Smooth Memoryless Time-Varying Control of a Class of Chained Nonholonomic Systems with Input Delays ⋆

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Abstract

In this paper, the control design for a class of chained nonholonomic systems, consisting of a linear subsystem and a bilinear chained subsystem with input delays, using smooth memoryless time-varying feedback will be investigated. For the linear subsystem, leveraging certain properties of parametric Lyapunov equations, a smooth, memoryless nonhomogeneous control law that guarantees exponential convergence is proposed. Then the bilinear chained subsystem is transformed into a linear time-varying system through a linear time-varying state transformation. By exploiting the structural properties of nonholonomic systems and applying the Razumikhin stability theorem, smooth linear memoryless time-varying controllers are designed. Both state feedback and observer-based output feedback are considered. Finally, the effectiveness of the proposed methods is demonstrated through a model for an underactuated axisymmetric spacecraft.

Key words: Nonholonomic systems, linear time-varying systems, smooth finite-dimensional time-varying feedback, smooth memoryless time-varying feedback.

1 Introduction

Since nonholonomic systems were distinguished from general mechanical systems, their control problems have remained a prominent research topic [21]. In 1983, Brockett proposed a necessary condition for the existence of smooth time-invariant feedback for nonlinear systems, now known as Brockett's necessary condition [3]. Unfortunately, nonholonomic systems fail to satisfy this condition. Consequently, the research on the control of nonholonomic systems has focused on smooth time-varying feedback and non-smooth feedback approaches. Notably, many nonholonomic systems, such as two-wheeled robots, can be transformed into chained nonholonomic systems, making them the most extensively studied class of nonholonomic systems. For chained nonholonomic systems, various control strategies have been proposed. These include smooth time-varying feedback, such as sinusoid-based state feedback [4,24,32,26] and linear time-varying state feedback [31,30], as well as discontinuous time-invariant control laws [9], such as the σ-process method [2,13,15,20,38,39] and the logic-based switching method [12]. In this paper, we further investigate the control problem for chained nonholonomic systems.

Time delays frequently arise in many practical control systems, including long transmission lines in pneumatic systems, rolling mills, nuclear reactors, hydraulic systems, manufacturing processes, and digital control systems (see [5,17–19,34,46,47]). The presence of time delays can lead to significant performance degradation or even system instability. As time-delay systems belong to the class of functional differential equations, which are inherently infinite-dimensional, their analysis and synthesis have attracted significant research attention over the past three decades. Several approaches have been developed to address the stabilization of time-delay systems. The Lyapunov-Krasovskii functional-based method is one of the most efficient and widely used techniques (see, for example, [41]), however, it is generally applicable only to systems with small delays. Another effective approach is the infinite-dimensional predictor-based feedback method, which is capable of handling arbitrarily large delays [16–18,28,40]. Using

⋆ This work was supported in part by the National Natural Science Foundation of China for Distinguished Young Scholars under Grant 62125303, the National Natural Science Foundation of China-"Qisun Ye" Science Foundation under Grant U2441243, the Research Foundation–Flanders (FWO-Vlaanderen) project G092721N, the KU Leuven Research Council project C14/22/092, and the NSFC grants 62173111, and 62188101. The material in this paper was partially presented at the Joint IFAC Conference: SSSC-TDS-COSY, France, 2025 [45]. * Corresponding author.

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the input-state scaling technique, researchers have proposed infinite-dimensional predictor feedback laws for input-delay chained nonholonomic systems [7,27]. Similarly, the control of stochastic nonholonomic systems in chained form with time-varying state delays has been studied in [10]. Consensus-formation control problems for networked nonholonomic systems under communication delays have also been addressed in [29,33,35]. Despite the theoretical strengths of infinite-dimensional predictor feedback laws, their practical implementation remains challenging due to computational and numerical difficulties (see [23], [37], [47]). To ensure a safe implementation, a novel predictor method based on a state observer, capable of controlling and stabilizing all linear systems with arbitrary pure input delays, was proposed in [25]. In practice, it is often difficult to obtain the precise value of the delay. For general linear systems, an incomplete or imprecise knowledge of time delays may not be an obstacle to designing dynamic controllers, but this is not the case for nonholonomic systems since uncertain input delay is converted to structurally "uncertain" due to structural coupling for nonholonomic systems (see (25) and [6,42]). Consequently, developing memoryless controllers for general chained nonholonomic systems is essential.

In this paper, the smooth memoryless time-varying control problem for a class of chained nonholonomic systems with input delays will be investigated. The chained nonholonomic system can be decomposed into two components: a linear subsystem and a bilinear chained subsystem. For the linear subsystem, leveraging certain properties of parametric Lyapunov equations, a smooth memoryless nonhomogeneous control law that can achieve exponential convergence is proposed. Through a linear time-varying state transformation, the bilinear chained subsystem is transformed into a linear time-varying system. By exploiting the structural properties of nonholonomic systems and applying the Razumikhin stability theorem, smooth memoryless time-varying feedback controllers are designed.

The contributions of this paper are threefold.

(1) Firstly, different from traditional chained nonholonomic systems studied in works such as [11,14,24,32,43,44,49], which consist solely of kinematic equations, the chained nonholonomic systems examined in this paper incorporate both kinematic and dynamic equations. As a result, they are more general and better aligned with practical applications.
(2) Secondly, unlike existing traditional predictor methods for nonholonomic systems (such as [7,27,29]), which may be expensive or unsafe to implement (see [23,34,37]), and the finite-dimensional control method for nonholonomic systems in [44], where the controllers are memory-based–that is, they require precise time-delay information–the proposed controllers are memoryless, making them easier to implement and more practical.
(3) Thirdly, unlike traditional discontinuous control methods for nonholonomic systems (such as the σ-process method in [2,11,13,14,20,36,49]), which may lead to singularities when the initial condition approaches zero and cause chattering problems, the methods proposed in this paper are both global and smooth. Moreover, in contrast to smooth control methods for nonholonomic systems–such as the coordinate transformation approach (see [1] and related citations) and sine-function-based techniques (see [24] and related citations)–which are not easily applicable to full nonholonomic systems incorporating both kinematics and dynamics and are incapable of handling input delays, the proposed method can effectively address input delay problems.

2 Problem Formulation and Preliminaries

2.1 Problem Formulation

Consider the following chained nonholonomic system with input delays

$$\begin{cases} \dot{X}_0 = A_0 X_0 + B_0 u_0 (t-\tau_1), \\ \dot{X} = F(X_0)AX + bu(t-\tau_2), \\ y = \left[y_1^{\mathrm{T}}, y_2\right]^{\mathrm{T}} = \left[X_0^{\mathrm{T}}, X_{11}\right]^{\mathrm{T}} = \left[X_0^{\mathrm{T}}, cX\right]^{\mathrm{T}}, \end{cases} \tag{1}$$

where $(A_0, B_0) \in (\mathbf{R}^{m\times m}, \mathbf{R}^{m\times l})$ is controllable and all the eigenvalues of A_0 are zero,

$$F(X_0) = \mathrm{blkdiag}\left(\begin{bmatrix} X_{02} I_{n_1-1} & \mathbf{0} \\ \mathbf{0}^{\mathrm{T}} & 1 \end{bmatrix}, I_{n_2}\right), \tag{2}$$

$$A = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ & & & & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}. \tag{3}$$

$X_0 = [X_{01}, X_{02}, \ldots, X_{0m}]^{\mathrm{T}} \in \mathbf{R}^m$, $X = [X_1^{\mathrm{T}}, X_2^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^n, n = n_1+n_2, X_1 = [X_{11}, X_{12}, \ldots, X_{1n_1}]^{\mathrm{T}} \in \mathbf{R}^{n_1}$ and $X_2 = [X_{21}, X_{22}, \ldots, X_{2n_2}]^{\mathrm{T}} \in \mathbf{R}^{n_2}$ are state vectors, $y \in \mathbf{R}^{m+1}$ is output vector, $u_0 \in \mathbf{R}^l$ and $u \in \mathbf{R}$ are control inputs, τ_1 and τ_2 are unknown constants satisfying

$$0 \le \tau_1 \le D, \quad 0 \le \tau_2 \le D, \tag{4}$$

where D is a known positive constant.

Most studies on chain nonholonomic systems (such as [11,14,24,32,43,44,49]) focus exclusively on kinematic nonholonomic systems, characterized by $m = 1$ and $n_2 = 0$. However, practical nonholonomic systems typically consist of integrated systems that encompass both kinematics and dynamics, i.e., $m > 1$ and $n_2 > 0$ [31]. Hence, the system described in (1) is more general and better aligns with practical applications.

