NUMERICAL SOLUTIONS FOR RESONANTLY DAMPED MHD QUASI-MODES IN TWO-DIMENSIONAL CORONAL LOOPS

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ABSTRACT

The damping of coronal loop oscillations by means of resonant absorption of quasi-mode kink oscillations in cylindrical flux tubes supports the idea that resonant absorption may be a significant component operating in the observed fast decay of coronal loop oscillations. In this work, the classic straight, cylindrical, one-dimensional, flux tube model is extended by considering a two-dimensional equilibrium configuration with non-uniformity in both the axial and radial directions. The frequency and damping rate of fast kink quasi-modes has been computed by numerically solving the linear resistive magnetohydrodynamic (MHD) wave equations in flux tubes with thick non-uniform boundary layers. It is seen that longitudinal stratification produces and increase of the frequency and damping rate of fast kink-mode oscillations. However, when stratification is radially independent, the damping per period remains unchanged.

Key words: MHD; Sun: corona; Sun: coronal loops; Sun: oscillations.

1. INTRODUCTION

Transverse coronal loop oscillations triggered by explosive events, such as flares or filament eruptions, were first observed by the EUV telescope on board the TRACE satellite (Aschwanden et al. 1999; Nakariakov et al. 1999). Since then, several similar events have been reported and thoroughly studied (Aschwanden et al. 2002; Schrijver et al. 2002). Regarding the nature of these oscillations, and from a theoretical point of view, they have been interpreted in terms of the MHD fast kink-mode of a cylindrical coronal flux tube at the fundamental harmonic by Nakariakov et al. (1999) and Nakariakov & Ofman (2001).

Most of the coronal loops that exhibit oscillations have been found to be strongly damped, typically having an exponential decay time of a few oscillatory periods. This observational fact might have some important physical consequences, both for coronal seismology, i.e. the probing of unknown coronal physical properties by means of the study of the oscillations of its magnetic structures (Uchida 1970; Roberts et al. 1984), as for wave based coronal heating theories (Poedts & De Groof 2004), once we have identified the correct damping mechanism. Nevertheless, the cause of the fast damping of transverse coronal loop oscillations is still a matter of considerable debate. Currently, we can find several competing mechanisms: non-ideal effects such as viscous and ohmic damping, optically thin radiation and thermal conduction; lateral wave leakage due to the curvature of the loops (Roberts 2000); mechanisms based on the topology of magnetic field lines, such as footpoint motions near separatrices (Schrijver & Brown 2000); footpoint leakage through the chromospheric density gradient (De Pontieu et al. 2001; Ofman 2002); phase mixing of Alfvén waves (Heyvaerts & Priest 1983; Nakariakov et al. 1999; Ofman & Aschwanden 2002); and, finally, resonant absorption of waves (Goossens et al. 2002; Ruderman & Roberts 2002; Aschwanden et al. 2003). First observational tests with TRACE data revealed that the scaling law of the damping time as a function of other physical parameters, such as the loop length and the oscillation period, favours the phase mixing mechanism (Ofman & Aschwanden 2002). However, Goossens et al. (2002) pointed out that resonant absorption by damping of quasi-mode oscillations gives a perfect explanation of the fast decay of the observed coronal loops, if the inhomogeneity length scale is allowed to vary from loop to loop. An additional attraction of this mechanism is that quasi-mode damping is fully consistent with the current estimates of very large coronal Reynolds numbers of the order of 10^{14}.

Goossens et al. (2002) also stressed the fact that the assumption of a "thin" boundary layer is not fully consistent with the values they obtained for a collection of observed loops oscillations. For this reason, Van Doorsselaere et al. (2004) computed the frequency and damping rate of quasi-modes by relaxing the assumption of a "thin" boundary layer. They found that for low density contrast and large inhomogeneity length scales, as observed in oscillating coronal loops (Aschwanden et al. 2003), the numerically computed damping rates can deviate by up to 25% from the approximate results predicted by the "thin" boundary formula.

