Arno Berger and Theodore P. Hill. *An Introduction to Benford's Law*. Princeton University Press, Princeton (2015). viii+248 pages. Hardcover, ISBN 978-0-691-16306-2 (hbk), GBP 52.00. `f = r528`

Steven S. Miller (Ed.). *Benford's Law: Theory and Applications*. Princeton University Press, Princeton (2015). xxvi+438 pages. Hardcover, ISBN 978-0-691-14761-1 (hbk), GBP 52.00.

These books are complementary in the sense that the first gives a mathematical introduction to Benford's law and the second collects a number of research papers on the theory and applications of the same.

Benford's law should actually be called after Simon Newcomb because he noticed in 1881 that the logarithmic tables were more worn out for the small digits than for the larger. This was only rediscovered by Benford in 1938. The law says that it is most likely that the first significant digit in a set of numbers is 1 (30.10%) and the probability of the other leading digits decreases exponentially until digit 9 with only 4.57% probability.

This curious event is investigated on a sound stochastic basis in the introductory book. The precise law says that the probability that the first digit is d_1, the second d_2 etc. until the mth digit is d_m is given by $\log_{10}(1 + (\sum_{j=1}^{m} 10^{m-j} d_j)^{-1})$. Some sequences obey the law, others don't, so it is interesting to know if, and how well the law is satisfied. This is characterized by a number Δ which is the largest deviation from the ideal distribution. Moreover the law holds for an infinite set of numbers, so there is dependence on the size of the data set.

It requires some flexibility to get used to Berger and Hill's strange notation of real numbers that get only 4 digits like $\pi = 3.141$ or even only 2 decimal digits for Δ, so that $\Delta = 0.00$ actually means Δ is between 0.00 and 0.01. The introduction includes invariance properties of the law, and the generalization from the discrete law to the real valued analogs in one and more dimensions and a short chapter on the possible applications. So it requires some background in probability theory, and the reader should be ready to deal with dynamical systems, differential equations, and random processes.

The impatient reader can switch to chapter two in the second book where Berger and Hill give a summary of their book on some 40 pages. The editor Miller adds in chapter three his arguments based on Fourier analysis to show that most sequences will (almost) satisfy Benford's law. Many other specialists on the topic contribute the 16 remaining papers. That concerns theoretical issues such as convergence, but mostly applications in accounting and voting systems, in economics and finance, psychology, games, clinical data, and medical images. An additional chapter provides exercises for all the previous chapters. See also the editor's website `web.williams.edu/Mathematics/sjmiller/public_html/-benford/` for supplementary material. Note that although Benford's law is rather popular among mathematical hobbyists, it requires a good mathematical training to embark on these two books. They really require a sound mathematical background. This shows that Benford's law has matured and is now taken much more seriously than it was before.

An online bibliography on Benford's law from the authors of the first book is available at `www.benfordonline.net`. Another recent book on this topic by A.E. Kossovsky is *Benford's Law* (World Scientific, 2014) but this is much less mathematical and more speculative.

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