In this paper, we are interested in the following questions.

Problem 1 *We design a smooth memoryless time-varying control law for $u_0(t)$ and $u(t)$ such that, for any initial condition, the state and control of the closed-loop system converge to zero asymptotically.*

Problem 2 *We design an observer-based smooth memoryless time-varying output feedback control law such that, for any initial condition, the state and control of the closed-loop system converge to zero asymptotically.*

Remark 1 *The state X_{02} in $F(X_0)$ is not a special case but represents an arbitrary element in X_0, as the equations of the X_0 subsystems can always be reordered.*

2.2 Preliminaries

Consider the parametric Lyapunov equation (PLE)

$$\mathcal{A}^{\mathrm{T}}P + P\mathcal{A} - P\mathcal{B}\mathcal{B}^{\mathrm{T}}P = -\gamma P, \tag{5}$$

and its dual form

$$\mathcal{A}Q + Q\mathcal{A}^{\mathrm{T}} - Q\mathcal{C}^{\mathrm{T}}\mathcal{C}Q = -\gamma Q, \tag{6}$$

where $\gamma > 0$ is a constant. Let $(\mathcal{A}, \mathcal{B}) \in (\mathbf{R}^{N\times N}, \mathbf{R}^{N\times L})$ be controllable and $(\mathcal{A}, \mathcal{C}) \in (\mathbf{R}^{N\times N}, \mathbf{R}^{M\times N})$ be observable. It has been shown in [47] that the PLEs (5) and (6) have (unique) positive definite solution $P(\gamma)$ and $Q(\gamma)$ respectively if and only if $\gamma > -2\min_{i=1,2,\ldots,N}\{\mathrm{Re}(\lambda_i(\mathcal{A}))\}$. Next, we will recall some existing results for the PLEs (5) and (6).

Lemma 1 *[47,48] Let $(A_0, B_0) \in (\mathbf{R}^{m\times m}, \mathbf{R}^{m\times l})$ be defined as in (1) and $\gamma > 0$. Denote $P_0(\gamma)$ as the unique positive definite solution to (5) for $(\mathcal{A}, \mathcal{B}) = (A_0, B_0)$. The following properties hold:*

(1) The closed-loop system matrix $A_0 - B_0B_0^{\mathrm{T}}P_0(\gamma)$ satisfies, for $i = 1, 2, \ldots, m$,

$$\mathrm{Re}\left\{\lambda_i(A_0 - B_0B_0^{\mathrm{T}}P_0(\gamma))\right\} = -\gamma. \tag{7}$$

(2) The positive definite matrix $P_0(\gamma)$ satisfies the inequality

$$A_0^{\mathrm{T}}P_0(\gamma)A_0 \le 3m^2\gamma^2P_0(\gamma). \tag{8}$$

(3) The positive definite matrix $P_0(\gamma)$ satisfies

$$\mathrm{tr}(B_0^{\mathrm{T}}P_0(\gamma)B_0) = m\gamma. \tag{9}$$

(4) In particular, if $(\mathcal{A}, \mathcal{B}, \mathcal{C}) = (A, b, c) \in (\mathbf{R}^{n\times n}, \mathbf{R}^{n\times 1}, \mathbf{R}^{1\times n})$ as defined in (3), the corresponding positive definite matrix $P(\gamma)$ can be determined as follows

$$P(\gamma) = \gamma L_nP_nL_n, \tag{10}$$

and

$$Q(\gamma) = \gamma^{2n-1}L_n^{-1}Q_nL_n^{-1}, \tag{11}$$

where $L_n = L_n(\gamma) = \mathrm{diag}([\gamma^{n-1}, \gamma^{n-2}, \ldots, \gamma, 1])$, $P_n = P(1)$, and $Q_n = Q(1)$.

(5) The matrices P_n, Q_n, P_n^{-1}, and Q_n^{-1} are similar to each other, and thus

$$\begin{aligned}\lambda_{\max}(P_n) = \lambda_{\max}(Q_n) = \lambda_{\min}^{-1}(P_n) = \lambda_{\max}(Q_n^{-1})\\ = \lambda_{\min}^{-1}(Q_n) = \lambda_{\min}^{-1}(Q_n^{-1}) \triangleq \Lambda.\end{aligned} \tag{12}$$

3 Design of the Control Law for $u_0(t)$

For convenience, X_0-subsystem in (1) is rewritten as follows

$$\dot{X}_0 = A_0X_0 + B_0u_0\left(t - \tau_1\right). \tag{13}$$

Let $\alpha > 0$ be a constant. According to (5) with $(\mathcal{A}, \mathcal{B}, P, \gamma) = (A_0, B_0, P_0, 3\alpha)$, we can get that

$$\begin{aligned}&\left(\alpha I_m + A_0 - B_0B_0^{\mathrm{T}}P_0(3\alpha)\mathrm{e}^{\alpha\tau_1}\right)^{\mathrm{T}}P_0(3\alpha)\\ &+ P_0(3\alpha)\left(\alpha I_m + A_0 - B_0B_0^{\mathrm{T}}P_0(3\alpha)\mathrm{e}^{\alpha\tau_1}\right)\\ =&2\alpha P_0(3\alpha) + A_0^{\mathrm{T}}P_0(3\alpha)\\ &+ P_0(3\alpha)A_0 - 2P_0(3\alpha)B_0B_0^{\mathrm{T}}P_0(3\alpha)\mathrm{e}^{\alpha\tau_1}\\ \le&2\alpha P_0(3\alpha) + A_0^{\mathrm{T}}P_0(3\alpha)\\ &+ P_0(3\alpha)A_0 - P_0(3\alpha)B_0B_0^{\mathrm{T}}P_0(3\alpha)\\ &- P_0(3\alpha)B_0B_0^{\mathrm{T}}P_0(3\alpha)\left(\mathrm{e}^{\alpha\tau_1} - 1\right) \le -\alpha P_0(3\alpha),\end{aligned}$$

which implies that $\alpha I_m + A_0 - B_0B_0^{\mathrm{T}}P_0(3\alpha)\mathrm{e}^{\alpha\tau_1}$ is Hurwitz and invertible. Let $k \in \mathbf{N}$, $h \in \{0, 1\}$, and denote

$$\begin{aligned}&E = [E_1, E_2, \ldots, E_m]^{\mathrm{T}}\\ &= (-1)^h\alpha^k(\alpha I_m + A_0 - B_0B_0^{\mathrm{T}}P_0(3\alpha)\mathrm{e}^{\alpha\tau_1})^{-1}B_0\mathrm{e}^{\alpha\tau_1}\mathbf{1}_l,\end{aligned} \tag{14}$$

where $\mathbf{1}_l = [1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbf{R}^l$.

Assumption 1 *There exist $k \in \mathbf{N}$, $h \in \{0, 1\}$, $\hat{\alpha} > 0$, and functions $E_{2\min} = E_{2\min}(\alpha)$ and $E_{2\max} = E_{2\max}(\alpha)$ from $[0,\ \hat{\alpha}]$ to $\mathbf{R}^+$ such that $E_2 = E_2(\tau_1, \alpha) \in [E_{2\min}(\alpha), E_{2\max}(\alpha)]$ for all $\alpha \in [0,\ \hat{\alpha}]$ and $\lim_{\alpha\to 0^+} E_{2\min}(\alpha)/E_{2\max}(\alpha) = 1$.*

Remark 2 *The validity of the above assumption depends on the structures of A_0 and B_0. For common integrator systems, from (14), when $h = 0$ and $k = m-1$, we can calculate*

$$E = \frac{1}{(-1)^m \left(\mathrm{e}^{-\alpha\tau_1} - 1\right) + 2^m} \times \left[(-1)^1 \tfrac{1}{\alpha},\, (-1)^2,\, \cdots,\, (-1)^m \alpha^{m-2} \right]^{\mathrm{T}},$$

which satisfies the above assumption.