Our aim in this paper is to improve our theoretical knowledge on the damping of coronal loop oscillations by resonant absorption by considering two-dimensional equilibrium configurations with non-uniformity of the equilibrium density in both the radial and longitudinal (i.e. along the field lines) directions, in which constitutes an extension to the numerical study of Van Doorsselaere et al. (2004), for the damping of fast kink-modes of one-dimensional highly inhomogeneous coronal loop models. To this end, the frequency and damping rate of the fundamental fast kink-mode is computed, in fully resistive MHD, for a wide range of loop parameter values.

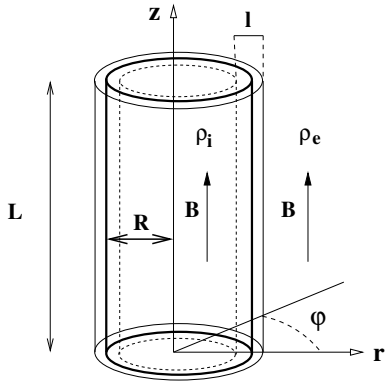


Figure 1. Sketch of the equilibrium configuration representing a straightened coronal flux tube of length L and radius R modelled as a density enhancement. The magnetic field is uniform and parallel to the z-axis and the whole configuration is invariant in the φ-direction. The density varies in a non-uniform boundary layer of length l from a constant internal value, ρ_i to a constant external value in the coronal environment, ρ_e.

2. EQUILIBRIUM CONFIGURATION

We consider that our coronal loop may be modelled by means of the classical straight cylindrically symmetric flux tube. In a system of cylindrical coordinates (r, φ, z) with the z-axis coinciding with the axis of the cylinder (loop) the magnetic field is then pointing in the z-direction (see Fig. 1). In the cold plasma approximation, $\beta = 0$, the equilibrium condition implies that the magnetic field is uniform, $\mathbf{B} = B\,\hat{\mathbf{e}}_z$, and also that the density, $\rho(r,z)$, or Alfvén speed, $v_{\rm A}(r,z)$, profiles can be chosen arbitrarily. Here, we consider that the equilibrium density is given by a two-dimensional function of the form

$$\rho(r,z) = \rho(r)\left[1 - \alpha\,\sin\left(\frac{\pi z}{L}\right)\right], \qquad (1)$$

with a dependence in the radial direction and a dependence in the longitudinal direction. The parameter α controls the gradient of stratification along field lines. For the radial dependence of the density in Eq. (1) we follow Ruderman & Roberts (2002) and assume a continuous sinusoidal variation between the internal, ρ_i, and the external, ρ_e, values of the density in a non-uniform boundary layer of length l such that

$$\rho(r) = \left\{ \begin{array}{ll} \rho_i & \text{for } 0 \le r < R - \frac{l}{2}, \\ \frac{\rho_i}{2}\left[\left(1+\frac{1}{\zeta}\right) - \left(1-\frac{1}{\zeta}\right)\sin\frac{\pi(r-R)}{l}\right] & \\ \text{for } R - \frac{l}{2} \le r \le R + \frac{l}{2}\,, & \\ \rho_e & \text{for } r > R + \frac{l}{2}, \end{array} \right\}$$

