the recent literature it has been questioned whether the local backreaction of antibranes in flux throats can induce a perturbative brane-flux decay. This was briefly reviewed in section 4.5 when discussing the interpretation of singular three-form fluxes in the vicinity of $\overline{\text{D3}}$. Most solutions in the literature have flat worldvolume geometries and noncompact transversal spaces. In this chapter we consider what happens when the worldvolume is AdS and the transversal space is compact. We show that in these circumstances polarisation into a D5 smoothens the flux singularity, which is an indication that brane-flux decay is prevented. This is consistent with the fact that the cosmological constant would be less negative after brane-flux decay. In the case of flat worldvolume our model allows for the possibility of perturbative brane-flux decay through the NS5 channel. Our results extend recent results [14, 140] on AdS_7 solutions from D6-branes to AdS_{p+1} solutions from Dp-branes. We show that supersymmetry of the AdS solutions depend on p nontrivially. This chapter is a reprint of [113].

6.1 Introduction

In recent years there has been an active study of supergravity solutions that feature Dp-branes locally surrounded by fluxes that induce a delocalised Dp charge density of the opposite sign to the brane charge. These different signs

can be seen in the Bianchi identity:

$$\mathrm{d}F_{8-p} = H \wedge F_{6-p} + Q\delta_{8-p}\,. \tag{6.1}$$

The solutions of interest are such that the orientation of the first form on the RHS of (6.1) is opposite to the second one. For this reason we name those branes "antibranes" where the "anti" refers to the charge being opposite to the charge density in the fluxes.

Solutions with this distinctive property can be categorised into two classes: 1) Non-supersymmetric solutions with flat Dp worldvolume and 2) solutions with a Dp worldvolume that is AdS and a transversal space that is potentially compact. The AdS solutions can be supersymmetric [14], but do not need to be [140].

Examples of the first kind are the non-compact geometries in which supersymmetry (SUSY) is broken by the brane, with the prime example being anti D3-branes in the Klebanov-Strassler throat [162, 31, 29] first studied in [143]. When combined with orientifolds and quantum corrections there is a belief that it can be made into a compactification geometry for which the SUSY-breaking branes are used to uplift AdS vacua to dS vacua [142] or as a base for brane inflation [141].

Examples of the second kind are warped AdS_7 vacua in massive IIA sugra built from spacetime-filling D6-branes and a transversal space that is conformal to an S^3. These solutions were first uncovered in [50, 51], but their supersymmetry together with more details was properly understood in [14]. Furthermore, these solutions provide a concrete gravity dual to six-dimensional $(1,0)$ SCFT's [109].

Both classes of solutions feature a peculiar property that has been the origin of an ongoing debate: due to the differences in charges, the fluxes are electromagnetically (and gravitationally) attracted to the branes in such a way that a singular flux cloud is formed around the branes [91, 50, 52, 163]. This was first uncovered in [162, 31] and by now a vast literature on this exists, with a formal proof for this unavoidable singularity presented in [111, 48][1]. Since fluxes can materialise into actual branes [143], one is tempted to conclude that a singular, or large, flux pile-up leads to a quick annihilation of the flux with the antibrane, possibly making the solution perturbatively unstable [52, 84] (see also [163] for recent comments on this). This picture is strengthened by the absence of regular solutions at finite temperature with flat worldvolume [26, 25, 48].

[1] Around orientifolds this phenomenon does not occur since they repel the flux gravitationally as much as they attract it electromagnetically [49]. This nicely fits together with orientifold compactifications where orientifolds in fluxes of opposite charge are the basic ingredients [115].

Recently this interpretation of a perturbative decay has been challenged by some good arguments [163, 127]. First of all it has been claimed that the singularity will get resolved by stringy corrections in such a way that the resulting flux clumping is small enough for at least a single antibrane to be meta-stable [163]. This can very well be correct and is currently under investigation, but it would clearly be more gratifying if the singularity can be resolved at supergravity length scales, such that the original arguments for antibrane meta-stability [143] are applicable. Interestingly a resolution at sufficiently large length scales has been argued by Hartnett in [127]. The basic claim of [127] is that the nogo-theorem for finite temperature resolutions of [48] can be circumvented. Secondly a simplified trick was found to understand the local geometry of localised antibranes and it seems to indicate that a Polchinski-Strassler (PS) type of singularity resolution [177] will take place that dilutes the flux clumping strongly enough to prevent direct brane-flux decay. This is in contrast with earlier investigations of a possible PS resolution that turned out not to work [36, 33], and according to [127] the reason for this is the use of smeared antibranes[2] instead of localised ones. It would be interesting to verify this explicitly. In this chapter we do consider smeared antibranes but we demonstrate that, when they live in AdS space, their singularities do get resolved as opposed to flat space. In that sense there is no disagreement in the literature when it comes to flat space smeared anti Dp-branes or localised anti D6-branes: the absence of regular finite T solutions seems without doubt and perfectly in line with the 'no PS-resolution' results. Hence perturbative brane-flux decay might very well be what is going on in the flat space solutions, but is inconsistent with the AdS solutions. Indeed, upon brane-flux decay these would have a less negative cc and hence more energy, so one expects those solutions to be stable.

In the case of the anti D6-brane this has been understood by now. For the supersymmetric AdS_7 solutions it was found that the (anti)D6-branes polarise into spherical D8-branes [14, 140], which resolves the flux singularity since a charged sphere attracts the flux in a more delocalised fashion. This polarisation does not occur for the flat SUSY-breaking solutions [36] and we expect the latter solutions to decay perturbatively before the flux reaches the singular values. In this chapter we extend this picture to the other branes and uncover that a similar story holds (up to certain subtleties): the compact AdS_{p+1} solutions are such that the Dp-branes polarise into spherical D$(p+2)$-branes and brane-flux decay does not occur. For D3-branes brane-flux decay can be studied very explicitly since in that case it proceeds through brane polarisation into an NS5-brane [143] in a direction orthogonal to the D5 polarisation. For the other branes it is unclear what the explicit brane-flux decay mechanism is, most likely

[2]For the anti D6 solutions [51] no smearing was used and the absence of a PS resolution is not questioned in this particular case.

it is related to a T-dual version of spherical NS5-branes[3].

The rest of this chapter is organised as follows. In section 6.2 we discuss smeared anti D3 solutions and the corresponding AdS_4 vacua and in section 6.3 we verify whether polarising into spherical D5-branes resolves the singularity and whether brane-flux decay can be prevented. In section 6.4 we generalise the discussion to Dp-branes smeared over $(6-p)$ directions and we conclude in section 6.5. In appendix C we have added a discussion of the constraints that SUSY puts on the AdS solutions in this chapter.

6.2 Anti D3 solutions

In this section we describe the compact AdS_4 solutions build from anti D3-branes whose RR tadpole is canceled by 3-form fluxes and non-compact anti D3 solutions with flat worldvolumes that do not require an RR tadpole cancellation. The existence of the AdS_4 solutions found here were established in the limit of smeared antibranes in [188] (and generalised to other dimensions in [49]). We have not yet made an attempt to construct the solutions with fully localised branes but we expect them to be contained in the analysis of [92]. The reader interested in checking the calculations will find use of Appendix A.3 that fixes notation and conventions.

6.2.1 Ansatz

We consider the following Ansatz for the metric:

$$\mathrm{d}s^2 = \mathrm{e}^{2A}\mathrm{d}s^2_{\mathrm{AdS}_4} + \mathrm{e}^{2B}\mathrm{d}s^2_{S^3} + \mathrm{e}^{2C}\left(\mathrm{d}\rho^2 + \mathrm{e}^{2D}\mathrm{d}\Omega^2\right), \tag{6.2}$$

where $\mathrm{d}s^2_{\mathrm{AdS}_4}$, $\mathrm{d}s^2_{S^3}$ and $\mathrm{d}\Omega^2$ are the metrics for AdS_4, the three-sphere S^3 and S^2, the two-sphere, respectively. These metrics are chosen such that the corresponding unwarped Ricci tensors take the canonical form:

$$\hat{R}_{\mu\nu} = \Lambda\hat{g}_{\mu\nu}\ , \quad \hat{R}_{mn} = 2\hat{g}_{mn}\ , \quad \hat{R}_{ij} = \hat{g}_{ij}\ . \tag{6.3}$$

Here the hatted variables denote the unwarped quantities:

$$\mathrm{d}s^2_{\mathrm{AdS}_4} = \hat{g}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu\ , \qquad \mathrm{d}s^2_{S^3} = \hat{g}_{mn}\mathrm{d}x^m\mathrm{d}x^n\ , \quad \mathrm{d}\Omega^2 = \hat{g}_{ij}\mathrm{d}x^i\mathrm{d}x^j\ , \tag{6.4}$$

these equations also serve to fix the index conventions that we use throughout the chapter. The warp factors A, B, C and D are only functions of the coordinate

[3]This was later understood [114] as was explained in the previous chapter (ch. 5).

ρ and so the Einstein equations reduce to ordinary differential equations for the warp factors. We parametrize our nonzero fluxes as follows:

$$\begin{aligned} F_3 &= M \mathrm{vol}_3 \ , \\ H &= -\lambda \mathrm{e}^{\phi} \star_6 F_3 \ , \\ F_5 &= (1 + \star_{10}) \star_6 \mathrm{e}^{-4A} \mathrm{d}\alpha \ . \end{aligned} \tag{6.5}$$

Like the warp factors, λ, α and ϕ are functions of the ρ coordinate, whereas M is a constant topological flux quantum. The volume form vol_3 is the volume form on the unit three-sphere. The equations of motion turn into a system of coupled ordinary differential equations (ODE's) and are written out explicitly in appendix C.1. From the equation of motion for H it immediately follows that:

$$\alpha = \lambda \mathrm{e}^{4A} \ . \tag{6.6}$$

In the case where λ is ± 1 the 3-form fluxes combine into ISD or anti-ISD flux as in [115] but this is not true in general due to the non-trivial 5-form field strength. The Bianchi identity for F_5 takes the form:

$$\mathrm{d}F_5 - H \wedge F_3 = N_{\mathrm{D3}} \mu_3 \delta_6 \ , \tag{6.7}$$

where δ_6 is a 6-form with delta function support on the worldvolume of the branes. Notice that we have added an integer N_{D3} to allow for a stack of D3-branes. Integrating the Bianchi identity over the internal space, we obtain the tadpole cancellation condition:

$$\int F_3 \wedge H = Q \ , \tag{6.8}$$

where Q denotes the total brane charge. It is clear that even though we have included the effect of D3-branes in the Bianchi identity (6.7), the form of the metric (7.22) does not allow for fully localised branes. Indeed the branes are smeared over the three-sphere, but are localised at $\rho = 0$.

6.2.2 Fully smeared 3-branes

A fully smeared limit of the solution can be obtained by replacing the delta function in equation (6.7) with its integrated average:

$$\delta_6 \to \mathrm{vol}_6 \ , \tag{6.9}$$

where the internal volume form vol_6 is unwarped and normalized to 1. This solution has previously been described in [188] (see also [49]). The smearing

has the effect of restoring symmetry in the internal manifold so it reduces to an exact product space of two spheres. This means that $e^{2D} = \sin^2\rho$ and all other warp factors and functions are constant. Therefore we find that F_5 vanishes and the function λ introduced in equation (6.5) takes a constant value $\lambda = 1$. This corresponds precisely to the combined 3-form flux:

$$G_3 = F_3 - ie^{-\phi}H \ , \tag{6.10}$$

being imaginary self-dual, i.e. $\star_6 G_3 = -iG_3$. The size of the 3-form flux is fixed by (6.7) to be:

$$Q = g_s|F_3|^2 \ . \tag{6.11}$$

The charge conjugated solution would have $Q = -g_s|F_3|^2$ and $\lambda = -1$ which corresponds to anti-ISD fluxes. The expression for the *warped* cosmological constant is:

$$e^{-2A}\Lambda = -\frac{1}{4}Q \ , \tag{6.12}$$

which shows that if the brane charge Q would decrease, the total vacuum energy would increase. So already here we notice that a decay process which eliminates Q against M such that the tadpole cancelation condition (6.8) is still satisfied cannot occur.

6.2.3 Compact AdS solutions

The fully smeared solutions make it clear that compactness is only possible when the worldvolume of the brane is AdS. Flat solutions are necessarily noncompact. It is well-known that for noncompact solutions there is no relation between the cc of the base space and the fluxes. In compact solutions the size of the cc is determined by the energy, that in turn is determined by the fluxes and the branes. In this subsection we look at compact AdS solutions.

We derive the relation between the cc and the fluxes using the results of [111]. This computation requires the gauge potential C_4. We can choose a gauge for C_4 for which the external part, C_4^{ext}, vanishes at the position of the branes, $\rho = 0$:

$$C_4 = B \wedge C_2 - (\alpha - \alpha_0)\text{vol}_4 \ . \tag{6.13}$$

Recall that volume forms such as vol_4 are always defined without warp factors. The equation for the cosmological constant of the external spacetime can be expressed as follows:

$$\Lambda = \frac{1}{4V_6}\left[N_{\text{D3}}S_{\text{loc}} + \frac{1}{V_4}\int H \wedge \left(e^{-\phi} \star_{10} H + F_3 \wedge C_4^{\text{ext}}\right)\right] \ , \tag{6.14}$$

where V_4 and V_6 are "volumes" of the external and internal spaces, defined as follows:

$$V_4 = \int \mathrm{vol}_4, \qquad V_6 = \int \star_6 \, \mathrm{e}^{2A} \ . \tag{6.15}$$

The gauge choice for C_4 is such that, on-shell, the first term in the bracket drops out and we are left only with the flux integral in the second term. This reduces to:

$$\Lambda = \frac{\alpha_0}{4V_6} \int H \wedge F_3 \ = -\frac{Q\alpha_0}{4V_6} \ , \tag{6.16}$$

where we made use of (6.8). Crucially this shows that the function α takes a nonvanishing value at the position of a D3-brane, this means that λ which relates F_3 and H in (6.5) has the asymptotic behaviour close to the brane:

$$\lambda \xrightarrow[\rho\to 0]{} -\frac{4V_6\Lambda}{Q} \, \mathrm{e}^{-4A} \ . \tag{6.17}$$

This obviously blows up since the warp factor e^{2A} vanishes in the vicinity of the brane. Combined with the fact that F_3 is constant 3-form flux, this leads to the, by now well-known fact that the energy density of H has a singular behaviour close to the brane.

In deriving (6.16) we have assumed that the solution is compact, although no proof for this exists, apart from the observation that the fully smeared solution is compact. To show that compact solutions can exist one should numerically evaluate the coupled ODE's from appendix C.1, which we have not done and leave for future investigation. We verify in appendix C.2, that unlike the AdS_7 solution from anti D6-branes, the compact AdS_4 solution from partially smeared anti D3-branes cannot be supersymmetric.

6.2.4 noncompact flat solutions

Compactness enforces the solutions to be AdS but once we give up compactness, we can consider flat solutions. Flat solutions are interesting from the point of view of antibrane SUSY breaking [159, 143, 142]. Antibrane SUSY breaking, at least for anti D3-branes, can be studied explicitly in the Klebanov-Strassler (KS) throat [148]. The KS throat is supersymmetric and regular at the tip and therefore it makes a perfect background to add the singular SUSY breaking source at the tip. The full solution is out of reach and most likely will remain out of reach, but when the anti D3-branes are smeared over the tip, the equations of motion, that describe the backreaction become ODE's and approximate solutions have been found [31, 28]. Close to the tip the details of the singular flux clumping are nicely captured by a much simpler background [25], which

is the T-dual to the flat anti D6 solution [51]. The only difference from our previous Ansatz are the curvature of the metric factors:

$$\mathrm{d}s^2 = \mathrm{e}^{2A}\mathrm{d}s^2_{\mathrm{Mink}_4} + \mathrm{e}^{2B}\mathrm{d}s^2_{\mathbb{T}^3/S^3} + \mathrm{e}^{2C}\left(\mathrm{d}\rho^2 + \mathrm{e}^{2D}\mathrm{d}\Omega^2\right), \tag{6.18}$$

where $\mathrm{d}s^2_{\mathrm{Mink}_4}$, $\mathrm{d}s^2_{\mathbb{T}^3/S^3}$ and $\mathrm{d}\Omega^2$ are the metrics for Mink_4, the three-torus $\mathbb{T}^3$ or the 3-sphere S^3 and two-sphere S^2 respectively. The solutions with the torus factor can be obtained from T-duality of the anti D6 solution. When it comes to the physics of flux clumping there is no real difference between the solutions with the torus factor and the solutions with the S^3 factor. The solutions with the S^3 factor are however more insightful since there is a more explicit picture for brane-flux annihilation in that case [143].

Also here it can easily be demonstrated that λ blows up and hence there is singular flux clumping [25]. When it comes to the local physics associated to the flux clumping, this model captures exactly the same physics as the model with anti D3's smeared over the tip of the S^3 in KS; for instance it can be shown that the polarisation potential for spherical D5-branes in both cases is almost identical [33]. In the next section we compute the polarisation potential both for the polarisation into D5 and NS5-branes.

Due to the noncompactness, we can decouple the size of the cc from the brane charges and the flux quanta, we are free to choose a value for the cc. It is not necessary to take it to be exactly zero. The main result of this chapter is that when the cc is set by the brane charges and fluxes, polarisation will occur. When the cc is parametrically different, which can be done for non compactifications, or in KKLT-type scenarios[4] [142], then it does not occur and the flux singularity remains unresolved.

6.3 Spherical 5-branes

Whenever one considers p-branes in background with F_{6-p}-flux there is the possibility that the p-branes polarise into a spherical $(p+2)$-brane carrying $(p+2)$-brane dipole charge and the original p-brane monopole charges induced by gauge fluxes on the worldvolume [166]. This possibility is enhanced by the flux singularity discussed in the previous section. In the case at hand one expects two polarisation channels: a spherical D5 wrapping a contractible S^2 inside the M_3 that is threaded by H and a spherical NS5 wrapping a contractible S^2 inside the S^3 filled with F_3 (denoted S^3_F). If polarisation occurs, then the

[4] In KKLT type scenarios the cc can be decoupled from the KK scale due to orientifolds and quantum corrections.

flux singularity is *regularized* in one of two ways: either the spherical branes sit at a finite distance from the original D-brane charge and the flux clumping parameter λ takes a finite value throughout the solution, this is similar to what happened in [177], or the anti D3-branes polarise to an NS5-brane wrapping an S^2 inside the S^3 with a runaway potential towards the other pole of the S^3. This latter possibility results in annihilation of flux against the anti D3-brane charge as we will explain in more detail below.

In this chapter we follow a procedure for computing the polarisation potentials that has become standard, but is not without danger of not being fully waterproof. The procedure aims to take into account the backreaction of the branes in computing the polarisation potential because some polarisation phenomena are invisible in the probe limit. Since the polarisation potential necessarily relies on probe brane actions the actual computation is in between probe and full backreaction. The idea is the following: If one wants to know whether N anti D3-branes polarise into D5-branes, one can work in the limit where $N-p$ antibranes backreact and p are treated as probe. For large N and with $p \ll N$ this should be a valid assumption. Hence one can use the probe action for p D3-branes and study their polarisation process. A caveat could be that the probe calculation differs if instead of backreacting $N-p$ antibranes one uses the backreaction of spherical D5-branes carrying $N-p$ anti D3 charges. If for instance the latter computation reveals a meta-stable position for polarisation of the p probes one might take it as a sign that the full stack of D3-branes polarises into a meta-stable state. To our knowledge it has so far been consistent to work in the situation in which one regards the backreaction of the $N-p$ D3-branes. A rationale for this could be that if the p D3 probes do not polarise into a meta-stable position when the antibranes backreact, then neither will they if one considers the backreaction of spherical D5 probes. The reason is that one could think of the polarisation to proceed stepwise. We nonetheless emphasise that this should be better understood.

Finally we mention that for a compact solution, M_3 takes the form of a three-sphere which we denote by S^3_H. S-duality interchanges the roles of D5 and NS5 as well as the fluxes F_3 and H. So performing an S-duality effectively smears the D3-branes over S^3_H and localises them inside S^3_F. We therefore expect the fully localised solution (that is localised on $S^3_H \times S^3_F$) to polarise into a web of (p,q) 5-branes as in [92]. In what follows we discuss both channels.

6.3.1 D5 polarisation

There are generically two ways of showing that a Dp-brane polarizes to a D$(p+2)$-brane; either by considering the non-abelian Dp-brane action as in

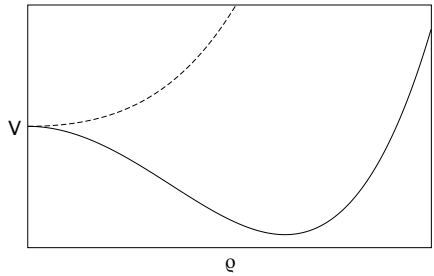


Figure 6.1: The potential V (6.20) for probe D5-branes in the background of D3-branes. The dashed line shows the polarisation potential when the external space is flat while the solid line displays the potential for AdS space, for which the probes are tachyonic near $\bar{\rho} = 0$. A stable state exists at a finite distance away from the tip.

[166], or by considering the probe action of a D$(p+2)$-brane in the Dp-brane background [177]. In this chapter we take the latter approach. The required terms of the D5-brane action in Einstein frame are:

$$S_{\rm D5} = \mu_5 \int \left\{ -\mathrm{e}^{-\phi} \sqrt{-\det(\mathrm{e}^{\phi/2} G - \mathcal{F})} - C_6 + \mathcal{F} \wedge C_4 \right\} , \tag{6.19}$$

where $\mathcal{F} = \mathcal{B} - \mathcal{F}$. We will take $F = n\pi \mathrm{vol}_2$ and expand the D5-brane action for large n for which polarisation is preferred and then look at the behaviour close to $\rho = 0$. Instead of working through the computation here, we state the result and in section 6.4.3 we present a more general computation for Dp-branes polarising into $D(p+2)$-branes, which includes this case by putting $p = 3$. For AdS_4 external space we find:

$$V \propto \left(4\Lambda + \frac{1}{2} k_0^2 \right) \bar{\rho}^2 - 2 k_0 \bar{\rho}^3 + 3 \bar{\rho}^4 , \tag{6.20}$$

where $\bar{\rho}$ is a dimensionless distance from the D3-branes and is defined in section 6.4.3. The cosmological constant Λ is normalized to -3 for AdS external space and vanishes of course if the external space is flat. The numerical value k_0 is directly related to Λ but with a proportionality factor that depends on the details of the solution. We can however estimate the value of k_0 by smearing for which we find $k_0 = \sqrt{6}$ (cf. section 6.4.3) and the potential takes the form in figure 6.1.

What we find is that the D5 polarisation occurs for the AdS solutions, but that it cannot happen for the noncompact Minkowski solutions. This is in perfect agreement with what has been found for the D6 solutions of [50, 14] as shown in [36, 140] (see also [14, 109]).

The polarisation into (meta-)stable spherical D5-branes smoothens out the singular pile-up of the three-form fluxes. In other words, λ remains finite throughout the solution. The only singularities are the expected ones in the metric and form fields that comes from the localised charge and tension of (spherical) branes. This smoothening can be deduced quite easily by repeating the computation of the size of λ at the boundary of the spherical D5-brane. One simply needs to use the near-brane expansion of a 5-brane that is smeared over 3-directions. This computation is completely analogues to what has been done in [36] (equations 2.16 and 2.17).

6.3.2 NS5 polarisation and flux decay

Our AdS_4 solution is constructed by smearing the D3-branes over S^3_F which makes it unclear how brane polarisation can proceed inside S^3_F. For the polarisation into D5-branes to occur it was necessary to have a solution localised in M_3 because the localised solution backreacts in such a way that the profiles of the background supergravity fields induce a minimum in the D5 polarisation potential at a finite size for the spherical 5-brane. But the "dual" channel does not need localised branes, one just has to keep in mind that the brane polarisation computation is a *probe* computation and probes are localised. It turns out that the probe computation parallels those done in [143, 52, 84]).

Whereas the physics of the D5 polarisation channel is to resolve the flux clumping, the physics of the NS5 channel is brane-flux decay [143]. If the D3-branes polarise into NS5-branes that either tunnel or move perturbatively to the North Pole of the S^3_F one can verify that its monopole charge has shifted to $M-p$ instead of $-p$ D3 charges. The interpretation of this is that M units of 3-form flux materialised into M physical D3-branes that consequently annihilate with the p anti D3-branes to leave $M-p$ D3-branes in the process.

The NS5 potential is calculated by considering the worldvolume action of an NS5-brane (treated as a probe) in the background of D3-branes and fluxes. The main difference to the computation done in [143] has to do with the Wess-Zumino (WZ) action for the NS5-brane. The complete NS5-brane action for $C_0 = 0$ takes the form[5]:

$$-\mu_{\rm NS5}\int \mathrm{e}^{-2\phi}\sqrt{-\det\left(\mathrm{e}^{\phi/2}G-\mathrm{e}^{\phi}\mathcal{F}\right)}+\mu_{\rm NS5}\int\left(B_6+\mathcal{F}\wedge C_4\right)\ , \qquad (6.21)$$

The world volume field strength F enters through the combination $\mathcal{F}=F-C_2$. To evaluate the WZ action we need to find B_6. We have parametrized H as

[5]The sign if the WZ action is fixed by demanding that a spherical NS5-brane induces a D3 brance charge, i.e. that the sign of the $\int F\wedge C_4$ matches the sign of the WZ action for the D3-branes above.

$-\lambda e^{\phi} \star_6 F_3$ and the dual flux is defined as $H_7 = e^{-\phi} \star H$. The gauge potential for H_7 can be defined through the Bianchi identity $dH_7 = -F_5 \wedge F_3$. Evaluating the right hand side for our Ansatz we determine:

$$H_7 = dB_6 - C_4 \wedge F_3 \ . \tag{6.22}$$

From this we find B_6 by writing:

$$dB_6 = e^{-\phi} \star_{10} H + C_4 \wedge F_3 = (\lambda e^{4A} - \alpha + \alpha_0) \text{vol}_4 \wedge F_3 = \alpha_0 \text{vol}_4 \wedge F_3 \,, \tag{6.23}$$

such that:

$$B_6 = \alpha_0 \text{vol}_4 \wedge C_2 \ , \tag{6.24}$$

which implies that the WZ Lagrangian takes the form:

$$B_6 + \mathcal{F} \wedge C_4 = (\alpha C_2 - (\alpha - \alpha_0) F) \wedge \text{vol}_4 \ . \tag{6.25}$$

We also need an expression for C_2 that correctly reproduces the constant F_3-flux via $F_3 = dC_2$:

$$C_2 = \frac{M}{2} \left(\psi - \frac{1}{2} \sin(2\psi) \right) \text{vol}_2 \ , \tag{6.26}$$

where ψ is used as the third Euler angle on the smeared three-sphere. Remember that the D3-branes are localised on M_3 at $\rho = 0$ (cf. equation (7.22)). The polarisation potential depends on ψ but takes a different form depending on the position on M_3. We will denote the potential by $V_\rho(\psi)$. Finally we let $F_2 = \pi p \ \text{vol}_2$ where p sets the D3-brane charge of the probe. The full polarisation potential is obtained by dividing the NS5 action by $(-\mu_{\text{NS5}} M)$ and relevant volume factors, and the result is:

$$\begin{aligned} V_\rho(\psi) \quad &= \quad e^{4A} \sqrt{\frac{1}{M^2} e^{4B-\phi} \sin^4 \psi + \frac{1}{4} \left(2\pi \frac{p}{M} - \psi + \frac{1}{2} \sin(2\psi) \right)^2} \\ &\quad - \frac{\alpha}{2} \left(\psi - \frac{1}{2} \sin(2\psi) \right) - \pi(\alpha - \alpha_0) \frac{p}{M} . \end{aligned} \tag{6.27}$$

This potential is valid for either the AdS or flat D3-brane solutions discussed above. The main difference when analysing the potential lies in the fact that the D3-branes in AdS polarize immediately to D5-branes.

Let us discuss the potential in the flat case for which we have shown that the D3-branes do *not* polarise into D5-branes. Even though e^{4A} vanishes as $\rho \to 0$, α stays constant. This means that the first term in (6.27) vanishes as $\rho \to 0$ but the second term does not. The potential therefore reduces to:

$$V_{\rho=0}(\psi) = -\frac{\alpha_0}{2} \left(\psi - \frac{1}{2} \sin(2\psi) \right) \ . \tag{6.28}$$

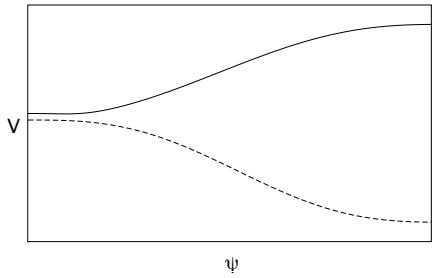


Figure 6.2: The polarisation potential for spherical NS5-branes. The dashed line is a plot of the potential (6.27) for flat external space and at $\rho = 0$. The solid line is evaluated for AdS external space with $\lambda_{\max} = 1/2$.

The number α_0 is finite and positive, just as for the compact AdS solution. This can be understood from studying the F_5 Bianchi identity which leads to a strong constraint on the form of α when combined with D3-brane boundary conditions and the asymptotic behaviour of the fields far away from the D3s. The analysis is completely analogous to the one done in [51] in the case of D6-branes. Hence the NS5 potential for flat branes indicate a perturbative brane-flux decay as shown in figure 6.2. Note that the potential (6.28) is completely independent of the ratio p/M, this shows that no matter how small the $\overline{\text{D3}}$ charge is carried by the probe, the NS5-brane will always be pushed away by the divergent H-flux at the tip $\rho \to 0$.