Theorem 1 *Let Assumption 1 be satisfied and define $k \in \mathbf{N}$, $h \in \{0, 1\}$ accordingly. Let $\alpha > 0$ and $\gamma_0 = 3\alpha$ be two constants. Consider the following smooth time-varying nonhomogeneous feedback*

$$u_0(t) = -B_0^{\mathrm{T}} P_0(\gamma_0) X_0(t) + \beta \mathrm{e}^{-\alpha t} \mathbf{1}_l, \tag{15}$$

where $\beta = -(-1)^h \alpha^{k+1} / E_{2\max}$. Then, there exists a sufficiently small positive constant $\alpha_* = \alpha_*(D)$ such that, for any initial condition and $\alpha \in (0, \alpha_*)$, the state and control of closed-loop system consisting of (13) and (15) converge to zero exponentially. In particular, the state $X_{02}(t)$ can be expressed as

$$X_{02}(t) = \mathrm{e}^{-\alpha t} \left(Z_{02}(t) + \alpha\rho \right), \tag{16}$$

where $\rho = E_2 / E_{2\max} \in [E_{2\min}/E_{2\max}, 1]$, and $Z_{02}(t)$ is a time-varying function satisfying $\lim_{t\uparrow\infty} Z_{02}(t) = 0$ for any initial condition.

Proof. From Assumption 1 we impose $\alpha_* < \hat{\alpha}$. To simplify notations we let $P_0 = P_0(\gamma_0)$. The closed-loop system consisting of (13) and (15) can be expressed as

$$\dot{X}_0 = A_0 X_0 - B_0 B_0^{\mathrm{T}} P_0(\gamma_0) X_0 (t - \tau_1) + B_0 \beta \mathrm{e}^{-\alpha(t-\tau_1)} \mathbf{1}_l. \tag{17}$$

Denote

$$Z_0 = \mathrm{e}^{\alpha t} X_0 + \varepsilon, \tag{18}$$

where $\varepsilon = E\beta(-1)^h/\alpha^k$ with E being defined in (14). Then in view of (17), we can get

$$\begin{aligned} \dot{Z}_0 =& \alpha Z_0 + \left(A_0 - B_0 B_0^{\mathrm{T}} P_0(\gamma_0) \mathrm{e}^{\alpha\tau_1} \right) Z_0 \\ &+ B_0 B_0^{\mathrm{T}} P_0(\gamma_0) \mathrm{e}^{\alpha\tau_1} D_{\tau_1}(Z_0), \end{aligned} \tag{19}$$

where $D_{\tau_1}(Z_0) = Z_0(t) - Z_0(t-\tau_1)$ and we have used

$$\left(-\alpha I_m - A_0 + B_0 B_0^{\mathrm{T}} P_0(\gamma_0) \mathrm{e}^{\alpha\tau_1} \right) \varepsilon + B_0 \beta \mathrm{e}^{\alpha\tau_1} \mathbf{1}_l = 0.$$

Choose the Lyapunov function $V_0(Z_0) = Z_0^{\mathrm{T}} P_0 Z_0$, whose time-derivative along the trajectory of closed-loop system (19) can be written as

$$\begin{aligned} \dot{V}_0(Z_0) \le& 2\alpha Z_0^{\mathrm{T}} P_0 Z_0 + Z_0^{\mathrm{T}} \left(A_0^{\mathrm{T}} P_0 + P_0 A_0 \right) Z_0 \\ &- 2\mathrm{e}^{\alpha\tau_1} Z_0^{\mathrm{T}} P_0 B_0 B_0^{\mathrm{T}} P_0 Z_0 \\ &+ 2\mathrm{e}^{\alpha\tau_1} Z_0^{\mathrm{T}} P_0 B_0 B_0^{\mathrm{T}} P_0 D_{\tau_1}(Z_0) \\ \le& 2\alpha Z_0^{\mathrm{T}} P_0 Z_0 + Z_0^{\mathrm{T}} \left(A_0^{\mathrm{T}} P_0 + P_0 A_0 - P_0 B_0 B_0^{\mathrm{T}} P_0 \right) Z_0 \\ &- \mathrm{e}^{\alpha\tau_1} Z_0^{\mathrm{T}} P_0 B_0 B_0^{\mathrm{T}} P_0 Z_0 + 2\mathrm{e}^{\alpha\tau_1} Z_0^{\mathrm{T}} P_0 B_0 B_0^{\mathrm{T}} P_0 D_{\tau_1}(Z_0) \\ \le& 2\alpha Z_0^{\mathrm{T}} P_0 Z_0 - \gamma_0 Z_0^{\mathrm{T}} P_0 Z_0 \\ &+ \mathrm{e}^{\alpha\tau_1} D_{\tau_1}^{\mathrm{T}}(Z_0) P_0 B_0 B_0^{\mathrm{T}} P_0 D_{\tau_1}(Z_0) \\ \le& 2\alpha Z_0^{\mathrm{T}} P_0 Z_0 - \gamma_0 Z_0^{\mathrm{T}} P_0 Z_0 + m\gamma_0 \mathrm{e}^{\alpha\tau_1} D_{\tau_1}^{\mathrm{T}}(Z_0) P_0 D_{\tau_1}(Z_0), \end{aligned} \tag{20}$$

where we have used (5) with $(\mathcal{A}, \mathcal{B}) = (A_0, B_0)$, Young's inequality, and (9).

Let $V_0(Z_0(t+\upsilon)) < pV_0(Z_0(t)), \upsilon \in [-2D, 0]$, $t \in [0, \infty)$, where $p > 1$ is any constant. Integrating both sides of (19) gives

$$\begin{aligned} &D_{\tau_1}(Z_0) = Z_0(t) - Z_0(t-\tau_1) = \int_{t-\tau_1}^{t} \mathrm{d}Z_0(s) \\ &= \int_{t-\tau_1}^{t} (\alpha Z_0(s) + A_0 Z_0(s) - B_0 B_0^{\mathrm{T}} P_0 \mathrm{e}^{\alpha\tau_1} Z_0(s-\tau_1)) \mathrm{d}s. \end{aligned}$$

From Jensen inequality (see Lemma C.1 in [47]), (4), (8), and (9), it follows that

$$\begin{aligned} &D_{\tau_1}^{\mathrm{T}}(Z_0) P_0 D_{\tau_1}(Z_0) \\ \le& 3\alpha^2 \left(\int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s) \mathrm{d}s \right) P_0 \left(\int_{t-\tau_1}^{t} Z_0(s) \mathrm{d}s \right) \\ &+ 3 \left(\int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s) A_0^{\mathrm{T}} \mathrm{d}s \right) P_0 \left(\int_{t-\tau_1}^{t} A_0 Z_0(s) \mathrm{d}s \right) \\ &+ 3\mathrm{e}^{2\alpha\tau_1} \left(\int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s-\tau_1) P_0 B_0 B_0^{\mathrm{T}} \mathrm{d}s \right) \\ &\times P_0 \left(\int_{t-\tau_1}^{t} B_0 B_0^{\mathrm{T}} P_0 Z_0(s-\tau_1) \mathrm{d}s \right) \\ \le& 3\alpha^2 \tau_1 \int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s) P_0 Z_0(s) \mathrm{d}s \\ &+ 3\tau_1 \int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s) A_0^{\mathrm{T}} P_0 A_0 Z_0(s) \mathrm{d}s \\ &+ 3\tau_1 \mathrm{e}^{2\alpha\tau_1} \int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s-\tau_1) P_0 B_0 B_0^{\mathrm{T}} \\ &\times P_0 B_0 B_0^{\mathrm{T}} P_0 Z_0(s-\tau_1) \mathrm{d}s \\ \le& 3\alpha^2 \tau_1 \int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s) P_0 Z_0(s) \mathrm{d}s \\ &+ 9m^2 \gamma_0^2 \tau_1 \int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s) P_0 Z_0(s) \mathrm{d}s \\ &+ 3\tau_1 \mathrm{e}^{2\alpha\tau_1} m^2 \gamma_0^2 \int_{t-\tau_1}^{t} Z_0^{\mathrm{T}}(s-\tau_1) P_0 Z_0(s-\tau_1) \mathrm{d}s \\ \le& 3\alpha^2 D^2 p V_0(Z_0) + \left(9 + 3\mathrm{e}^{2\alpha D} \right) m^2 \gamma_0^2 D^2 p V_0(Z_0). \end{aligned}$$