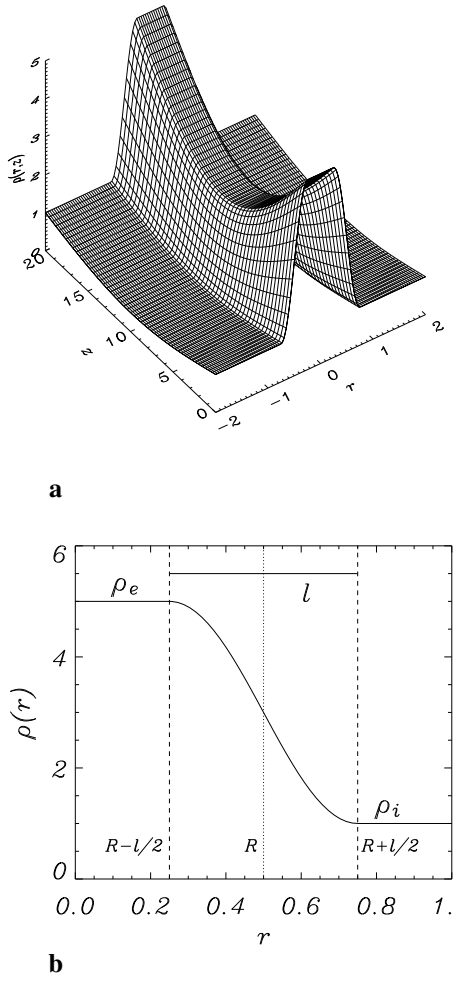


Figure 2. **a** *Surface plot of the equilibrium density, $\rho(r,z)$ given by Eq. (1) in a coronal loop with $\zeta = 5$, $l/R = 1$ and $\alpha = 0.5$.* **b** *Radial dependence on the density at the footpoint $z = 0$ showing the locations of the inner radius, $R - l/2$, and the outer radius, $R + l/2$.*

where $\zeta = \rho_i/\rho_e$ is the density contrast. The corresponding two-dimensional distribution of the equilibrium density, for a given value of the density contrast and stratification parameter, is shown in Fig. 2a. Fig. 2b shows the radial dependence of the density in half the width of the tube for a case in which the length scale of the inhomogeneity equals the loop radius.

3. LINEAR MHD WAVES

The previous equilibrium is then perturbed in order to study the small amplitude oscillations of the system. The perturbed quantities are Fourier analysed in time and in the ignorable φ-direction by assuming a dependence of the form $\exp^{i(m\varphi - \omega t)}$. Then, the linear resistive MHD equations are obtained which together with the appro-

priate boundary conditions form an eigenvalue problem. Here m (an integer) is the azimuthal wave number and ω the frequency. The $m = 1$ mode corresponds to the fundamental fast kink-mode, the only oscillatory mode that displaces the axis of the tube. As was pointed out by Goossens et al. (2002), in the presence of a non-uniform boundary layer, kink mode oscillations have always their frequency in the Alfvén continuum and, hence, are damped quasi-modes. The real part of the frequency is related with the oscillatory period, $\omega_{\rm R} = 2\pi/{\rm P}$, whilst the damping is expressed by a negative imaginary part of the frequency, $\omega_{\rm I} = -1/\tau_{\rm d}$.

4. NUMERICAL METHOD

Analytical solutions for the eigenmode problem do not exist in general for non-uniform equilibrium models, except possibly for special choices of the equilibrium profiles (Van Doorsselaere et al. 2004). It is important to emphasise that the assumption of a "thin boundary" is essential for analytical development. The consideration of "thick" boundaries yields strong damping rates, and in this case, the eigenvalue problem has to be solved numerically. For this reason, we use a numerical code (POLLUX) to solve the linear resistive MHD equations.

Nevertheless, analytical or semi-analytical theory is still possible when looking to the effect(s) of longitudinal stratification on the frequency and damping of coronal loop oscillations. In particular, by generalising the "thin boundary theory" to two-dimensional equilibrium configurations, Andries et al. (2004) have shown that the frequency and damping rate of the ideal quasi-mode is independent of stratification in the longitudinal direction, provided stratification is the same inside and outside the coronal loop. An important purpose of this work is to check whether this theoretical prediction is confirmed in fully resistive two-dimensional numerical calculations.

