Optimization of large-scale systems with model reduction

Yao Yue, Karl Meerbergen
Department of Computer Science
Katholieke Universiteit Leuven
Celestijnenlaan 200A
3001 Heverlee, Belgium
Email: {yao.yue,karl.meerbergen}@cs.kuleuven.be

1 Introduction

Numerical parameter studies of acoustic problems arising from applications such as airplane engines and insulation panels along motor ways or in houses are usually extremely expensive, since for each parameter value, an entire frequency response function needs to be computed. The computation of a single frequency response function for fixed parameter values is by itself already quite expensive. Parameter studies are often carried out in order to choose the 'optimal' values of the parameters.

The computational cost for the frequency response function has been dramatically reduced by a factor of ten or more by using model order reduction methods. The goal of model order reduction (MOR) is to construct a low order model from the original large-scale model to solve the original problem more efficiently. It has been successfully applied to many different applications such as (vibro) acoustics, circuit simulation and controller design. However, little work has been done to introduce MOR in optimization.

Our research group has recently started research on exploiting the properties of model reduction methods in optimization problems arising from acoustics. This talk presents our first results.

2 Motivation

Optimizing the design parameter of insulating material covering the faces of a closed acoustic cavities serves as a good example. In this problem, we want to decrease the maximum noise level at a given point as much as possible by choosing the optimum design parameter. This is a minimax optimization problem.

This problem is often solved numerically using finite element or finite difference discretizations, which leads to large-scale sparse linear systems. Optimization of this problem typically needs to solve a large number of such linear systems, which is extremely expensive. Therefore, it is reasonable to use MOR methods to decrease the time spent in solving the linear systems. The fact that the same reduced model can solve the original linear system with different parameter values would drastically reduce the optimization time.

3 Our work

For simplicity, we concentrate on line search based optimization methods for now. Two types of Krylov based MOR methods will be investigated for reducing the original problem: single parameter reduction such as SOAR [1] and multi-parameter reduction such as PIMTAP [2]. The idea of using SOAR in optimization comes from the fact that once the search direction is determined, all the parameters could be represented by a single parameter and we can use SOAR to do MOR on this parameter. Furthermore, we prove that the gradient and Hessian of the points on the search direction could also be computed with the SOAR reduced model. PIMTAP, on the other hand, does reduction on all the parameters in the original model. Like in the SOAR case, the PIMTAP reduced model could also be used directly to compute gradient and Hessian. The advantage of PIMTAP based MOR is that it could compute all the points near the interpolation point rather than only the ones on the search direction. This make it possible to reuse the same PIMTAP based reduced model in several line search iterations. In conclusion, all data needed by optimization could be computed with the reduced model and MOR should drastically the optimization time.

In this paper, we first compare SOAR and PIMTAP in accuracy and performance. Then, we compare the minimax optimization problem with no reduction, with SOAR and with PIMTAP to study the effect of introducing model reduction techniques in minimax optimization. Numerical results show that using MOR in optimization could not only save a lot of time in solving linear equations, but also accelerate the optimization convergence rate in most cases.

References

[1] Zhaojun Bai, Karl Meerbergen, and Yangfeng Su, *Arnoldi methods for structure-preserving dimension reduction of second-order dynamical systems*, Dimension reduction of large-scale systems, Springer, 2005, pp. 173–189.

[2] Yung-Ta Li, Zhaojun Bai, Yangfeng Su, and Xuan Zeng, *Model order reduction of parameterized interconnect networks via a Two-Directional Arnoldi process*, Computer-Aided Design of Integrated Circuits and Systems, IEEE Transactions on **27** (2008), no. 9, 1571–1582.