Substituting this expression in (20) yields

$$\dot{V}_0(Z_0) \leq -\alpha(1 - 3\alpha \mathrm{e}^{\alpha D} D^2 pm\gamma_0 - 27\mathrm{e}^{\alpha D}\varphi_1\alpha^2)V_0(Z_0),$$

where $\varphi_1 = \left(9 + 3\mathrm{e}^{2\alpha D}\right) m^3 D^2 p$ and $\gamma_0 = 3\alpha$. Clearly, there exists a sufficiently small positive constant $\alpha_* = \alpha_*(D)$ such that, for all $\alpha \in (0, \alpha_*)$, $\dot{V}_0(Z_0(t)) \leq -\alpha\varepsilon_0 V_0(Z_0(t))$, where ε_0 is a positive constant. Then the global stability of the transformed closed-loop system (19) is guaranteed by the Razumikhin stability theorem, namely, $\lim_{t\uparrow\infty} Z_0(t) = 0$. Therefore, in view of (18), we can rewrite X_0 as $X_0(t) = \mathrm{e}^{-\alpha t}(Z_0(t) - \varepsilon)$, where $\varepsilon = E\beta(-1)^h/\alpha^k$ and thus $X_{02}(t) = \mathrm{e}^{-\alpha t}(Z_{02}(t) - \beta(-1)^h E_2/\alpha^k)$, which proves (16). The proof is finished by noting that $u_0(t)$ also converges to zero exponentially. ■

Remark 3 *Similar to the control laws in [8,31,50], the controller (15) includes a nonhomogeneous term $\beta\mathrm{e}^{-\alpha t}\mathbf{1}_l$. The purpose of introducing this term is to ensure that $X_{02}(t)$ maintains the form given in (16) even when $X_{02}(s) = 0$, $s \in [-D, 0]$. Traditional control strategies (e.g., [2,11,13,15,49]) typically address the scenario $X_{02}(0) = 0$ by employing a switching control to drive $X_{02}(t)$ away from the origin. Once $X_{02}(t_1) \neq 0$ at some $t_1 > 0$, a designed control strategy is then applied. In contrast, the method proposed in the theorem is smooth and guarantees global exponential convergence of both the state and the control input to zero.*

4 Design of the Control Law for $u(t)$

Consider the linear time-varying state transformation

$$z(t) = T(t)\begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}, \tag{21}$$

where

$$T(t) = \mathrm{blkdiag}\left(L_{n_1}(\mathrm{e}^{\alpha t}),\ \frac{1}{\alpha^{n_2}} L_{n_2}(\alpha)\right), \tag{22}$$

$$L_n(\gamma) = \mathrm{diag}(\gamma^{n-1}, \gamma^{n-2}, \ldots, \gamma, 1). \tag{23}$$

By using (21), (1) can be rewritten as

$$\begin{aligned}\dot{z} =& \frac{\mathrm{d}T(t)}{\mathrm{d}t} X + T(t)(F(X_0)AX + bu(t-\tau_2)) \\ =& \frac{\mathrm{d}T(t)}{\mathrm{d}t} T^{-1}(t)z + T(t)F(X_0)AX + T(t)bu(t-\tau_2) \\ =& \frac{\mathrm{d}T(t)}{\mathrm{d}t} T^{-1}(t)z + F(X_0)T(t)AT^{-1}(t)z \\ &+ \frac{1}{\alpha^{n_2}} bu(t-\tau_2),\end{aligned} \tag{24}$$

substituting (16) into which yields

$$\begin{aligned}\dot{z}(t) =& \alpha Az(t) + \alpha(\rho-1)A_3 z(t) + \alpha A_2 z(t) \\ &+ Z_{02}(t)A_3 z(t) + \frac{1}{\alpha^{n_2}} bu(t-\tau_2),\end{aligned} \tag{25}$$

where

$$A_2 = \mathrm{blkdiag}\left(\mathrm{diag}(n_1-1,\, n_1-2,\, \ldots,\, 0),\ \mathbf{0}_{n_2\times n_2}\right),$$

and

$$A_3 = \left[\begin{array}{c|c|c} \mathbf{0}_{(n_1-1)\times 1} & I_{(n_1-1)\times(n_1-1)} & \\ \hline \mathbf{0}_{1\times 1} & \mathbf{0}_{1\times(n_1-1)} & \mathbf{0}_{n_1\times n_2} \\ \hline \mathbf{0}_{n_2\times n_1} & & \mathbf{0}_{n_2\times n_2} \end{array}\right].$$

4.1 State Feedback

Next, we will design state feedback controller for such linear time-varying system. The solution to Problem 1 can be stated as follows.

Theorem 2 *Let Assumption 1 be satisfied, and define $k \in \mathbf{N}$, $h \in \{0, 1\}$ accordingly. Let $\alpha > 0$, $\gamma_0 = 3\alpha$, and γ be constants satisfying*

$$\gamma \geq 8\sqrt{\Lambda\lambda_{\max}(A_2 P_n A_2)}, \tag{26}$$

where Λ is defined in (12). Then there exists a sufficient small positive constant $\alpha_* = \alpha_*(D)$ such that, for any initial condition, the states X_0, X and controls u_0, u of the closed-loop system consisting of (1), smooth memoryless time-varying controllers (15) and

$$u(t) = -\alpha^{n_2+1} b^{\mathrm{T}} P(\gamma) T(t) X(t), \quad \alpha \in (0, \alpha_*), \tag{27}$$

with $P(\gamma)$ being the unique positive solution to (5) with $(\mathcal{A}, \mathcal{B}) = (A, b)$ and $T(t)$ being defined in (22), converge to zero asymptotically.

Proof. Let $P = P(\gamma)$ and $\rho_{\min} = E_{2\min}/E_{2\max}$. The closed-loop system consisting of (25) and (27) can be expressed as

$$\begin{aligned}\dot{z} =& \alpha Az + \alpha(\rho-1)A_3 z + \alpha A_2 z \\ &+ Z_{02}(t)A_3 z - \alpha bb^{\mathrm{T}} Pz(t-\tau_2),\end{aligned} \tag{28}$$

which can be also rewritten as

$$\begin{aligned}\dot{z} =& \alpha\left(A - bb^{\mathrm{T}}P\right)z + \alpha(\rho-1)A_3 z + \alpha A_2 z \\ &+ Z_{02}(t)A_3 z + \alpha bb^{\mathrm{T}} P D_{\tau_2}(z),\end{aligned} \tag{29}$$

where $D_{\tau_2}(z) = D_{\tau_2}(z(t)) = z(t) - z(t-\tau_2)$.

Choose the Lyapunov function $V(z) = z^{\mathrm{T}}Pz$, whose time-derivative along the trajectory of closed-loop system (29) can be calculated as

$$\begin{aligned}
\dot{V}(z) =& -\alpha\gamma z^{\mathrm{T}}Pz - \alpha z^{\mathrm{T}}Pbb^{\mathrm{T}}Pz + 2\alpha z^{\mathrm{T}}PA_2z \\
&+2\alpha(\rho-1)z^{\mathrm{T}}PA_3z + 2\alpha z^{\mathrm{T}}Pbb^{\mathrm{T}}PD_{\tau_2}(z) \\
&+2Z_{02}(t)z^{\mathrm{T}}PA_3z \\
\leq& -\alpha\gamma z^{\mathrm{T}}Pz - \alpha z^{\mathrm{T}}Pbb^{\mathrm{T}}Pz + \alpha k_1 z^{\mathrm{T}}A_2PA_2z \\
&+\alpha\frac{1}{k_1}z^{\mathrm{T}}Pz + 2\alpha(\rho-1)z^{\mathrm{T}}PA_3z \\
&+2\alpha z^{\mathrm{T}}Pbb^{\mathrm{T}}PD_{\tau_2}(z) + 2Z_{02}(t)z^{\mathrm{T}}PA_3z \\
\leq& -\alpha\gamma z^{\mathrm{T}}Pz + \alpha D_{\tau_2}^{\mathrm{T}}(z)Pbb^{\mathrm{T}}PD_{\tau_2}(z) \\
&+\alpha k_1\gamma z^{\mathrm{T}}A_2L_nP_nL_nA_2z + \alpha\frac{1}{k_1}z^{\mathrm{T}}Pz \\
&+\alpha(1-\rho_{\min})\gamma z^{\mathrm{T}}Pz \\
&+\alpha(1-\rho_{\min})z^{\mathrm{T}}A_3^{\mathrm{T}}L_nP_nL_nA_3z \\
&+|Z_{02}(t)|\gamma z^{\mathrm{T}}Pz + |Z_{02}(t)|z^{\mathrm{T}}A_3^{\mathrm{T}}L_nP_nL_nA_3z \\
\leq& -\alpha\gamma z^{\mathrm{T}}Pz + \alpha\lambda_A D_{\tau_2}^{\mathrm{T}}(z)PD_{\tau_2}(z) \\
&+\alpha k_1\Lambda\lambda_{\max}(A_2P_nA_2)z^{\mathrm{T}}Pz + \alpha\frac{1}{k_1}z^{\mathrm{T}}Pz \\
&+\alpha(1-\rho_{\min})\gamma\lambda_{\max}\left(A_3^{\mathrm{T}}P_nA_3\right)\Lambda z^{\mathrm{T}}Pz \\
&+\alpha(1-\rho_{\min})\gamma z^{\mathrm{T}}Pz + |Z_{02}(t)|\gamma z^{\mathrm{T}}Pz \\
&+|Z_{02}(t)|\gamma\lambda_{\max}\left(A_3^{\mathrm{T}}P_nA_3\right)\Lambda z^{\mathrm{T}}Pz, \qquad (30)
\end{aligned}$$