In POLLUX, finite elements are used for the discretisation in the radial direction. In the longitudinal direction, the perturbed quantities are represented by a finite number of Fourier modes of the form

$$f\,(r,z) = \sum_{n=-M}^{+M} f_n(r)\; e^{\imath\pi n z/L},$$

where n, an integer, is the longitudinal mode number. We use dimensionless variables by normalising all lengths to the cylinder radius, R, whereas the density and the magnetic field are normalised to their respective equilibrium values on the cylinder axis. Herewith, speeds are normalised to the Alfvén speed on axis. The output of the code, then, consists on the frequency and damping rate of resistive eigenmodes.

5. NUMERICAL RESULTS

In this section, numerical results to the linearised resistive MHD equations are shown. Our purpose is to compute the frequency and damping rate of fast kink quasi-modes when longitudinal stratification is taken into account. Following the procedure used by Van Doorsselaere et al. (2004), numerical solutions to the linearised resistive MHD wave have been obtained for a wide range of loop parameter values trying to cover the physical and geometrical properties of most of the coronal loops observed by Aschwanden et al. (2002).

The free parameters in our problem and the ranges of variation selected in our computations are; the thickness of the inhomogeneous layer ($l/R \in [0.0 - 2.0]$), the inverse aspect ratio of the tube ($\epsilon = 2\pi R/L \in [0.02 - 0.18]$), which yields $R/L \simeq 0.003$ for the longest tube and $R/L \simeq 0.02$ for the shortest one, the density contrast ($\zeta \in [1.5 - 5.0]$) and the longitudinal stratification coefficient ($\alpha \in [0.0 - 1.0]$). Another important parameter is resistivity, η. For practical purposes, in the numerical computations, resistivity has to be small enough to assure that the damping rate is independent of dissipation (Poedts & Kerner 1991), but large enough that the number of grid-points suffices to resolve the resonant layer.

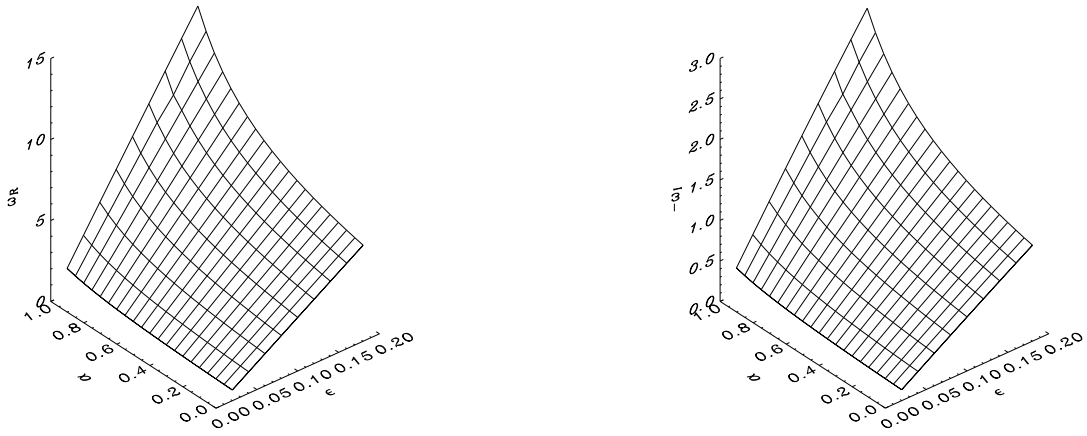
We start by considering coronal loops with fixed thickness of the boundary layer and fixed density contrast and look for the dependence of the frequency and damping rate on the stratification parameter, α, and on the inverse aspect ratio of the loop, ϵ (or equivalently, the length of the loop L). In particular, we have chosen coronal loops with thick non-uniform boundaries, that equal the radius of the loop, and with high density contrast. Figs. 3a and b show that, as expected, both the real part of the frequency as well as the damping rate increase for decreasing length of the loop (increasing values of ϵ). This is a well known result in previous studies with one-dimensional loop models. These figures also show that both the frequency and damping rate are strongly dependent on the stratification parameter. Thus, stratification in the longitudinal direction produces a decrease in the oscillatory period and an increase in the damping rate.

