The Square Root of 2: A dialogue concerning a number and a sequence, Praxis Publishing, 2006 (256 p.) by *David Flannery*

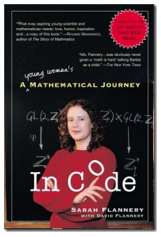




David Flannery

The author David Flannery is an Irish mathematician, who used to tease his eldest daughter Sarah with puzzles and he coached her in solving mathematical problems that she loved doing. So she made a science project on cryptography as a teenager, by which she won in 1999 first a local competition, then on a national level and eventually she won the internationa *Young Scientist Award*. This was interesting enough for the media to pick up the story. Shortly after that she related her story in a book *In code* that was published in 2001. It was not only biographical, but it also told about her love for mathematics and her dedication to solving problems. Some puzzles were included for amusement of the reader and it became a popular math book that had some international success.

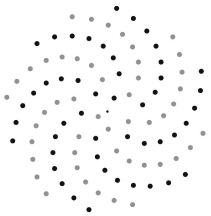
Sarah was the main author of *In Code*, but David certainly has helped. It might have been because of the success of that book that David engaged on the project of writing the present book about the square root of two. It is another math book for a broad audience that is written in a classic style of dialogues. Dialogues between master and student is a technique of teaching that goes back to an antique Greek tradition, and that has been also used by Galileo. For Galileo, it was more a way of letting his characters formulate provocative statements about the heliocentric system that would have been dangerous for him if he had formulated them more directly. In *The square root of 2* we follow the chatty conversation between two people in a master-student relation where the teacher is coaching the student towards discovering some mathematics about numbers and sequences, and the student is clearly finding joy in that. This is most probably inspired by how he has coached Sarah when she was young.





Sarah & D. Flannery

Drawing the diagonal of a square with side 1 gives $\sqrt{2}$, but what is this number? First rational approximations are tried. Integer solutions to the Pell equations $m^2 = 2n^2 \pm 1$ give rational approximations m/n, in fact a sequence of alternating upper and lower bounds $\frac{1}{1}$, $\frac{3}{2}$, $\frac{7}{5}$, $\frac{17}{12}$, $\frac{41}{29}$, $\frac{99}{70}$, $\ldots$; $\frac{m}{n} \to \frac{m+2n}{m+n}$ involving the Fibonacci sequence and recurrence. This gradually leads to continued fractions and the Heron sequence $a \to \frac{1}{2}(a + \frac{2}{a})$ to approximate $\sqrt{2}$ which is actually Newton's method. Grouping several iteration steps into one gives convergence acceleration methods. The above sequence is also the sequence of best approximations: $\frac{m}{n}$ is a better approximation than any rational approximant with a smaller denominator. Here Flannery introduces some notes on Ramanujan and Gauss (both were good at spotting patterns). Summing formulas and formulas for triangular numbers are derived. Irrationality is illustrated graphically by drawing dots at the points given in polar coordinates by $r = \sqrt{n}$ and $\theta = n\phi\pi$ with $\phi = \sqrt{2}$ resulting in spirals. This holds for any other irrational number ϕ, for example the golden ratio or its inverse showing the spiral patterns seen in sunflowers.



There are a few notes with additional literature or historical remarks, like that the notation $\sqrt{\ }$ was introduced by Christoff Rudolff in 1525 and derived from the letter 'r' for radix. The book makes some interesting links between simple mathematical elements from number theory, but everything remains at a really low level. For example it is explained that $\neq$ means not equal and $\approx$ means approximately equal. In my opinion the book is somewhat long winding and after a while it is really tempting to leaf through or skip a number of pages. If could be used as a guideline if the reader wants to coach a promising young pupil, or it could be a read assignment to be explored by a young teenager with some interest in mathematics.

Adhemar Bultheel