where $\lambda_A = b^{\mathrm{T}}Pb = n\gamma$, $k_1 > 0$, we have used $L_nA_3 = \gamma A_3L_n$, (9), (10) and (5) with $(\mathcal{A},\mathcal{B}) = (A,b)$, and Young's inequality. Integrating both the left and right sides of (28) yields

$$\begin{aligned}
D_{\tau_2}(z(t)) = \int_{t-\tau_2}^{t} &\left(\alpha Az(s) + \alpha(\rho-1)A_3z(s) + \alpha A_2z(s)\right. \\
&\left.-\alpha bb^{\mathrm{T}}Pz(s-\tau_2) + Z_{02}(s)A_3z(s)\right)\mathrm{d}s.
\end{aligned}$$

By using Jensen inequality (see Lemma C.1 in [47]), (8), (10), and (12), we can get

$$\begin{aligned}
&D_{\tau_2}^{\mathrm{T}}(z)PD_{\tau_2}(z) \\
\leq& 5\alpha^2\tau_2\int_{t-\tau_2}^{t} z^{\mathrm{T}}(s)A^{\mathrm{T}}PAz(s)\mathrm{d}s \\
&+5\alpha^2(\rho-1)^2\tau_2\int_{t-\tau_2}^{t} z^{\mathrm{T}}(s)A_3^{\mathrm{T}}PA_3z(s)\mathrm{d}s \\
&+5\alpha^2\tau_2\int_{t-\tau_2}^{t} z^{\mathrm{T}}(s)A_2^{\mathrm{T}}PA_2z(s)\mathrm{d}s \\
&+5\alpha^2\tau_2\int_{t-\tau_2}^{t} z^{\mathrm{T}}(s-\tau_2)Pbb^{\mathrm{T}}Pbb^{\mathrm{T}}Pz(s-\tau_2)\mathrm{d}s \\
&+5\tau_2\int_{t-\tau_2}^{t} Z_{02}^2(s)z^{\mathrm{T}}(s)A_3^{\mathrm{T}}PA_3z(s)\mathrm{d}s \\
\leq& 5\alpha^2D\lambda(\gamma)\int_{t-\tau_2}^{t} z^{\mathrm{T}}(s)Pz(s)\mathrm{d}s \\
&+5\alpha^2D(1-\rho_{\min})^2\lambda_{\max}\left(A_3^{\mathrm{T}}P_nA_3\right) \\
&\times\Lambda\gamma^2\int_{t-\tau_2}^{t} z^{\mathrm{T}}(s)Pz(s)\mathrm{d}s \\
&+5\alpha^2D\lambda_{\max}(A_2P_nA_2)\Lambda\int_{t-\tau_2}^{t} z^{\mathrm{T}}(s)Pz(s)\mathrm{d}s \\
&+5\alpha^2\lambda_A^2D\int_{t-\tau_2}^{t} z^{\mathrm{T}}(s-\tau_2)Pz(s-\tau_2)\mathrm{d}s \\
&+5D\lambda_{\max}\left(A_3^{\mathrm{T}}P_nA_3\right)\Lambda\gamma^2\int_{t-\tau_2}^{t} Z_{02}^2(s)z^{\mathrm{T}}(s)Pz(s)\mathrm{d}s, \qquad (31)
\end{aligned}$$

where $\lambda(\gamma) = 3n^2\gamma^2$. Let $p > 1$ be any constant. Since $\lim_{\alpha\to 0^+}\rho_{\min} = 1$, we can choose a sufficiently small positive constant $\alpha_* = \alpha_*(D)$ such that for $\alpha \in (0,\alpha_*)$, $\varepsilon(\alpha) \leq \gamma/4$, where

$$\begin{aligned}
\varepsilon(\alpha) =& \lambda_A\alpha^2\pi_0 + (1-\rho_{\min})\gamma(1+\lambda_{\max}(A_3^{\mathrm{T}}P_nA_3)\Lambda), \\
\pi_0 =& 5D^2\lambda(\gamma)p + 5(1-\rho_{\min})^2D^2\lambda_{\max}(A_3^{\mathrm{T}}P_nA_3)\Lambda\gamma^2p \\
&+5D^2\lambda_{\max}(A_2P_nA_2)\Lambda p + 5\lambda_A^2D^2p.
\end{aligned}$$

Since $\lim_{t\uparrow\infty} Z_{02}^2(t) = 0$, let t_0 be the smallest positive constant such that $\vartheta(t) \leq \gamma/4$ for all $t \geq t_0$ where

$$\begin{aligned}
\vartheta(t) =& 5\lambda_AD^2p\lambda_{\max}\left(A_3^{\mathrm{T}}P_nA_3\right)\Lambda\gamma^2\max_{s\in[0,D]} Z_{02}^2(t-s) \\
&+\frac{\gamma}{\alpha}|Z_{02}(t)| + \frac{\gamma}{\alpha}\lambda_{\max}\left(A_3^{\mathrm{T}}P_nA_3\right)\Lambda|Z_{02}(t)|.
\end{aligned}$$

Let $V(z(t+\upsilon)) < pV(z(t))$, $\upsilon \in [-2D,0]$, $t \in [t_0,\infty)$. Then (31) can be calculated as

$$\begin{aligned}
&D_{\tau_2}^{\mathrm{T}}(z(t))PD_{\tau_2}(z(t)) \leq \alpha^2\pi_0V(z) \\
&+5\max_{s\in[0,D]} Z_{02}^2(t-s)D^2p\lambda_{\max}\left(A_3^{\mathrm{T}}P_nA_3\right)\Lambda\gamma^2V(z). \qquad (32)
\end{aligned}$$

Take $k_1 = \sqrt{\lambda_{\max}(A_2P_nA_2)\Lambda}$. Since $\vartheta(t) \leq \gamma/4$ and $\varepsilon(\alpha) \leq \gamma/4$ for all $\alpha \in (0,\alpha_*)$, (30) can be continued as

$$\begin{aligned}
&\dot{V}(z(t)) \\
\leq& -\alpha\left(\gamma - 2\sqrt{\Lambda\lambda_{\max}(A_2P_nA_2)} - \varepsilon(\alpha) - \vartheta(t)\right)V(z(t)) \\
\leq& -\alpha\left(\frac{1}{2}\gamma - 2\sqrt{\Lambda\lambda_{\max}(A_2P_nA_2)}\right)V(z(t)) \\
\leq& -\frac{1}{4}\alpha\gamma V(z(t)), \quad t \in [t_0,\infty).
\end{aligned}$$

Then the global stability of the transformed closed-loop system (28) is guaranteed by the Razumikhin stability theorem, namely, $\lim_{t\uparrow\infty} z(t) = 0$. Thus, in light of (21), we can get $\lim_{t\uparrow\infty} X(t) = \lim_{t\uparrow\infty} T^{-1}(t)z(t) = 0$. Finally, we end this proof by noting that $\lim_{t\uparrow\infty} u(t) = 0$. ■

Remark 4 *On the one hand, in the linear time-varying state feedback (27), that is $-\alpha^{n_2+1}b^{\mathrm{T}}P(\gamma)z(t)$, we need a sufficiently small constant α to eliminate the impact of the input delay. As a consequence, the system has the characteristics of low gain feedback with respect to state z. On the other hand, due to the nonholonomic characteristics of the system, we need a high gain characteristic to drive each state X to zero. In this sense, the controller $-\alpha^{n_2+1}b^{\mathrm{T}}P(\gamma)T(t)X(t)$ exhibits high gain characteristics with respect to state $X_1(t)$.*

4.2 Output Feedback

For clarity, the system (25) with output $y_2(t) = X_{11} = cX$ can be rewritten as

$$\begin{cases} \dot{z}(t) = \alpha Az(t) + \alpha(\rho-1)A_3z(t) + \alpha A_2z(t) \\ \qquad\quad +Z_{02}(t)A_3z(t) + \dfrac{1}{\alpha^{n_2}}bu(t-\tau_2), \\ y_2(t) = cX(t) = \mathrm{e}^{-\alpha(n_1-1)t}z_1(t) = \mathrm{e}^{-\alpha(n_1-1)t}cz(t). \end{cases} \tag{33}$$

With the above preparation, we can give the following results.