An additional interesting variable is the normalised damping rate, $-\omega_{\rm I}/\omega_{\rm R}$, which is a dimensionless observable quantity related with the damping per period,

$$-\frac{\omega_{\rm I}}{\omega_{\rm R}} = \left(\frac{1}{2\pi}\right)\left(\frac{P}{\tau_{\rm d}}\right).$$

Fig. 4 shows that the normalised damping rate is independent of stratification and that a slightly smaller damping rate can be expected when shorter loops are considered.

Next, we consider coronal loops with a fixed length and density contrast and look for the dependency of the frequency and the damping rate on the thickness of the inhomogeneous layer, l/R, and the stratification parameter, α. Figs. 5a and b show that both the frequency and the damping rate have a strong dependence on the stratification parameter. Note also, in Fig. 5b, that there is no damping when $l = 0$, i.e. a discontinuity on the density connects the internal and external parts of the loop.



a **b**

Figure 3. **a** *Frequency and* **b** *damping rate as a function of the inverse aspect ratio, ϵ, and longitudinal stratification parameter, α, for coronal loops with fixed inhomogeneity length scale ($l/R = 1.0$) and fixed density contrast ($\zeta = 5.0$).*

Previous analytical studies of coronal loops in equilibrium states with "thin" non-uniform boundary layers give a linear dependence of the damping rate on l/R. Following Van Doorsselaere et al. (2004), we can also compare the normalised damping rate, $-\omega_{\rm I}/\omega_{\rm R}$, with the expected value under the "thin tube and thin boundary" (tttb) approximation defined in one-dimensional equilibrium states by means of the factor $q_{\rm tttb}$ as follows

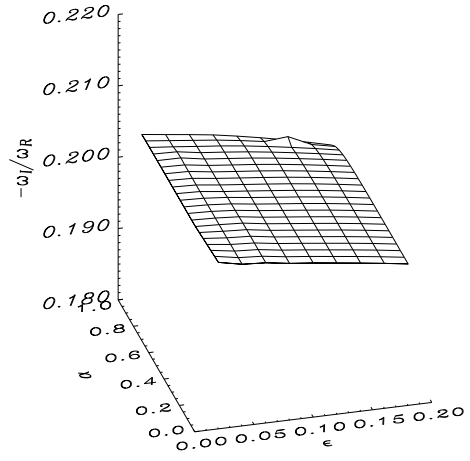


Figure 4. Normalised damping rate as a function of the inverse aspect ration, ϵ, and the stratification parameter, α for coronal loops with the same parameter values as in Fig. 3

$$\left(\frac{\omega_i}{\omega_r}\right) = -q_{\rm tttb}\frac{1}{4}\left(\frac{l}{R}\right)\left(\frac{\zeta-1}{\zeta+1}\right). \qquad (2)$$

Fig. 5c shows this quantity as a function of the stratification parameter and the thickness of the boundary layer. We see, again, that the normalised damping rate is independent of stratification. Also, we confirm the result obtained by Van Doorsselaere et al. (2004) that "thin tube and thin boundary" theory underestimates the damping rate even for moderate values of the inhomogeneity length scale.

Finally, we compute the combined effects of the density contrast and stratification by considering coronal loops with fixed length and fixed thickness of the inhomogeneous layer. Figs. 6a and b display the frequency and damping rate as a function of these two variables. We firmly confirm the strong dependence of the frequency and damping rate on the stratification parameter. By plotting the normalised damping rate (see Fig. 6c), we can see that the density contrast strongly determines the damping of coronal loop oscillations.

A simple relation that predicts the number of oscillations is given by $N = \tau_{\rm d}/P$. Fig. 6d shows that, as expected from the previous discussion, this quantity is independent of stratification, but strongly depends on the loop density contrast in such a way that, for the limits of density contrast considered through this work, coronal loop oscillations are very rapidly damped in a range of oscillatory periods from 0.78 to 2.85.