For an AdS_4 external space the situation is different, the polarisation of the D3-branes to D5's regularizes the singularity in λ such that the both terms in equation (6.27) play a role. Brane-flux decay depends on the relative size of the two terms in the potential. The first term effectively pulls the NS5 probe towards the D5-branes while the second term pushes the probe away. We have seen that for the fully smeared solution brane-flux decay cannot occur for a very simple reason; the energy of the vacuum would increase. We expect the same result to hold in the partially localised case (and in the fully localised case). In order to confirm this we would have to scan the potential V_ρ for all values of ρ and show that the gravitational pull of the first term outweighs the electromagnetic push of the second one. It would then be enough to find an upper bound on the function λ and show that it is less than the smeared value $\lambda = 1$. For the AdS vacua, we have plotted the expected qualitative behavior for the NS5 potential in figure 6.2.

6.4 Anti Dp solutions

The D3-brane solutions discussed so far belong to a class of Dp-brane solutions studied first in [49], where the smeared limit and flat limit were explored. Here we study the setup in more generality.

The metric takes the form:

$$\mathrm{d}s^2 = \mathrm{e}^{2A}\mathrm{d}s^2_{\mathrm{AdS}_{p+1}} + \mathrm{e}^{2B}\mathrm{d}s^2_{S^{6-p}} + \mathrm{e}^{2C}\mathrm{d}s^2_{M_3}\,, \quad \mathrm{d}s^2_{M_3} = \mathrm{d}\rho^2 + \mathrm{e}^{2D}\mathrm{d}\Omega^2\ , \tag{6.29}$$

where $\mathrm{d}s^2_{\mathrm{AdS}_{p+1}}$, $\mathrm{d}s^2_{S^{6-p}}$ and $\mathrm{d}\Omega^2$ are the metrics for AdS_{p+1}, the $(6-p)$-sphere S^{6-p} and the two-sphere S^2 respectively. Once again we choose the metrics such that the corresponding unwarped Ricci tensors take the canonical form:

$$\hat{R}_{\mu\nu} = \Lambda\ \hat{g}_{\mu\nu}\ , \quad \hat{R}_{mn} = (5-p)\ \hat{g}_{mn}\ , \quad \hat{R}_{ij} = \hat{g}_{ij}\ . \tag{6.30}$$

The hatted variables denote the unwarped quantities, and:

$$\mathrm{d}s^2_{\mathrm{AdS}_{p+1}} = \hat{g}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu\ , \quad \mathrm{d}s^2_{S^{6-p}} = \hat{g}_{mn}\mathrm{d}x^m\mathrm{d}x^n\ , \quad \mathrm{d}\Omega^2 = \hat{g}_{ij}\mathrm{d}x^i\mathrm{d}x^j\ . \tag{6.31}$$

The warp factors A, B, C and D are only functions of the coordinate ρ and so the Einstein equations reduce to ordinary differential equations for the warp factors. We parametrize our fluxes as follows:

$$\begin{aligned} H &= -\lambda \mathrm{e}^{\frac{p+1}{4}\phi} \star_{9-p} F_{6-p}\ , \\ F_{6-p} &= M \mathrm{vol}_{S^{6-p}}\ , \\ F^{\perp}_{8-p} &= \mathrm{e}^{-(p+1)A-\frac{p-3}{2}\phi} \star_{9-p} \mathrm{d}\alpha\ . \end{aligned} \tag{6.32}$$

The functions λ and α as well as the dilaton are assumed, like the warp factors, to depend only on the ρ coordinate. The volume form vol_{6-p} is the unwarped volume form on the $(6-p)$-sphere. The superscript $\perp$ on F_{8-p} indicates the fact that we are specifying the components transverse to the brane worldvolume. For $p=3$ the components of F_5 along the worldvolume are also nonzero by the selfduality condition. From the equation of motion for H we determine:

$$\alpha = -(-1)^p\lambda \mathrm{e}^{(p+1)A+\frac{p-3}{4}\phi} = -(-1)^p\lambda\beta\ , \tag{6.33}$$

where the function β is defined by the second equality. The only unsatisfied form field equation is the F_{8-p} Bianchi identity, which takes the form:

$$\mathrm{d}F_{8-p} - H\wedge F_{6-p} = 2\tilde{\kappa}^2_{10} N_{\mathrm{D}p}\mu_p\delta_{9-p}\ . \tag{6.34}$$

6.4.1 The AdS curvature

Once again we can relate the zero point value of the function α to the AdS curvature via the relation given in [111]. For the Ansatz specified above we find:

$$\frac{p-1}{2}\Lambda = \frac{1}{4V_{p+1}}\left[N_{\mathrm{D}p}S_{\mathrm{loc}} + \frac{1}{V_{9-p}}\int H\wedge\left(\mathrm{e}^{-\phi}\star_{10}H - \sigma(F_{6-p})\wedge C_{p+1}\right)\right] , \tag{6.35}$$

from which we can get rid of the first term by a gauge choice for C_{p+1} along the worldvolume:

$$C^{\parallel}_{p+1} = -(\alpha-\alpha_0)\sigma(\mathrm{vol}_{p+1}) , \tag{6.36}$$

where α_0 is the value of α at $\rho = 0$. Putting this together we get:

$$\frac{p-1}{2}\Lambda = \frac{\alpha_0}{4V_{9-p}}\int H\wedge F_{6-p} = -\frac{\alpha_0 Q}{4V_{9-p}} , \tag{6.37}$$

where we used equation (6.34) to evaluate the integral and:

$$V_{9-p} = \int \star_{9-p}\,\mathrm{e}^{(p-1)A} . \tag{6.38}$$

Since the warped volume V_{9-p} is not known a priori, the equation (6.37) does not fix the value of α_0. However, we now know that for non-vanishing cosmological constant, the value of α_0 must be a strictly positive number. This fact enables us to conclude that a singularity is developed in the energy density of H just as for the special case $p = 3$ discussed above. Since the argument is completely analogous we do not repeat it here.

Concerning supersymmetry we explain in Appendix C.2 that compact AdS_p solutions cannot be supersymmetric for $p = 3, 4, 5$, when an anti Dp singularity is assumed at $\rho = 0$.

6.4.2 Flat solutions

As with the anti D3-branes, we can also investigate flat solutions, or solutions with a nonzero cc, but whose value is not fixed by compactness and hence decoupled from the scale set by the brane charges and the fluxes. The metric Ansatz is simply the generalisation of the Ansatz used for anti D3-branes:

$$\mathrm{d}s^2 = \mathrm{e}^{2A}\mathrm{d}s^2_{\mathrm{Mink}_{p+1}} + \mathrm{e}^{2B}\mathrm{d}s^2_{\mathbb{T}^{6-p}/S^{6-p}} + \mathrm{e}^{2C}\mathrm{d}s^2_{M_3}, \ . \tag{6.39}$$

These solutions describe anti Dp-branes smeared over the $\mathbb{T}^{6-p}/S^{6-p}$. The solutions with the torus factor can be obtained from T-duality of the anti D6 solution [51].

6.4.3 Brane polarisation

We now compute the potential for a probe D$(p+2)$-brane in the background of $N_{\mathrm{D}p}$ Dp-branes. This is a standard computation that we repeat for completeness and we find agreement with the results of [36, 140] for $p=6$ and [33] for $p=3$.

The probe action in this case is:

$$S_{\mathrm{D}(p+2)} = -\mu_{p+2}\int\left\{\mathrm{e}^{-\phi}\sqrt{-\det(\mathrm{e}^{\phi/2}G-\mathcal{F})}-(-1)^p\sigma(C_{p+3}-\mathcal{F}\wedge C_{p+1})\right\}\,, \tag{6.40}$$

where as before $\mathcal{F}=B-F$ and F is the world volume field strength. We take $F=\pi n\ \mathrm{vol}_2$ and expand the action for large n to obtain:

$$V\propto(\pi n-b)L_{\mathrm{D}p}+(-1)^p\gamma+\frac{\beta\mathrm{e}^{\phi+4C+4D}}{2(\pi n-b)}\,, \tag{6.41}$$

where:

$$B=b(\rho)\ \mathrm{vol}_2,\quad C_{p+3}=\gamma(\rho)\ \sigma(\mathrm{vol}_{p+1})\wedge\mathrm{vol}_2\,, \tag{6.42}$$

and:

$$L_{\mathrm{D}p}(\rho)=\beta-(-1)^p(\alpha-\alpha_0)=\beta(1+\lambda)+(-1)^p\alpha_0\,. \tag{6.43}$$

The functions $b(\rho)$ and $\gamma(\rho)$ are determined by employing the definition of H and F_{p+4} in terms of their potentials and comparing to the anzats. The result is:

$$b'(\rho) = M\alpha\mathrm{e}^{\phi-(p+1)A-(6-p)B+3C+2D}\,, \tag{6.44}$$

$$\gamma'(\rho) = \left(\beta^2-\alpha(\alpha-\alpha_0)\right)M\mathrm{e}^{\phi-(p+1)A-(6-p)B+3C+2D}\,. \tag{6.45}$$

We will use these equations to determine the behaviour of b and γ close to the Dp-branes. Before we expand the fields we present a differential equation for $L_{\mathrm{D}p}$ which is obtained by combining the external Einstein equation with the dilaton equation and the Bianchi identity (6.34). The equation is remarkably simple, and in particular no source terms appear:

$$\nabla^2L_{\mathrm{D}p}-\beta^{-1}(\nabla L_{\mathrm{D}p})^2=\beta\left[(p+1)\Lambda\mathrm{e}^{-2A}+\frac{(1+\lambda)^2}{2}\mathrm{e}^{\frac{p-1}{2}\phi}|F_{6-p}|^2\right]\,. \tag{6.46}$$

We are now in position to expand the fields close to the Dp-branes so as to obtain an expression for the potential V close to the branes. We use the standard boundary conditions of the fields close to a Dp-brane. This is the expected

behaviour based for example on the analysis in [51]:

$$\begin{aligned}
\mathrm{e}^{2A} &\approx \rho^{\frac{7-p}{8}}(a_0 + a_1\rho) \ , \\
\mathrm{e}^{2B} &\approx \rho^{\frac{-1-p}{8}}(b_0 + b_1\rho) \ , \\
\mathrm{e}^{2C} &\approx \rho^{\frac{-1-p}{8}}(c_0 + c_1\rho) \ , \\
\mathrm{e}^{2D} &\approx \rho^2(1 + d_1\rho) \ , \\
\mathrm{e}^{2\phi} &\approx \rho^{\frac{p-3}{2}}(f_0 + f_1\rho) \ , \\
L_{\mathrm{D}p} &\approx \rho(l_0 + \rho l_1) \ ,
\end{aligned} \tag{6.47}$$

the last expansion in this list is determined by noting that we chose the gauge for C_{p+1} such that the constant part of $L_{\mathrm{D}p}$ vanishes. The constants a_0, b_0, c_0 and f_0 can be rewritten in terms of the number of Dp-branes $N_{\mathrm{D}p}$ and string coupling g_s by studying the flat p-brane solutions. The near brane behaviour of α is given by:

$$\alpha \approx \alpha_0 + \rho\alpha_1 \ . \tag{6.48}$$

Below we need the first term in the expansion of β:

$$\beta \approx \rho\beta_0 = \rho f_0^{\frac{p-3}{8}} a_0^{\frac{p+1}{2}} \ . \tag{6.49}$$

Expanding the equation (6.46) to leading order we immediately find $l_0 = 0$ which implies:

$$\beta_0 = (-1)^p \alpha_1 \ . \tag{6.50}$$

The next order coefficient is:

$$6l_1 = \frac{c_0\beta_0}{a_0}\left((p+1)\Lambda + \frac{1}{2}(\lambda_0\beta_0 M)^2 f_0^{\frac{1}{2}} a_0^{-p} b_0^{p-6}\right) \ . \tag{6.51}$$

Expanding the gauge potentials gives:

$$b(\rho) \approx \frac{1}{2}\rho^2\alpha_0 M f_0^{\frac{1}{2}} a_0^{-\frac{p+1}{2}} b_0^{-\frac{6-p}{2}} c_0^{\frac{3}{2}} \ , \tag{6.52}$$

$$\gamma(\rho) \approx -\frac{1}{3}\rho^3\alpha_0\alpha_1 M f_0^{\frac{1}{2}} a_0^{-\frac{p+1}{2}} b_0^{-\frac{6-p}{2}} c_0^{\frac{3}{2}} \ . \tag{6.53}$$

Using these results it is straightforward to write down the first few terms of the expansion of the brane potential:

$$V \approx \frac{\beta_0(\pi n)^3}{6a_0^2 f_0^{\frac{1}{2}}}\left\{\left((p+1)\Lambda + \frac{1}{2}k_0^2\right)\bar{\rho}^2 - 2k_0\bar{\rho}^3 + 3\bar{\rho}^4\right\} \ , \tag{6.54}$$

where:

$$\bar{\rho} = \frac{\sqrt{c_0 a_0} f_0^{\frac{1}{4}}}{\pi n} \rho \quad \text{and} \quad k_0 = (\alpha_0 M) a_0^{-\frac{p}{2}} b_0^{-\frac{6-p}{2}} f_0^{\frac{1}{4}} \ . \tag{6.55}$$

We now see that the polarisation potential (6.54) is fully determined as soon as the constant k_0 is given.

Besides the extremum at the origin, this potential has at most two more extrema at:

$$\bar{\rho} = \frac{k_0}{2} \pm \sqrt{-\frac{k_0^2}{12} - \frac{(p+1)\Lambda}{3}} \ , \tag{6.56}$$

which shows that polarisation occurs in AdS space when:

$$-\Lambda \geq \frac{k_0^2}{4(p+1)} \ . \tag{6.57}$$

One of our key results is that polarisation for solutions where the cc is parametrically small is impossible. This in particular includes the flat solutions which obviously do not polarise by equation (6.56).

Let us now estimate k_0 for the AdS solutions. The second-order equations of motion can unfortunately not help without solving the system completely. We can however estimate k_0 by solving the equation (6.46) in the fully smeared limit. This amounts to putting the derivatives to zero and replacing λ with its smeared value [49]:

$$\lambda \to \frac{p-1}{2} \ . \tag{6.58}$$

We then find:

$$\frac{1}{2} k_0^2 \to \frac{p(p-1)^2}{p+1} \ , \tag{6.59}$$

which can be inserted into the polarisation potential (6.54). The potential is easily verified from equation (6.57) to allow for a polarisation at a finite value of $\bar{\rho}$.

In this section we used two expansions, one for large n and another for small radius ρ. Clearly we can choose ρ small to justify the near-brane expansion and $N_{\mathrm{D}p}$ large to stay within the probe approximation, $n \ll N_{\mathrm{D}p}$. Within this regime a minimum of the D$(p+2)$ potential can be deduced in the following way. Since the WZ term becomes less important when we move away from the anti Dp, the DBI term dominates forcing the potential upward. By checking whether the D$(p+2)$ potential decreases away from the point $\rho = 0$ we deduce the existence of a minimum. Obviously, a more careful analysis is needed to quantitatively trust the calculation from the previous section up to the minimum of the potential but this is unnecessary to verify brane polarisation.

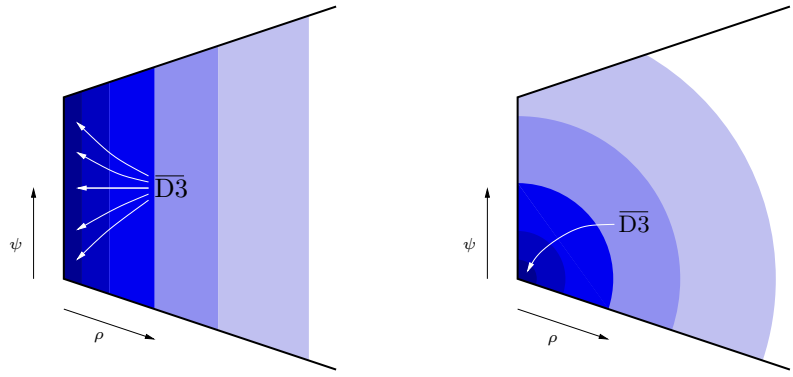


Figure 6.3: The difference between the flux clumping between antibranes smeared over the tip of the KS throat (left) and antibranes localised at the South Pole of the tip (right). At every point in the ψ-direction we suppressed the two-sphere (with zero size at South and North Pole, $\psi = 0, \pi$, and maximum size in between). The flux clumping on the right is less severe in the middle of the ψ direction, such that the force that pushes the NS5 towards the North Pole is less and could be small enough to create a classical barrier.

6.5 Conclusion

We have shown that there is a consistent picture for the flux singularities associated with anti Dp solutions that are smeared over $6-p$ compact directions. These solutions come in two types: compact AdS_{p+1} solutions and noncompact flat solutions and both feature singular fluxes that partially screen the antibrane charges. We have found a story similar to what happened for localised anti D6-branes [14, 140, 36]: the flat solutions do not polarise into spherical branes whereas the AdS solutions do. As a consequence the AdS solutions have regular flux clouds in the supergravity limit. Compact AdS vacua have a cosmological constant related to the energy in such a way that brane-flux decay would *increase* the energy. This is opposite to the flat solution, where brane-flux decay lowers the energy. So either flux clouds that are singular at the sugra level initiate perturbative brane-flux decay [52, 84] or brane polarisation has to occur in order to resolve the flux singularity. We have found exactly that. Concerning the flat solutions we also find consistency with [52, 84]: the flux clumping is too large and causes perturbative brane-flux decay such that the smooth solution is expected to be time dependent. This is in agreement with the absence of regular finite temperature solutions [48, 26, 25, 127].

An important restriction of our work is the smearing of the antibranes over the

internal S^{6-p} or $\mathbb{T}^{6-p}$. It is important to investigate brane polarisation and brane-flux decay for fully localised branes. This becomes especially important in case one looks at backgrounds with very few, or even a single, SUSY-breaking antibrane. For a single antibrane it is clearly not physical to smear the charges over a compact submanifold and it is exactly in the regime of a single brane that it has been argued that the meta-stable states are most likely to exist [143, 163]. An important argument here in favor of meta-stability is that the flux clumping effects of [52, 84] are only relevant at the South Pole of the S^3 and once the NS5-brane moves away from the South Pole the forces that push it over the equator are diluted[6] as depicted in figure 6.3, also see [70].

At first sight this allows a classical barrier against brane-flux decay and would seem consistent with a recent claim that regular finite temperature solutions can exist if the antibranes are fully localised [127]. If this is the case, it is a good indication that there is no tachyonic mode that describes the onset of brane-flux decay and it would therefore be most important to find a full proof of the claims in [127].

The sensitivity of antibrane uplifting to instabilities arises through the use of a warped throat that redshifts string-scale energies down to much smaller energy scales. This locally creates a lack of scale-separation and various modes can mix with Kaluza–Klein (KK) modes that become light [6]. The flux-clumping instabilities, if present, are an example of this effect since flux gradients correspond to KK modes and mix with the modes that correspond to the NS5 position. Checking full stability remains therefore a subtle issue for scenarios that are based on antibrane uplifting. One could for instance worry about the tachyonic modes found in [34] or even Gregory-Laflamme-like instabilities in the screened anti D3-brane [81].

Finally, there is a black hole analogue to antibrane SUSY breaking in flux backgrounds, which are near-extremal micro-state geometries build from meta-stable supertubes [38, 39]. One could worry whether these constructions also feature the problem of enhanced decay due to flux-clumping like effects. We have good reasons to believe this is not the case and hope to report on this in the near future.

[6] We like to thank U. Danielsson, D. Junghans and the authors of [163] for discussions on that matter.

Chapter 7

Non-SUSY fractional branes

This chapter is a reprint of [154] where we construct a non-supersymmetric deformation of the Klebanov-Strassler background [148] (see section 3.2.1). We consider a simplified Ansatz for supergravity solutions describing fractional p-brane solutions (throat geometries supported by fluxes) for various p. For $p = 3$ the Ansatz captures the Klebanov-Tseytlin (KT) solution [149][1]. This solution was described in section 3.2.1. The equations of motion can be derived from an effective action by performing a dimensional reduction to a flat domain wall geometry in $(p+2)$-dimensions. We find an interesting deformation of the known superpotential defining the SUSY domain wall flow. The deformation parameter breaks supersymmetry but still preserves the property that a test Dp-brane feels no force inside the throat. After the publication of this work it was observed in [45] that the non-SUSY deformation presented in this chapter corresponds to explicit supersymmetry breaking in the dual field theory. Moreover, the deformation in this chapter was recently found to survive the Klebanov-Strassler resolution [153]. This makes the background potentially interesting for studying non-SUSY cascading gauge theories holographically.

7.1 Introduction

One of the pressing problems in string phenomenology and formal string theory is to increase our understanding of controlled mechanisms to break supersymmetry (SUSY). In the supergravity limit of string theory one can attempt to construct

[1] Recall that Klebanov-Tseytlin agrees with the UV of the Klebanov-Strassler background [148].

explicit solutions describing SUSY-breaking phenomena. A popular method relies on adding SUSY-breaking branes (antibranes) to SUSY backgrounds whose warping can redshift the SUSY-breaking sources and hence control their backreaction [159, 143, 15, 142]. In the context of the AdS/CFT correspondence such supergravity solutions might capture the elusive dynamical SUSY-breaking in strongly coupled gauge theories. Away from the decoupling limit, in compact models, the previously proposed methods for SUSY-breaking could lead to a landscape of meta-stable de Sitter vacua in string theory [142].

Recently these SUSY-breaking mechanisms have been the subject of heavy scrutiny because of certain, potentially unphysical, singularities found in the supergravity solutions, see for instance [31, 162, 32] for the earliest comments. These singularities arise because of a no-force condition that is being violated: the fluxes in the throat geometries feel a force that draws them towards the SUSY-breaking brane at the tip of the warped throat [49, 50, 91]. Indeed, the specific form of the singularity is such that the density in the fluxes diverges near the source. The orientation of the fluxes is furthermore such that it describes an infinite pile-up of brane charge dissolved in fluxes with a sign opposite to that of the SUSY-breaking brane. The force on the fluxes can be thought of as the force on a test D-brane inside the throat since fluxes act in a way very similar to a smooth distribution of branes.

One reason to worry about the singular flux density is that high flux densities, when they grow indefinitely, trigger perturbative brane-flux annihilation, turning the background perturbatively unstable [52, 84]. To circumvent these issues we are interested in solutions that break SUSY but at the same time still preserve the no-force condition on the fluxes or test Dp-branes. This is by no means a unique way to circumvent the singularities inherent to antibrane solutions, but the no-force condition is one of the reasons why supersymmetric backgrounds are easier controllable. Non-SUSY solutions sharing the no-force property might be the first place to look for controlled SUSY-breaking.

In this chapter we are able to find *first-order* ("fake SUSY like") equations for such solutions. It is intriguing that we can find explicit first-order flows that break SUSY but preserve the no-force condition. As it turns out, our new solutions indeed evade the singular flux-pile up that are typical to solutions that break the no-force condition. The case of $p = 3$ is analysed in some detail and the SUSY solution is the so-called Klebanov-Tseytlin (KT) solution [149]. The solutions we find are distortions of the KT solution that are controlled by a deformation parameter. The UV behavior of the solutions is identical to KT and some of the new solutions have also a very similar IR region, which contains a naked singularity. In case SUSY is not broken it is well known that this singularity can be resolved either by deforming the conifold [148] or by putting the system at finite temperature [7]. We leave the question whether

the singularity can be resolved for future work, though we initiate an attempt to describe the finite temperature solution through first-order equations.

Non-SUSY no-force solutions have already occurred in the literature. Such a background was for instance constructed in [99], but the approach of [99] differs from ours since it leads to second-order equations of motion[2].

This chapter is organised as follows. In Section 7.2 we review the relation between first-order flow equations, Hamilton-Jacobi theory and fake SUSY applied to domain wall flows in theories of gravity coupled to scalars subject to some scalar potential. We use this formalism to describe the new SUSY-breaking fractional branes in section 7.3. To apply our formalism to this set-up we first dimensionally reduce the 10-dimensional geometry to a flat domain wall Ansatz. We discuss the absence of a force on a test Dp-brane inside the throat. Section 7.4 extends the analysis to finite temperature and we conclude in Section 7.5. In appendix D we briefly discuss the curved (AdS-sliced) domain walls of the theories we have investigated and their lift to 10 dimensions.

7.2 Effective actions and first-order flows

In gravity theories, coupled ordinary first-order equations (flow equations) can arise whenever the Ansätze contain sufficient spacetime symmetries (e.g. spherical black holes, FLRW cosmologies, domain walls). The equations of motion can then be recast into ordinary differential equations of the standard Hamiltonian form which can be derived from a one-dimensional effective action:

$$I = \int \mathrm{d}\rho \Big(\frac{1}{2} G_{ij}(q) \dot{q}^i \dot{q}^j - \mathcal{V}(q) \Big) \,. \tag{7.1}$$

The canonical variables q^i $(i = 1, \ldots, N)$ are the undetermined functions of a single coordinate, ρ, in the solution Ansätze. The conditions for preserving some amount of SUSY then become first-order flow equations, typically of the form of an autonomous system:

$$\dot{q}^i = F^i(q_1, \ldots, q_N) \,, \tag{7.2}$$

where F^i are certain functions of the canonical variables. Such first-order integrations can sometimes be found for solutions that break SUSY as has been observed over the last 10 years for black holes, domain walls, cosmologies and general, non-SUSY, p-brane solutions. In fact, many of these examples are related by dimensional reduction, which then relate the corresponding first-order

[2]More well known are IIB flux solutions on conformal Calabi-Yau spaces with ISD 3-form fluxes that have a $(0,3)$-component [115].

equations. When a p-brane is reduced over its spatial worldvolume it becomes a black hole (0-brane). If, instead, it is reduced over the angles in the transversal space it becomes a domain wall.

It has been observed by several authors that the first-order equations (7.2) are instances of the Hamilton-Jacobi equation in classical mechanics [86, 190, 9, 201]. This implies that the first-order equations (7.2) are equivalent to gradient flow equations:

$$\dot{q}^i = G^{ij} \frac{\partial \mathcal{S}}{\partial q^j} \,, \tag{7.3}$$

where the gradient function $\mathcal{S}(q)$ is *Hamilton's principal function*. In the supergravity literature this gradient function is sometimes called *fake superpotential*. This is, however, a slight abuse of language since SUSY-like equations (7.2) are only equivalent to Hamilton-Jacobi equations when the principal function factorises [201] as further reviewed in Section 7.2.2. To avoid confusion, from here on we will use the abbreviation $\mathcal{S}$-function.

7.2.1 Algebraic Hamilton-Jacobi theory

We first revise some aspects of Hamilton-Jacobi (HJ) theory. Thanks to diffeomorphism covariance it is always possible to parametrise the gravitational Ansätze of interest such that neither the metric G_{ij} nor the potential $\mathcal{V}$ in (7.1) have explicit ρ-dependence. Hamilton's principal function then takes the form $\mathcal{S}(q^i) - E\rho$, where the first term is ρ-independent and E is a constant, which has the interpretation of energy for ordinary dynamical systems[3]. The $\mathcal{S}$-function then satisfies the following Hamilton-Jacobi (HJ) equation:

$$\frac{1}{2} G^{ij} \frac{\partial \mathcal{S}}{\partial q^i} \frac{\partial \mathcal{S}}{\partial q^j} + \mathcal{V} = E \,. \tag{7.4}$$

The non-dynamical part of the Einstein equations enforces an algebraic "zero energy" constraint for the gravitational systems of interest. For FLRW cosmologies and domain walls this nicely translates into $E = 0$ in (7.4). Spherical black holes, on the other hand, have $E = 0$ for extremal solutions and nonzero E for non-extremal ones, where E becomes proportional to ST, the product of entropy and temperature[4].

[3]For a separated solution $\mathcal{S}(q^i, t) = \mathcal{S}'(q^i) - Et$, the function $\mathcal{S}'$ is usually called the *characteristic* function. However, in all of the example we will consider $E = 0$, and so we will still refer both to $\mathcal{S}$ and $\mathcal{S}'$ as the principal function and use the same notation for the two of them.

[4]To be more precise, E is zero both for extremal and non-extremal solutions but the non-extremal black holes have a constant term in the potential of (7.4) which can then be reinterpreted as the energy [138].

The canonical momenta p_i associated to the canonical variables in (7.1) are defined by $p_i = G_{ij}\dot{q}^j$. The HJ equations of motion then take the form:

$$p_i = \frac{\partial \mathcal{S}}{\partial q^i}\,, \tag{7.5}$$

which are thus equivalent to the flow equations (7.3).