Theorem 3 *Let Assumption 1 be satisfied and define $k \in N$, $h \in \{0,1\}$ accordingly. Let $\alpha > 0$ and γ be constants satisfying*

$$\gamma \geq 16\max\{\sqrt{\Lambda\lambda_{\max}(A_2P_nA_2)}, \sqrt{\Lambda\lambda_{\max}(A_2Q_n^{-1}A_2)}\}. \tag{34}$$

Consider the observer-based smooth linear memoryless time-varying output feedback

$$\begin{cases} \dot{\xi}(t) = \alpha A\xi(t) + \alpha A_2\xi(t) + \frac{1}{\alpha^{n_2}}bu(t) \\ \qquad\quad +\alpha Q(\gamma)c^{\mathrm{T}}(\mathrm{e}^{\alpha(n_1-1)t}y_2(t) - c\xi(t)), \\ u(t) = -\alpha^{n_2+1}b^{\mathrm{T}}P(\gamma)\xi(t), \end{cases} \tag{35}$$

where $P(\gamma)$ and $Q(\gamma)$ are the unique positive solutions to (5) and (6) respectively, with $(\mathcal{A},\mathcal{B},\mathcal{C}) = (A,b,c)$. Then there exists a sufficiently small constant $\alpha_* = \alpha_*(D) > 0$, such that, for any initial condition and $\alpha \in (0,\alpha_*)$, the states X_0, X, ξ, and controls u_0, u of the closed-loop system consisting of (1), (15) and (35) converge to zero asymptotically.

Proof. Let $P = P(\gamma)$ and $Q = Q(\gamma)$. Denote $e = z - \xi$. Then the closed-loop system consisting of (33) and (35) can be written as

$$\begin{cases} \dot{e} = \alpha\left(A - Qc^{\mathrm{T}}c\right)e + \alpha\left(\rho-1\right)A_3z + \alpha A_2e \\ \qquad\quad +Z_{02}(t)A_3z + \alpha bb^{\mathrm{T}}P\left(\xi(t) - \xi(t-\tau_2)\right), \\ \dot{\xi} = \alpha\left(A - bb^{\mathrm{T}}P\right)\xi + \alpha A_2\xi + \alpha Qc^{\mathrm{T}}ce. \end{cases} \tag{36}$$

Choose the Lyapunov function

$$V(\xi(t),e(t)) = \xi^{\mathrm{T}}P\xi + \eta\gamma^{2n}e^{\mathrm{T}}Q^{-1}e,$$

where $\eta = 4\sigma_1$ and $\sigma_1 = ncQ_nP_nQ_nc^{\mathrm{T}}$. Let $p > 1$ be a constant. Since $\lim_{\alpha\to 0^+}\rho_{\min} = 1$, where $\rho_{\min} = E_{2\min}/E_{2\max}$, we can choose a sufficiently small positive constant $\alpha_* = \alpha_*(D)$ such that for any $\alpha \in (0,\alpha_*)$,

$$\max\{\eta_1,\eta_2\} \leq \frac{1}{8}\gamma, \tag{37}$$

where

$$\eta_1 = 2\eta\gamma^2a_1\left(1-\rho_{\min}\right) + \eta\gamma^2\Lambda n\rho_1\alpha, \tag{38}$$

$$\eta_2 = \mu_1 + \alpha + \eta\gamma^2\Lambda n\rho_1\alpha, \tag{39}$$

$$\mu_1 = (1-\rho_{\min})\left(1 + 2\gamma^2a_1\Lambda^2\right), \tag{40}$$

$$\rho_1 = 4D^2\lambda(\gamma)p + 4D^2\lambda_A^2p + 4pD^2\frac{cQPQc^{\mathrm{T}}}{\eta\gamma^{2n}\lambda_{\min}(Q^{-1})} + 4D^2p\lambda_{\max}(A_2P_nA_2)\Lambda, \tag{41}$$

with $a_1 = \lambda_{\max}(A_3^{\mathrm{T}}Q_n^{-1}A_3)\Lambda$, $\lambda(\gamma) = 3n^2\gamma^2$, and $\lambda_A = n\gamma$. Since $\lim_{t\uparrow\infty}|Z_{02}(t)| = 0$, let t_{00} be the smallest positive constant such that

$$\rho_2|Z_{02}(t)| \leq \frac{\gamma\alpha}{8}, \quad t \geq t_{00}, \tag{42}$$

with

$$\rho_2 = 1 + 2\eta\gamma^2a_1 + 2\gamma^2a_1\Lambda. \tag{43}$$

Let

$$V(\xi(t+\upsilon),e(t+\upsilon)) < pV(\xi(t),e(t)),$$

where $\upsilon \in [-2D,0]$ and $t \in [t_{00},\infty)$, then we have, for all $\upsilon \in [-2D,0]$ and $t \in [t_{00},\infty)$,

$$\xi^{\mathrm{T}}(t+\upsilon)P\xi(t+\upsilon) < pV(\xi(t),e(t)), \tag{44}$$

and

$$\eta\gamma^{2n}e^{\mathrm{T}}(t+\upsilon)Q^{-1}e(t+\upsilon) < pV(\xi(t),e(t)). \tag{45}$$

Next we need to prove that there exists a positive constant ε, such that, for $t \geq t_{00}$, $\dot{V}(\xi(t),e(t)) \leq -\varepsilon V(\xi(t),e(t))$.

Take the Lyapunov function $V_\xi = V_\xi(\xi) = \xi^{\mathrm{T}}P\xi$, whose time-derivative along the trajectory of closed-loop system (36) can be written as

$$\begin{aligned} \dot{V}_\xi \leq& -\alpha\gamma\xi^{\mathrm{T}}P\xi + 2\alpha\xi^{\mathrm{T}}PQc^{\mathrm{T}}ce + 2\alpha\xi^{\mathrm{T}}PA_2\xi \\ \leq& -\alpha\gamma\xi^{\mathrm{T}}P\xi + \alpha k_1\gamma\xi^{\mathrm{T}}P\xi + \frac{\alpha}{k_1\gamma}e^{\mathrm{T}}c^{\mathrm{T}}cQPQc^{\mathrm{T}}ce \\ &+\alpha k_2\xi^{\mathrm{T}}P\xi + \alpha\frac{1}{k_2}\xi^{\mathrm{T}}A_2PA_2\xi, \end{aligned} \tag{46}$$

where we have used (5) with $(\mathcal{A},\mathcal{B}) = (A,b)$ and Young's inequalities with $k_i > 0$, $i = 1,2$. On the one hand, according to [48], we can get $e^{\mathrm{T}}c^{\mathrm{T}}cQPQc^{\mathrm{T}}ce \leq \gamma^{2n+2}\sigma_1 e^{\mathrm{T}}Q^{-1}e$. On the other hand, it follows from (10) and (12) that

$$\begin{aligned}\xi^{\mathrm{T}}A_2PA_2\xi &= \xi^{\mathrm{T}}A_2\gamma L_nP_nL_nA_2\xi\\ &= \gamma\xi^{\mathrm{T}}L_nA_2P_nA_2L_n\xi\\ &\leq \lambda_{\max}\left(A_2P_nA_2\right)\Lambda\xi^{\mathrm{T}}P\xi.\end{aligned}$$

Then (46) can be continued as

$$\begin{aligned}\dot{V}_\xi \leq& -\alpha\gamma\xi^{\mathrm{T}}P\xi + \alpha k_1\gamma\xi^{\mathrm{T}}P\xi + \frac{\alpha}{k_1}\gamma^{2n+1}\sigma_1 e^{\mathrm{T}}Q^{-1}e\\ &+\alpha k_2\xi^{\mathrm{T}}P\xi + \alpha\frac{\lambda_{\max}\left(A_2P_nA_2\right)\Lambda}{k_2}\xi^{\mathrm{T}}P\xi. \qquad (47)\end{aligned}$$