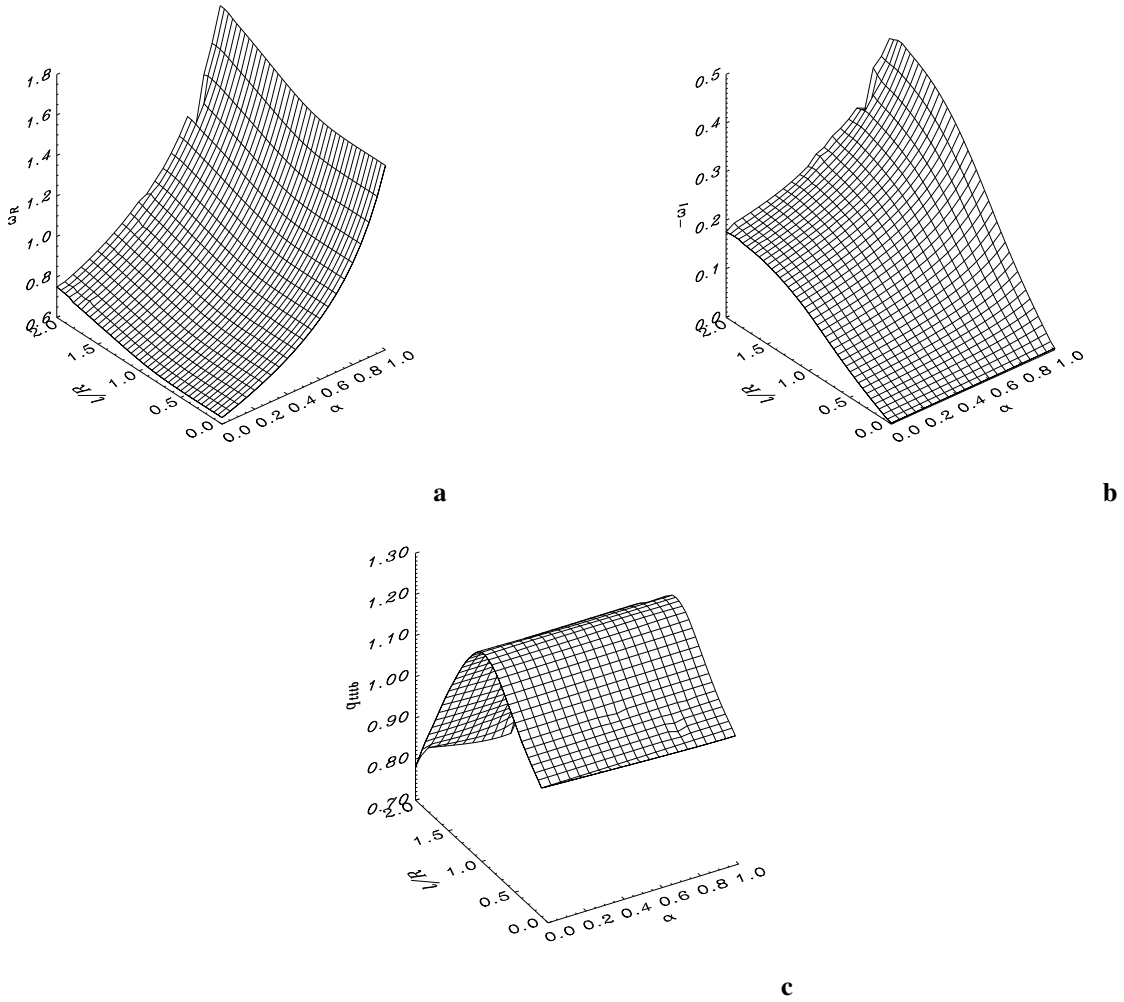


Figure 5. **a** *Frequency,* **b** *damping rate and* **c** $q_{\rm tttb}$ *factor as a function of the stratification parameter and the thickness of the inhomogeneous boundary layer for coronal loops with fixed length ($\epsilon = 0.02$) and density contrast ($\zeta = 4.0$).*

