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THE CONVERGENCE OF ZERO SEQUENCES IN THE INDETERMINATE RATIONAL STIELTJES MOMENT PROBLEM

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Abstract

In the strong or two-point Stieltjes moment problem, one has to find a positive measure on $[0, \infty)$ for which infinitely many moments are prescribed at the origin and at infinity. Here we consider a multipoint version in which the origin and the point at infinity are replaced by sequences of points that may or may not coincide. In the indeterminate case, two natural solutions μ_0 and μ_∞ exist that can be constructed by a limiting process of approximating quadrature formulas. The supports of these natural solutions are disjoint (with possible exception of the origin). The support points are accumulation points of sequences of zeros of even and odd indexed orthogonal rational functions. These functions are recursively computed and appear as denominators in approximants of continued fractions. They replace the orthogonal Laurent polynomials that appear in the two-point case. In this paper we consider the properties of these natural solutions and analyse the precise behaviour of which zero sequences converge to which support points. This generalizes results that were obtained for the strong Stieltjes moment problem.

For relevant discussion of the strong Stieltjes moment problem see [1], [2], [3], [8], [9], [10], [11]. For the rational moment problem see [4], [5], [6], [7] and further references found there. In this contribution we shall in particular generalize the results of [3].

Keywords: strong Stieltjes moment problem, orthogonal rational functions, natural solutions, indeterminate moment problem, multipoint approximation, continued fractions.

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