Denote $D_{\tau_2}(\xi(t)) = \xi(t) - \xi(t-\tau_2)$, then (36) can be re-expressed as

$$\begin{aligned}\dot{e} =&\alpha\left(A - Qc^{\mathrm{T}}c\right)e + \alpha\left(\rho-1\right)A_3z\\ &+\alpha A_2e + Z_{02}(t)A_3z + \alpha bb^{\mathrm{T}}PD_{\tau_2}(\xi). \qquad (48)\end{aligned}$$

By integrating both the left and right sides of the formula (36), we can get

$$\begin{aligned}D_{\tau_2}(\xi(t)) =&\alpha\int_{t-\tau_2}^{t}\left(A\xi(s) - bb^{\mathrm{T}}P\xi(s)\right.\\ &\left.+A_2\xi(s) + Qc^{\mathrm{T}}ce(s)\right)\mathrm{d}s. \qquad (49)\end{aligned}$$

Therefore, in light of Jensen inequality (see Lemma C.1 in [47]), (8), (9), (12), (44), (45), and (49) we can get

$$\begin{aligned}&D_{\tau_2}^{\mathrm{T}}(\xi)PD_{\tau_2}(\xi) \leq 4\alpha^2D\int_{t-\tau_2}^{t}\xi^{\mathrm{T}}(s)A^{\mathrm{T}}PA\xi(s)\mathrm{d}s\\ &+4\alpha^2D\int_{t-\tau_2}^{t}\xi^{\mathrm{T}}\left(s\right)Pbb^{\mathrm{T}}Pbb^{\mathrm{T}}P\xi\left(s\right)\mathrm{d}s\\ &+4\alpha^2D\int_{t-\tau_2}^{t}e^{\mathrm{T}}\left(s\right)c^{\mathrm{T}}cQPQc^{\mathrm{T}}ce\left(s\right)\mathrm{d}s\\ &+4\alpha^2D\int_{t-\tau_2}^{t}\xi^{\mathrm{T}}\left(s\right)A_2PA_2\xi\left(s\right)\mathrm{d}s\\ \leq&4\alpha^2D\lambda(\gamma)\int_{t-\tau_2(t)}^{t}\xi^{\mathrm{T}}(s)P\xi(s)\mathrm{d}s\\ &+4\alpha^2D\lambda_A^2\int_{t-\tau_2}^{t}\xi^{\mathrm{T}}\left(s\right)P\xi\left(s\right)\mathrm{d}s\\ &+4\alpha^2D\frac{cQPQc^{\mathrm{T}}}{\lambda_{\min}\left(Q^{-1}\right)}\int_{t-\tau_2}^{t}e^{\mathrm{T}}\left(s\right)Q^{-1}e\left(s\right)\mathrm{d}s\\ &+4\alpha^2D\lambda_{\max}\left(A_2P_nA_2\right)\Lambda\int_{t-\tau_2}^{t}\xi^{\mathrm{T}}\left(s\right)P\xi\left(s\right)\mathrm{d}s\\ \leq&\alpha^2\rho_1V\left(\xi(t),e\left(t\right)\right),\end{aligned}$$

where ρ_1 is defined in (41).

Take another Lyapunov function $V_e = V_e(e) = e^{\mathrm{T}}Q^{-1}e$, whose time-derivative along the trajectory of closed-loop system (48) can be written as

$$\begin{aligned}\dot{V}_e \leq& -\alpha\gamma e^{\mathrm{T}}Q^{-1}e + 2\alpha(\rho-1)e^{\mathrm{T}}Q^{-1}A_3z + 2\alpha e^{\mathrm{T}}Q^{-1}A_2e\\ &+2Z_{02}(t)e^{\mathrm{T}}Q^{-1}A_3z + 2\alpha e^{\mathrm{T}}Q^{-1}bb^{\mathrm{T}}PD_{\tau_2}(\xi),\end{aligned} \qquad (50)$$

where we have used (6) with $(\mathcal{A},\mathcal{C}) = (A,c)$. On the one hand, by using Young's inequality, (11), $L_nA_3 = \gamma A_3L_n$, and $L_nb = b$, we can get

$$\begin{aligned}2e^{\mathrm{T}}Q^{-1}A_3z &\leq e^{\mathrm{T}}Q^{-1}e + z^{\mathrm{T}}A_3^{\mathrm{T}}Q^{-1}A_3z\\ &= e^{\mathrm{T}}Q^{-1}e + \frac{1}{\gamma^{2n-1}}z^{\mathrm{T}}A_3^{\mathrm{T}}L_nQ_n^{-1}L_nA_3z\\ &= e^{\mathrm{T}}Q^{-1}e + \frac{\gamma^2}{\gamma^{2n-1}}z^{\mathrm{T}}L_nA_3^{\mathrm{T}}Q_n^{-1}A_3L_nz\\ &\leq e^{\mathrm{T}}Q^{-1}e + \frac{\lambda_{\max}\left(A_3^{\mathrm{T}}Q_n^{-1}A_3\right)\Lambda}{\gamma^{2n-2}}z^{\mathrm{T}}Pz,\end{aligned}$$

and

$$\begin{aligned}&2e^{\mathrm{T}}Q^{-1}bb^{\mathrm{T}}PD_{\tau_2}\\ \leq&\alpha e^{\mathrm{T}}Q^{-1}e + \frac{1}{\alpha}D_{\tau_2}^{\mathrm{T}}Pbb^{\mathrm{T}}Q^{-1}bb^{\mathrm{T}}PD_{\tau_2}\\ \leq&\alpha e^{\mathrm{T}}Q^{-1}e + \frac{\Lambda}{\alpha\gamma^{2n-1}}D_{\tau_2}^{\mathrm{T}}Pbb^{\mathrm{T}}bb^{\mathrm{T}}PD_{\tau_2}\\ \leq&\alpha e^{\mathrm{T}}Q^{-1}e + \frac{\Lambda n\gamma}{\alpha\gamma^{2n-1}}D_{\tau_2}^{\mathrm{T}}PD_{\tau_2}\\ \leq&\alpha e^{\mathrm{T}}Q^{-1}e + \frac{\Lambda n\gamma}{\gamma^{2n-1}}\alpha\rho_1V\left(\xi(t),e\left(t\right)\right).\end{aligned}$$

On the other hand, in view of Young's inequality with $k_3 > 0$, (10) and (11), we can obtain

$$\begin{aligned}&2e^{\mathrm{T}}Q^{-1}A_2e\\ \leq&k_3e^{\mathrm{T}}Q^{-1}e + \frac{1}{k_3}e^{\mathrm{T}}A_2Q^{-1}A_2e\\ =&k_3e^{\mathrm{T}}Q^{-1}e + \frac{1}{k_3\gamma^{2n-1}}e^{\mathrm{T}}A_2L_nQ_n^{-1}L_nA_2e\\ =&k_3e^{\mathrm{T}}Q^{-1}e + \frac{1}{k_3\gamma^{2n-1}}e^{\mathrm{T}}L_nA_2Q_n^{-1}A_2L_ne\\ \leq&k_3e^{\mathrm{T}}Q^{-1}e + \frac{\lambda_{\max}(A_2Q_n^{-1}A_2)\Lambda}{k_3\gamma^{2n-1}}e^{\mathrm{T}}L_nQ_n^{-1}L_ne\\ =&k_3e^{\mathrm{T}}Q^{-1}e + \frac{\lambda_{\max}(A_2Q_n^{-1}A_2)\Lambda}{k_3}e^{\mathrm{T}}Q^{-1}e,\end{aligned}$$

and

$$\begin{aligned}z^{\mathrm{T}}Pz \leq& 2\xi^{\mathrm{T}}P\xi + 2e^{\mathrm{T}}Pe = 2\xi^{\mathrm{T}}P\xi + 2e^{\mathrm{T}}\gamma L_nP_nL_ne\\ \leq& 2\xi^{\mathrm{T}}P\xi + 2\Lambda^2e^{\mathrm{T}}\gamma L_nQ_n^{-1}L_ne\end{aligned}$$

$$= 2\xi^{\mathrm{T}} P\xi + 2\Lambda^2\gamma^{2n} e^{\mathrm{T}} Q^{-1} e.$$

Then (50) can be continued as

$$\begin{aligned}\dot{V}_e \leq & -\alpha\left(\gamma - \mu_1 - k_3 - \frac{\lambda_{\max}(A_2 Q_n^{-1} A_2)\Lambda}{k_3} - \alpha\right) e^{\mathrm{T}} Q^{-1} e \\ & + 2\alpha\left(1-\rho_{\min}\right)\frac{a_1}{\gamma^{2n-2}}\xi^{\mathrm{T}} P\xi \\ & + \left|Z_{02}(t)\right| e^{\mathrm{T}} Q^{-1} e + 2\left|Z_{02}(t)\right|\frac{a_1}{\gamma^{2n-2}}\xi^{\mathrm{T}} P\xi \\ & + 2\left|Z_{02}(t)\right|\gamma^2 a_1 \Lambda^2 e^{\mathrm{T}} Q^{-1} e \\ & + \frac{\Lambda n\gamma}{\gamma^{2n-1}}\alpha^2\rho_1 V\left(\xi(t), e\left(t\right)\right), \end{aligned} \tag{51}$$

where μ_1 is defined in (40).