6. SUMMARY AND CONCLUSIONS

In this work, numerical solutions for resonantly damped MHD fast kink quasi-mode oscillations have been computed in two-dimensional cylindrical models of solar magnetic coronal loops. To this end, the classical cylindrical flux tube model has been considered with a two-dimensional equilibrium in which the density is allowed to vary both in the radial and longitudinal directions. Also, as in Van Doorsselaere et al. (2004), the assumption of a thin boundary layer that connects the external and internal parts of the tube has been removed. By solving the linearised resistive MHD wave equations a parametric numerical study of the fast kink-mode frequency and damping rate has been performed for a wide range of values for the thickness of the inhomogeneous layer, the density contrast, the length of the tube and the stratification coefficient.

Previous semi-analytical results obtained in ideal MHD, by means of a perturbation analysis by Andries et al. (2004), have been confirmed in fully resistive two-dimensional computations in the sense that longitudinal stratification of the density produces and increase of the frequency and damping rate of coronal loop oscillations. On the other hand, the normalised damping rate, a quantity that is related with the observable damping per period, remains unchanged, at least when stratification is the same inside and outside the coronal flux tube ($\alpha \neq \alpha(r)$) (see also Andries et al. 2004). Also, we have confirmed that the classical "thin tube and thin boundary" approximation underestimates the damping rate for moderate values of l/R.

Finally, we have seen that the density contrast is a very important parameter, in such a way that large contrast loops get damped in less that a period.