HJ theory implies that a first-order integration always exists locally. These flow equations (7.3) may be integrated to obtain a solution to the equations of motion. However, the original purpose of HJ theory is not to integrate first-order equations but rather solve them algebraically. Recall that the most general solution to the HJ equation depends on N constants of motion α_i. For convenience we identify α_N as E. The HJ formalism then implies that the quantities:

$$\beta^i \equiv \frac{\partial \mathcal{S}\left(q^k, \alpha_k\right)}{\partial \alpha_i}\,, \qquad i = 1, \ldots, N\,, \tag{7.6}$$

are also constants of motion: $\dot{\beta}^i = 0$. Consequently (7.6) and (7.3) become a set of algebraic equations for $q^i(\rho)$ and $p_i(\rho)$ (or, equivalently $\dot{q}^i(\rho)$) in terms of α_i's and β^i's, which are in turn determined by the initial conditions. The physical meaning of the constants α_i's and β^i's is easy to understand by recalling that Hamilton's principal function is in fact a generating function for a certain canonical transformation $\left(q^i, p_i\right) \to \left(Q^i, P_i\right)$. The HJ equation (7.4) for $\mathcal{S}$ then implies that the new Hamiltonian $\tilde{H}\left(Q^i, P_i\right)$ identically vanishes, guaranteeing that $\dot{P} = 0$ and $\dot{Q} = 0$. If the $\mathcal{S}$-function depends on q^i's and P_i's, then α_i and β^i correspond to P_i and Q^i respectively.

In practice, finding a complete solution of the HJ equation (7.4) is rather rare and one can typically only hope to find the dependence on a few α_i's out of the full set of N constants. If, for instance, $\mathcal{S}(q^i, \alpha_1)$ is available, then we end up with one algebraic equation, namely (7.6) for $i = 1$, together with $N - 1$ first-order differential equations (7.3), which we still have to solve.

For completeness, let us mention that starting from $\mathcal{S}\left(q^i, \alpha_i\right)$ we can always introduce a new generating function $\widetilde{\mathcal{S}}\left(q^i, \beta^i\right)$ by performing a Legendre transformation on the α_i's:

$$\widetilde{\mathcal{S}}\left(q^i, \beta^i\right) = \mathcal{S}\left(q^i, \alpha_i\right) - \beta^j \alpha_j\,, \tag{7.7}$$

where β^i on the right hand side is defined by (7.6)[5]. Although the new principal function, $\widetilde{\mathcal{S}}\left(q^i, \beta^i\right)$, seems to provide a new solution of the HJ equation, the

[5]One can verify that the new $\mathcal{S}$-function solves the original HJ equation simply by noticing that:

$$\left.\frac{\partial \mathcal{S}}{\partial q^i}\right|_{\alpha_j} = \left.\frac{\partial \widetilde{\mathcal{S}}}{\partial q^i}\right|_{\beta_j}\,, \tag{7.8}$$

final solution of the equations of motion is merely a reparametrization of the solution obtained using the original principal function $\mathcal{S}\left(q^i, \alpha_i\right)$.

7.2.2 Relation with fake supersymmetry

The prime goal of this subsection is to clarify the connection between the superpotential approach and the Hamilton-Jacobi formalism applied to gravity systems. The discussion is based on [86, 201, 93].

Consider the following action of gravity coupled to n scalar fields in D spacetime dimensions:

$$S = \int \sqrt{-g}\Big\{R - \frac{1}{2}h_{ab}\partial\phi^a\partial\phi^b - V(\phi)\Big\} \qquad a, b = 1, \ldots, n\,. \tag{7.9}$$

Such theories can support nontrivial domain wall geometries which are solutions to the following metric Ansatz:

$$\mathrm{d}s_D^2 = f(z)^2\mathrm{d}z^2 + g(z)^2\mathrm{d}\Sigma_k^2\,, \tag{7.10}$$

where the $(D-1)$-dimensional metric Σ_k^2 is either Minkowski ($k=0$), AdS ($k=-1$) or dS ($k=+1$). In what follows we consider Minkowski-sliced domain walls, also known as flat domain walls. The scalar fields are considered to depend on the coordinate z which parametrizes the distance from the wall located at some fixed value $z = z_0$. The function $f(z)$ in the metric is a gauge choice and can (locally) be chosen at will. It is well known that for any function $\mathcal{W}$ that obeys:

$$V = \frac{1}{2}h^{ab}\partial_a\mathcal{W}\partial_b\mathcal{W} - \frac{D-1}{4(D-2)}\mathcal{W}^2\,, \tag{7.11}$$

the following first-order flows [90] (where a dot denotes a z-derivative):

$$f^{-1}\dot{\phi}^a = -h^{ab}\partial_b\mathcal{W}\,, \tag{7.12}$$

$$f^{-1}\frac{\dot{g}}{g} = \frac{\mathcal{W}}{2(D-2)}\,, \tag{7.13}$$

solve the second-order equations of motion. This function $\mathcal{W}$ is called a *fake superpotential* when it is not related to supersymmetry and for supersymmetric solutions it corresponds to the genuine *superpotential*.

As discussed in various papers [86, 9, 201, 190] these first-order equations are a manifestation of the Hamilton-Jacobi formalism for one-dimensional

which also shows that the physical meanings of α_i's and β^i's is reversed under the transformation.

Hamiltonian systems. The Hamiltonian system is defined by the second-order differential equations, which can be derived from the following effective action obtained from the Ansätze (7.9), (7.10):

$$I = \int \mathrm{d}\rho \left\{ \frac{g^{D-1}}{f} \left(-\frac{1}{2} h_{ab} \dot{\phi}^a \dot{\phi}^b + (D-1)(D-2) \left(\frac{\dot{g}}{g} \right)^2 \right) - f g^{D-1} V \right\} . \tag{7.14}$$

The variable f is algebraic (since it stems from a degree of freedom that can be gauged away) and it enforces the zero energy constraint $E = 0$. One must choose a gauge for f to regard this as a Hamiltonian system in $n+1$ variables g, ϕ^i and impose the zero energy constraint separately. When Hamilton's principal function $\mathcal{S}(q)$ factorises as follows [201]:

$$\mathcal{S} = g^{D-1} \mathcal{W}(\phi) \,, \tag{7.15}$$

the HJ equations (7.3, 7.4) with $E = 0$ reproduces exactly the fake SUSY first-order flow equations (7.11-7.13), independently of the chosen gauge for f [6]. When such solutions have a holographic dual, the first-order HJ equation (7.3) is dual to the RG-flow equations of the gauge theory [86]. Whether this factorisation can always occur is a subtle issue [201] and is clarified in [93].

7.3 Fractional branes as domain walls

In this section we study Ansätze that captures p-branes magnetically charged under the F_{8-p} RR field strength along with extra NSNS H_3 flux and RR F_{6-p} flux. The latter fluxes can be combined into a $(9-p)$-form $H_3 \wedge F_{6-p}$ that acts as a smooth magnetic source together with the singular p-brane source:

$$\mathrm{d}F_{8-p} = H_3 \wedge F_{6-p} + \delta_{9-p}(\mathrm{D}p) \,. \tag{7.16}$$

Such solutions play a prominent role for non-conformal holographic backgrounds or, when the Dp-brane is replaced by an Op-plane [53], serve as standard flux compactifications. Maybe the most well known example is the Klebanov–Strassler geometry [148] for $p = 3$, but here we study more symmetric examples for $p = 3$ [149] and for general p [132, 137, 53].

The way we proceed is to first reduce the brane solutions over the angles in their transversal space to a flat domain wall in $(p+2)$-dimensions. This allows us to write down the effective action (7.1) in a simple manner and in doing so we also make contact with the concept of fake SUSY, which was historically

[6] Notice that f in (7.14) can be absorbed in a proper ρ-coordinate redefinition.

developed for the special case of domain walls [105]. We find new first-order flow equations that preserve one interesting property of the SUSY solutions: the force on a probe Dp-brane inside the throat vanishes for any position of the probe.

Not every new domain wall flow that solves the equations of motion is necessarily a physical solution. For domain walls in 5D (i.e. $p = 3$) we investigate the solutions in some details and observe that they all posses a naked singularity at small radius whereas the large radius behaviour is identical to the SUSY Klebanov-Tseytlin (KT) solution. The KT solution also has a naked singularity but it is known to be resolved when the conifold is deformed [148], yielding the Klebanov-Strassler (KS) solution. There exists a simple criterion [122] for naked singularities to be admissible. Assuming that the singularities have certain brane interpretation, at finite temperature the solution must develop a horizon. Hence, the criterion states that admissible singularities must be cloaked by a horizon at sufficiently high temperature. This will briefly be discussed in Section 7.4.

In the next subsection we present the 10-d Ansatz of IIA/IIB sugra that describes our solutions.

7.3.1 Dissolved fractional branes (branes in throats)

In 10-d Einstein frame we consider the following Ansatz:

$$\mathrm{d}s_{10}^2 = e^{2A(\rho)}\mathrm{d}s_{p+1}^2 + e^{2B(\rho)}\mathrm{d}s_{9-p}^2\,. \tag{7.17}$$

The worldvolume metric is assumed to be flat, $\mathrm{d}s_{p+1}^2 = \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$, whereas the transversal space is assumed to be a cone over a base space, Σ_{8-p}, with the following metric:

$$\mathrm{d}s_{9-p}^2 = \mathrm{d}\rho^2 + e^{2C(\rho)}\left[g_{ij}^\Sigma(\psi)\mathrm{d}\psi^i\mathrm{d}\psi^j\right], \tag{7.18}$$

where ρ is the radial coordinate and the metric g_{ij}^Σ is assumed to be Einstein:

$$R_{ij}^\Sigma = (7-p)g_{ij}^\Sigma\,, \tag{7.19}$$

such that for $e^{2C} = \rho^2$ the metric $\mathrm{d}s_{9-p}^2$ is Ricci flat. The field content that supports the metric is given by:

$$\begin{aligned} H_3 &= \mathrm{d}B_2 = b'(\rho)\mathrm{d}\rho\wedge\epsilon_2 \\ F_{6-p} &= m\star_{8-p}\epsilon_2 \\ F_{8-p} &= Q\epsilon_{8-p} + B_2\wedge F_{6-p}\,, \end{aligned} \tag{7.20}$$

where $\star_{8-p}$ is the Hodge star on the base space Σ_{8-p}, m is some constant which measures the fractional fluxes and ϵ_2 is a 2-form on the cone base. The constant Q can be absorbed in the definition of $b(\rho)$ but it will be useful to keep Q explicit in order to discuss the limit $m \to 0$ where it becomes related to the brane charge. The field equations require that the two-form ϵ_2 is closed and co-closed. The fields only depend on ρ. As usual there is the subtlety that the Ansatz for F_5 has to be changed to:

$$F_5 = (1 + \star_{10})(Q\epsilon_5 + B_2 \wedge F_3)\,. \tag{7.21}$$

The known solutions are given by [132, 53]:

$$\mathrm{d}s_{10}^2 = H^{\frac{p-7}{8}} \mathrm{d}s_{p+1}^2(x^a) + H^{\frac{p+1}{8}} \mathrm{d}s_{9-p}^2(y^i)\,, \tag{7.22}$$

$$\mathrm{e}^{\phi} = g_s H^{\frac{3-p}{4}}\,, \tag{7.23}$$

$$F_{8-p} = -g_s^{\frac{3-p}{4}} \star_{9-p} (\partial_i H \mathrm{d}y^i)\,, \tag{7.24}$$

$$F_{6-p} = g_s^{-\frac{p+1}{4}} \star_{9-p} H_3\,, \tag{7.25}$$

where, again, the transversal and world-volume metrics are Ricci flat. The Hodge star $\star_{9-p}$ is defined with respect to the flat metric $\mathrm{d}s_{9-p}^2(y^i)$. For $p = 1, 2, 4, 6$, we have explicitly:

$$H = C_2 + \frac{\tilde{Q}}{r^{7-p}} - \frac{g_s^{\frac{p-1}{2}}\, m^2}{2(p-3)(p-5)}\, \frac{1}{r^{10-2p}}\,, \tag{7.26}$$

where C_2 is arbitrary. For asymptotically (Ricci) flat backgrounds we can set $C_2 = 1$ and in the decoupling limit C_2 is taken to be zero. $\tilde{Q}$ determines the charge of the p-brane at the tip (but such an interpretation is subtle) and, up to factors and constants, it agrees with the Q introduced in (7.20). We use r as the notation for the radial coordinate, instead of ρ, when we employ the gauge choice:

$$e^{2C(r)} = r^2\,, \tag{7.27}$$

in the cone-metric (7.18).

According to [132] there exist no solutions with $p = 5$. For $p = 3$ the last term becomes a logarithm:

$$H = C_2 + \frac{\tilde{Q}}{r^4} + \frac{g_s m^2}{16\, r^4}\Big(4\, \ln r + 1\Big)\,. \tag{7.28}$$

Supersymmetry requires the choice of a suitable conical internal space. For D3-branes ($p = 3$) the metric $\mathrm{d}\Omega_5$ should be that of an Einstein-Sasaki space, such as the conifold $T^{1,1}$ and this solution was first described by Klebanov and Tseytlin in [149] and will be referred to as KT in what follows.

For $p = 6$ no SUSY is possible with this Ansatz. A more general solution for $p = 6$ was found in [137] and is characterised by the following choice for H:

$$H(r, z) = 1 + \frac{\tilde{Q}}{r} - \frac{1}{6} g_s^{\frac{5}{2}} m^2 r^2 + cz \,, \tag{7.29}$$

where z is a Cartesian spatial coordinate on the D6 worldvolume, m is the Romans mass and c a constant. When m and c both vanish this is the standard extremal D6 solution in IIA supergravity, which preserves 1/2 of the supersymmetry. When $m \neq 0$ it was shown that maximally 1/4 of the supersymmetry could be preserved if $c = m$.

7.3.2 The lower-dimensional theory

The metric on the $(9 - p)$-dimensional space transversal to the brane can be written in polar coordinates and for p-branes with sufficient symmetries the fields only depend on the radial coordinate and not on the angles. The angles themselves parameterise a space Σ_{8-p} with Einstein metric of positive curvature. It is natural to perform a dimensional reduction over the Einstein space such that the p-brane has but a single transversal direction (the radial direction) in the reduced theory that lives in $p + 2$ spacetime dimensions. This way p-branes reduce to domain walls. If the reduced theory is a maximal gauged supergravity, the 1/2 SUSY domain walls lift to 1/2 SUSY p-branes in the 10-dimensional theory, see [41] for a list of explicit examples.

The reason for reducing the 10-dimensional brane solutions to flat domain walls in $p + 2$ dimensions is mainly practical; the Kaluza–Klein reduction offers a useful form for the effective action for the unfixed degrees of freedom in the action. It also allows us to make contact with the concept of fake supersymmetry, which is usually defined for domain wall space-times. Although in this chapter we will only barely touch upon the issue of fake supersymmetry.

The Ansatz for a dimensional reduction over Σ_{8-p} is simply a rewriting of the metric Ansatz (7.17) for the fractional brane solutions:

$$\mathrm{d}s_{10}^2 = e^{2\alpha\varphi} \mathrm{d}s_{p+2}^2 + e^{2\beta\varphi} \mathrm{d}\Sigma_{8-p}^2 \,. \tag{7.30}$$

This is not the most general Ansatz but describes a consistent truncation for three dynamical scalar fields in $p+2$ dimensions: ϕ, φ, b. If we choose:

$$\beta = -\frac{p\alpha}{8-p}\,, \qquad \alpha^2 = \frac{8-p}{16p}\,, \tag{7.31}$$

the reduced theory is in Einstein frame and the scalar φ is canonically normalised. The reduced action is:

$$I = \int \sqrt{-g_{p+2}}\Big\{R_{p+2} - \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{\frac{4p\alpha}{8-p}\varphi-\phi}(\partial b)^2 - V(\varphi,\phi,b)\Big\}\,, \tag{7.32}$$

with the following scalar potential:

$$\begin{aligned} V(\varphi,\phi,b) = &-(8-p)(7-p)e^{\frac{16\alpha}{8-p}\varphi} + \frac{1}{2}m^2 e^{\frac{2\alpha}{8-p}(8+5p-p^2)\varphi+\frac{(p-1)}{2}\phi} \\ &+ \frac{1}{2}(Q+mb)^2 e^{2(p+1)\alpha\varphi+\frac{(p-3)}{2}\phi}\,. \end{aligned} \tag{7.33}$$

Flat domain walls

If we take a flat domain wall metric (7.10) for $\mathrm{d}s^2_{p+2}$ then one can indeed map the fractional Dp Ansatz (7.17, 7.20) to (7.30), i.e. when the domain wall is uplifted to 10-d. The known SUSY fractional Dp solutions discussed in (7.22 - 7.25) can be obtained from the following principal function:

$$\mathcal{S}_0(g,\phi,\varphi,b) = g^{p+1}\Big((Q+mb)\,e^{\frac{(p-3)}{4}\phi+(p+1)\alpha\varphi} \pm 2(8-p)e^{\frac{8}{8-p}\alpha\varphi}\Big)\,. \tag{7.34}$$

This principal function is indeed of the fake SUSY form (7.15) such that the term between brackets defines the superpotential $\mathcal{W}(\phi,\varphi,b)$ (7.15):

$$\mathcal{W}(\phi,\varphi,b) = (Q+mb)\,e^{\frac{(p-3)}{4}\phi+(p+1)\alpha\varphi} \pm 2(8-p)e^{\frac{8}{8-p}\alpha\varphi}\,. \tag{7.35}$$

One can explicitly check that $\mathcal{W}$ obeys (7.11) with $D = p+2$ for the system defined in (7.32), (7.33).

Curved domain walls

One could also wonder about the curved (AdS-sliced) domain walls of the reduced theories in $p+2$ dimensions. Since we have not yet found first-order flow equations for all p (for $p=6$ this has already been found in [14]). We rather discuss the would-be solutions in general in the appendix to demonstrate how their lift predicts a generalisation of a set of warped *compact* AdS vacua of recent interest [50, 51, 14, 140, 109, 80].

7.3.3 Non-SUSY solutions

The main result of this chapter is that we have found a second *family* of solutions of the Hamilton-Jacobi equation:

$$\mathcal{S}_1(c) = g^{p+1}\Bigg((Q+mb)\, e^{\frac{(p-3)}{4}\phi+(p+1)\alpha\varphi} \\ \pm 2(8-p)\, e^{\frac{8}{8-p}\alpha\varphi}\cosh\left(\gamma_1\phi+\gamma_2\varphi+\gamma_3\ln(g)+c\right)\Bigg), \tag{7.36}$$

where:

$$\gamma_1 = \frac{(7-p)}{4}\sqrt{\frac{p+1}{2(8-p)}}\,, \qquad \gamma_2 = -4\frac{(p-3)}{(7-p)}\gamma_1\alpha\,, \qquad \gamma_3 = \frac{\gamma_2}{\alpha}\,, \tag{7.37}$$

and c is an arbitrary real constant. Obviously c is an example of the constant α_1 we introduced in Section 7.2. Part of the inspiration that led us to this solution of the Hamilton-Jacobi equation comes from the example of a free-scalar field in AdS whose superpotential is of the cosh-form. The parameter c should correspond to a Noether charge associated to a symmetry of the effective action. We have not yet been able to identify this symmetry.

It is not immediately clear that the principal function $\mathcal{S}_1(c)$ describes solutions that can be seen as one-parameter deformations of the SUSY solutions described by $\mathcal{S}_0$ since there is no limit for the integration constant c for which $\mathcal{S}_1(c)$ coincides with $\mathcal{S}_0$. However, we can look at the Legendre transform, (7.7), which generates an equivalent $\mathcal{S}$-function, $\mathcal{S}_2(d)$, as explained in Section 7.2.1:

$$\begin{aligned}\mathcal{S}_2(d) \;=\; & (Q+mb)\, g^{p+1}\, e^{\frac{(p-3)}{4}\phi+(p+1)\alpha\varphi} \pm 2\sqrt{d^2+(8-p)^2 g^{2(p+1)} e^{\frac{16}{8-p}\alpha\varphi}} \\ & +2d\left[\gamma_1\phi+(\gamma_2+\frac{8\alpha}{8-p})\varphi+(\gamma_3+p+1)\ln(g)\right] \\ & -2d\ln\left(\sqrt{d^2+(8-p)^2 g^{2(p+1)} e^{\frac{16}{8-p}\alpha\varphi}}\pm d\right),\end{aligned} \tag{7.38}$$

where d is analogous to β_1 in (7.6). This one-parameter family $\mathcal{S}_2(d)$, with the continuous parameter d, is connected to the SUSY solutions described by $\mathcal{S}_0$ by taking $d=0$, i.e. $\mathcal{S}_2(d=0)=\mathcal{S}_0$. The first-order flows from $\mathcal{S}_1(c)$ and $\mathcal{S}_2(d)$ can be mapped to each other by substituting c as a function of d and the other fields: $c = c(d,\phi,\varphi,b,g)$.

In the absence of H_3 and F_{6-p} flux ($m = 0$) we can obtain explicit solutions and they coincide with the non-SUSY brane solutions constructed earlier by Lu and Pope in [155]. For the latter solutions it was indeed observed that they could be found from a first-order formulation [138] and our results can be seen as an extension that includes extra form fluxes. In the decoupling limit of the $p = 3$ solution we expect that our new solutions coincide with the ones of [120].

(Fake) supersymmetry

The known SUSY solutions are found from the principle function $\mathcal{S}_0$[7], hence the new principle functions $\mathcal{S}_1(c)$ (or equivalently $\mathcal{S}_2(d)$) give rise to non-SUSY solutions. We therefore can verify whether the new solutions are fake SUSY, prior to solving the first-order Hamilton-Jacobi flow equations. Earlier we mentioned that fake SUSY is usually defined as a factorisation of the principle function (7.15). The principle function $\mathcal{S}_1(c)$ in (7.36) factorises in the sense (7.15) when $p = 3$ such that we may naively speak of a fake superpotential. However, the fully equivalent principal function $\mathcal{S}_2$ in (7.38) does not factorise and one would correspondingly not speak of fake SUSY. This confusion is related to the subtlety pointed out in [201] and can be resolved by a more careful definition of the concept of fake supersymmetry [93].

Explicit analysis for $p = 3$

We have not been able to find analytic solutions for all fields to the first-order flow equations for these new principal functions for generic values of c. However, having the first-order equations at hand a numerical integration becomes rather straightforward. We now present some analytic and numerical results for $p = 3$ from the 10-d point of view for easy comparison to the known fractional brane solutions [149, 132, 53]. We focus on solutions in the decoupling limit, by which we mean that the large r behavior corresponds with that of solution (7.28) with $C_2 = 0$.

To compare the new solutions with the known SUSY ones (7.28), we are interested in the behaviour of A, B, b in (7.17, 7.20) and the dilaton ϕ. We employ the standard gauge for the radial coordinate r in (7.27), used in (7.28)[8]. The flow equations derived from (7.34, 7.36, 7.38) can be rewritten to the 10-d

[7] To be more precise, the solutions defined by $\mathcal{S}_0$ are SUSY for a proper choice of the metric $\mathrm{d}\Omega_{8-p}$.

[8] This corresponds to the choice $f(r) = r^{-1} e^{\frac{8\beta}{p}\varphi}$.

field basis, for $\mathcal{S}_0$ this becomes:

$$A' = \frac{(Q+mb)e^{-4B}}{4r^5} \quad , \quad b' = m\frac{e^{\phi}}{r}, \tag{7.39}$$

$$B' = -\frac{(Q+mb)e^{-4B}}{4r^5} \quad , \quad \phi' = 0. \tag{7.40}$$

Here a prime indicates differentiation w.r.t. r. This is the SUSY case and it is clear that we can impose Ricci flat conditions at large r. The flow can be integrated to give (7.22, 7.23, 7.24, 7.25, 7.28). The non-SUSY flows, defined by $\mathcal{S}_1(c)$ only differ in the equations for B and ϕ while the equations for A and b remain unchanged. Using $\mathcal{S}_1(c)$ one finds:

$$B' = -\frac{(Q+mb)e^{-4B}}{4r^5} + \frac{1}{r}\left(\cosh\left(\sqrt{\frac{2}{5}}\phi + c\right) - 1\right), \tag{7.41}$$

$$\phi' = -\frac{2\sqrt{10}}{r}\sinh\left(\sqrt{\frac{2}{5}}\phi + c\right).$$

Additionally, HJ theory provides us with an algebraic equation (7.6) relating the constants c and d:

$$d = -5e^{4A+4B}r^4\sinh\left(\sqrt{\frac{2}{5}}\,\phi + c\right). \tag{7.42}$$

We will now solve the equations arising from $\mathcal{S}_1$ and compare it to the SUSY solutions (7.21, 7.22, 7.23 7.28). In our conventions we have $Q > 0$ such that we end up with the standard D3-brane solution in the limit $m \to 0$[9]. The non-SUSY dilaton equation can be straightforwardly integrated to give:

$$e^{\phi(r)} = e^{\phi_\infty}\left(\frac{r^4 - r_s^4}{r^4 + r_s^4}\right)^{\sqrt{\frac{5}{2}}}. \tag{7.43}$$

Here we identified c in $\mathcal{S}_1$ with the value of the dilaton at $r \to \infty$:

$$c = -\sqrt{\frac{2}{5}}\phi_\infty\,, \tag{7.44}$$

and r_s is an integration constant corresponding with the minimal value of the radial coordinate r. With this choice of the sign in front of r_s^4, the string

[9] A negative Q could be interpreted as an orientifold solution since one obtains an orientifold solution in flat space in the limit $m = 0$.

coupling goes to zero for $r \to r_s$, while for the opposite sign it diverges at $r = r_s$. In what follows we will stick with the former option. In the SUSY case the dilaton is obviously constant, which is reproduced for $r_s = 0$.

Next, we solve for e^{A+B} using the A' and B' equations in (7.39) and (7.41) respectively. For the SUSY background $A + B = 0$ everywhere while the non-SUSY solution gives:

$$e^{4(A(r)+B(r))} = 1 - \left(\frac{r_s}{r}\right)^8 . \tag{7.45}$$

This result suggests that we have a singularity at $r = r_s$ as long as r_s does not vanish and we have verified that this is indeed the case. Again, for $r_s = 0$ we reproduce the SUSY solution. One can verify that $d = 10r_s^4$.

The solution of the $b(r)$ equation of motion is simple but does not have a neat analytic form. For large r the function $b(r)$ diverges as in the KT solution:

$$b(r) \approx m e^{\phi_\infty} \ln r + \dots , \tag{7.46}$$

and near $r = r_s$ the function $b(r)$ is finite:

$$b(r) = b_s + \mathcal{O}\left((r - r_s)^{\sqrt{\frac{5}{2}}+1}\right) . \tag{7.47}$$

Although b_s is clearly a gauge constant and the full 3-form flux H_3 does not depend on it, a shift in b_s nevertheless corresponds to a *large* gauge transformation, since within our Ansatz $\int_{S^2, r=\text{const}} B_2 = b(r)$, where S^2 is the 2-sphere on the conifold base. The full 10-d background will therefore be sensitive to the choice of b_s similar to what happens in the SUSY KT geometry.

Finally we are in a position to study the behaviour of e^{-4A} such that the full solution is understood. The relevant equation is:

$$\left(e^{-4A}\right)' = -\frac{(Q + mb(r))}{r^5} e^{-4(A+B)} = -(Q + mb(r)) \frac{r^3}{r^8 - r_s^8} . \tag{7.48}$$

In the far UV ($r \to \infty$) the solution has the familiar form in agreement with the SUSY case:

$$e^{-4A} \approx \frac{1}{4} m^2 e^{\phi_\infty} \cdot \frac{\ln r}{r^4} + \mathcal{O}\left(r^{-4}\right) , \tag{7.49}$$

but the IR behaviour ($r = r_s$) depends on the sign of $Q + mb(r_s)$. Loosely speaking the combination $Q + mb(r)$ can be interpreted as an effective charge density. Since the solutions are such that an effective no-force condition holds (see below), this effective charge is also an effective tension. Let us define r_e as the radius where $Q + mb(r_e) = 0$. At $r = r_e$ the effective tension of the

"would be" brane changes sign and gravity becomes repulsive. In the SUSY case the singularity is located at a radius smaller than r_e, let us call this radius r_*. The non-SUSY solutions, on the other hand, allow to shift the radius of the singularity r_s such that both solutions with $r_e > r_s$ and $r_e < r_s$ exist.

These two possibilities lead to a very different behaviour near the singularity. If we reach the singularity before $Q + mb(r)$ changes sign, $r_e < r_s$, then (7.48) shows that e^{-4A} diverges as:

$$e^{-4A} \approx -(Q + mb(r_s)) \ln(r - r_s) + \dots . \tag{7.50}$$

Note that while the metric is singular, the string coupling e^{ϕ}, goes to zero at $r = r_s$. This behaviour is new and not present in the SUSY case.

If, on the contrary, $r_e > r_s$ then $Q + mb(r)$ turns negative in the IR and this forces $e^{-4A} \to 0$, even before we reach $r = r_s$. This behaviour is very similar to the SUSY case. The radius, $r_*(r_s, r_e)$, at which e^{-4A} becomes zero is defined through:

$$e^{-4A(r_*(r_s,r_e))} = \int_{r_*(r_s,r_e)}^{\infty} \mathrm{d}r (Q + mb(r)) \frac{r^3}{r^8 - r_s^8} = 0 \,, \tag{7.51}$$

and satisfies $r_s < r_*(r_s, r_e) < r_e$. Note that (7.51) only makes sense if $Q + mb(r)$ becomes negative. This shows that whenever $r_e > r_s$, we find a singularity at $r_*(r_s, r_e)$ but, contrary to the $r_e < r_s$ case, the dilaton is well behaved (finite) at the singularity. This singularity is also very similar to the KT singularity of the SUSY case, which is confirmed by a numerical treatment which also shows that $r_*(r_s, r_e)$ agrees with r_* in the limit $r_s \to 0$. The singularity at r_s is no longer present as the geometry only exists for $r > r_*(r_s, r_e)$.