Take $k_1 = 1/2$, $k_2 = \sqrt{\Lambda\lambda_{\max}(A_2 P_n A_2)}$, and $k_3 = \sqrt{\Lambda\lambda_{\max}(A_2 Q_n^{-1} A_2)}$. From (47), (51), and $\eta = 4\sigma_1$, it follows that

$$\begin{aligned} & \dot{V}\left(\xi, e\right) \\ \leq & -\alpha\left(\frac{1}{2}\gamma - 2\sqrt{\Lambda\lambda_{\max}\left(A_2 P_n A_2\right)} - \eta_1\right)\xi^{\mathrm{T}} P\xi \\ & - \alpha\eta\gamma^{2n}\left(\frac{1}{2}\gamma - 2\sqrt{\lambda_{\max}(A_2 Q_n^{-1} A_2)\Lambda} - \eta_2\right) e^{\mathrm{T}} Q^{-1} e \\ & + \eta\gamma^{2n}\left|Z_{02}(t)\right| e^{\mathrm{T}} Q^{-1} e + 2\eta\gamma^2\left|Z_{02}(t)\right| a_2 \xi^{\mathrm{T}} P\xi \\ & + 2\eta\left|Z_{02}(t)\right|\gamma^{2n+2} a_2 \Lambda^2 e^{\mathrm{T}} Q^{-1} e, \end{aligned}$$

where η_1 and η_2 are defined in (38) and (39). According to (34), (37), and (42), for all $\alpha \in (0, \alpha_*)$ and $t \in [t_{00}, \infty)$,

$$\begin{aligned} & \dot{V}(\xi(t), e(t)) \\ \leq & -\alpha\left(\frac{1}{2}\gamma - 2\sqrt{\Lambda\lambda_{\max}\left(A_2 P_n A_2\right)} - \eta_1\right)\xi^{\mathrm{T}} P\xi \\ & - \alpha\eta\gamma^{2n}\left(\frac{1}{2}\gamma - 2\sqrt{\Lambda\lambda_{\max}\left(A_2 Q_n^{-1} A_2\right)} - \eta_2\right) e^{\mathrm{T}} Q^{-1} e \\ & + \frac{\gamma\alpha}{8} V(\xi(t), e(t)) \leq -\frac{1}{8}\gamma\alpha V(\xi(t), e(t)). \end{aligned}$$

Then the global stability of closed-loop system (28) is guaranteed by the Razumikhin stability theorem, namely, $\lim_{t\uparrow\infty}\xi(t) = 0$ and $\lim_{t\uparrow\infty} e(t) = 0$. Then in view of (21) and $e = z - \xi$, we can get $\lim_{t\uparrow\infty} z(t) = \lim_{t\uparrow\infty}(\xi(t)+e(t)) = 0$ and $\lim_{t\uparrow\infty} X(t) = \lim_{t\uparrow\infty} T^{-1}(t) z(t) = 0$. The proof is finished by noting that $\lim_{t\uparrow\infty} u(t) = 0$. ■

Remark 5 *The coupling constraints in nonholonomic systems have consistently posed challenges in controller design. In this paper, we transform these specific structural coupling constraints into advantages by designing an appropriate nonhomogeneous controller. By carefully designing the controller $u_0(t)$, the nonholonomic system is transformed into linear time-varying systems (28) and (36), with a parameter α in the system matrix αA, whose choice is crucial in handing the input delay. Under the low-gain approach, to deal with larger delays, a smaller value of α should be selected.*

5 Numerical Example

Consider the kinematics and dynamics of an underactuated axisymmetric spacecraft model proposed in [22,50],

$$\begin{cases} \dot{\xi}_1 = \frac{1}{2}\omega_1, & \dot{\omega}_1 = \frac{T_1(t-\tau_1)}{J}, \\ \dot{\xi}_2 = \frac{1}{2}\omega_2, & \dot{\omega}_2 = \frac{T_2(t-\tau_2)}{J}, \\ \dot{\xi}_3 = \frac{1}{2}\xi_1\omega_2 - \frac{1}{2}\xi_2\omega_1, & \end{cases} \tag{52}$$

where $\xi = [\xi_1, \xi_2, \xi_3]^{\mathrm{T}}$ denotes the Rodrigues parameter, $\omega = [\omega_1, \omega_2, \omega_3]^{\mathrm{T}}$ denotes the angular velocity vector, $T_1(t)$ and $T_2(t)$ denote the control torques, J denotes the principal moments of inertia, which, together with $X_{02} = \omega_1/2, X_{21} = -\omega_2$, $X_{01} = \xi_1$, $X_{12} = -2\xi_2$, and $X_{11} = \xi_3 - \xi_1\xi_2$, can be rewritten as

$$\begin{cases} \dot{X}_{01} = X_{02}, & \dot{X}_{11} = X_{02}X_{12}, \\ \dot{X}_{02} = u_0(t-\tau_1), & \dot{X}_{12} = X_{21}, \\ & \dot{X}_{21} = u(t-\tau_2), \end{cases}$$

where $u_0(t-\tau_1) = T_1(t-\tau_1)/(2J)$ and $u(t-\tau_2) = -T_2(t-\tau_2)/(2J)$. From Remark 2, we can calculate that $E_{2\max} = 1/(3+\mathrm{e}^{-\alpha D})$ and $E_{2\min} = 1/4$, and we choose the parameter $\beta = -\alpha^2/E_{2\max}$ as in Theorem 1. Here, we consider only the state feedback case, meaning that $u(t)$ is given by (27) in Theorem 2. For simulation, take the initial condition $[X_{01}(0), X_{02}(0)] = [0.1, 0.02]$ and $[X_{11}(0), X_{12}(0), X_{21}(0)] = [0, 0.2, 0]$, $\tau_1 = 0.2$, $\tau_2 = 0.5$, $D = 0.5$, $J = 15\mathrm{kg}\cdot\mathrm{m}^2$, and $\gamma = 1$. The states and controllers are plotted in Fig. 1 for different values of α. From the above figure, we can observe that the proposed method can indeed drive the states and controllers to zero. In addition, the smaller α is, the slower is the convergence speed. In practical applications, a suitable choice of α should balance delay robustness, convergence speed, and overshoot in the system response.

6 Conclusion

In this paper, the smooth memoryless time-varying control problem for a class of chained nonholonomic systems, consisting of a linear subsystem, a bilinear subsystem, and an integrator chain subsystem with input delays, has been addressed. This paper employs a low-gain approach to address the smooth control problem of nonholonomic systems with input delays. Further studies may focus on the control of nonholonomic systems by using safe predictor methods, such as [25].

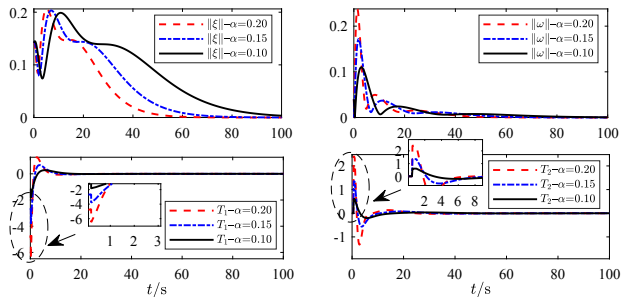


Fig. 1. The state responses and control signals for system (52) (The units of ω_1 and ω_2 are radians per second (rad/s), while the units of T_1 and T_2 are newton-meters ($\mathrm{N}\cdot\mathrm{m}$))

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