Sum stories. Equations and their origins, *Robin Wilson*, Oxford University Press, 2025 (368 p.) isbn: 9780199607907 (ebk)

Many books have been written that tell the story of mathematics and its history for a broad audience. They differ by the selection of topics, how much or how deeply the mathematics are explored, and the general framework that is used to organize the material. The present book is yet another collection, using 18 basic formulas as hooks to hang up a story told in as many chapters about the history and importance of the formula, and hence about the history and evolution of mathematics.

To elaborate on each of these formulas in detail would make this review too long, but they range from $1 + 1 = 2$, which is about the very origin of counting and the different number systems that have been used by different cultures, up to Whitehead and Russell's proof of $1 + 1 = 2$ in the *Principia Mathematica*. The last chapter is an explanation of $\aleph_0 + \aleph_0 = \aleph_0$ introducing countability, cardinal numbers, and the continuum hypothesis.

The geometry of the Greek antiquity is covered in the next chapter, starting with stories on the sum of the angles of a triangle being 180 degrees or π radians and an excursion about the parallel hypothesis and non-Euclidean geometries. There is of course discussion of the Pythagorean theorem in two and more dimensions and its analogue in Fermat's last theorem regarding $a^n + b^n = c^n$, as well as the formulas for the circumference and area of a circle. The latter two involve π, that most remarkable number in mathematics which turns up in so many unexpected places.

Then there are a number of chapters on formulas related to the origins of algebra and how geometry was gradually decoupled from the formulas, where the problem was to provide solution formulas for quadratic, cubic, and quartic polynomial equations (including the well known controversy between Tartaglia and Cardano) and later about higher degree polynomials. The fundamental theorem of algebra and the work of Abel and Galois showed that no such formulas exist for degree 5 or higher.

The equation $\phi^2 = \phi + 1$ introduces the golden ratio, Fibonacci numbers and their generalizations (the Lucas numbers) and their fascinating relation to Pascal's triangle, including an excursion on tessellations and Penrose tilings. A similar equation $x^2 = x$ is used to justify the introduction of Boole algebra (19th century), set theory and switching circuits.

There are also many stories for the previous 17th-18th centuries which I quickly mention: Prime numbers (those named after Mersenne, Fermat,

and Euler), combinatorics (as related to probability), and Euler's formula for polyhedra $V - E + F = 2$ (where the letters represent the number of vertices, edges and faces).

With Descartes, algebra meets geometry as functions are represented by curves with their tangents referring to rate of change. This perspective triggered Newton and Leibniz to consider infinitesimal changes and to develop analysis. Although discovered independently, it became a famous priority fight.

On the calculus side Briggs and Napier proposed the logarithmic function, and taking the square root of a negative number became acceptable after the introduction imaginary and complex numbers. Hamilton then generalized the idea to quaternions. Also, complex numbers have a convenient geometric interpretation as a point in the 2-D plane. The relation $e^{ix} = \cos(x) + i\sin(x)$ easily implies de Moivre's formulas, and $e^{i\pi} = -1$ is considered the most beautiful formula in mathematics.

Another story describes Euler's solution to be the Basel problem, showing that $\sum_{n\geq 1} \frac{1}{n^2} = \frac{\pi^2}{6}$. This is directly connected to Riemann's zeta function $\zeta(s) = \sum_{n\geq 1} \frac{1}{n^s}$. The location of all its poles (formulated in the Riemann hypothesis) is still an open problem with deep consequences for the distribution of prime numbers.

The penultimate chapter introduces Mandelbrodt's fractal geometry.

Choosing simple formulas as titles of the chapters is an original approach that works nicely. It allows Wilson to concisely catch the topic that he wants to elaborate on and it may arouse curiosity in the reader. The order of the formulas being connected to some problem or mathematician(s) makes sense historically. However, the material covered in the corresponding chapter may include the past and future of the problem and is not always restricted to a particular period. Wilson seamlessly interweaves short biographical aspects and history, with the explanation of the mathematics and how the topic contributed to advance mathematics.

I hope to have illustrated that the book contains many interesting and entertaining stories that span large historical periods and cover a broad spectrum of mathematical subjects, but in the end, these are all stories about people who have contributed in their own way to build and extend this masterpiece of the human culture that we call mathematics.

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