In the limiting case, $r_s = r_e$, the function e^{-4A} approaches a finite value at $r = r_s$.

The above discussion is confirmed by a numerical treatment of the first-order flows. We focused on the solutions for which the string coupling decreases in the IR. A non-SUSY solution with $r_e > r_s$ is shown in Figure 7.1. It is indeed very similar to the KT solution. Figure 7.2 shows a non-SUSY solution with $r_e < r_s$, where e^{2A} is forced to go to zero according to (7.50). The pictures are plots of the solutions that asymptote to the KT background, which means that the solutions are asymptotically AdS (up to the usual logarithm).

Physical interpretation?

If we ignore the issues of the singularities for a moment, we can compute physical properties of these solutions. One such a nontrivial property that we have found

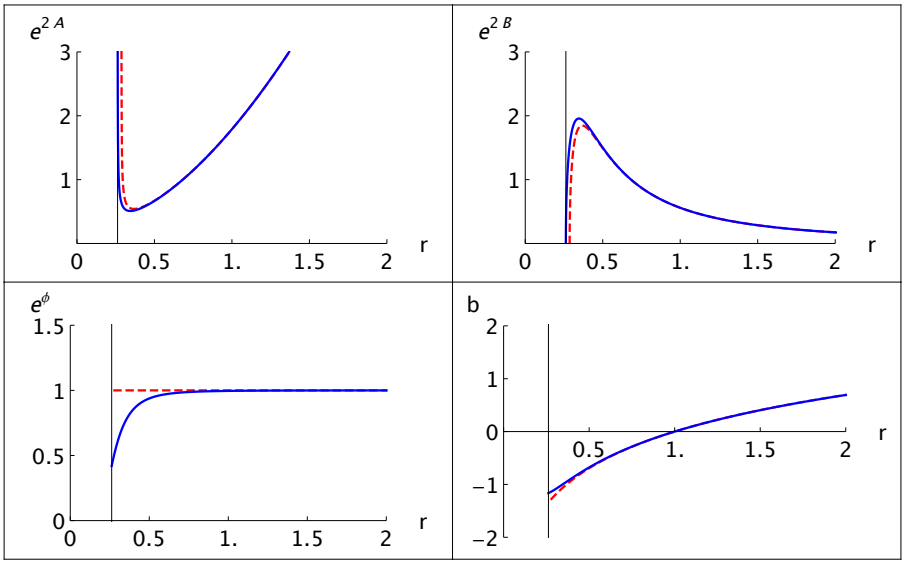


Figure 7.1: *Solutions to the flow equations which follow from $\mathcal{S}_0$ (dashed curve) and $\mathcal{S}_1$ (solid curve) with $r_e > r_s$ for $p = 3$ and $m = 1$, $Q = 1$. Initial conditions where chosen such that the singularity of the SUSY case (dashed) is located at $r_* \approx 0,286505$ and $r_e \approx 0,367879$. For the non-SUSY solution (solid) we have $r_s \approx 0,188588$, $r_e \approx 0,34466$ and a singularity forms at $r_*(r_s, r_e) \approx 0,262082$. The vertical line denotes the point where the non-SUSY solution develops a singularity.*

is that the force on a probe p-brane identically vanishes for all solutions that satisfy the first-order equations given by $\mathcal{S}_0$ and $\mathcal{S}_1(c)$ (or, equivalently, $\mathcal{S}_2(d)$). To see this explicitly we recall the potential on a probe p-brane:

$$\begin{aligned} V &= -T_p \int_{p+1} e^{\frac{([p-3])}{4}\phi} \sqrt{|g_{p+1}|} + Q_p \int_{p+1} C_{p+1} \\ &= -T_p e^{\frac{([p-3])}{4}\phi(r)} e^{(p+1)A(r)} + Q_p \mathcal{C}(r)\,. \end{aligned} \tag{7.52}$$

where we introduced the function $\mathcal{C}(r)$ defined via $C_{p+1} = \mathcal{C}(r)\epsilon_{p+1}$. In our conventions the test brane has $T_p = Q_p$. For all the solutions obeying the first-order flows derived from $\mathcal{S}_0$, $\mathcal{S}_1$ and $\mathcal{S}_2$ the following Hodge-duality relation:

$$\star_{9-p} H_3 = \mathrm{e}^{\frac{p+1}{4}\phi} F_{6-p} \tag{7.53}$$

is satisfied. By the virtue of the H equation of motion, this implies that, up to a constant:

$$\mathcal{C}(r) = e^{(p+1)A + \frac{p-3}{4}\phi}\,. \tag{7.54}$$

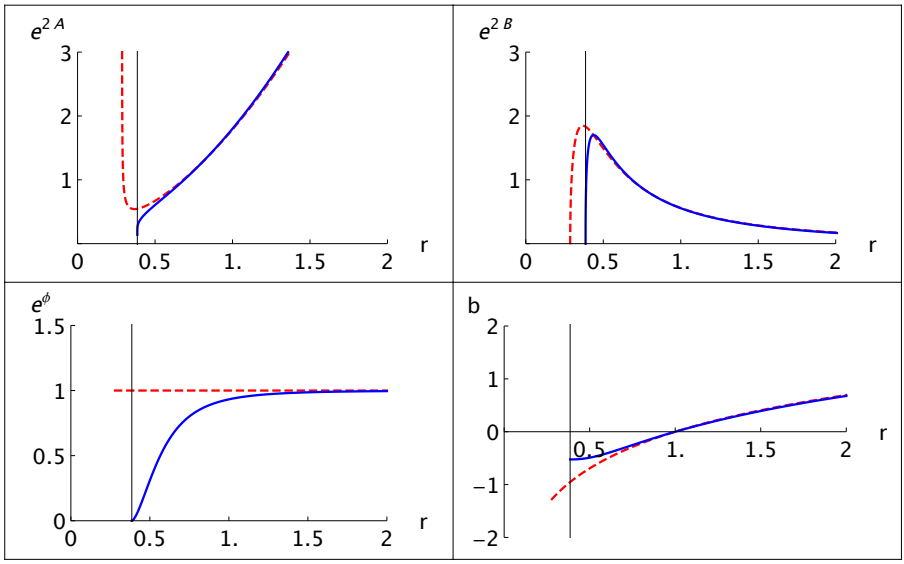


Figure 7.2: *The solutions to the flow equations which follow from $\mathcal{S}_0$ (dashed curve) and $\mathcal{S}_1$ (solid curve) with $r_e < r_s$ for $p = 3$ and $m = 1$, $Q = 1$. Initial conditions where chosen such that the singularity of the SUSY case (dashed) is located at $r_* \approx 0,286505$ and $r_e \approx 0,367879$. For the non-SUSY solution (solid) we have $r_s \approx 0,385706$ and $r_e \approx 0,385133$. The vertical line denotes the point where the non-SUSY solution develops a singularity.*

When this is plugged into the potential for a test Dp-brane (7.52) we find that it vanishes identically. This behavior is well known for the SUSY solutions described by $\mathcal{S}_0$. What it physically means is that the charges and tension that reside in the fluxes (originating from the dissolved fractional branes) are of equal magnitude such that the gravitational attraction cancels exactly the Coulomb-like repulsion [49]. This behavior is typical for SUSY solutions and we find it surprising that it extends to these new solutions.

Note that a second class of solutions exist in which we insert a minus sign in (7.53) as follows $\star_{9-p} H_3 = -\mathrm{e}^{\frac{p+1}{4}\phi} F_{6-p}$. Such solutions will then possess a no-force condition on anti Dp-branes, which are defined by $T_p = -Q_p$ in our conventions.

Concerning the singularity at small radius, we have little insight to offer. The singular behavior for $r_e > r_s$ is similar to the singular behavior of the KT solution, but, unlike the KT solution, we do not yet know whether these solutions are any physical. According to Gubser's criterion for "acceptable" singularities [122], if the systems is put at finite-temperature by introducing a blackening factor into the Ansatz, the singularity should get cloaked by the

horizon to be acceptable. This could be a test, which is however numerically involved, and in Section 7.4 we show that at least a class of finite T solutions can be found from explicit first-order flows that are deformations of the first-order flows we have found here.

But since one class of the solutions has an IR singularity which resembles very closely the singularity of the KT solution, we expect that the deformation of the conifold, as in [148], will resolve the IR singularity. We leave this for future investigation. To check this interesting possibility one has to enlarge our Ansatz as we explain below. Most important for the physical interpretation of the solution and the question of the significance of the singularity in the IR, would be a holographic interpretation along the techniques of [16, 17].

More general Ansätze

Deformations of the SUSY KT background typically involve the running of a $T^{1,1}$ modulus w, defined as follows [62]:

$$\mathrm{d}s^2_{T^{1,1}} = e^{8w}e^2_\psi + e^{-2w}\left(e^2_{\theta_1} + e^2_{\phi_1} + e^2_{\theta_2} + e^2_{\phi_2}\right), \tag{7.55}$$

with $e_\psi, e_{\theta_{1,2}}, e_{\phi_{1,2}}$ a standard labelling of the Maurer-Cartan forms on $T^{1,1}$. Solutions on the deformed conifold, that resolve the KT singularity, require further moduli to be turned on, as follows [149]:

$$\mathrm{d}s^2_{T^{1,1}} = e^{8w}e^2_\psi + e^{-2w}\left(e^{y_1}[e^2_{\theta_1} + e^2_{\phi_1}] + e^{y_2}[e^2_{\theta_2} + e^2_{\phi_2}]\right). \tag{7.56}$$

Principle functions (superpotentials) are known for these more general situations but we have not been able to find extensions that would reproduce the principle functions $\mathcal{S}_1$ or $\mathcal{S}_2$ of equations (7.36, 7.38) upon truncation of the extra moduli.

7.4 Towards first-order flows at finite temperature

In this section we demonstrate how our simplified fractional brane Ansätze allow a finite temperature extension that is still described by a first-order flow equation. The first-order flow equations we find are restricted to the blackening of the simplified Ansatz (7.30) and we have not yet been able to extend the flow to the more general Ansätze used in (7.55) for the KT black brane when $p = 3$ [62, 7]. The latter solutions necessarily include the dynamical shape modulus e^w of the $T^{1,1}$, see equation (7.55). Since our non-SUSY solutions have the same UV as KT we expect the singularity to only get cloaked by a finite temperature

horizon if the relevant $T^{1,1}$ modulus is included. This seems to be confirmed by a brief analysis of our first-order finite temperature flow.

7.4.1 Black domain walls

Acceptable singularities should develop a horizon at sufficiently large temperatures [122]. This can be analysed by introducing a blackening factor h^2 in front of the time component in the metric Ansatz. As before we take the action of D-dimensional gravity coupled to scalar fields subject to a potential (7.9) and we are interested in the following Ansatz for a domain wall at finite temperature:

$$\mathrm{d}s_D^2 = f(z)^2\mathrm{d}z^2 + g(z)^2(-h(z)^2\mathrm{d}t^2 + \mathrm{d}\vec{x}_{D-2}^2)\,. \tag{7.57}$$

If we choose a field basis in which g is replaced by $G \equiv g^{D-1}h$ the effective action takes a simple form:

$$S = \int \mathrm{d}z \left\{ \frac{G}{f}\left[\frac{(D-2)}{(D-1)}\left(\frac{G'^2}{G^2} - \frac{h'^2}{h^2}\right) - \frac{1}{2}h_{ab}\phi'^a\phi'^b\right] - GfV(\phi)\right\}\,, \tag{7.58}$$

The h EOM can be integrated (in any gauge) to obtain:

$$\frac{Gh'}{fh} = v\,, \tag{7.59}$$

where v is constant. This shows that finite temperature extension of domain walls have a very universal character. The constant v can be interpreted as a Noether charge for the radial flow as was observed earlier in [7]. Here we solve for h explicitly. For instance, in the gauge $f = G$ we have:

$$h = \exp(vz + C)\,, \tag{7.60}$$

where C is a constant which we can take to be zero by shifting z. This eliminates one unknown completely. If the field h is integrated out in the effective action we obtain a new effective action but this time the Hamiltonian constraint does not set the energy equal to zero but rather proportional to v^2, in complete analogy with the non-extremality parameter v for spherical black holes[10]. The analogy even extends further since, as for black holes, the non-extremality parameter turns out to be proportional to the product sT with s the entropy density of the domain wall when regarded as a black brane in 10-d and T the temperature [7][11].

[10] To be more precise, the Hamiltonian constraint still sets the energy in (7.4) equal to zero, but integrating out the h-EOM introduces a constant term in the potential proportional to v^2 which can then be reinterpreted as the energy.

[11] Spherical black hole solutions of 4D gravity coupled to massless scalars and Abelian vector fields have the following metric: $\mathrm{d}s^2 = -\mathrm{e}^{2U}\mathrm{d}t^2 + e^{-2U}\left(\mathrm{e}^{-4A(\tau)}\mathrm{d}\tau^2 + \mathrm{e}^{-2A(\tau)}\mathrm{d}\Omega_2^2\right)$ where $e^A = v^{-1}\sinh(v\tau)$ and $v = 2ST$.

Let us now briefly discuss a finite temperature extensions of the non-SUSY solutions presented in section (7.3.3).

7.4.2 Non-SUSY at finite temperature

Interestingly one can find explicit first-order flows for the finite T solutions, similar to what has been noticed for black holes [164] and for black branes in flat space [138]. To illustrate this we take $p = 3$ and h_{ab}, $V(\phi)$ in (7.58) are defined as in (7.32, 7.33). The Hamilton-Jacobi equation for the principle function at finite temperature is:

$$\begin{aligned} &\frac{G^2}{3}(\partial_G \mathcal{S})^2 - \frac{h^2}{3}(\partial_h \mathcal{S})^2 - \frac{1}{2}(\partial_\varphi \mathcal{S})^2 \\ &- \frac{1}{2}(\partial_\phi \mathcal{S})^2 - \frac{1}{2}e^{-\sqrt{\frac{3}{5}}\varphi+\phi}(\partial_b \mathcal{S})^2 + G^2 V = 0\,. \end{aligned} \tag{7.61}$$

We have found the following solution:

$$\begin{aligned} \mathcal{S}_T &= G\left(mbe^{\sqrt{\frac{5}{3}}\varphi} \pm 10e^{\frac{2}{\sqrt{15}}\varphi}\cosh\left[\mathcal{P}\right]\right)\,, \\ \mathcal{P} &= \frac{1}{\sqrt{5}}\left(\sqrt{3}\ln(h)\sin\alpha_1 + \sqrt{2}\phi\cos\alpha_1 + \alpha_2\right)\,. \end{aligned} \tag{7.62}$$

For $\alpha_1 = 0$ we recover our previous principle function at zero temperature (7.36). Let us now briefly discuss the finite temperature solution. We have two free parameters or constants of motion in $\mathcal{S}_T$, α_1 and α_2. HJ theory gives us two algebraic equations, namely the following constants of motion (7.6):

$$\begin{aligned} \beta_1 &= \frac{\partial \mathcal{S}_T}{\partial \alpha_1} \\ &= \pm 10 G e^{\frac{2}{\sqrt{15}}\varphi}\sinh\left[\frac{1}{\sqrt{5}}\left(\sqrt{3}\ln(h)\sin\alpha_1 + \sqrt{2}\phi\cos\alpha_1 + \alpha_2\right)\right]\cdot \\ &\qquad \cdot\frac{1}{\sqrt{5}}\left(\sqrt{3}\ln(h)\cos\alpha_1 - \sqrt{2}\phi\sin\alpha_1\right)\,, \end{aligned} \tag{7.63}$$

$$\begin{aligned} \beta_2 &= \sqrt{3}\frac{\partial \mathcal{S}_T}{\partial \alpha_2} \\ &= \pm 10\sqrt{\frac{3}{5}} G e^{\frac{2}{\sqrt{15}}\varphi}\sinh\left[\frac{1}{\sqrt{5}}\left(\sqrt{3}\ln(h)\sin\alpha_1 + \sqrt{2}\phi\cos\alpha_1 + \alpha_2\right)\right]\,. \end{aligned} \tag{7.64}$$

From this we deduce that:

$$h = e^{\sqrt{\frac{2}{3}}\phi \tan\alpha_1 + \frac{\beta_1}{\beta_2}\sec\alpha_1} \,. \tag{7.65}$$

If we use this relation in $\mathcal{S}_T$ then it is obvious that the remaining first-order equations are very similar to the $\mathcal{S}_1$ flow at zero temperature. The analysis of section 7.3.3 can be repeated to find that a horizon can only form in combination with a singularity such that it cannot get cloaked from the perspective of the $\mathcal{S}_T$ flow. This is expected since the known finite temperature KT extension requires the $T^{1,1}$ modulus e^w introduced in (7.55) to have a nontrivial flow [7]. Nevertheless, it is interesting that explicit solutions to the HJ equation can be found even at finite temperature. The non-SUSY solutions could have a finite temperature extension similar to the known KT extension if the relevant $T^{1,1}$ modulus is included. However, finding first-order equations for such systems is nontrivial and we leave this for future work.

7.5 Discussion

We have found new SUSY-breaking solutions in type IIA/B supergravity describing throat geometries supported by fractional p-brane fluxes. The SUSY-breaking is such that the large radius limit of the solutions is the same as the SUSY solutions [132] and the force on a probe Dp-brane inside the throat vanishes. Solutions of this kind could be useful as holographic backgrounds, especially when $p = 3$, where the solution describes a deformation of the Klebanov-Tseytlin background [149], dual to a cascading $SU(N) \times SU(N+M)$ SUSY gauge theory. It is of general interest to extend holographic studies of strongly coupled gauge theories to situations in which SUSY is broken. On the supergravity side this is typically done using a perturbative approach [56] that can capture small deviations from the SUSY solution.

In this chapter we have shown that one can do better and can integrate the second-order equations to first-order integrations for deformations that preserve the no-force condition on a test Dp-brane inside the throat. We have organised the computations using an effective one-dimensional action that is naturally obtained after reducing the fractional branes to Minkowski-sliced domain wall solutions in $p+2$ dimensions. As a side comment we have suggested (see appendix D) that the AdS-sliced domain walls of the same $(p+2)$-dimensional theories lift to a class of new AdS compactifications. Only when $p = 6$ have these AdS compactifications been found explicitly [51, 14, 140] and they appear to be SUSY for certain choices of integration constants [14, 140].

Concerning the newly found non-SUSY deformations of the fractional branes of [132] we have not yet explored their physical meaning. Like the SUSY backgrounds, naked singularities arise at small distances and this could be a worry. We found two classes of solutions and for one class the singularity is in all respects the same as the KT singularity. As future work one might attempt to find a finite temperature regularisation which would make the solutions physically acceptable [122]. We have initiated such a search by providing explicit solutions to the Hamilton-Jacobi equation at finite temperature. Unfortunately, our solutions most likely do not contain enough shape moduli of the $T^{1,1}$ in order to allow for smooth geometries. Another possible direction is to find a (non-supersymmetric) deformation of the $6d$ conifold geometry that renders the 10-d solution regular exactly as it happens for the (supersymmetric) Klebanov-Strassler background. In fact, the gauge theory analysis of the non-SUSY no-force solution of [99] shows that their supergravity background describes only one of two UV perturbations related by the $\mathcal{Z}_2$ symmetry of the dual quiver gauge theory. It is plausible that (the regular version of) the solution we found in this chapter is exactly the missing gravity background mentioned in [99].

One can also compute correlation functions of the dual gauge theory holographically to shed light on the relevance of the new supergravity solutions we have found [16, 17].

After the publication of this work it was observed in [45] that for $p = 3$ the non-SUSY deformation presented in this chapter corresponds to explicit supersymmetry breaking in the dual field theory. Moreover, the deformation in this chapter ($p = 3$) was recently found to survive the Klebanov-Strassler resolution [153]. This makes the background potentially interesting for studying non-SUSY cascading gauge theories holographically.

Chapter 8

Holography, Brane Intersections and Six-dimensional SCFTs

This chapter is a reprint of [55] where we study supersymmetric intersections of NS5-, D6- and D8-branes in type IIA string theory. We focus on the supergravity description of this system and identify a "near horizon" limit in which we recover the recently classified supersymmetric seven-dimensional AdS solutions of massive type IIA supergravity. Using a consistent truncation to seven-dimensional gauged supergravity we construct a universal supersymmetric deformation of these AdS vacua. In the holographic dual six-dimensional (1,0) superconformal field theory this deformation describes a universal RG flow on the tensor branch of the vacuum moduli space triggered by a vacuum expectation value for a protected scalar operator of dimension four.

8.1 Introduction

Six-dimensional interacting SCFTs provide an interesting and exotic corner of the landscape of consistent QFTs. Early hints for their existence came from studying the low-energy dynamics of brane intersections in string and M-theory [187, 61, 126]. It is believed that the list of six-dimensional $\mathcal{N} = (2, 0)$ SCFTs is exhausted by the theories labeled by the ADE algebras. The kaleidoscope of $\mathcal{N} = (1, 0)$ SCFTs appears to be much richer and a full classification of such

theories is still lacking. Recently there has been a revival in this area sparked by advances in F-theory [131, 89, 129] and holographic constructions [109, 14, 13, 182, 12], as well as our better understanding of the anomaly polynomials of six-dimensional supersymmetric theories [168][1]. This renewed interest is well justified, since understanding the structure of six-dimensional interacting CFTs is bound to teach us important lessons about the mysterious theory living on the world-volume of M5-branes. In addition, compactifications of six-dimensional theories lead to new insights into the physics of lower-dimensional QFTs and the dualities that they enjoy.

Our interest here is in the class of linear quiver six-dimensional SCFTs introduced in [61, 126] and explored recently with new tools by Gaiotto and Tomasiello [109]. In field theory language this setup is the six-dimensional analog of the usual Hanany-Witten type linear quivers which are well-studied in the context of three-dimensional $\mathcal{N} = 4$ [125] and four-dimensional $\mathcal{N} = 2$ [206] theories. One starts with a particular brane intersection of NS5-, D6-, and D8-branes in type IIA string theory in which the branes share five flat spatial and one temporal direction[2]. When the NS5-branes are separated along the worldvolume of the D6-branes one has a description of the low-energy theory as a six-dimensional quiver gauge theory. Each segment of n D6-branes leads to an $SU(n)$ gauge group. The D8-branes transverse to each segment add "flavor" hyper multiplets in the fundamental of the gauge group, while the NS5-branes cary bi-fundamental hyper multiplets. The relative separation between the NS5-branes is controlled by the real scalar in a six-dimensional tensor multiplet. When the vacuum expectation value for this scalar vanishes the NS5-branes coincide and one finds an interacting $\mathcal{N} = (1, 0)$ SCFT. It was argued in [109] that when the number of NS5-branes is large these SCFTs admit a dual holographic description in terms of type IIA supergravity on the AdS_7 backgrounds classified and studied in [14]. These AdS_7 solutions are constructed directly in type IIA supergravity without any direct reference to the underlying brane construction [14]. While there is substantial evidence for the validity of the holographic duality proposed in [109] (see for example [75]) we believe that there is room for improvement.

The "gold standard" of the AdS/CFT correspondence is the duality between type IIB string theory on the $AdS_5 \times S^5$ background and the $\mathcal{N} = 4$ SYM theory [157]. The key to understanding this duality is provided by the underlying D3-branes. To obtain the $AdS_5 \times S^5$ solution of type IIB string theory in the supergravity limit one starts from the asymptotically flat space solution describing N coincident D3-branes, which in turn can be thought of as an

[1] See [198] for a review on these recent developments with a more exhaustive list of references, and [46] for a Lagrangian-based approach to classifying anomaly-free six-dimensional supersymmetric QFTs.

[2] This is summarized in table 8.1 below.

extremal black brane. Then one takes an appropriate near-horizon limit to isolate the $AdS_5 \times S^5$ region. The same procedure can be applied to D3-branes at singular CY three-folds and it leads to a plethora of AdS_5/CFT_4 holographically dual pairs. The AdS_7/CFT_6 duality studied in [109] is on a different footing. The reason is that the AdS_7 solutions of [14] have not been shown to arise from some type of near-horizon limit of intersecting brane solutions in massive type IIA supergravity. The goal of our work is to fill in this gap.

Our starting point is a careful analysis of the system of BPS equations derived by Imamura in [135]. These equations control supersymmetric solutions of massive type IIA supergravity which should describe the backreaction of a system of intersecting NS5-, D6-, and D8-branes. Finding solutions to these non-linear partial differential equations in general is a non-trivial problem. We make progress using several different approaches. First, we impose an Ansatz for all background fields of type IIA supergravity which is invariant under the isometries of AdS_7. Upon a judicious choice of coordinates this leads to a drastic simplification and the BPS equations reduce to a simple system of coupled ordinary differential equations which we solve explicitly. In this way we recover the supersymmetric AdS_7 solutions classified in [14]. Equipped with these explicit solutions we then proceed to study deformations which break the isometries of AdS_7 and are holographically dual to supersymmetric RG flows in the $\mathcal{N} = (1, 0)$ SCFTs of [109]. An important technical ingredient in our analysis is the existence of a consistent truncation of massive type IIA supergravity to minimal seven-dimensional gauged supergravity established in [170]. The holographic RG flows of interest are particularly simple analytic solutions of this seven-dimensional supergravity which can be readily uplifted to ten or eleven dimensions. The uplifted backgrounds in turn provide nontrivial examples of explicit analytic solutions to the non-linear PDEs of [135]. These backgrounds can be interpreted as sourced by smeared NS5-branes in type IIA supergravity with a particular charge density controlled by the conformal symmetry breaking parameter in the dual RG flow. Equipped with some intuition from these explicit solutions we are able also to construct more general supersymmetric backgrounds in type IIA supergravity with vanishing Romans mass. They correspond to a general charge distribution of NS5-branes along a stack of D6-branes.

In addition to understanding how the AdS_7 solutions of [14] arise as the particular brane intersections suggested by the field theory construction of [109] a further motivation for our work is to study supersymmetric deformations of these six-dimensional SCFTs using holography. The deformations of AdS_7 mentioned above, correspond to supersymmetric RG flows in the dual SCFT triggered by a dimension four scalar operator. This operator is the lowest component in the energy-momentum tensor multiplet and is thus present in

every $\mathcal{N} = (1, 0)$ SCFT. In harmony with the results in [73] we find that the only possible supersymmetric and Lorentz-invariant deformation of the $\mathcal{N} = (1, 0)$ SCFT at hand is realized by turning on a vacuum expectation value (vev) for this operator. This vev parametrizes a particular direction in the tensor branch of the $\mathcal{N} = (1, 0)$ SCFT. Our holographic construction suggests that such RG flows on the tensor branch, at least in some appropriate large N limit, have a universal nature which is independent of the details of the six-dimensional theory.

We start our exploration in the next section by reviewing the salient features of the 6d $\mathcal{N} = (1, 0)$ SCFTs arising from intersecting D6-, NS5-, and D8-branes in type IIA string theory. In Section 8.3 we switch gears to supergravity to discuss the intersecting brane Ansatz and BPS equations of [135] and show how the AdS_7 solutions of [14, 182] arise as solutions of these equations. Section 8.4 is devoted to a construction of an explicit supergravity solution which is holographically dual to a particular tensor branch deformation in the six-dimensional SCFTs. In Section 8.5 we discuss a new type IIA supergravity solution which describes an intersection of NS5- and D6-branes and relate it to the discussion in Section 8.4. We conclude with a brief summary of our results and possible directions for future study in Section 8.6. Appendix E contains our conventions, some details on the derivation of the AdS_7 solutions of interest, and an explicit relation between the BPS equations derived in [135] and those of [14, 182].

Note added: After the submission of this manuscript to the arXiv we became aware of the work in [156] which has partial overlap with our results in Section 3.

8.2 Brane intersections and six-dimensional SCFTs

We are interested in six-dimensional $\mathcal{N} = (1, 0)$ supersymmetric QFTs. These theories preserve eight real chiral supercharges and the R-symmetry group is $SU(2)$. The supersymmetric multiplets are the usual vector and hyper multiplets familiar from theories with eight supercharges in three and four dimensions, as well as the more exotic tensor multiplet. The only bosonic field in the vector multiplet is the gauge field A_μ with field strength $F_{\mu\nu}$. Therefore, in contrast to three and four-dimensional supersymmetric theories, there is no Coulomb branch of the vacuum moduli space since there are no scalars in the vector multiplet. In the hyper multiplet we have four real scalars. These parametrize the Higgs branch which has a structure similar to the one of four-dimensional

$\mathcal{N} = 2$ theories. The tensor multiplet contains one real scalar field, ϕ, and a two-form tensor potential, $b_{\mu\nu}$, with a self-dual field strength, $h_{\mu\nu\rho}$. The vacuum expectation value of the scalar, ϕ, in the tensor multiplet parametrizes a branch of the vacuum moduli space called the tensor branch. This will play an important role in our story. To illustrate how this works schematically we present the relevant terms of the bosonic Lagrangian for an Abelian tensor multiplet coupled to a gauge field:

$$\mathcal{L} \supset \phi \mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu}) + \partial_\mu\phi\partial^\mu\phi + h_{\mu\nu\rho}h^{\mu\nu\rho} + \star\,(b \wedge \mathrm{Tr}\,(F \wedge F)) \ . \tag{8.1}$$

The operator ϕ is gauge invariant and its classical scaling dimension is 2. Its vacuum expectation value, $\langle\phi\rangle$ parametrizes the tensor branch of the moduli space. Here we have restricted ourselves to one tensor multiplet for simplicity. The vev $\langle\phi\rangle$ can be thought of as the effective gauge coupling $\langle\phi\rangle \sim 1/g^2_{YM}$ and the singular point $\langle\phi\rangle = 0$ should be analyzed with care. Crucial insight from string theory suggests that the limit $\langle\phi\rangle \to 0$ often corresponds to a critical point of the renormalization group flow and thus an interacting SCFT [187]. In fact to the best of our knowledge all known examples of interacting six-dimensional CFTs are supersymmetric and arise from suitable constructions in string, M-, or F-theory.