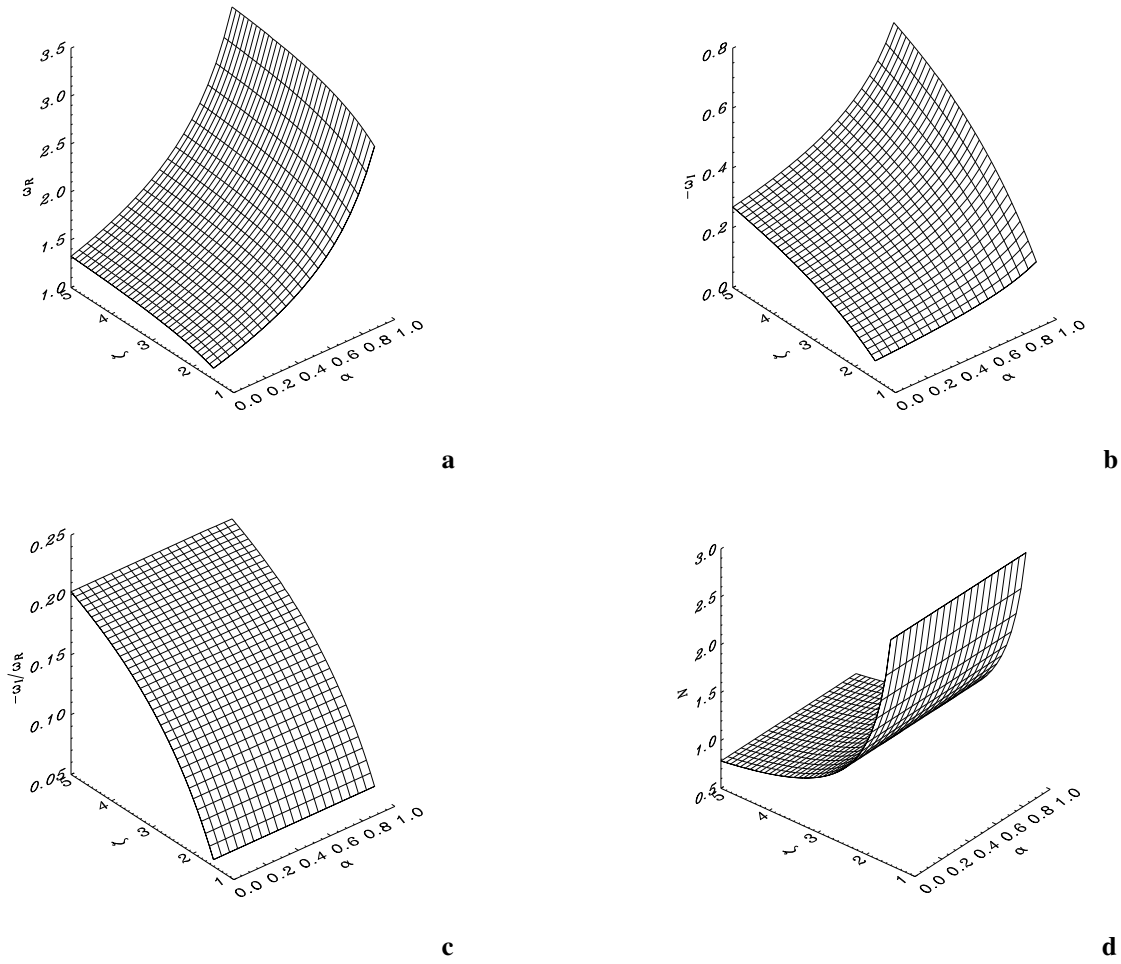


Figure 6. **a** *Frequency,* **b** *damping rate,* **c** *normalised damping rate and* **d** *number of oscillation periods, $N = \tau_{\rm d}/P$, as a function of the stratification parameter and the density contrast for coronal loops with a fixed length ($\epsilon = 0.04$) and fixed thickness of the boundary layer ($l/R = 1$).*

REFERENCES

Andries, J., Goossens, M., Hollweg, J. V., Arregui, I., & Van Doorsselaere, T. 2004, submitted

Aschwanden, M. J., De Pontieu, B., Schrijver, C. J., & Title, A. M. 2002, Solar Phys., 206, 99

Aschwanden, M. J., Fletcher, L., Schrijver, C. J., & Alexander, D. 1999, ApJ, 520, 880

Aschwanden, M. J., Nightingale, R. W., Andries, J., Goossens, M., & Van Doorsselaere, T. 2003, ApJ, 598, 1375

De Pontieu, B., Martens, P. C. H., & Hudson, H. S. 2001, ApJ, 558, 859

Goossens, M., Andries, J., & Aschwanden, M. J. 2002, A&A, 394, L39

Heyvaerts, J. & Priest, E. R. 1983, A&A, 117, 220

Nakariakov, V. M. & Ofman, L. 2001, A&A, 372, L53

Nakariakov, V. M., Ofman, L., DeLuca, E. E., Roberts, B., & Davila, J. M. 1999, Science, 285, 862

Ofman, L. 2002, ApJL, 568, L135

Ofman, L. & Aschwanden, M. J. 2002, ApJL, 576, L153

Poedts, S. & De Groof, A. 2004, these proceedings

Poedts, S. & Kerner, W. 1991, Phys. Rev. Let., 66, 2871

Roberts, B. 2000, Solar Phys., 193, 139

Roberts, B., Edwin, P. M., & Benz, A. O. 1984, ApJ, 279, 857

Ruderman, M. S. & Roberts, B. 2002, ApJ, 577, 475

Schrijver, C. J., Aschwanden, M. J., & Title, A. M. 2002, Solar Phys., 206, 69

Schrijver, C. J. & Brown, D. S. 2000, ApJ, 537, L69

Uchida, Y. 1970, Publ. Astron. Soc. of Japan, 22, 341

Van Doorsselaere, T., Andries, J., Poedts, S., & Goossens, M. 2004, ApJ, 606, 1223