Stabilisation of time-delay systems using nonsmooth optimisation techniques

Joris W. Vanbiervliet
Department of Computer Science, Katholieke Universiteit Leuven, Belgium
joris.vanbiervliet@cs.kuleuven.be
Joint work with : Wim Michiels, Stefan Vandewalle

This talk addresses the stabilisation of the general linear time-delay system

$$\dot{x}(t) = \sum_{i=0}^{m} A_i x(t-\tau_i) + \sum_{j=0}^{n} B_j u(t-\tau_j), \tag{1}$$

controlled by a linear static state feedback $u(t) = Kx(t)$. Here, $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^p$ represent the respective state and input vector, A_j, B_j and K are matrices of appropriate dimensions and $\tau_i, \tau_j \geq 0$ are the fixed delays.

The closed-loop system is stable if the real part of the rightmost eigenvalue, or spectral abscissa, $\alpha(K)$ is strictly negative. The proposed stabilisation method consists of solving directly the optimisation problem

$$\min_K \alpha(K) = \min_K \sup_\lambda \{\Re(\lambda) : \lambda \text{ is an eigenvalue}\} \tag{2}$$

in function of the controller parameters contained in K. This is a hard problem for two reasons. Firstly, time-delay systems exhibit infinitely many eigenvalues, whereas only a finite number of degrees of freedom is available. Secondly, the objective function $\alpha(K)$ is a nonsmooth and even non-Lipschitz function of the parameters in K, and its minimisers are likely to be found in points of nondifferentiability. This prohibits the use of standard optimisation algorithms.

We therefore use the recently developed gradient sampling algorithm as presented in [1], a bundle gradient optimisation method that is able to find minima of nonsmooth functions. It has already been successfully applied to finite-dimensional eigenvalue optimisation problems, where the dimension of the system is much larger than the number of controller parameters. Considering the infinite nature of time-delay systems, our results can be regarded as an extension to an infinite-dimensional setting.

To evaluate the objective function $\alpha(K)$ itself, we have DDE-BIFTOOL at our disposal, a MATLAB package for the bifurcation analysis of nonlinear delay equations. It includes a routine that, using a linear multi-step approach, is able to compute the stability determining (i.e. rightmost) eigenvalues of a linear or linearised time-delay system with fixed delays, from which the value of $\alpha(K)$ is of course easily obtained.

We illustrate the effectiveness of our method by applying it to several numerical examples.

References

[1] J.V. Burke, A.S. Lewis, and M.L. Overton (2005), "A robust gradient sampling algorithm for nonsmooth, nonconvex optimization´´, SIAM Journal on Optimization, 15(3), 751-779.