The six-dimensional supersymmetric theories of interest to us are the linear quivers introduced in [61, 126] and further studied in [109]. These are six-dimensional cousins of the three- and four-dimensional linear quiver gauge theories with eight supercharges [125, 206]. The six-dimensional gauge theories describe the low-energy dynamics of a system of NS5-, D6-, and D8-branes in flat space arranged according to the diagram in table 8.1[3]. The six-dimensional vector multiplets of the gauge theory arise from the worldvolume dynamics of the D6-branes. The gauge group is $SU(n)$ for a segment of n D6-branes in the z direction. The D8-branes intersect the D6-branes at isolated points on the z line and lead to hypermultiplets in the fundamental representation of the gauge group. The NS5-branes are point-like on the line parametrized by z. Each NS5-brane leads to a bi-fundamental hypermultiplet associated with the two stacks of D6-branes that end on the given NS5-brane. In addition each pair of NS5-branes contains a tensor multiplet. The vev for the real scalar field in this multiplet corresponds to the distance between the NS5-branes in the z direction. In general there are many such NS5-branes with generic values of these real vevs. This situation corresponds to a general point on the tensor branch of the six-dimensional theory and is illustrated by the diagram in figure 8.1. When the NS5-branes coincide all the tensor multiplet scalars have a vanishing vev and one is at the origin of the tensor branch where it is expect that a strongly

[3]One could also introduce appropriate orientifold planes in this construction while still preserving $\mathcal{N} = (1, 0)$ supersymmetry. See [109] for more details.

interacting SCFT resides[4]. This is illustrated by the diagram in figure 8.2. These SCFTs are strongly coupled and evade a Lagrangian description. In a suitable limit when the number of coinciding NS5-branes is large it was argued in [109] that these SCFTs are dual to the supersymmetric AdS_7 solutions of massive type IIA supergravity found in [14].

	t	x^1	x^2	x^3	x^4	x^5	z	r	θ	ϕ
NS5	○	○	○	○	○	○				
D6	○	○	○	○	○	○	○			
D8	○	○	○	○	○	○		○	○	○

Table 8.1: The brane intersection in type IIA string theory that leads to the SCFTs and supergravity solutions of interest in this work.

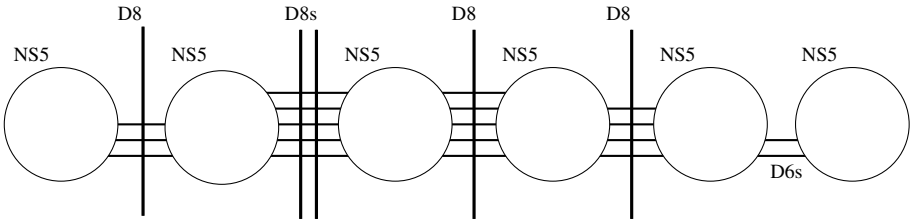


Figure 8.1: An illustrative example of the system of intersecting branes discussed in the main text and in table 8.1.

In the absence of a Lagrangian it is often instructive to adopt an algebraic approach to study SCFTs. Every six-dimensional $\mathcal{N} = (1, 0)$ SCFT should contain an energy-momentum tensor which belongs to a particular short multiplet of the $OSp(8|2)$ superconformal algebra. The bosonic content of the energy-momentum tensor multiplet is:[5] a scalar operator, $\mathcal{O}$, of conformal dimensions 4 which is neutral under the R-symmetry; the $SU(2)$ R-current, $\mathcal{J}_\mu$, which has conformal dimension 5 and is in the spin-1 representation of $SU(2)$; another operator of dimension 5, $\mathcal{S}^+_{\mu\nu\rho}$, which transforms as a self-dual 3-form under the six-dimensional Lorentz group and is neutral under the R-symmetry; and the energy-momentum tensor, $\mathcal{T}_{\mu\nu}$, which is a symmetric rank two tensor of conformal dimension 6 and is neutral under the R-symmetry. It was shown in [73] using superconformal algebraic methods that there are no supersymmetric, Lorentz-invariant, relevant or marginal deformations of

[4] In the absence of D8-branes these six-dimensional theories are the same as the $\mathcal{N} = (1, 0)$ theories of type (A_N, A_k) obtained by placing N M5-branes on a $\mathbb{Z}_k$ singularity in M-theory. Here N and k are the numbers of NS5- and D6-branes, respectively.

[5] See for example table 31 in [73].

$\mathcal{N} = (1, 0)$ superconformal theories (see also [1, 64]). Thus the only possible Lorentz invariant supersymmetric RG flows in such SCFTs are obtained by vevs, i.e. by moving on the vacuum moduli space. This moduli space consists of two branches - the tensor branch where the $SU(2)_R$ symmetry is unbroken and the Higgs branch where it is broken. For a recent review and references to the original literature see [72]. All known six-dimensional interacting SCFTs have a tensor branch. This state of affairs is similar to the situation in four-dimensional interacting $\mathcal{N} = 2$ SCFTs which all appear to have a Coulomb branch. In general the tensor branch is multi-dimensional. For example in the linear quiver gauge theories discussed above each pair of NS5-branes carries a tensor multiplet and thus adds one real dimension to the tensor branch. In anticipation of the supergravity results in Section 8.4 we should point out that the holographic RG flows discussed there describe some particular direction in this multi-dimensional tensor branch. This direction is singled out since it is parametrized by the vev for the dimension 4 scalar operator $\mathcal{O}$ discussed above.

After this short foray into the world of six-dimensional theories with $\mathcal{N} = (1, 0)$ supersymmetry it is time to move to a more detailed discussion of their dual supergravity description.

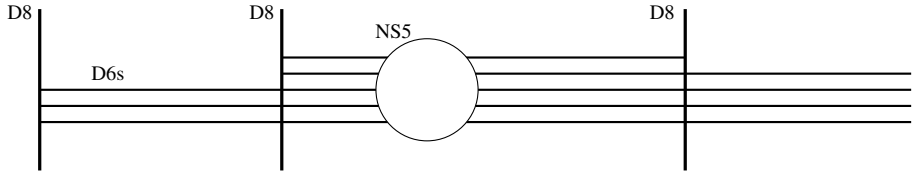


Figure 8.2: A brane configuration that should be described by the conformal limit of a linear quiver gauge theory.

8.3 Supergravity description

Our goal is to construct supersymmetric solutions of massive type IIA supergravity [181] which describe the backreaction of the system of intersecting NS5-, D6- and D8-branes presented in table 8.1. This problem was addressed by Imamura in [135] and below we will heavily exploit his results. Starting from the brane intersection in table 8.1, we impose Poincaré invariance along the shared worldvolume of the branes spanned by $t, x^1, \ldots, x^5$ and unbroken $SO(3)$ isometry along a two-sphere parametrized by the angles θ and ϕ. All background fields in the supergravity theory are in general non-trivial functions of the coordinates r and z. Type IIA supergravity has a number of form fields

which are also assumed to respect the Poincaré symmetry and $SO(3)$ isometry of the metric. These are the RR 2-form F_2 which has legs along θ and ϕ and the NSNS 3-form H which has both $r\theta\phi$ and $z\theta\phi$ components. It was argued in [135] that both the rz component of F_2 and the entire RR 4-form F_4 vanish. Finally in order to preserve 1/4 of the maximal supersymmetry one has to impose that the supersymmetry variations of the type IIA gravitino and dilatino vanish subject to the following projection conditions:

$$\epsilon^2 = \Gamma_{r\theta\phi}\epsilon^1 \ , \qquad \epsilon^2 = \Gamma_z\epsilon^1 \ . \tag{8.2}$$

Here $\epsilon^{1,2}$ are 16-component Majorana-Weyl spinors. The first relation in (8.2) is the familiar spinor projection satisfied by the supersymmetry parameter of D6-branes in flat space, whereas combining the two equations in (8.2) gives the analogous spinor projector for NS5-branes. The resulting BPS equations can be solved and lead to the field configuration[6]:

$$\mathrm{d}s^2 = S^{-1/2}\mathrm{d}s_6^2 + K\left[S^{-1/2}\mathrm{d}z^2 + S^{1/2}(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2)\right] \ , \tag{8.3}$$

$$\mathrm{e}^{2\phi} = g_s^2 K S^{-3/2} \ , \tag{8.4}$$

$$F_2 = -r^2 g_s^{-1}\partial_r S \ \mathrm{vol}_2 \ , \tag{8.5}$$

$$H = -r^2\left[\partial_r K\mathrm{d}z - \partial_z(KS)\mathrm{d}r\right]\wedge \mathrm{vol}_2 \ , \tag{8.6}$$

where $\mathrm{d}s_6^2$ is the flat metric on six-dimensional Minkowski space, $\mathrm{d}\Omega_2^2$ and vol_2 are the Einstein metric and the volume form on the round two-sphere[7] S^2. The functions S and K depend on r and z and can be thought of as the "harmonic functions" associated with D6- and NS5-branes respectively. The Bianchi identities for F_2 and H:

$$\mathrm{d}F_2 - MH = 0 \ , \qquad \mathrm{d}H = 0 \ , \tag{8.7}$$

imply three partial differential equations for S and K:

$$\begin{aligned} \partial_z S - Mg_s K &= 0 \ , \\ \triangle_3 S + Mg_s\partial_z(KS) &= 0 \ , \\ \triangle_3 K + \partial_z^2(KS) &= 0 \ , \end{aligned} \tag{8.8}$$

[6] We work in string frame. Our supergravity conventions can be found in chapter 2 and appendix E.1.

[7] It is compatible with supersymmetry to replace S^2 with $\mathbb{RP}^2$. We thus have this freedom for all supergravity solutions discussed below.

where $\triangle_3 = r^{-2}\partial_r r^2 \partial_r$. For non-vanishing Romans mass, $M \neq 0$, this system can be rewritten as a single non-linear equation for the function S:

$$\triangle_3 S + \frac{1}{2}\partial_z^2 S^2 = 0 \,. \tag{8.9}$$

Given a solution to this equation the function K is then determined through the first equation in (8.8). For $M = 0$ one finds that $\partial_z S = 0$ and the last two equations in (8.8) have to be solved as a coupled system.

The system of equations in (8.8) is in general non-linear which is a well-known feature of the BPS equations controlling brane solutions of massive type IIA supergravity (see for example [137]). In the limit of vanishing Romans mass, $M = 0$, the system in (8.8) becomes linear and should describe backreacted NS5-D6-brane solutions. At this point it is worth presenting some simple well-known solutions of IIA supergravity in the massless limit that fit into this general discussion:

- The solution corresponding to a stack of N_6 D6 branes localized at $r = 0$ is given by:

$$M = 0 \,, \qquad K = 1 \,, \qquad S = 1 + \frac{N_6 g_s}{4\pi r} \,. \tag{8.10}$$

- The solution corresponding to a stack of N_5 NS5-branes localized at $z = r = 0$ is given by:

$$M = 0 \,, \qquad K = 1 + \frac{N_5 g_s}{4\pi^2 (r^2 + z^2)} \,, \qquad S = 1 \,. \tag{8.11}$$

- The solution corresponding to a stack of N_6 branes localized at $r = 0$ and NS5 branes smeared along z with density ρ_5 is:

$$M = 0 \,, \qquad K = 1 + \frac{\rho_5}{4\pi r} \,, \qquad S = 1 + \frac{N_6 g_s}{4\pi r} \,. \tag{8.12}$$

Thus in the massless limit of type IIA supergravity the function K can be thought of as the harmonic function associated with the NS5-branes and the function S the one associated with the D6-branes.

8.3.1 AdS_7 solutions

As reviewed in Section 8.2 one can obtain interacting six-dimensional $(1, 0)$ SCFTs from the intersection of NS5-, D6- and D8-branes in type IIA string

theory summarized in table 8.1. It is thus natural to expect that the system of BPS equations (8.8) admits AdS_7 solutions which provide a dual holographic description of these interacting SCFTs. In this section we determine the conditions on the functions $K(r,z)$ and $S(r,z)$ under which the system of equations (8.8) leads to AdS_7 solutions.

The strategy is to combine the coordinates z and r to form the radial coordinate of AdS_7 which we call ρ. We use the following parametrization of the metric on AdS_7:

$$\mathrm{d}s_7^2 = \frac{1}{(g\rho)^2}\mathrm{d}\rho^2 + (g\rho)\mathrm{d}s_6^2 \ , \tag{8.13}$$

where $\mathrm{d}s_6^2$ is the flat Minkowski metric as in (8.3), and g is related to the AdS radius L through $L = 2/g$. The other independent combination of r and z will form a coordinate which we call α. This coordinate, combined with the coordinates on the two-sphere $\mathrm{d}\Omega_2^2$ in (8.3), forms a three-dimensional space $\mathcal{M}_3$. Upon finding an explicit solution for the metric one then has to properly analyze the global properties of $\mathcal{M}_3$ in order to understand the physics of the AdS_7 solution.

In appendix E.2 we summarize the analysis of equations (8.8) which ensures that the background fields of type IIA supergravity in (8.3)-(8.6) obey the isometries of AdS_7. The upshot is that one finds the following relation between the radial variable of AdS ρ and the coordinates (r,z):

$$\rho^{-1} = g^3(z^2 + 4r^2 S)K \ . \tag{8.14}$$

In addition one finds that the functions S and K must satisfy the following differential constraints:

$$\begin{aligned} 2S + 2r\partial_r S + z\partial_z S = 0 \ , \\ 3K + 2r\partial_r K + z\partial_z K = 0 \ , \\ -z\partial_r K + 2r\partial_z(KS) = 0 \ . \end{aligned} \tag{8.15}$$

The first two equations in (8.15) can be integrated to give:

$$K = \frac{2}{z^3}G(r/z^2) \ , \qquad\qquad S = \frac{1}{2g^2 r}y(r/z^2) \ , \tag{8.16}$$

where y and G are so far undetermined functions of the variable r/z^2. One can then show that the internal coordinate α is also a function of r/z^2. It proves convenient to use the following parametrization of α:

$$\alpha \equiv 4g^2\left(g^2 + 2\frac{r}{z^2}y(r/z^2)\right)G(r/z^2) = \frac{2gz}{\rho} \ , \tag{8.17}$$

In addition it is beneficial to define the following function of r/z^2:

$$\beta(\alpha) \equiv \frac{r}{2g^2z^2}\alpha(r/z^2)^2 \ . \tag{8.18}$$

It is important to emphasize that since α depends only on r/z^2 from now on we will consider β and y to be a function solely of α. After all of these coordinate changes and redefinitions one can show that the system of BPS equations in (8.8) together with the constraints in (8.15) reduce to the following pair of simple ODEs:

$$\begin{aligned} 2y(\alpha)y'(\alpha) - Mg_s =& 0 \ , \\ 2y(\alpha)\beta'(\alpha) + \alpha =& 0 \ , \end{aligned} \tag{8.19}$$

where the prime denotes a derivative with respect to α. Our analysis so far has shown that a solution to the equations in (8.19) together with the definitions in (8.16), (8.17), and (8.18) leads to an AdS_7 solution to the system of BPS equations in (8.8). In fact, the metric and background fields of type IIA supergravity can now be written explicitly in terms of α, y, and β:

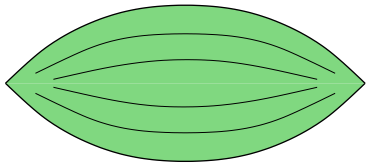
$$\begin{aligned} \mathrm{d}s^2 =& \sqrt{\frac{\beta}{y}}\left(\mathrm{d}s_7^2 + \frac{1}{g^2\beta y}\left(\frac{\mathrm{d}\alpha^2}{4} + \frac{(\beta y)^2}{\alpha^2+4y\beta}\mathrm{d}\Omega_2^2\right)\right) \ , \\ \mathrm{e}^{4\phi} =& \frac{16g^4g_s^4\beta^3}{y^3(\alpha^2+4y\beta)^2} \ , \\ F_2 =& \frac{1}{2g^2g_s}\left(y + \frac{Mg_s\beta\alpha}{\alpha^2+4y\beta}\right)\mathrm{vol}_2 \ , \\ H =& \frac{\beta}{2g^2y(\alpha^2+4y\beta)}\left(3y - \frac{2Mg_s\beta\alpha}{\alpha^2+4y\beta}\right)\mathrm{d}\alpha\wedge\mathrm{vol}_2 \ . \end{aligned} \tag{8.20}$$

It is worth pointing out that the general conditions for the existence of supersymmetric AdS_7 solutions of type II supergravity were first derived in [14][8]. In appendix E.3 we show that the background in (8.20) together with the differential equations in (8.19) provide a solution to the system of differential equations derived in [14].

We end this section with a discussion on the solutions of the differential equations in (8.19) and their interpretation in terms of branes[9]. In the absence of D8-brane

[8]Similar solutions were studied also in earlier work [51] where the authors write down a general AdS_7 Ansatz in massive type IIA supergravity and find non-supersymmetric solutions of this type.

[9]Analytic AdS_7 solutions were constructed also in [182, 13] and further analyzed in [75]. Further numerical analysis of such solutions can be found in [14].



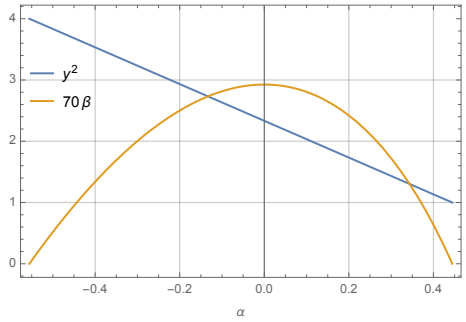


Figure 8.3: The solution (8.22-8.23) with $Mg_s = -3$, $c_1 = 7/3$ and $c_2 = -6$. The coordinate range for α is $[-5/9, 4/9]$. Notice that y^2 is a decreasing function of α because of the negative mass and that it takes non-zero values at both poles indicating the presence of D6-branes at both poles.

charge we have $M = 0$ and the solution to (8.19) is:

$$\beta = \frac{1}{4y}(c_2^2 - \alpha^2) \ , \qquad y = \frac{g^2 N_6 g_s}{2\pi} \equiv \sqrt{c_1} \ , \tag{8.21}$$

where c_1 and c_2 are integration constants. One can show that this is the dimensional reduction of the well-known $AdS_7 \times S^4/\mathbb{Z}_{N_6}$ supersymmetric background of eleven-dimensional supergravity to type IIA supergravity[10]. The coordinate range for α is determined by positivity of $y\beta$ which shows that $-c_2 \leq \alpha \leq c_2$. The solution possesses a D6-brane singularity at $\alpha = \pm c_2$ and the D6 charge at these points is determined by the value of y there as shown in (8.21). The NS5 brane charge is controlled by the parameters c_2 and g which is related to the AdS_7 scale via $g = 2/L$.

In general, for non-vanishing Romans mass, $M \neq 0$, y^2 is a linear function of α:

$$y^2 = Mg_s\alpha + c_1 \ , \tag{8.22}$$

where c_1 is an integration constant. One can then solve the second equation in (8.19) in terms of a cubic polynomial in y:

$$\beta = \frac{P(y)}{3(Mg_s)^2} \ , \quad \text{where} \quad P(y) = -y^3 + 3c_1 y + c_2 \ , \tag{8.23}$$

where c_2 is another integration constant. The same principles hold here as for the massless solution. The coordinate range of α is determined by the positivity

[10]The $\mathbb{Z}_{N_6}$ orbifold acts on S^4 in a way that preserves 16 of the 32 supercharges of $AdS_7 \times S^4$.

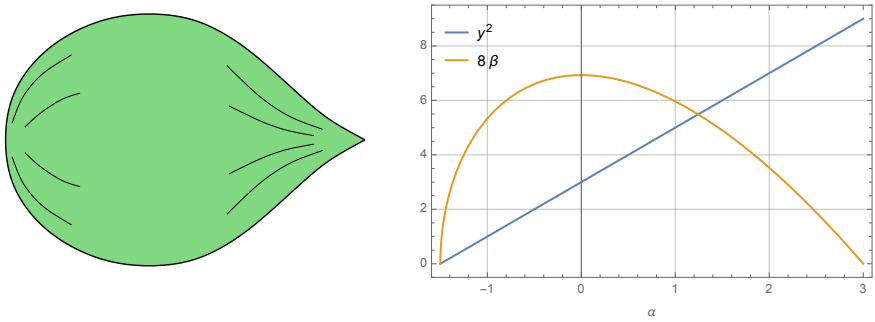


Figure 8.4: An example of the solution of (8.22-8.23) with $Mg_s = 2$, $c_1 = 3$ and $c_2 = 0$. The geometry has a stack of D6 branes at one pole, $\alpha = 3$, but is regular at the other pole, $\alpha = -3/2$. The function $y(\alpha)$ has a non-zero value at $\alpha = 3$ but vanishes at $\alpha = -3/2$ indicating that only one of the poles has D6 branes.

of the function $y(\alpha)\beta(\alpha)$, i.e. the positivity of the polynomial $yP(y)$. When $P(y)$ has positive discriminant, $\Delta \equiv 27(4c_1^3 - c_2^2)$, it has two non–negative roots, and y takes values between these roots. This solution also possesses D6-brane singularities at the ends of the coordinate range and the D6 charge is determined by the value of y at the singularity (See figure 8.3 for an example). In the special case when $c_2 = 0$, one of the roots of $P(y)$ is at $y = 0$. In this case the D6 charge there vanishes and the metric is regular (See figure 8.4). If the discriminant Δ is negative the polynomial $P(y)$ has only one real root and the coordinate range is between $y = 0$ and the root of $P(y)$, where one again finds a localized D6-brane singularity. This guarantees that the metric has the correct signature and the dilaton is real. This solution is once again singular at $y = 0$, however in this case the singularity is an O6-plane. Finally, if one has $\Delta = 0$, then one finds $c_2^2 = 4c_1^3$ and $c_1 > 0$[11]. In this case $P(y)$ has a double root at $y = -\sqrt{c_1}$ and a single root at $y = 2\sqrt{c_1}$. Imposing that the dilaton is real and the correct signature of the metric leads to the range $y \in [0, 2\sqrt{c_1}]$. At $y = 0$ one has an O6-plane singularity and the singularity at $y = 2\sqrt{c_1}$ corresponds to a localized D6-brane.

Finally, local solutions with different mass parameters M can be patched together after imposing continuity of α, β and y. The patching surfaces where the value of M changes discontinuously are D8-brane singularities [135, 14]. In fact these D8-branes are dielectric, they carry D6 charge and can be understood through the Myers effect [166] as polarized D6-branes as a result of the H-flux in the background [140]. The supergravity solution is built by specifying the values of the mass parameter M, the integration constants c_1, c_2 and the coordinate

[11] The case $c_1 = c_2 = 0$ leads to an unphysical solution.

endpoints α_- and α_+ for each region of constant mass parameter. We label these constants in each region by the superscript (i) where i runs over the number of regions n. An overall shift in the coordinate α together with the constants $c_1^{(i)}$ enables us to shift the coordinate range and hence $\alpha_-^{(1)}$ can be chosen to take any convenient value. The other parameters $\alpha_\pm^{(i)}$ are related by the continuity constraint $\alpha_+^{(i)} = \alpha_-^{(i-1)}$. The total number of constants to be specified a priori is $4n$. Imposing continuity of y and β leads to $2n-2$ constraint equations which in turn reduces the number of free parameters in the solution to $2n+2$. The physical quantities determined by these constants are the n mass parameters $M^{(i)}$, the n dielectric D6 charges embedded in the D8-branes and the two D6 charges at end points, $\alpha_-^{(1)}$ and $\alpha_+^{(n)}$, of the α interval. An example of a solution with two D8-branes in shown in figure 8.5.

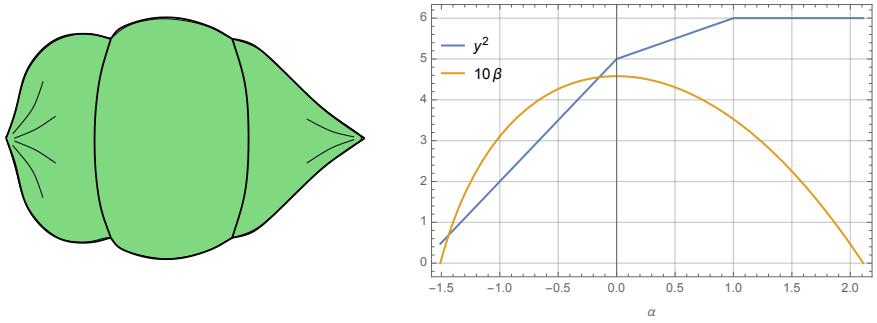


Figure 8.5: A solution of (8.19) with two D8 brane singularities. The mass parameters are $M^{(1)}g_s = 3$, $M^{(2)}g_s = 1$ and $M^{(3)}g_s = 0$ and determine the slope of the linear function y^2. The other integration constants are $c_1^{(1)} = c_1^{(2)} = c_1^{(3)} - 1 = 5$, $c_2^{(1)} = -10$ and $\alpha_+^{(1)} = \alpha_+^{(2)} - 1 = 0$. The remaining constants can be obtained by the continuity of α, β and y. The coordinate α ranges from $\alpha_-^{(1)} \approx -1.51$ to $\alpha_+^{(3)} \approx 2.11$. The reason, only approximate values are given is that these are obtained by setting $\beta(\alpha) = 0$ and are therefore solutions to cubic and quadratic equations respectively.

8.4 Holographic RG flows

After having shown how to construct the supergravity AdS_7 solutions dual to the six-dimensional SCFTs discussed in Section 8.2 we are now ready to study a class of deformations of these theories which have a universal supergravity description. These deformations are described by a particular vacuum expectation value (vev) in the field theory that parametrizes a direction in the tensor branch of the vacuum moduli space. Constructing the gravitational dual description of this

deformation directly in type IIA supergravity is in general a hard task. Here we sidestep this difficulty by exploiting a seven-dimensional effective supergravity description. It was shown in [170] (see also [11]) that supersymmetric vacua of type IIA supergravity of the kind discussed in Section 8.3.1 admit a consistent truncation to a simple seven-dimensional theory known as minimal seven-dimensional supergravity. It is important to emphasize that the details of the particular AdS_7 vacuum of IIA supergravity are not visible in the seven-dimensional theory and are encoded in the way one uplifts seven-dimensional solutions to ten dimensions.

As we show below the universal tensor branch deformation of the SCFT is described by a simple supersymmetric domain wall solution of the seven-dimensional supergravity. Similar domain wall "Coulomb branch" flow solutions and their holographic interpretation were studied in [152, 106, 77]. In particular in [77] (see also [18]) the authors focused on domain wall solutions of the maximal seven-dimensional $SO(5)$ gauged supergravity. Thus the solutions they studied are holographically dual to deformations of the interacting $(2,0)$ SCFT living on the worldvolume of coincident M5-branes. The solution we describe below can be obtained as a limit of the solutions of [77] since the seven-dimensional minimal gauged supergravity is a consistent truncation of the maximal theory studied in [77].

The bosonic sector of minimal supergravity in seven dimensions consists of the metric, a real scalar λ, a 3-form $\mathcal{A}_3$ with field strength $\mathcal{F}_4$ and three gauge fields $\mathcal{A}_1^I$, with field strengths $\mathcal{F}_2^I$, transforming in the adjoint of $SU(2)$. The bosonic action was originally derived in [200]. Here we use the conventions[12] of [170]:

$$S = \int d^7x\sqrt{-g_7}\left\{R_7 - \frac{1}{2}|\mathrm{d}\lambda|^2 - V(\lambda) - \frac{1}{2}X^4|\mathcal{F}_4|^2 - \frac{1}{2}X^{-2}\mathrm{Tr}(|\mathcal{F}_2|^2)\right\}$$
$$+ \frac{1}{2}\int [\mathrm{Tr}(\mathcal{F}_2\wedge\mathcal{F}_2) - g\mathcal{F}_4]\wedge\mathcal{A}_3 \ , \tag{8.24}$$

where:

$$V(\lambda) \equiv -\frac{1}{2}g^2\left(8X^2 + 8X^{-3} - X^{-8}\right) \quad \text{and} \quad X \equiv \mathrm{e}^{\frac{\lambda}{\sqrt{10}}} \ . \tag{8.25}$$

The potential can be written in terms of a superpotential as:

$$V = \frac{1}{2}\left(\partial_\lambda W\right)^2 - \frac{3}{10}W^2 \ , \tag{8.26}$$

[12] We have fixed $h = \frac{g}{2\sqrt{2}}$ in the notation of [170].

where we have defined the superpotential:

$$W \equiv g\left(4\,\mathrm{e}^{\frac{\lambda}{\sqrt{10}}} + \mathrm{e}^{-\frac{4\lambda}{\sqrt{10}}}\right) . \tag{8.27}$$

There are two AdS_7 vacua of this theory which can be found by solving the equation $\partial_\lambda V = 0$. If an AdS_7 vacuum in addition obeys the relation $\partial_\lambda W = 0$ it preserves some supersymmetry. The vacuum at:

$$\lambda = 0 , \qquad V(0) = -\frac{15}{2}g^2 , \tag{8.28}$$

is supersymmetric and thus perturbatively stable. The dimensionless mass of the scalar λ around this vacuum is $m^2L^2 = -8$ where $L = 2/g$ is the AdS_7 scale. This mass is above the BF bound $m^2_{BF}L^2 = -9$ as required for perturbative stability. Using the standard holographic relation:

$$\Delta(\Delta - 6) = m^2L^2 , \tag{8.29}$$

we can conclude that the operator $\mathcal{O}_\lambda$ dual to the scalar λ in the supersymmetric 6d SCFT has dimension $\Delta = 4$. In fact $\mathcal{O}_\lambda$ is the same as the scalar operator, called $\mathcal{O}$ in Section 8.2, in the energy-momentum tensor multiplet and it exists in every $(1, 0)$ SCFT. The $SU(2)$ gauge symmetry is preserved in this vacuum and it is mapped, via the standard holographic dictionary, to the $SU(2)$ R-symmetry in the dual SCFT.

