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Approximation theory and applications. Piecewise linear and generalized functions. (English)

Zbl 08010774

Amsterdam: Elsevier/Academic Press (ISBN 978-0-443-29141-8/pbk; 978-0-443-29142-5/ebook). xi, 248 p. (2025).

The main message of the book is that many systems in economics, science, and engineering problems are not always smooth, but they can experience impulsive peaks and sudden disruptive changes. These are represented in the model by generalized functions like Dirac delta functions, Heaviside step functions, block functions or more generally by piecewise linear functions. To approximate these models, the classical approximation techniques based on smooth functions like Fourier analysis or polynomial or spline approximation are not able to deal with these because of overshoots like in the Gibbs phenomenon near the discontinuities.

In a first chapter, a survey is given of types of approximations (Fourier, splines, polynomials,...) using different criteria (sup norm, least squares, interpolation, L^p,...), in different function spaces (continuous, integrable,..) and some theoretical background (orthogonal system, Weierstrass theorem, Lebesgue constant,...). Wavelets and fractal modelling are avoided though.

The second chapter is the most mathematical one. It illustrates how and why the classical approximation methods fail and the author's solution is proposed, which consists in using a recursively generated approximation. For example, define $A(x) = \sin(\pi/2x)$ then if is proved that by iterating A defining $f_n(x) = A^{\circ n}(\sin(x))$, this will converge to $\text{sign}(x)$ and estimates of the error are derived. Basically this generates the Haar wavelet. Similar results are obtained for the Dirac delta (a derivative of the previous), the block function and other singular generalized functionals (step function, pulse train, staircase, transition functions,...) and a number of applied examples.

The next two chapters form a collection of examples of engineering practice where this idea is applied, and showing how the methods perform: signal processing (signal recovery from phonograph records, ECG signals,...) and mechanics (structural mechanics, gear design, freewheel mechanism, suspension of cars,...). The last chapter describes the basics of a new macroeconomic model to predict, detect, and deal with all these sudden changes. This relies strongly on statistics, factor analysis, etc., applied to data from the European economy. These applied chapters mainly consist of the description of the application, needed to arrive at the intended use of the approximation ideas.

This book can be used as a textbook for some *capita selecta* course for engineering students, but it is essentially a round-up of the papers that the author has published on this topic. As one would expect in lecture notes, each chapter ends with a set or review questions and some problems to solve, but in the applications chapters there are still some phrases like "in this paper" while one would expect "in this chapter" and there are also some minor typos that may have resulted from copy-pastes.

Reviewer: Adhemar Bultheel (Leuven)

MSC:

41-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to approximations and expansions
41Axx	Approximations and expansions
46F99	Distributions, generalized functions, distribution spaces
65Z05	Applications to the sciences
70-10	Mathematical modeling or simulation for problems pertaining to mechanics of particles and systems
93-10	Mathematical modeling or simulation for problems pertaining to systems and control theory
91B62	Economic growth models
91B64	Macroeconomic theory (monetary models, models of taxation)
91B84	Economic time series analysis

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Keywords:

iterated approximation; disconinuities; mechanical engineering; economics; signal processing; macroeconomics model

Full Text: DOI

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