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Data-Driven Linearization of Nonlinear Finite Element Analyses

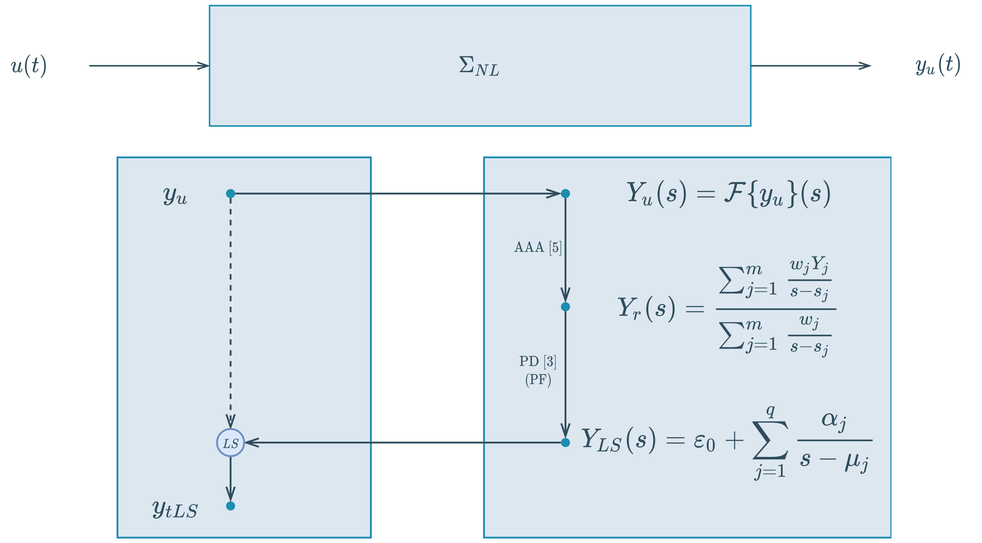
Introduction

Goal : MOR of nonlinear finite element analysis, for applications like Digital Twins of mechatronic systems

Properties : MOR needs to be

- Non-intrusive, the FOM is a black box
- Data-driven, use data to generate the ROM
- Time-domain, where the data comes from
- Ideally, with no control on input
- Generate a linear ROM, to allow for linear Kalman filtering

Solution : Not linearization of the whole FOM, but find a linear ROM which approximates $y_u(t)$, output of the FOM from the input $u(t)$: this is the Time-domain Least Squares AAA method, **tLS-AAA**



$$\delta(t) \longrightarrow \boxed{\Sigma_{tLS} : \begin{cases} M_r \ddot{\vec{x}}_r(t) + E_r \dot{\vec{x}}_r(t) + A_r \vec{x}_r(t) = B_r u(t) \\ y_{tLS}(t) = \varepsilon_0 \delta(t) + C_r^T \vec{x}_r(t) + D_r^T \dot{\vec{x}}_r(t) \end{cases}} \longrightarrow y_{tLS}(t)$$

$$\min_{\varepsilon_0,\{\alpha_j\},\{\beta_k\},\{\gamma_k\}} \left\| y_u(t) - \left(\varepsilon_0 \delta(t) + \sum_{j=1}^{q_r} \alpha_j e^{\mu_j t} + \sum_{m=1}^{l} \beta_k e^{\lambda_k t} \cos(\omega_k t) + \sum_{m=1}^{l} \gamma_k e^{\lambda_k t} \sin(\omega_k t) \right) \right\|_2$$

Numerical Results

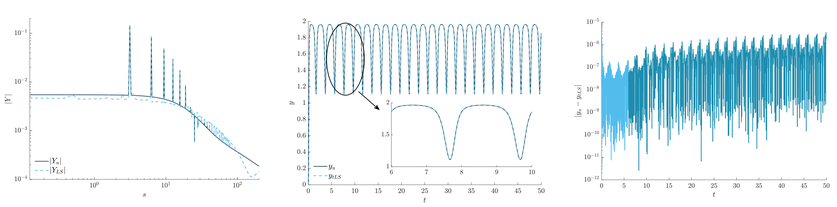
For each numerical test case we show:

- Y_u and its LS-AAA approximation Y_{LS} in the frequency domain
- y_u and its tLS-AAA approximation y_{tLS} in the time domain
- y_{tLS} approximation error; in light blue, the data used to build Σ_{tLS}

Chafee-Infante

$$\begin{cases} c_t - c_{xx} = \lambda c \left(1 - c^2\right) \\ c(0, x) = 0 \\ c(t, 0) = u(t) \\ c_x(t, 1) = 0 \end{cases}$$

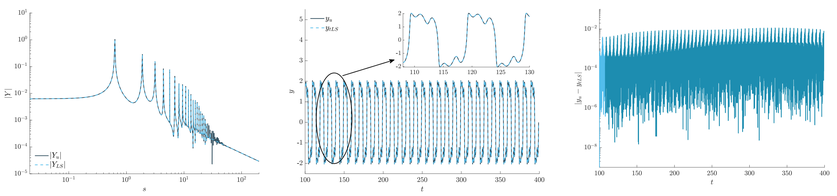
$$u(t) = 30(\sin(\pi t) + 1)$$



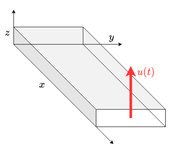
Van Der Pol

$$\ddot{y} - \lambda \left(1 - y^2\right) \dot{y} + y = u(t)$$

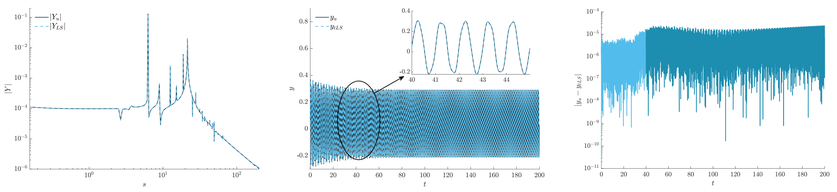
$$\lambda = 5, \quad u(t) = 7 \cos(\pi t)$$



Cantilever Beam



$$u(t) = 10^3 \sin(2\pi t)$$



Other MOR Methods

Time-domain Loewner Framework [**peherstorfer˙data-driven˙2017**]

- Quadratic-Bilinear ROM
- Requires access to input

Operator Inference [1]

- Quadratic-Bilinear ROM
- Requires access to state vector

Time-domain Vector Fitting [4]

- Linear ROM
- Computational costly, needs poles initialization

Future Developments

Issue : Σ_{tLS} is built for a specific $u(t)$: how to deal with different inputs? Is a new ROM required every time?

Restriction : Focusing on the case of periodic inputs, and more specifically to monotonic ones, we can find a way to solve this issue

Solution : There is a parallel between Σ_{tLS} and the Volterra series expansion. Inspired by these similarities, we build a parametric ROM with the input frequency ω as a parameter.

References

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