The other AdS_7 vacuum of the minimal gauged supergravity is at:

$$\lambda = \lambda_* = -\frac{2}{\sqrt{10}}\log(2) , \qquad V(\lambda_*) = -5 \times 2^{3/5}g^2 . \tag{8.30}$$

The AdS_7 scale is $L_* = 2^{1/5} \times 3^{1/2}/g$ and one finds that the mass of λ is $m^2_*L^2_* = 12$. This means the the scalar operator in the dual CFT is irrelevant with conformal dimension $\Delta_* = 3 + \sqrt{21} \approx 7.58$. This vacuum does not preserve any supersymmetry and is perturbatively stable within the minimal seven-dimensional supergravity as well as in the supergravity theory discussed in [11]. It is however a perturbatively unstable vacuum of the maximal seven-dimensional $SO(5)$ gauged supergravity as shown in [171]. This vacuum will not play any further role in our discussion.

The domain wall solution we are interested in can be derived by setting the gauge fields and the 3-form in (8.24) to zero and using a standard domain wall Ansatz for the metric and scalar field:

$$\mathrm{d}s^2_7 = \mathrm{d}\eta^2 + \mathrm{e}^{2\mathcal{A}(\eta)}\mathrm{d}s^2_6 , \qquad \lambda(\eta) . \tag{8.31}$$

We would like to emphasize an important point for our further analysis. Any solution of the minimal seven-dimensional supergravity of the form (8.31) can

be uplifted to a solution of massive type IIA supergravity using the results in [170, 11]. There is some freedom in the way this uplift is performed which is encoded in the cubic polynomial $P(y)$ introduced in eq. (8.23). As explained there, $P(y)$ is only piecewise cubic and the singularities of $P(y)$ determine the location of D8 branes where the mass parameter changes value. Here we will stick to a fixed mass, M, and will choose $P(y)$ to be a cubic polynomial. The extension to include D8 branes is straight forward. Using the results in [170] adapted to our notation we find that the full type IIA supergravity background corresponding to a seven-dimensional solution of the type (8.31) is:

$$\begin{aligned}
\mathrm{d}s^2 &= \sqrt{\frac{\beta}{Xy}}\left\{\mathrm{d}s_7^2 + \frac{X^3}{g^2\beta y}\left(\frac{\mathrm{d}\alpha^2}{4} + \frac{(\beta y)^2}{\alpha^2+4X^5y\beta}\,\mathrm{d}\Omega_2^2\right)\right\},\\
e^{4\phi} &= \frac{16g^4g_s^4\beta^3X^4}{y^3(\alpha^2+4X^5y\beta)^2}\ ,\\
F_2 &= \frac{1}{2g^2g_s}\left(y + \frac{Mg_s\beta\alpha}{\alpha^2+4X^5y\beta}\right)\,\mathrm{vol}_2\ ,\\
H &= \frac{\beta}{2g^2y(\alpha^2+4X^5y\beta)}\Bigg\{(2X^5+1)y\mathrm{d}\alpha\\
&\qquad - 2\alpha\frac{(2-X^5)Mg_s\beta\mathrm{d}\alpha + 2\mathrm{d}\left(y^2\beta(X^5-1)\right)}{\alpha^2+4X^5y\beta}\Bigg\}\wedge\mathrm{vol}_2\ ,
\end{aligned} \tag{8.32}$$

where $\mathrm{d}s_7^2$ is the metric in (8.31) and X is the scalar field as defined in (8.25). The functions y and β satisfy the same equations (8.19) as for the undeformed AdS_7 backgrounds. For a fixed mass M they are given by (8.22) and (8.23).

To find supersymmetric domain wall solutions of the form (8.31) we plug this Ansatz in the supersymmetry variations of the seven-dimensional theory and find that any background of this type should obey the following differential equations:

$$\begin{aligned}
\frac{\mathrm{d}\lambda}{\mathrm{d}\eta} &= -\frac{\partial W}{\partial\lambda}\ ,\\
\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\eta} &= \frac{1}{10}W\ .
\end{aligned} \tag{8.33}$$

To solve this system of equations we find it convenient to perform the following change of variables:

$$\mathrm{d}\eta = \frac{\rho\mathrm{d}\rho}{g((\rho^2-\ell_1^2)^4(\rho^2-\ell_5^2))^{1/5}}\ . \tag{8.34}$$

With this at hand one can then solve the system of equations in (8.33) analytically. We will omit the derivation here and only quote the result using notation which fits in the general framework studied in [77]. The non-trivial fields are:

$$\begin{aligned} \mathrm{d}s_7^2 &= \frac{1}{(g\rho)^2(H_1^4H_5)^{2/5}}\mathrm{d}\rho^2 + (g\rho)(H_1^4H_5)^{1/10}\mathrm{d}s_6^2 \ , \\ X(\rho)^5 &= \frac{H_5}{H_1} \ , \end{aligned} \tag{8.35}$$

where:

$$H_1(\rho) = 1 - \frac{\ell_1^2}{\rho^2} \ , \quad H_5(\rho) = 1 - \frac{\ell_5^2}{\rho^2} \ . \tag{8.36}$$

Note that we have used the notation of [77] which is adapted to treating similar domain walls in the maximal seven-dimensional $SO(5)$ gauged supergravity. In particular we have set $\ell_1 = \ell_2 = \ell_3 = \ell_4$ in the notation of [77] thereby making four of the five scalars considered there equal[13]. We choose to present the seven-dimensional domain wall solution in this language in order to make contact with the uplifted supergravity solution discussed in Section 8.4.1 below.

In the limit $\rho \to \infty$ the metric reduces to AdS_7 in the vacuum (8.28), it is convenient to change coordinates in this limit:

$$g\rho = \mathrm{e}^{g\eta} \ , \tag{8.37}$$

such that the metric takes the form:

$$\mathrm{d}s_7^2 = \mathrm{d}\eta^2 + \mathrm{e}^{2\eta/L}\mathrm{d}s_6^2 \ , \quad \text{where} \quad L = \frac{2}{g} \ . \tag{8.38}$$

The canonically normalized scalar field in this limit has the expansion:

$$\lambda = g^2\frac{2}{\sqrt{10}}(\ell_1^2 - \ell_5^2)\mathrm{e}^{-4\eta/L} + g^4\frac{1}{\sqrt{10}}(\ell_1^4 - \ell_5^4)\mathrm{e}^{-8\eta/L} + \cdots \ . \tag{8.39}$$

Since the operator dual to λ is of dimension 4, the coefficient of $\mathrm{e}^{-4\eta/L}$ is proportional to the vev, v, of the dual operator where:

$$v \equiv g^2\frac{2}{\sqrt{10}}(\ell_1^2 - \ell_5^2) \ . \tag{8.40}$$

The source is given by the coefficient of $\mathrm{e}^{-2\eta/L}$ in the UV expansion of the scalar (8.39), and hence vanishes. The fact that the source term in (8.39)

[13] We have taken the integration constants in (8.36) to be negative to make the singularity at the end of the flow apparent.

vanishes is in harmony with the results of [73] where it was shown that the only supersymmetric relevant deformations of six-dimensional SCFT are given by vevs.

It is clear that at values of ρ where either H_1 or H_5 vanishes, the metric is singular. The range of the coordinate ρ is therefore set by the larger of the two integration constants ℓ_1^2 and ℓ_5^2. Without loss of generality we can choose ℓ_1 and ℓ_5 to be positive and thus we find the coordinate range:

$$\max\{\ell_1, \ell_5\} \leq \rho \leq \infty \ . \tag{8.41}$$

The nature of the curvature singularity encountered at the minimum value of ρ depends on which of the two integration constants, ℓ_1 or ℓ_5, is greater. We explore both possibilities below. When $\ell_1 < \ell_5$ the metric locally takes the form as $\rho \to \ell_5$:

$$\mathrm{d}s_7^2 \approx \mathrm{d}\zeta^2 + \sqrt{-v}\,(10\ g\zeta)^{1/8}\,\mathrm{d}s_6^2 \ , \tag{8.42}$$

where we changed coordinates as follows:

$$\rho - \ell_5 = -\frac{v}{\ell_5}\left(\frac{8}{5g}\right)^{3/4}\zeta^{5/4} \ . \tag{8.43}$$

When $\ell_1 > \ell_5$ the metric locally takes the form:

$$\mathrm{d}s_7^2 \approx \mathrm{d}\zeta^2 + \sqrt{v}\left(\frac{2}{5}\right)^{7/4} g^2\zeta^2\mathrm{d}s_6^2 \ , \tag{8.44}$$

where we have defined:

$$\rho - \ell_1 = \frac{\sqrt{10}\ v}{\ell_1}\,(2g)^3\left(\frac{\zeta}{5}\right)^5 \ . \tag{8.45}$$

Finally when $\ell_1 = \ell_5$ the solution trivializes. The scalar is constant, $X = 1$, and the metric is that of the supersymmetric AdS_7 vacuum in (8.28).

The metrics in (8.42) and (8.44) have a curvature singularity and are therefore hard to interpret in the realm of classical supergravity. Fortunately holography and string theory have offered insights into this type of singularities. In particular there are two well-known criteria for deciding which curvature singularities arising in similar holographic domain walls are acceptable [122, 160]. The criterion in [122] states that a singularity is acceptable only if the scalar potential in (8.26) is bounded from above. It is easy to verify that this the case only when $\ell_1 \geq \ell_5$. The Maldacena-Nuñez criterion states that for acceptable singularities in string theory the g_{tt} component of the ten-dimensional Einstein frame metric should be bounded above as the singularity is approached. The results in

Section 8.4.1 and Section 8.4.2 show that applying this criterion again leads to the condition $\ell_1 \geq \ell_5$ for a physically acceptable singularity. From now on we will therefore take $\ell_1 \geq \ell_5$ which, using (8.40), is equivalent to:

$$v \geq 0 \ . \tag{8.46}$$

This result is in harmony with the field theory discussion below (8.1). The parameter v is dual to the vev of a scalar operator that parametrizes a particular direction on the tensor branch. When $v \neq 0$ one may think of this vev as the effective gauge coupling on some locus of the tensor branch, $v \sim 1/g_{YM}^2$. The constraint $v \geq 0$ therefore agrees with this intuition since it implies that the effective couplig g_{YM}^2 is positive.

We would like to end this section with a technical comment that will play a role in the subsequent discussion. As explained in [77], the smaller of the integration constants $\ell_{1,5}$ can be shifted to zero by a redefinition of the coordinate ρ. This amounts to shifting all the integration constants by the smallest one. We will make use of this result to eliminate the constant ℓ_5. The result of this choice is that the seven-dimensional metric and scalar take the same form as before (8.35), but the functions H_1 and H_5 are now:

$$H_1(\rho) = 1 - \frac{\ell_1^2 - \ell_5^2}{\rho^2} = 1 - \frac{\sqrt{10}\ v}{2(g\rho)^2} \ , \qquad H_5(\rho) = 1 \ , \tag{8.47}$$

where we have also made use of (8.40).

8.4.1 Uplift to eleven dimensions

Before interpreting the domain wall in terms of intersecting NS5- and D6-branes in massive type IIA, we first review how the solution can be uplifted to eleven dimensions and interpreted as a distribution of M5-branes. The eleven dimensional metric takes the standard M5 brane form [77] (see also [18]):

$$\mathrm{d}s_{11}^2 = h^{-1/3}\mathrm{d}s_6^2 + h^{2/3}\mathrm{d}s_5^2 \ , \tag{8.48}$$

where:

$$h^{-1} = (g\rho)^3 (H_1 H_2 H_3 H_4 H_5)^{1/2} \sum_{i=1}^{5} H_i^{-1} \mu_i^2 \ , \tag{8.49}$$

$$\mathrm{d}s_5^2 = \sum_{i=1}^{5} \left(H_i^{-1} \mu_i^2 \mathrm{d}\rho^2 + \rho^2 H_i \mathrm{d}\mu_i^2 \right) \ . \tag{8.50}$$

For the domain wall (8.35) the harmonic functions are $H_5 = 1$ and $H_1 = H_2 = H_3 = H_4$ is given in (8.47). The coordinates μ_i parametrize a four-sphere and satisfy $\sum_{i=1}^{5} \mu_i \mu_i = 1$. By a change of coordinates $y_i \equiv \rho\sqrt{H_i}\mu_i$ the five dimensional metric $\mathrm{d}s_5^2$ can be made manifestly flat, $\mathrm{d}s_5^2 = \mathrm{d}y_i \mathrm{d}y_i$. It is simple to verify that the function:

$$h = \frac{4g\rho}{(\sqrt{10}\ v - 2g^2\rho^2)(\sqrt{10}\ v\mu_5^2 - 2g^2\rho^2)} \ , \tag{8.51}$$

is harmonic, up to isolated singularities, in the five-dimensional space spanned by $(y_1, y_2, y_3, y_4, y_5)$. These singularities determine a distribution of M5-branes:

$$-\triangle_5 h = \sigma_{\mathrm{M5}} \ , \tag{8.52}$$

where σ_{M5} is the charge density of this distribution. The charge density was determined in [77] (using the techniques of [152]) to be:

$$\sigma_{\mathrm{M5}} = \left(\frac{2\pi}{g}\right)^2 \left(\frac{4}{10v^2}\right)^{1/4} \Theta\left(\sqrt{10}\ v - 2g^2 y_5^2\right) \delta^{(4)}(y_1, y_2, y_3, y_4) \ . \tag{8.53}$$

Given this charge density the harmonic function h can be written as:

$$h = \int E_{\mathrm{M5}}(\vec{y} - \vec{y}')\sigma_{\mathrm{M5}}(\vec{y}')d\vec{y}' \ , \tag{8.54}$$

where $E_{\mathrm{M5}} \equiv (8\pi^2(y_i y_i)^{3/2})^{-1}$ is the fundamental solution to the Laplace equation in the flat five-dimensional space spanned by y_i.

The eleven-dimensional domain wall solution presented above should be the gravitational dual to a particular direction in the tensor branch of the six-dimensional $(2,0)$ superconformal theory of type A_{N_5} that lives on the worldvolume of N_5 M5 branes. It should also capture an analogous locus on the tensor branch of the $(1,0)$ cousins of this $(2,0)$ SCFTs which are obtained by placing coincident M5 branes at ADE singularities. See [89] for a recent discussion of the tensor branch of these $(1,0)$ SCFTs. It will be very interesting to make the correspondence between holography and field theory on this branch of the moduli space more precise.

8.4.2 Uplift to type IIA supergravity

The seven-dimensional domain wall flow solution in (8.35) can also be uplifted to massive type IIA supergravity via the uplift formulas presented in (8.32) which were derived in [170] (see also [11]). The uplifted solutions can be cast

into the "intersecting brane" form (8.3)-(8.6) for which the metric takes the form:

$$\mathrm{d}s^2 = S^{-1/2}\mathrm{d}s_6^2 + K\left[S^{-1/2}\mathrm{d}z^2 + S^{1/2}(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2)\right] \ . \tag{8.55}$$

The only technical task is to determine how the coordinates r and z which are natural in (8.3)-(8.6) get mapped to the coordinates ρ and α in the uplift formulas (8.32).

We start by analyzing this ten-dimensional uplifted solution for vanishing Romans mass, $M = 0$. Then we have:

$$S(r) = \frac{N_6 g_s}{4\pi r} \ , \tag{8.56}$$

where the parameter N_6 appears as a free constant compatible with the uplift formulas, i.e. it is not determined in terms of any quantity in the seven-dimensional flow solution. The function K is:

$$K = \frac{16 g^3 \rho}{(\sqrt{10}\ v - 2g^2\rho^2)(\sqrt{10}\ v\alpha^2 - 2c_2^2 g^2 \rho^2)} \ , \tag{8.57}$$

where $\alpha \in [-c_2, c_2]$ was defined in (8.17) and ρ is the seven-dimensional radial coordinate as in (8.35). With this at hand we find the following relation between the coordinates (r, z) and (ρ, α):

$$r = (\sqrt{10}\ v - 2g^2\rho^2)\frac{\pi(\alpha^2 - c_2^2)}{8g^4 N_6 g_s} \ , \qquad z = \frac{\rho\alpha}{2g} \ . \tag{8.58}$$

As in the case of M5 branes (8.54) we can express the harmonic function K in terms of a convolution with the fundamental solution of the last equation in (8.8):

$$K = \left(\frac{N_6 g_s}{4\pi}\right) \int \frac{1}{4\pi((z - z')^2 + 4S(r)r^2)^{3/2}} \sigma_{\mathrm{NS5}}(z')\mathrm{d}z' \ , \tag{8.59}$$

where the charge density is defined as:

$$\sigma_{\mathrm{NS5}}(z) \equiv \left(\frac{2\pi}{g}\right)^3 \left(\frac{4}{10v^2}\right)^{1/4} \frac{1}{(c_2 N_6 g_s)^2} \Theta\left(\sqrt{10}\ v - 32 g^3 z^2\right) \ . \tag{8.60}$$

This in turn implies that equation (8.57) provides a solution to the following equation:

$$-\triangle_3 K - S(r)\partial_z^2 K = \sigma_{\mathrm{NS5}}(z) \ . \tag{8.61}$$

This equation is simply the last equation in (8.8) with a non-trivial source provided by the NS5 charge density $\sigma_{\mathrm{NS5}}(z)$. It should be noted that this solution

is not asymptotically flat in ten dimension. Having found the charge density $\sigma_{\rm NS5}$, a full ten-dimensional asymptotically flat solution will be determined in Section 8.5.

We now move to the case in which $M \neq 0$. For the domain wall (8.35) we can express the uplift in terms of the harmonic functions S and K as for the massless case. Once again the D6 "harmonic function" takes a simple form:

$$S = \frac{y}{2g^2 r} \ . \tag{8.62}$$

However, the function K takes a substantially more complicated form:

$$K = \frac{g^3 (12 M g_s)^2 \rho}{(\sqrt{10}\ v - 2g^2\rho^2)(\sqrt{10}\ v P'(y)^2 - 2g^2\rho^2(P'(y)^2 + 12 y P(y)))} \ . \tag{8.63}$$

Here y and ρ are related to the coordinates r and z through:

$$r = -(\sqrt{10}\ v - 2g^2\rho^2)\frac{P(y)}{12(M g_s g)^2} \ , \qquad z = -\frac{\rho P'(y)}{6 M g_s g} \ . \tag{8.64}$$

We should emphasize that the notation we are using here is similar to the one used for the ten-dimensional AdS_7 solutions in Section 8.3.1 since the domain wall solutions at hand are deformations of these AdS_7 vacua controlled by the parameter v. In particular from the three equations in (8.15) only the first one is obeyed by the domain wall with $v \neq 0$ and the other two are broken. This also implies that the function S still has the same form as in equation (8.16) as is evident from (8.56) and (8.62) above.

8.5 Asymptotically flat brane intersections for $M = 0$

In this section we focus on massless type IIA supergravity and find explicitly a supergravity solution that completes the intersecting brane solution found in the previous section to a ten-dimensional asymptotically flat background. For the M5 brane solution in eleven–dimensional supergravity, this task is easily accomplished simply by adding a constant to the harmonic function in (8.54):

$$h = 1 + \int E_{\rm M5}(\vec{y} - \vec{y}')\sigma_{\rm M5}(\vec{y}')d\vec{y}' \ . \tag{8.65}$$

In type IIA supergravity in the presence of both NS5- and D6-branes the situation is more complicated. Naively one is inclined to "add 1" to both

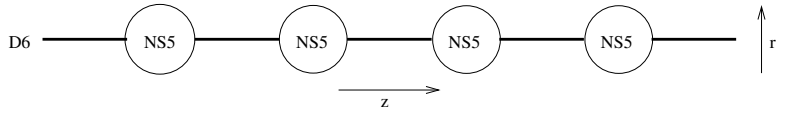


Figure 8.6: The explicit realisation of intersecting branes in type IIA with $M = 0$. A single stack of N_6 D6-branes fills the z-direction while NS5-branes are scattered over the same direction.

functions S and K in order to recover the elementary D6- and NS5-brane solutions in (8.10) and (8.11). However this procedure does not lead to a solution since the system of BPS equations in (8.8) are coupled and nonlinear. In the massless limit, $M = 0$, of type IIA supergravity the problem however reduces to a linear one which we solve below.

Inspired by the solution obtained by uplift in (8.56)-(8.60) we study the intersection of D6 and NS5 branes for which the NS5 are localized at $r = 0$ but spread along the z-direction. The stack of D6-branes is kept localized at $r = 0$ (see figure 8.6). A solution of this type but with a single stack of NS5-branes was constructed previously in [69]. We will start by reviewing that solution and then extend it to a distribution of NS5-branes. Remember that the PDEs in (8.8) are obtained as a result of the Bianchi identities. Let us set $M = 0$ and write these Bianchi identities with explicit brane sources:

$$\begin{aligned} \mathrm{d}F_2 &= -N_6\delta(r)\ r^2\mathrm{d}r\wedge\mathrm{vol}_2\ , \\ \mathrm{d}H &= -N_5\delta(r)\delta(z)\ r^2\mathrm{d}z\wedge\mathrm{d}r\wedge\mathrm{vol}_2\ . \end{aligned} \tag{8.66}$$

The effect of adding explicit brane sources on the right hand side of the Bianchi identities is the following modification of the PDEs in (8.8):

$$\begin{aligned} \partial_z S &= 0\ , \\ -\triangle_3 S &= g_s N_6\delta(r)\ , \\ -\triangle_3 K - \partial_z^2(KS) &= N_5\delta(r)\delta(z)\ . \end{aligned} \tag{8.67}$$

The delta functions serve to fix boundary values of S and K when an explicit solution is written down. The function S is independent of z and is found to be:

$$S = a_1^2 + \frac{N_6 g_s}{4\pi r}\ , \tag{8.68}$$

where a_1 controls part of the asymptotic behavior of the solution. Notice that the uplift of the seven-dimensional domain wall solution lead to the function S

in (8.56), i.e. to $a_1 = 0$. Here we will explore the more general situation with $a_1 \neq 0$.

The general system of equations in (8.8) possesses two scaling symmetries. These symmetries act on the fields and coordinates as follows:

$$\begin{aligned} r \to r' = s^2 r \ , \qquad & z \to z' = t^2 z \ , \\ x^\mu \to x'^\mu = s^{-1} t x^\mu \ , \qquad & g_s \to g_s' = s^{-2} t^4 g_s \\ S \to S' = s^{-4} t^4 S(r,z) \ , \qquad & K \to K' = s^{-2} t^{-2} K(r,z) \ , \end{aligned} \tag{8.69}$$

where t, s are arbitrary real numbers. One of these scaling symmetries can be used to set $a_1 = 1$ (as long as $a_1 \neq 0$) which we will do from now on. Later on we will be interested in exploring the limit $a_1 \to 0$ which can be achieved by taking the limit $r \to 0$ while keeping g_s finite. With S at hand the function K then satisfies a *linear* PDE:

$$-\triangle_3 K - S(r)\partial_z^2 K = N_5 \delta(r)\delta(z) \ . \tag{8.70}$$

To solve this equation we proceed by a Fourier transform along the z-coordinate:

$$-\triangle_3 \hat{K} + S(r)\lambda^2 \hat{K} = \frac{N_5}{\sqrt{2\pi}}\delta(r) \ , \tag{8.71}$$

where we have set:

$$K(r,z) = a_2^2 + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{K}(r,\lambda) \mathrm{e}^{i\lambda z} \mathrm{d}\lambda \ , \tag{8.72}$$

where a_2 is a constant that will ultimately also control the asymptotic behavior of the solutions. We will use the second scaling symmetry of the system to set $a_2 = 1$ (again assuming that $a_2 \neq 0$). The homogeneous solution to (8.71) is:

$$\begin{aligned} \hat{K} \ = \ & b_1(\lambda) \mathrm{e}^{-|\lambda| r} U\left(1 + \frac{N_6 |\lambda| g_s}{8\pi}, 2, 2|\lambda| r\right) \\ & + b_2(\lambda) \mathrm{e}^{-|\lambda| r} {}_1F_1\left(1 + \frac{N_6 |\lambda| g_s}{8\pi}, 2, 2|\lambda| r\right) \ , \end{aligned} \tag{8.73}$$

where U and ${}_1F_1$ are hypergeometric function. The second term diverges for large r and so we must set $b_2(\lambda) = 0$. Once $b_1(\lambda)$ has been determined the full solution is written entirely in terms of U which is defined by:

$$U(a,b,z) \equiv \frac{1}{\Gamma(a)} \int_0^\infty \mathrm{e}^{-z\tau} \tau^{a-1} (1+\tau)^{b-a-1} \mathrm{d}\tau \ , \quad a > 0 \ . \tag{8.74}$$

We can determine $b_1(\lambda)$ by integrating (8.71) in a ball of radius ϵ and taking $\epsilon \to 0$:

$$-4\pi \lim_{\epsilon\to 0} \int_0^\epsilon r^2(\triangle_3 \hat{K} - S(r)\lambda^2 \hat{K})\mathrm{d}r = \frac{N_5}{\sqrt{2\pi}} \ . \tag{8.75}$$

Only the first term on the left hand side gives a finite contribution as $\epsilon \to 0$ which results in the following equation for $b_1(\lambda)$:

$$b_1(\lambda) = \lambda^2 \frac{N_5 N_6 g_s}{\sqrt{2\pi}\ 16\pi^2} \Gamma\left(\frac{N_6 g_s |\lambda|}{8\pi}\right) \ , \tag{8.76}$$

where $\Gamma(x)$ is the Euler gamma function. It is easy to see that the limit for which N_5 vanishes gives the solution for N_6 D6-branes given in (8.10). A slightly more involved limit is $N_6 \to 0$ for which $S \to 1$ and:

$$K \to 1 + \frac{N_5}{4\pi^2(r^2+z^2)} \ , \tag{8.77}$$

which is the harmonic function for a collection of NS5 branes in (8.11).

We now explore the "near-horizon" limit $r \to 0$ while keeping g_s finite[14]. The scaling symmetries (8.69) show that in order to keep g_s finite, r/z^2 must also remain finite in this limit. This is in good agreement with the analysis in appendix E.2 which shows that for supersymmetric AdS_7 solutions the background fields depend nontrivially only on the combination r/z^2. In this limit $S(r)$ reduces to:

$$S(r) \to \frac{N_6 g_s}{4\pi r} \ . \tag{8.78}$$

We should expect that K also reduces to its AdS_7 form (8.16). To evaluate K in the $r \to 0$ limit we use a convenient expansion of the U-function in terms of the Bessel functions $\mathcal{K}_n$ for large a [196]. The first term in this expansion is:

$$U(a,b,z) \approx 2\frac{\mathrm{e}^{z/2}}{\Gamma(a)} \left(\frac{z}{a}\right)^{(1-b)/2} \mathcal{K}_{1-b}(2\sqrt{az}) \ . \tag{8.79}$$

Using this in (8.73) we obtain:

$$\hat{K}(r,\lambda) \to |\lambda| \frac{N_5 N_6 g_s}{\sqrt{2\pi}\ 16\pi^2} \sqrt{\frac{N_6 g_s}{4\pi r}} \mathcal{K}_1\left(\sqrt{\frac{N_6 g_s r}{\pi}}|\lambda|\right) \ , \tag{8.80}$$

which has the Fourier transform:

$$K(r,z) \to \left(\frac{N_6 g_s}{4\pi}\right)^2 \frac{N_5}{4\pi(z^2+4S(r)r^2)^{3/2}} \ . \tag{8.81}$$

[14]What we refer to as a "near horizon" limit can be thought of as a limit in which one zooms in on the NS5 branes in a controlled manner.

This solution can now be compared to the pure massless AdS_7 solution in (8.21) and indeed we find that (8.81) can be written as:

$$4y\beta = \left(\frac{N_5 y^2}{8\pi}\right)^2 - \alpha^2 \ , \tag{8.82}$$

where $y = N_6 g_s g^2/2\pi$. This then shows that the full solution:

$$K = 1 + \frac{N_5 N_6 g_s}{32\pi^3} \int_{-\infty}^{\infty} \lambda^2 \Gamma(\tfrac{N_6 g_s |\lambda|}{8\pi}) \mathrm{e}^{-|\lambda| r + i\lambda z} U(1 + \tfrac{N_6 |\lambda| g_s}{8\pi}, 2, 2|\lambda| r) \mathrm{d}\lambda \ , \tag{8.83}$$

which describes an intersection of NS5 and D6 branes has an AdS_7 space as its "near-horizon" geometry.

We can construct even more general solutions with continuous NS5 charge distribution σ_{NS5} on the z-axis. To do this we have to modify the right hand side of equation (8.70) to:

$$-\triangle_3 K - S(r)\partial_z^2 K = \sigma_{\mathrm{NS5}}(z) \ . \tag{8.84}$$

Since we have already given the solution for which $\sigma_{\mathrm{NS5}}(z)$ is a delta function in (8.83), we already know the fundamental solution, or Green's function, for the operator $-\triangle_3 - S(r)\partial_z^2$. The homogeneous problem at hand is linear and thus we can use the standard theory of Green's functions to write the solution to the inhomogeneous equation (8.84). The solution is given by convolution of the Green's function with $\sigma_{\mathrm{NS5}}(z)$. Fourier transform maps convolution to simple multiplication and so the solution is:

$$K(r,z) = 1 + \tfrac{N_6 g_s}{32\pi^3} \int_{-\infty}^{\infty} \lambda^2 \Gamma(\tfrac{N_6 g_s |\lambda|}{8\pi}) U(1 + \tfrac{N_6 |\lambda| g_s}{8\pi}, 2, 2r|\lambda|) \hat{\sigma}_{\mathrm{NS5}}(\lambda) \mathrm{e}^{-|\lambda| r + i\lambda z} \mathrm{d}\lambda \ , \tag{8.85}$$

where $\hat{\sigma}_{\mathrm{NS5}}$ is the Fourier transform of σ_{NS5}:

$$\sigma_{\mathrm{NS5}}(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{\sigma}_{\mathrm{NS5}}(\lambda) \mathrm{e}^{i\lambda z} \mathrm{d}\lambda \ . \tag{8.86}$$

In the "near-horizon" limit $r \to 0$ we recover the solution (8.59) obtained in previous section.

We have thus illustrated how one can construct explicit solutions of type IIA supergravity (with $M = 0$) which are asymptotically flat, describe the NS5-D6 brane intersection of interest and have a "near-horizon" AdS_7 limit. Ideally we would like to be able to do the same for the more general NS5-D6-D8 brane intersection system. However this problem is much more difficult. The cause

of trouble are as usual the D8-branes. Due to their presence we have $M \neq 0$ and thus we cannot hope for an asymptotically flat region of space-time. In addition for $M \neq 0$ the BPS equations in (8.8) can be combined into a single non-linear equation for the function S:

$$\triangle_3 S + \frac{1}{2}\partial_z^2(S^2) = 0 \ . \tag{8.87}$$

This is a non-linear PDE for which we were not able to find the general solution. We found a particular solution of this equation in (8.62)-(8.64) by uplifting the seven-dimensional domain-wall background. However due to the non-linear nature of the problem we cannot use this solution as a seed to construct more general solutions by superposition.

8.6 Conclusions

The three main results of our work can be summarized as follows. First, after carefully studying the BPS equations of massive type IIA supergravity which describe NS5-D6-D8-brane intersections we were able to recover the plethora of AdS_7 vacua classified in [14]. This is a satisfying result and provides additional strong evidence that the supergravity AdS_7 solutions of [14] are indeed dual to the six-dimensional $\mathcal{N} = (1, 0)$ SCFT studied in [109]. Furthermore we utilized a consistent truncation of massive type IIA supergravity to the minimal seven-dimensional gauged supergravity to construct an explicit analytic supersymmetric domain wall solution. This supergravity background can be interpreted holographically as describing a supersymmetric RG flow on the tensor branch of the six-dimensional theory. The flow is triggered by the scalar operator of dimension four which resides in energy-momentum tensor multiplet. Finally, we employed the linear structure of the BPS equations of type IIA supergravity with vanishing Romans mass to construct an explicit supergravity solution which describes the configuration of NS5- and D6-branes schematically presented in figure 8.6. Our results lead to many open questions both in supergravity and in field theory and we summarize some of the more pressing ones below.

An important open problem in supergravity is how to find an explicit solution similar to the one in Section 8.5 which describes the brane intersection of NS5- and D6-branes in the presence of non-vanishing D8-brane charge, $M \neq 0$ (and with the possible addition of O6-planes). This solution should admit a "near-horizon" limit in which one recovers the analytic AdS_7 solutions presented in Section 8.3.1. This problem is challenging for at least two reasons. The BPS equations in (8.8) with $M \neq 0$ are non-linear and one cannot readily find

explicit solutions. In addition, due to the presence of D8-branes, one should not expect the background of interest to be asymptotic to flat space and it is a priori not clear what is the correct asymptotic behavior far away from the AdS_7 region.

The domain wall supergravity solutions discussed in Section 8.4 are certainly interesting holographically, however they present a challenge for supergravity. While we have argued that the solutions with $v > 0$ are physical and should be dual to a locus on the tensor branch of the six-dimensional SCFT, they are singular in both seven- and ten-dimensional supergravity. It is crucial to understand how to resolve this singularity since this has the potential to teach us interesting lessons about holography as well as about the mechanisms of singularity resolution in string theory. One possible resolution is that the smeared brane densities found in Section 8.4 localize to branes distributed on a line segment. Such dynamics was observed in the case of smeared NS5-branes where world-sheet instantons lead to clumping of the branes [199]. An alternative possibility is suggested by the fact that the holographic RG flow at hand preserve eight real supercharges and look similar in spirit to the one of the four-dimensional $\mathcal{N} = 2^*$ gauge theory which can be thought of as a mass deformation of $\mathcal{N} = 4$ SYM and has been studied extensively in holography and string theory in [139, 63, 101, 172, 177]. It will certainly be very interesting to settle this question.

The vev deformation described holographically by the supergravity domain wall solutions in Section 8.4 is clearly universal and calls for a better field theory understanding. The scalar operator which drives the flow belongs to the energy-momentum multiplet in the six-dimensional $(1, 0)$ SCFT. The supergravity solution suggests that all such SCFTs with holographic duals exhibit this supersymmetric RG flow on their tensor branch. It is certainly desirable to have a field theory understanding of this universal behavior. It will also be interesting to establish a connection between this RG flow on the tensor branch and the field theory and geometric results for similar RG flows in [136] and [130].

Finally it should be noted that the six-dimensional SCFTs dual to the AdS_7 vacua discussed in Section 8.3.1 admit twisted compactifications to two-, three-, and four-dimensional interacting CFTs with various amounts of supersymmetry [12, 13, 182]. It is natural to expect that these lower-dimensional supersymmetric CFTs will in turn have non-trivial vacuum moduli spaces. It will be very interesting to understand whether the "universal" tensor branch flow of the 6d theory "descends" to some interesting RG flow in the lower-dimensional theory.

We hope that further research will elucidate some of these interesting questions.

Chapter 9

Conclusions

In this thesis we studied the dynamics of branes and fluxes with special emphasis on antibranes and their role in supersymmetry breaking.

In chapter 5 we explained how branes and fluxes can annihilate in flux compactifications as was described for D3-branes and H-flux in [143]. From the tadpole condition it is clear that a change in brane-charge comes together with a change in flux quanta. We provide a brane description of the (external) spacetime Brown-Bunster bubbles that lower the flux numbers and furthermore identify the microscopic description of the flux decay in the internal spacetime. The latter involves brane polarisation. These dynamics are important to assess the stability of antibranes in flux backgrounds. Apart from explaining the general physical picture we have found a new family of local throat geometries with toroidal cycles in which probe antibranes can form metastable states. They are metastable because of a potential barrier that prevents perturbative brane-flux decay such that a tunnelling event is required to decay to a different state.

In chapter 6 we investigate the backreaction of antibranes in flux backgrounds and consider two scenarios. A pheno scenario where the antibrane worldvolume is flat and a second setting with AdS worldvolume. For flat worldvolume it is debated whether a metastable state can exist after backreaction due to perturbative brane-flux decay. We show that the perturbative decay can be prevented in the case of AdS worldvolume. The results generalise the supersymmetric AdS_7 solutions where anti D6-branes polarise into spherical D8-branes [14, 140], which are (meta)stable since a charged sphere attracts the flux in a more delocalised fashion. The polarisation does not occur for the flat $\overline{\text{D6}}$ SUSY-breaking solutions [36, 82] and we expect the latter solutions to decay

perturbatively. The results are an indication of the difficulty in constructing explicit pheno compactifications in string theory.

The non-SUSY fractional p-brane solutions described in chapter 7 form a naked singularity at small distances, much like their supersymmetric analogues [149]. For $p = 3$ the solution was shown to correspond to explicit SUSY breaking [45]. The singularity can be resolved [153] similar to the Klebanov-Strassler situation [148]. Consequently, the background describes the UV of explicit SUSY breaking in Klebanov-Strassler making it a potentially interesting holographic playground for studying non-SUSY cascading gauge theories.

Antibranes need not break supersymmetry. In massive type IIA there exist both SUSY and non-SUSY backgrounds that describe anti D6 and their polarisation into D8 [14, 140]. The BPS equations of massive type IIA supergravity which describe NS5-D6-D8-brane intersections are studied in chapter 8. We were able to recover the plethora of AdS_7 vacua classified in [14]. This is a satisfying result and provides additional strong evidence that the supergravity AdS_7 solutions of [14] are indeed dual to the six-dimensional $\mathcal{N} = (1, 0)$ SCFT studied in [109]. Furthermore, in chapter 8 we construct an explicit analytic supersymmetric domain wall solution describing a supersymmetric RG flow on the tensor branch of the six-dimensional theory. For vanishing Romans mass we construct an explicit supergravity solution which describes the configuration of NS5- and D6-branes schematically presented in figure 8.6.

We conclude that antibranes with AdS worldvolume can form metastable configurations. However, an explicit construction seems out of reach in phenomenologically interesting situations. As a result we have no concrete proof for a landscape of pheno flux vacua in the spirit of KKLT [142] which is still the most explicit string theory model of a small cosmological constant. Perhaps one should take into account the nontrivial topology of polarised branes to understand the stability analysis [70]. It has also been suggested that the metastable state exists for a single antibrane [163, 42, 2]. However, in this situation it is difficult to trust the supergravity approximation and we are in the regime of effective field theory where we have no perturbative way to address the smallness of the cosmological constant.

Appendix A

Useful background

This appendix includes some useful background material from general relativity, differential geometry and type II string theory (supergravity). A lot of the material covered in this appendix can be found in modern textbooks such as [107].

A.1 Differential geometry

This section fixes our conventions for Riemannian and Lorentzian geometry used in general relativity. For Lorentzian geometry we use the mostly plus convention. Define the metric as:

$$\mathrm{d}s^2 = G_{ij}(x)\mathrm{d}x^i\mathrm{d}x^j\,, \tag{A.1}$$

where $\mathrm{d}x^i\mathrm{d}x^j = \mathrm{d}x^i \otimes \mathrm{d}x^j$ is the symmetric tensor product of two coordinate basis one-forms. A p-form is formed using the antisymmetric or wedge product ($\wedge$), locally:

$$\omega^p = \frac{1}{p!}\left(\omega^p\right)_{i_1\cdots i_p}\mathrm{d}x^{i_1}\wedge\cdots\wedge\mathrm{d}x^{i_p}\,. \tag{A.2}$$

We will also use the exterior derivative (d):

$$\mathrm{d}\omega^p = \frac{1}{p!}\left(\partial_k(\omega^p)_{i_1\cdots i_p}\right)\mathrm{d}x^k\wedge dx^{i_1}\wedge\cdots\wedge\mathrm{d}x^{i_p}\,. \tag{A.3}$$

Note that $\mathrm{d}^2 = 0$. The Hodge star ($\star$) relates a p-form with a q-form where $q = n - p$ in n-dimensions:

$$\star\big(\mathrm{d}x^{i_1} \wedge \cdots \wedge \mathrm{d}x^{i_p}\big) = \frac{\sqrt{|G|}}{q!} G^{i_1 j_1} \cdots G^{i_p j_p} \mathrm{d}x^{k_1} \wedge \cdots \wedge \mathrm{d}x^{k_q} \epsilon_{k_1 \cdots k_q j_1 \cdots j_p} \,. \quad \text{(A.4)}$$

We can integrate over n-forms such as the volume-form $\star 1 = \sqrt{|G|}\mathrm{d}^n x$. From the above one can easily prove:

$$|\omega^p|^2 \;\star_{10} 1 = \star_{10} \omega^p \wedge \omega^p \,, \quad \text{(A.5)}$$

where $|\cdot|^2$ denotes the contraction with p (inverse) metrics. Furthermore, p-forms can be integrated over p-cycles. We will also need Stokes' theorem, familiar from electrodynamics:

$$\int_{\Sigma_p} \mathrm{d}\omega^{p-1} = \int_{\partial \Sigma_p} \omega^{p-1} \,, \quad \text{(A.6)}$$

where Σ_p is a p-cycle and ∂ is used to denote its boundary. To discuss curvature we define the affine connection:

$$\Gamma^k_{ij}(G) = \frac{1}{2} G^{kl} \left(\partial_i G_{jl} + \partial_j G_{il} - \partial_l G_{ij}\right) \,, \quad \text{(A.7)}$$

and the Riemann tensor as:

$$\mathcal{R}_{ij}{}^k{}_l(G) = \partial_i \Gamma^k_{jl} - \partial_j \Gamma^k_{il} + \Gamma^k_{im}\Gamma^m_{jl} - \Gamma^k_{jm}\Gamma^m_{il} \,. \quad \text{(A.8)}$$

Furthermore, define the Ricci tensor and Ricci scalar as:

$$\mathcal{R}_{ij} = \mathcal{R}_i{}^k{}_{jk} \,, \quad \mathcal{R} = G^{ij}\mathcal{R}_{ij} \,. \quad \text{(A.9)}$$

The vacuum Einstein equations can now be derived from the following action principle[1]:

$$S_{EH} = \frac{1}{8\pi G_N} \int \star\,(\mathcal{R} - 2\Lambda) = \frac{1}{2\kappa_n^2} \int \mathrm{d}^n x \sqrt{|G|}\,(\mathcal{R} - 2\Lambda) \,. \quad \text{(A.10)}$$

A space is said to be maximally symmetric if the following holds:

$$\mathcal{R}_{ijkl} = \zeta\left(G_{ik}G_{jl} - G_{il}G_{jk}\right) \,, \quad \mathcal{R}_{ij} = \zeta\,(n-1)\,G_{ij} \,, \quad \mathcal{R} = \zeta n(n-1) \,. \quad \text{(A.11)}$$

It can be shown that this is only possible with ζ constant, such that maximally symmetric spaces have constant Ricci scalar. Examples include Minkowski, AdS and dS in Lorentzian signature, for Euclidean signature we have the Wick rotated counterpart, e.g. for dS this gives a sphere. Maximally symmetric spaces are conformally flat and solve Einstein's equations resulting from the above action principle with $\Lambda = \frac{(n-1)(n-2)}{2}\zeta$.

[1] To make the variational problem well defined one should add the Gibbons-Hawking-York boundary term for manifolds with a boundary.

A.1.1 Anti-de Sitter spacetime

We will introduce $(n+1)$-dimensional anti-de Sitter space (AdS_{n+1}) as a maximally symmetric space with negative curvature $\zeta = -\frac{1}{L^2}$. It is a solution in the general theory of relativity. It can be defined as an embedding in $(n+2)$-dimensional flat space with signature $(-++\cdots+-)$

$$\left(x^0\right)^2 - \sum_{i=1}^{n} \left(x^i\right)^2 + \left(x^{n+1}\right)^2 = L^2. \tag{A.12}$$

The above embedding of AdS_{n+1} should help clarify that its isometry group is $SO(n,2)$. We will describe the universal covering space of AdS_{n+1} in different coordinate systems.

Global coordinates

We may introduce coordinates on the hypersurface (A.12)

$$\begin{aligned} x^0 &= L\cosh\chi\cos\tau, \\ x^i &= L\Omega_i \sinh\chi, \quad 1 \le i \le n, \quad \sum_i \Omega_i^2 = 1, \\ x^{n+1} &= L\cosh\chi\sin\tau, \end{aligned} \tag{A.13}$$

where $\chi \geq 0$ and $0 \leq \tau \leq 2\pi$. The Ω_i are a parametrization of an $(n-1)$-dimensional unit sphere. It is straightforward to check that this is a parametrization of (A.12). In fact they cover the whole space and are therefore called global coordinates. Using (A.12) we can find the line element in global coordinates of AdS_{n+1}

$$\mathrm{d}s^2_{AdS} = L^2\left(-\cosh^2\chi \mathrm{d}\tau^2 + \mathrm{d}\chi^2 + \sinh^2\chi \mathrm{d}\Omega^2_{n-1}\right). \tag{A.14}$$

Here we define $\mathrm{d}\Omega^2_{n-1} = \sum \mathrm{d}\Omega_i^2$, which corresponds to the metric of a $(n-1)$-dimensional unit sphere. It should be clear that AdS_{n+1} as defined above is unphysical due to the presence of closed timelike curves (τ is timelike and periodic). We restore causality by redefining τ such that it takes values $-\infty \leq \tau \leq \infty$. The space obtained in this way is the universal covering space of AdS. We will, from this point on make no distinction between AdS and its covering space. There is another often used coordinate system for the covering space of AdS_{n+1} called Poincaré coordinates. They only cover half of AdS, called the Poincaré patch.

Poincaré coordinates

We may introduce Poincaré coordinates $(u, t, y^1, \cdots, y^{n-1})$ on the hypersurface (A.12) in the following way

$$\begin{aligned} x^0 &= \frac{1}{2u}\left(1 + u^2(L^2 + y^2 - t^2)\right), \\ x^i &= Luy^i, \quad 1 \le i \le n-1, \\ x^n &= \frac{1}{2u}\left(1 - u^2(L^2 - y^2 + t^2)\right), \\ x^{n+1} &= Lut. \end{aligned} \tag{A.15}$$

Here, we used the notation $y^2 = \sum_{i=0}^{n-1}(y^i)^2$. The coordinate u satisfies $0 \le u$ and all other coordinates take values in $\mathbb{R}$. The line element in this Poincaré patch then becomes

$$\mathrm{d}s_P^2 = L^2\left[\frac{\mathrm{d}u^2}{u^2} + u^2\left(-\mathrm{d}t^2 + \mathrm{d}y^2\right)\right]. \tag{A.16}$$

Now introduce $z = 1/u$ and rename $y^i \to x^i$ to obtain

$$\mathrm{d}s_P^2 = \frac{L^2}{z^2}\left(\mathrm{d}z^2 - \mathrm{d}t^2 + \mathrm{d}x^2\right). \tag{A.17}$$

The Poincaré coordinates satisfy $0 \le z \le \infty$ and $-\infty \le x^i, t \le \infty$. We have one timelike coordinate t and n spacelike coordinates x^i, z. Depending on the sign of t we reach a future or past horizon when $z \to \infty$. On the other hand, there is a boundary at $z = 0$. This boundary is conformally related to Minkowski space and described by the coordinates t, x^i in the Poincaré patch.

A.1.2 de Sitter spacetime

We will introduce $(n+1)$-dimensional de Sitter space (dS_{n+1}) as a maximally symmetric space with positive curvature $\zeta = \frac{1}{L^2}$. It solves the Einstein equations in general relativity when a cosmological constant is included. It can be defined as an embedding in $(n+2)$-dimensional flat space with signature $(-++\cdots+)$

$$-\left(x^0\right)^2 + \sum_{i=1}^{n+1}\left(x^i\right)^2 = L^2. \tag{A.18}$$

The above embedding of dS_{n+1} should help clarify that its isometry group is $SO(n+1,1)$. covering space?

Global coordinates

We may introduce coordinates on the hypersurface (A.18)

$$\begin{aligned} x^0 &= L\sinh t, \\ x^i &= L\cosh(t/L)\Omega_i, \quad 1 \le i \le n, \quad \sum_i \Omega_i^2 = 1, \end{aligned} \tag{A.19}$$

where Ω_i parametrise an n-dimensional unit sphere and $-\infty \le t \le \infty$. In this way we can cover the whole space and the coordinates are therefore called global. The corresponding line element for dS_{n+1} in global coordinates is:

$$\mathrm{d}s^2_{dS} = -\mathrm{d}t^2 + L^2\cosh(t/L)^2\mathrm{d}\Omega_n^2\,. \tag{A.20}$$

As in the case of AdS there are coordinates analogous to the Poincaré coordinates (A.17), these and much more information can be found in [10].

A.2 Polyforms and the democratic formulation

It will be useful to introduce linear combinations of forms. We will use boldface to indicate the polyform:

$$\mathbf{F} = \sum_n F_n\,. \tag{A.21}$$

We will use this notation for the democratic formulation of the type II theory [40] where in the case of supergravity IIA (B) when F is a Ramond field strength, we need n even (odd) so we will appropriately restrict the summation. We double the number of gauge potentials and field strengths (e.g. n runs over 1, 3, 5, 7 and 9 in IIB) such that we can derive both the equations of motion and Bianchi identities from the action. Moreover, the action will no longer involve any RR gauge potentials. To describe the correct number of d.o.f. we need to impose the following duality relation:

$$\star\,\sigma\mathbf{F} = \mathbf{F}\,, \tag{A.22}$$

where σ is the reversal operator, on 1-forms $\sigma(\omega_1) = \omega_1$ and in general $\sigma(A\wedge B) = \sigma B \wedge \sigma A$. Clearly, σ only introduces a sign depending on the degree of the form it acts on:

$$\sigma(\omega_p) = (-1)^{\frac{p(p-1)}{2}}\omega_p\,. \tag{A.23}$$

We will also define the twisted exterior derivative d_H acting on polyforms as:

$$\mathrm{d}_H \cdot = \mathrm{d}\cdot - H\wedge\cdot\,, \tag{A.24}$$

which satisfies $\mathrm{d}_H^2 = 0$ if H is closed (Bianchi).

Using this formalism we may define the IIA and IIB theory simultaneously (just remember to stick to even and odd polyform field strengths). We have:

$$\mathbf{F} = \mathrm{d}_H \mathbf{C}\,, \tag{A.25}$$

Note that the C_4 gauge potential from equation (2.3) does not agree with the one introduced in this section but differs by a term $\frac{1}{2} B \wedge C_2$. The gauge transformations can be written as:

$$\delta \mathbf{C} = \mathrm{d}_H \boldsymbol{\lambda}\,. \tag{A.26}$$

The NSNS and RR string frame actions can be written as:

$$S = \frac{1}{2\tilde{\kappa}_{10}^2} \int e^{-2\Phi} \left(\star \mathcal{R} + 4 \star \mathrm{d}\Phi \wedge \mathrm{d}\Phi - \frac{1}{2} \star H \wedge H - \frac{1}{4} e^{2\Phi} \star \mathbf{F} \wedge \mathbf{F} \right)\,. \tag{A.27}$$

In the last term we implicitly take the integration over all of spacetime to project out only the 10-form part. We need to impose the H Bianchi $\mathrm{d}H = 0$ and duality $\star\sigma\mathbf{F} = \mathbf{F}$ on top of the eoms following from this action. The Bianchi identities and field strength equations of motion following from this action take a particularly compact form:

$$\mathrm{d}H = 0\,, \tag{A.28}$$

$$\mathrm{d}_H \mathbf{F} = 0\,, \tag{A.29}$$

$$\mathrm{d}\left(e^{-2\Phi} \star H\right) - \frac{1}{2} \left\langle \mathbf{F} \wedge \sigma\left(\mathbf{F}\right) \right\rangle_8 = 0\,, \tag{A.30}$$

$$\mathrm{d}_{-H} \star \mathbf{F} = 0\,. \tag{A.31}$$

The notation $\langle \cdot \rangle_p$ is used to project the polyform onto its p-form part. As a final remark, it is possible to add a 0-form field strength F_0 in the IIA theory. When F_0 is nonzero we call the corresponding supergravity massive IIA (mIIA). The added 0-form field strength acts as a cosmological constant and is often called the Romans parameter or Romans mass. In the case of massive IIA we have to change (A.25) to include Romans mass $F_0 = m$:

$$\mathbf{F} = \mathrm{d}_H \mathbf{C} + m e^B\,. \tag{A.32}$$

A.3 Type II in Einstein frame

This summarises the type II actions in Einstein frame[2]. The bulk type II action takes the form

$$S = \frac{1}{2\kappa_{10}^2}\int \star_{10}\left\{\mathcal{R} - \frac{1}{2}|\mathrm{d}\phi|^2 - \frac{1}{2}\mathrm{e}^{-\phi}|H|^2 - \frac{1}{4}\sum_n \mathrm{e}^{\frac{5-n}{2}\phi}|F_n|^2\right\} , \qquad (A.34)$$

where $\mathcal{R}$ is the curvature scalar of the Einstein frame metric g with mostly plus signature and the RR forms are rescaled as $F \to g_s^{-1}F$. Notice that we are using the democratic formulation of [40] which means that all RR field strengths F_n with $n = 1, 3, 5, 7$ and 9 in type IIB and $n = 0, 2, 4, 6, 8$ and 10 in type IIA appear in the action. In this way the bulk action does not contain any Chern-Simons terms but a duality relation between the RR fields,

$$\mathrm{e}^{\frac{5-n}{2}\phi}F_n = \star_{10}\sigma(F_{10-n}) , \qquad (A.35)$$

must be imposed on-shell by hand. The reversal operator σ was introduced in the previous section. Including the localized action for a Dp-brane

$$S_{\mathrm{loc}} = -\tau_p\int\left\{\mathrm{e}^{\frac{p-3}{4}\phi}\star_{p+1}1 + (-1)^p\sigma(C_{p+1})\right\} , \qquad (A.36)$$

we see that the Bianchi identity for F_{8-p}, the Einstein equation and the dilaton equation acquire correction due to the presence of the branes. In the localized action we have put the worldvolume field strength $\mathcal{F} = 0$. The fields appearing in the D-brane action are understood as the pull-backs of their bulk counterparts. The modified Bianchi identity for F_{8-p} is:

$$\mathrm{d}F_{8-p} - H\wedge F_{6-p} = 2\kappa_{10}^2\tau_p\delta_{9-p} , \qquad (A.37)$$

where δ_{9-p} denotes the $(9-p)$-form with delta distribution support on the brane worldvolume, i.e. it is the Poincaré dual of the worldvolume.

A.4 Buscher rules

The Buscher rules [66] are the supergravity description of T duality. They are constructed such that the reduction of IIA and IIB along the circular direction

[2]Recall equation (2.8):

$$g_{MN} = e^{-\frac{(\Phi-\Phi_0)}{2}}G_{MN} , \qquad (A.33)$$

and we put $\phi = \Phi - \Phi_0$.

give the same 9 dimensional action. Similarly, the effective D-brane actions are mapped onto each other.

It will be useful to parametrise the metric and fields in the following way:

$$\mathrm{d}s^2 = G_{yy}\left(\mathrm{d}y + g\right)^2 + G_{ab}\mathrm{d}x^a\mathrm{d}x^b\,, \tag{A.38}$$

$$B = \left(\mathrm{d}y + \frac{1}{2}g\right) \wedge b + \frac{1}{2}B_{ab}\mathrm{d}x^a \wedge \mathrm{d}x^b\,, \tag{A.39}$$

$$F_p = (\mathrm{d}y + g) \wedge F_p^{(y)} + F_p^{(\not{y})}\,, \tag{A.40}$$

where we defined the one-forms:

$$g = \frac{G_{ya}}{G_{yy}}\mathrm{d}x^a \quad , \qquad b = B_{ya}\mathrm{d}x^a\,. \tag{A.41}$$

The T dual along the y direction is then given by:

$$\mathrm{d}\tilde{s}^2 = \tilde{G}_{yy}\left(\mathrm{d}y + \tilde{g}\right)^2 + G_{ab}\mathrm{d}x^a\mathrm{d}x^b\,, \tag{A.42}$$

$$\tilde{B} = \left(\mathrm{d}y + \frac{1}{2}\tilde{g}\right) \wedge \tilde{b} + \frac{1}{2}B_{ab}\mathrm{d}x^a \wedge \mathrm{d}x^b\,, \tag{A.43}$$

$$\tilde{F}_p = (\mathrm{d}y + \tilde{g}) \wedge \tilde{F}_p^{(y)} + \tilde{F}_p^{(\not{y})}\,, \tag{A.44}$$

where

$$\tilde{G}_{yy} = \frac{1}{G_{yy}}\,, \quad \tilde{g} = -b\,, \quad \tilde{b} = -g\,, \tag{A.45}$$

$$\tilde{F}_p^{(y)} = F_{p-1}^{(\not{y})}\,, \quad \tilde{F}_p^{(\not{y})} = F_{p+1}^{(y)}\,, \tag{A.46}$$

and for the dilaton we have

$$\tilde{\Phi} = \Phi - \frac{1}{2}\log G_{yy}\,. \tag{A.47}$$

Appendix B

Different types of brane-flux decay

This appendix accompanies chapter 5 and is a reprint of the appendix in [114]. The discussion in chapter 5 focused on preserving the RR tadpole through a domain wall bubble nucleation, and we have argued that this is accomplished by internal dynamics involving D-branes, NS5-branes or a KK5-brane. This is relevant for compactifications/solutions where only one RR flux is turned on together with H-flux. In two dimensions we may encounter multiple RR-fluxes for which the NSNS tadpole becomes non-trivial as can be seen from the integrated B_2-equation of motion:

$$-\frac{1}{2}\int \langle F \wedge \sigma(F)\rangle_8 = 2\kappa_{10} Q_{\mathrm{F1}} \ , \tag{B.1}$$

where on the right hand side the total fundamental string charge enters. The F1 sources wrap the two dimensional spacetime but cannot appear in higher dimensions without breaking Lorenz invariance of spacetime. The decay of flux numbers in this case must involve the polarization of F1's to a relevant D-brane that flows across compact cycles in the internal space to give rise to the decay of F1 charge (see for instance [145]).

Although it is well known that D-branes source the fundamental string, it is useful to observe this effect from the worldvolume point of view. In our conventions the full WZ terms of the Dp-brane worldvolume action takes the form

$$-(-1)^p \mu_p \int \langle \sigma(\mathrm{e}^{-\mathcal{F}} \wedge C)\rangle_{p+1} \tag{B.2}$$

where $\mathcal{F}_2 = B_2 - F_2$ as before. Previously, we discussed how Dp-branes can polarize to a D$(p+2)$-brane via the standard Myers effect. The worldvolume gauge field of the D$(p+2)$-brane takes a monopole configuration that generates the brane charge of the original brane. But when a D-brane sources F1 charge, the worldvolume field strength $\mathcal{F}_2$ also takes a nontrivial configuration which is *electric* rather than magnetic. $\mathcal{F}_2$ is then such that the that the combination

$$\langle \mathrm{e}^{-\mathcal{F}} \wedge C \rangle_{p-1} \ ,$$

threads a topologically trivial $p-1$ cycle. The topology of the Dp-brane is that of the F1 it is sourcing times a topologically trivial $p-1$ cycle. Using the standard DBI action and the WZ terms written above one can derive the potential for such a brane in a given background, this was the approach taken in [147]. Coupling the WZ action above with the bulk kinetic term for the B-field we find the equation of motion

$$\mathrm{d}H_7 - \frac{1}{2}\langle F \wedge \sigma(F) \rangle_8 = -2\kappa_{10}(-1)^p \mu_p \langle \sigma(\mathrm{e}^{-\mathcal{F}} \wedge C) \rangle_{p-1} \wedge \delta_{9-p} \ . \tag{B.3}$$

The F1 charge of the D-brane is measured by the right hand side of this equation. Again we can motivate that the F1 charge changes as the D-brane travels across a compact $p-1$ cycle. Taking only the leading order term of the above coupling and assuming a spherical $p-1$ cycle for simplicity we see that

$$Q(\Psi = \pi) - Q(\Psi = 0) \propto \int_{\Psi=\pi} C_{p-1} - \int_{\Psi=0} C_{p-1} = \int F_p \ . \tag{B.4}$$

The accompanying BT bubble is a Domain wall D-brane of the same dimension as the brane that sources the F1 charge. So in summary the annihilation of RR-fluxes with fundamental string is closely related to the annihilation of RR-fluxes with D-branes discussed in section 5.3, the only difference is that the polarized brane that travels in the internal space has an electric worldvolume configuration instead of the magnetic one discussed in section 5.3. Otherwise the physics of the annihilation is completely analogous.

We have not covered all possible decay processes. But we emphasized those that are relevant for the most typical flux compactifications. More general possibilities can for instance involve supertubes [38].

Appendix C

Smeared antibranes polarise in AdS

This appendix accompanies chapter 6 and is a reprint of the appendices in [113].

C.1 Second-order equations

In this section we present the second-order differential equations for the Ansatz in section 6.2. The F_5 Bianchi identity implies

$$(\alpha' e^{3B+C+2D-4A})' = \lambda M^2 e^{\phi-3B+3C+2D} - N_{\rm D3}\mu_3\delta_6 \ . \tag{C.1}$$

We use a prime to denote a derivative with respect to ρ. The dilaton equation gives

$$\phi'' + (4A+3B+C+2D)'\phi' = \frac{1}{2}M^2 e^{\phi+2C-6B}(1-\lambda^2) \ . \tag{C.2}$$

We present the Einstein equation in the trace reversed form

$$R_{MN} = \hat{T}_{MN} \ . \tag{C.3}$$

The Ricci tensor is

$$\begin{aligned}
R_{\mu\nu} &= -e^{-2C}\left(4e^{2A}A'^2+e^{2A}\left(3B'+C'+2D'\right)A'+3e^{2C}+e^{2A}A''\right)e^{-2A}g_{\mu\nu}\,,\\
R_{ij} &= e^{-2C}\left(-3e^{2B}B'^2-4e^{2B}A'B'-e^{2B}\left(C'+2D'\right)B'+2e^{2C}-e^{2B}B''\right)e^{-2B}g_{ij}\,,\\
R_{\rho\rho} &= -4A'^2+4C'A'-3B'^2-2D'^2+3B'C'-2C'D'\\
&\quad -4A''-3B''-2C''-2D''\,,\\
R_{ab} &= \Big(1-e^{2D}\left(C'+D'\right)^2-4e^{2D}A'\left(C'+D'\right)-3e^{2D}B'\left(C'+D'\right)\\
&\quad -e^{2D}\left(D'^2+C'D'+C''+D''\right)\Big)e^{-2C-2D}g_{ab}\,.
\end{aligned} \tag{C.4}$$

The components of the trace-reversed energy-momentum tensor are

$$\begin{aligned}
\hat{T}_{\mu\nu} &= \tfrac{-1}{8}g_{\mu\nu}\Big((1+\lambda^2)M^2e^{\phi-6B}+2e^{-8A-2C}(\alpha'^2)+2\mu_3N\delta\Big)\,,\\
\hat{T}_{ij} &= \tfrac{1}{8}g_{ij}\Big(M^2(3-\lambda^2)e^{\phi-6B}+2(\alpha')^2e^{-8A-2C}\Big)\,,\\
\hat{T}_{\rho\rho} &= \tfrac{1}{2}\phi'^2+\tfrac{1}{8}e^{2C+\phi-6B}(3\lambda^2-1)M^2-\tfrac{1}{4}(\alpha)'^2e^{-8A}\,,\\
\hat{T}_{ab} &= \tfrac{1}{8}g_{ab}\Big(M^2(3\lambda^2-1)e^{\phi-6B}+2(\alpha')^2e^{-8A-2C}\Big)\,.
\end{aligned} \tag{C.5}$$

C.2 Supersymmetric AdS

The BPS-equations for our AdS solutions (6.29-6.32) are:

$$\begin{aligned}
16(\nabla\phi)^2 &= -\gamma^{-2}M^2\left(4\beta^{-2}\alpha^2-(p-1)^2\right)\\
&\quad +(p-3)^2\beta^{-2}(\nabla\alpha)^2\,,\\
(\nabla(4A+\phi))^2 &= -16e^{-2A}+\left[\gamma^{-2}M^2+\beta^{-2}(\nabla\alpha)^2\right]\\
\left(4\mathrm{e}^{-B}\pm\nabla(4B+\phi)\right)^2 &= \left[\gamma^{-2}M^2+\beta^{-2}(\nabla\alpha)^2\right]\\
(\nabla(3\phi-4C-4D))^2 &= \left(4e^{-D-C}-(p-2)\beta^{-1}\nabla\alpha\right)^2\\
&\quad +(p-2)^2\gamma^{-2}M^2\,,
\end{aligned} \tag{C.6}$$

where

$$\begin{aligned}\beta &= \mathrm{e}^{(p+1)A+\frac{p-3}{4}\phi} \ , \\ \gamma &= \mathrm{e}^{(6-p)B-\frac{p-1}{4}\phi} \ .\end{aligned} \tag{C.7}$$

In addition to these equations we must supplement also the Bianchi identity for F_{8-p} in equation (6.34). We have verified that this system reproduces the one presented in [14] for $p = 6$ for which no B warp factor is present. For $p = 5$ the system is also modified as the third equation takes the form

$$(\nabla(4B+\phi))^2 = \left[\gamma^{-2}M^2 + \beta^{-2}(\nabla\alpha)^2\right] \ . \tag{C.8}$$

This is due to the fact that for general p the metric has a factor S^{6-p} which for $p = 5$ is simply a circle and the associated Ricci tensor must vanish.

The first two BPS equations can be combined so that $\nabla\alpha$ does not appear,

$$(p-3)^2(\nabla(4A+\phi))^2 - 16(\nabla\phi)^2 = -16(p-3)^2\mathrm{e}^{-2A} + 4\gamma^{-2}M^2\left(\lambda^2 - p + 2\right) \ . \tag{C.9}$$

From the equations of motion one can deduce:

$$-4p(p-3)\mathrm{e}^{-2A} - 4(p-3)\nabla^2 A + (7-p)\nabla^2\phi = -2\gamma^{-2}(\beta^{-2}\alpha^2 - 1) \ . \tag{C.10}$$

Expanding these two equations around a Dp singularity, we obtain an expression for the constant k_0 which was introduced in section 6.4.3:

$$\frac{1}{2}k_0^2 = \frac{(p-4)(p-3)^2}{(p-5)} \ . \tag{C.11}$$

This equation has important consequences because for $p = 3$ and $p = 4$ we see that α_0 vanishes (cf. equation (6.55)). The fact that α_0 vanishes is however in direct contradiction with our previous result that α_0 is proportional to the non-zero cc. We must conclude that the assumption made when deriving the relation between α_0 and the cc; that the internal space is compact, is not true for $p = 3, 4$. This result only holds when we assume a Dp singularity at $\rho = 0$, a compact solution with no brane at the pole might of course exist.

Appendix D

Curved domain walls as warped AdS vacua?

This appendix accompanies chapter 7 and is a reprint of the appendices in [154].We consider the same $(p+2)$-dimensional theories (7.32, 7.33) but instead of flat domain walls we investigate AdS-sliced domain walls.

D.1 D6-branes

Let us first look at 6-branes. For the ordinary 6-brane it is known that it can be obtained as a domain wall in maximal 8-dimensional gauged sugra obtained from an S^2 reduction of the massless IIA sugra [183]. Here we discuss the extension to massive IIA which naturally leads to the fractional D6 solution which is not SUSY [137] (see the explanation around equation (7.29)). Whether the S^2 reduction of massive IIA also leads to a maximal gauged sugra is not known to us and under investigation. At this point we simply work with the bosonic Kaluza–Klein reduced theory obtained in (7.32, 7.33) without worrying about a possible fermionic extension that makes it into (a truncation of) a gauged supergravity in eight dimensions.

If we then consider a domain wall Ansatz with an AdS_7 worldvolume then a would-be solution, defined by a specific profile for f, g, φ, ϕ, b, lifts to the

following 10-dimensional solution

$$\begin{aligned} \mathrm{d}s_{10}^2 &= \mathrm{e}^{2\alpha\varphi(\theta)}[g^2(\theta)\mathrm{d}s_{AdS_7}^2 + f^2(\theta)\mathrm{d}\theta^2] + \mathrm{e}^{2\beta\varphi(\theta)}\mathrm{d}\Omega_2^2\,, \\ B_2 &= b(\theta)\epsilon_2\,, \\ F_2 &= F_0 B_2\,, \end{aligned} \tag{D.1}$$

where ϵ_2 is the volume element on the S^2 with standard normalised metric $\mathrm{d}\Omega_2^2$. This Ansatz captures the AdS_7 compactifications discussed first in [50, 51], which were further developed and properly understood in [14] (see also [140, 109]). In the latter references the above solution was not regarded as an AdS_7 sliced domain wall in 8 dimensions but as a warped AdS_7 compactification on a compact space that is conformal to an S^3:

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2A(\theta)}\mathrm{d}s_{AdS_7}^2 + \mathrm{e}^{2B(\theta)}\Big[\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\Omega_2^2\Big]\,. \tag{D.2}$$

Indeed, by properly choosing the gauge for $f(\theta)$ the metric in (D.2) can be mapped to (D.2).

Generically the lift of an AdS-sliced domain wall in some d-dimensional gauged sugra to 10 dimensions does not relate to a compactification because the dimensions perpendicular to the AdS slicing do not need to be compact. The fact that a compactification is possible was alluded to in [49, 50] where it was noticed that the presence of the Romans mass allowed the tadpole condition (Q_6 is the total 6-brane charge):

$$\int_{S^3} F_0 H = Q_6\,, \tag{D.3}$$

to be satisfied. A full proof that the space was topologically compact was presented in [14] by finding first-order equations for the solution instead of second-order equations allowing a numerical treatment of the solutions. The first-order integration was possible since [14] demonstrated, contrary to the claim in [50], that such an Ansatz could lead to SUSY solutions[1]. If the 8-dimensional theory can be seen as a consistent truncation of an 8-dimensional maximal sugra then we furthermore conjecture that the domain wall is also 1/2 SUSY from the point of view of the 8-dimensional theory. Hence, from the point of view of the curved domain wall, one expects to have a superpotential that provides the first-order equations for *curved* domain walls, see for instance [105].

[1] Apart from the SUSY solutions, reference [140] has given evidence for a 1-parameter family of non-SUSY solutions.

So far we have hinted that the AdS_7 solutions constructed entirely from a 10-dimensional viewpoint can be possibly described within 8-dimensional gauged supergravity as a (SUSY?) AdS-sliced domain wall solution. Nonetheless the natural lower-dimensional supergravity to look at should be 7-dimensional half-maximal gauged sugra in which the AdS_7 is the natural vacuum. It became a standard lore that flux compactifications to (SUSY) AdS vacua with space-filling branes lead to gauged supergravity theories in the lower dimension, *if* the branes are calibrated and smeared over the internal space [49]. The smearing procedure implies constant dilaton and the absence of warping. After such a smearing the AdS solution simplifies to $AdS_7 \times S^3$ [49] since both e^A and e^B become constant in that limit and F_2 vanishes. Since S^3 is a group manifold one furthermore expects that the smeared compactification gives rise to a half-maximal sugra in 7 dimensions. Reference [80] has demonstrated that this belief is not founded since this AdS_7 solution is a counterexample[2].

D.2 D*p*-branes

As mentioned above, when the 6-branes are smeared the solution becomes a simple unwarped $AdS_7 \times S^3$ solution. Similar vacua can be made in any dimension less than 7 as well [188, 49]: when a smeared Dp-brane charge is cancelled by a combination of H flux filling some S^3 and F_{6-p} flux filling an orthogonal S^{6-p} then one can solve the 10-d equations of motion if the lower-dimensional part is AdS_{p+1}[3]. The fully localised (i.e. back-reacted) solutions are probably out of reach since the equations for the warp factor, conformal metric factors, dilaton, and B-field profile will not be ODEs any more when $p \neq 6$. However, once the Dp-branes are smeared over the S^{6-p} one ends up in a situation which is practically analogues to the D6 AdS_7 solution discussed above. Such a set-up should be described by the following Ansatz:

$$\begin{aligned} \mathrm{d}s_{10}^2 &= \mathrm{e}^{2A(\theta)}\mathrm{d}s^2_{AdS_{p+1}} + \mathrm{e}^{2B(\theta)}\mathrm{d}\Omega^2_{6-p} + \mathrm{e}^{2C(\theta)}\Big[\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\Omega_2^2\Big]\,, \\ F_{8-p} &= B_2 \wedge F_{6-p}\,, \\ B_2 &= b(\theta)\epsilon_2\,, \\ F_{6-p} &= m\epsilon_{6-p}\,, \end{aligned} \tag{D.4}$$

[2]However, reference [144] mentions that the 7-dimensional gauged sugra might still exist.

[3]Only for $p = 5$ does this fails at first sight since the absence of curvature of the S^1 factor makes the Einstein equation in that direction inconsistent. However, one could expect that a certain fibration of S^1 over S^3 could work.

where A, B, C, b, ϕ all only depend on θ. The solutions are currently under investigation. Clearly also here the solutions can be described by an AdS-sliced domain wall after compactification over the S^{6-p} and further over the S^2 inside the S^3.

For these AdS_{p+1} solutions, we anticipate physics which is similar to the AdS_7 case [50, 14, 109, 140]. The arguments of [50] can directly be used in this situation to demonstrate that the solution should have a divergent $e^{-\phi}H^2$ density. From the analogy with the AdS_7 solution we expect that the singularity gets resolved by polarising the p-brane into a spherical $(p+2)$-brane wrapping a finite size 2-sphere inside the S^3. The non-compact "antibrane" solutions, that have a flat worldvolume Mink_{p+1} instead of AdS_{p+1} cannot be stabilised at finite radius and hence the singularity cannot be resolved that way [36, 33, 35, 34].

The case $p = 3$ is somewhat more intriguing. There one should be able to S-dualise the solution and smear the 3-brane over the S^3 filled with F_3 flux instead of H_3 flux. The 3-form singularity in this case might get resolved by a spherical NS5-brane. This leads us to further speculate that the solution with fully-localised 3-branes, which might be out of reach, would be similar to the (also hypothetical) Polchinski–Strassler (PS) solution [177] where a web of spherical D5/NS5-branes wrap 2 cycles inside the S^5 of the deformed $AdS_5 \times S^5$. The crucial difference with the PS configuration is that the (deformed) AdS_5 part of the PS solution is replaced by an AdS_4-sliced domain wall and the S^5 is replaced by some fibration of S^2 over S^3. The space transversal to the AdS_4 is also compact. In that sense one could think of the PS solution in terms of a flat domain wall in 5 dimensions where the space transversal to the wall is non-compact whereas the new solution is an AdS-sliced domain wall in 5 dimensions with a compact space transversal to the AdS_4 wall.

Similar results are expected for M2-branes with AdS_3 worldvolume [47], that can be regarded as a compactification of 11-dimensional supergravity down to 3 dimensions.

Appendix E

Holography, Brane Intersections and Six-dimensional SCFTs

This appendix accompanies chapter 8 and is a reprint of the appendices in [55].

E.1 Conventions and notation

The action of massive type IIA supergravity in string frame can be found in chapter 2. Throughout chapter 8 we work in string units with:

$$2\pi\alpha' = 1 \,, \tag{E.1}$$

which implies:

$$16\pi G_{10} = \frac{1}{2\pi} \,, \tag{E.2}$$

and the string coupling g_s is absorbed in e^{ϕ}. This gives particularly simple quantization conditions for the fluxes [176], namely:

$$M \,, \quad \int H \,, \quad \int (F_2 - MB) \,, \quad \int \left(F_4 - B \wedge F_2 + \frac{1}{2} MB \wedge B \right) \in \mathbb{Z} \,. \tag{E.3}$$

E.2 General AdS$_7$ solutions of type IIA supergravity

The general type IIA supergravity solution corresponding to the brane intersection of interest is given in (8.6) and is obtained by solving the system of equations in (8.8). These equations are also equivalent to:

$$\begin{aligned} &\partial_z L_S = M g_s L_K \ , \\ &\partial_r L_S + M g_s T = 0 \ , \\ &\partial_r L_K + \partial_z T = 0 \ , \end{aligned} \tag{E.4}$$

where we have defined:

$$\begin{aligned} &L_S \equiv (2 + 2r\partial_r + z\partial_z)S \ , \\ &L_K \equiv (3 + 2r\partial_r + z\partial_z)K \ , \\ &T \equiv -z\partial_r K + 2r\partial_z(KS) \ . \end{aligned} \tag{E.5}$$

We will now show that AdS_7 solutions of the ten-dimensional theory necessarily obey the equations:

$$L_S = L_K = T = 0 \ . \tag{E.6}$$

Furthermore, all AdS$_7$ solutions of the original system can be found in this way.

To find AdS_7 within our general Ansatz (8.3)-(8.6) we have to impose that the metric and all background fields are invariant under the isometries of AdS_7. To implement this we change coordinates from (z, r) to (ρ, α) where ρ is the radial coordinate of AdS_7 and α is a coordinate on the internal space. The metric and three form field strength take the following form:

$$\begin{aligned} \mathrm{d}s^2 =& S^{-\frac{1}{2}} \mathrm{d}s_6^2 + K[S^{-\frac{1}{2}}(\partial_\rho z)^2 + S^{\frac{1}{2}}(\partial_\rho r)^2]\mathrm{d}\rho^2 \\ &+ K[S^{-\frac{1}{2}}\partial_\rho z \partial_\alpha z + S^{\frac{1}{2}}\partial_\rho r \partial_\alpha r]\mathrm{d}\rho\mathrm{d}\alpha \\ &+ K[S^{-\frac{1}{2}}(\partial_\alpha z)^2 + S^{\frac{1}{2}}(\partial_\alpha r)^2]\mathrm{d}\alpha^2 + KS^{\frac{1}{2}} r(\rho,\alpha)^2 \mathrm{d}\Omega_2^2 \, , \\ H =& - r^2[\partial_r K \partial_\rho z - \partial_z(KS)\partial_\rho r]\mathrm{d}\rho \wedge \Omega_2 \\ &- r^2[\partial_r K \partial_\alpha z - \partial_z(KS)\partial_\alpha r]\mathrm{d}\alpha \wedge \Omega_2 \, . \end{aligned} \tag{E.7}$$

Invariance under the isometries of AdS_7 requires that the warp factor in front of the AdS_7 part of the metric can only depend on the internal coordinate α. In addition the three form field strength should only have legs in the internal

space. We work with the following metric on AdS_7:

$$\mathrm{d}s_7^2 = \frac{1}{(g\rho)^2}\mathrm{d}\rho^2 + (g\rho)\mathrm{d}s_6^2 \,. \tag{E.8}$$

Imposing invariance under the $SO(6,2)$ isometry group of this space leads to the following relations:

$$\begin{aligned}
S(\rho,\alpha) &= (g\rho)^{-2}e^{-4A(\alpha)}\,, && \text{(E.9)}\\
(g\rho)^{-2}e^{2A(\alpha)} &= K[S^{-\frac{1}{2}}(\partial_\rho z)^2 + S^{\frac{1}{2}}(\partial_\rho r)^2]\,, && \text{(E.10)}\\
0 &= K[S^{-\frac{1}{2}}\partial_\rho z\partial_\alpha z + S^{\frac{1}{2}}\partial_\rho r\partial_\alpha r]\,, && \text{(E.11)}\\
P(\alpha) &= K[S^{-\frac{1}{2}}(\partial_\alpha z)^2 + S^{\frac{1}{2}}(\partial_\alpha r)^2]\,, && \text{(E.12)}\\
Q(\alpha) &= KS^{\frac{1}{2}}r(\rho,\alpha)^2\,, && \text{(E.13)}\\
0 &= \partial_r K\partial_\rho z - \partial_z(KS)\partial_\rho r\,. && \text{(E.14)}
\end{aligned}$$

Here we have defined the warp factor in front of AdS_7 metric in (E.8) to be $e^{2A(\alpha)}$ and $P(\alpha)$, $Q(\alpha)$ are so far undetermined functions that only depend on α. We furthermore have to impose that the dilaton (8.4) depends only on α. This condition, combined with (E.9), fixes the ρ dependence of the function $K(\rho,\alpha)$ to be:

$$e^{\phi} = g_s K^{\frac{1}{2}}S^{-\frac{3}{4}} \quad \rightarrow \quad K = g_s^{-2}(g\rho)^{-3}e^{-6A(\alpha)+2\phi(\alpha)}\,. \tag{E.15}$$

From equation (E.13) we immediately see that:

$$r(\rho,\alpha) = (g\rho)^2 f_1(\alpha)\,, \tag{E.16}$$

where $f_1(\alpha) = g_S e^{4A-\phi}Q^{-\frac{1}{2}}$ is a nonconstant function of α. Using this in turn allows one to rewrite equation (E.10):

$$(\partial_\rho z)^2 = g_s^2 e^{6A(\alpha)-2\phi(\alpha)} - 4g^2 e^{-4A(\alpha)}f_1(\alpha)^2 \equiv g_1(\alpha)^2\,, \tag{E.17}$$

where $g_1(\alpha)$ is defined to notational brevity. With this at hand we can find the ρ dependence of $z(\rho,\alpha)$ to be:

$$z = \rho g_1(\alpha) + g_2(\alpha)\,. \tag{E.18}$$

In (E.18) we have allowed for an arbitrary function $g_2(\alpha)$, however it is easy to show that g_2 has to be a constant. Indeed, from (E.11) and (E.18) one finds:

$$0 = g_1(\alpha)\,(\rho g_1(\alpha)' + g_2(\alpha)') + e^{-4A(\alpha)}2g^2\rho f_1(\alpha)f_1(\alpha)'\,, \tag{E.19}$$

which is only consistent if $g_1(\alpha)' \neq 0$ and $g_2(\alpha)' = 0$. The shift symmetry in z allows us to safely put $g_2 = 0$. Combining (E.16) and (E.18) one then finds:

$$\frac{r}{z^2} = \frac{g^2 f_1(\alpha)}{g_1(\alpha)^2} \,, \tag{E.20}$$

which in turn implies that α has to be a function of $\frac{r}{z^2}$. Furthermore, equation (E.10) can be used to define ρ implicitly, this yields the following relations:

$$\alpha = \alpha(r/z^2) \,, \qquad \rho^{-1} = g^3 K(z^2 + 4r^2 S) \,. \tag{E.21}$$

Moreover, using $S \sim \rho^{-2}$ and $K \sim \rho^{-3}$ one can show that:

$$\begin{aligned} L_S &\equiv 2S + 2r\partial_r S + z\partial_z S = 0 \,, \\ L_K &\equiv 3K + 2r\partial_r K + z\partial_z K = 0 \,. \end{aligned} \tag{E.22}$$

We still need to impose the condition that the three form field strength have legs only along the internal space. This is is given by (E.14), which in turn leads to:

$$T \equiv -z\partial_r K + 2r\partial_z(KS) = 0 \,, \tag{E.23}$$

The only thing left to show is that the conditions in (E.11) and (E.12) are satisfied. For this we need to invert the following Jacobian:

$$\frac{\partial(\rho,\alpha)}{\partial(z,r)} = \begin{pmatrix} \partial_z \rho & \partial_r \rho \\ -2\frac{r}{z^3}\alpha' & \frac{1}{z^2}\alpha' \end{pmatrix} \,, \tag{E.24}$$

where α' denotes the derivative of α with respect to r/z^2 and one can use the relations:

$$\begin{aligned} \partial_z \rho &= -\rho^2 \left(\rho^{-1} \frac{\partial_z K}{K} + g^3 K(2z + 4r^2 \partial_z S) \right) \,, \\ \partial_r \rho &= -\rho^2 \left(\rho^{-1} \frac{\partial_r K}{K} + g^3 K(8rS + 4r^2 \partial_r S) \right) \,. \end{aligned} \tag{E.25}$$

Using (E.22) one finds that:

$$\frac{\partial(z,r)}{\partial(\rho,\alpha)} = \frac{z^3 \rho}{\alpha'} \begin{pmatrix} \frac{1}{z^2}\alpha' & -\partial_r \rho \\ 2\frac{r}{z^3}\alpha' & \partial_z \rho \end{pmatrix} \,. \tag{E.26}$$

This equation can then be used to show the validity of (E.11, E.12). This concludes the prove that all supersymmetric AdS_7 solutions of type IIA supergravity should obey the constraints in (E.22, E.23). In Section 8.3.1 we show how to explicitly solve these constraints and find all of these AdS_7 solutions analytically.

E.3 Comparison to the results in the literature

In this appendix we show that the general system of equations for supersymmetric AdS_7 backgrounds of massive type IIA supergravity derived in [14] is solved by the background in (8.20) together with the equations in (8.19). In order to match the conventions used in this chapter we flip the signs of M and F_2 appearing in the system of [14]. We must also take $g = 2$ since in [14] the authors fix the radius of AdS_7 to be $L = 2/g = 1$. The AdS_7 solutions of [14] (in string frame) are given by:

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{e}^{2A}\left(\mathrm{d}s^2_{\mathrm{AdS}_7} + \mathrm{d}s^2_{M_3}\right) \ , \\ \mathrm{d}s^2_{M_3} &= (1-x^2)\left(\frac{16}{(4x+M\mathrm{e}^{A+\phi})^2}\mathrm{d}A^2 + \frac{1}{16}\mathrm{d}\Omega_2^2\right) \ , \\ F_2 &= \frac{\sqrt{1-x^2}}{16}\mathrm{e}^{A-\phi}(xM\mathrm{e}^{A+\phi}+4)\,\Omega_2 \ , \\ H &= -\frac{1}{4}\mathrm{e}^{2A}(1-x^2)^{\frac{3}{2}}\frac{6-xM\mathrm{e}^{A+\phi}}{4x+M\mathrm{e}^{A+\phi}}\mathrm{d}A\wedge\Omega_2 \ . \end{aligned} \tag{E.27}$$

This constitutes a supersymmetric background of massive type IIA supergravity provided that the dilaton $\phi(A)$ and the function $x(A)$ satisfy the pair of coupled ordinary differential equations:

$$\begin{aligned} \partial_A\phi &= 5-2x^2+\frac{8x(x^2-1)}{4x+M\mathrm{e}^{A+\phi}} \ , \\ \partial_A x &= 2(x^2-1)\frac{4-xM\mathrm{e}^{A+\phi}}{4x+M\mathrm{e}^{A+\phi}} \ . \end{aligned} \tag{E.28}$$

We find that the background above agrees with our expression in (8.20) if we set:

$$\begin{aligned} \mathrm{e}^{2A} &= \sqrt{\frac{\beta}{y}} \ , \\ x^2 &= \frac{\alpha^2}{\alpha^2+4y\beta} \ , \\ \mathrm{d}A &= -\frac{1}{8y}\left(\frac{\alpha}{\beta}-\frac{Mg_s}{y}\right)\mathrm{d}\alpha \ . \end{aligned} \tag{E.29}$$

Furthermore, both differential equations are solved provided the equations in (8.19) are obeyed.

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