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Lectures on Monte Carlo theory. (English) Zbl 08092770

Probability Theory and Stochastic Modelling 108. Cham: Springer (ISBN 978-3-032-01189-3/hbk; 978-3-032-01192-3/pbk; 978-3-032-01190-9/ebook). xvi, 632 p. (2025).

The book is an introduction to the theory, the methods, and applications of Monte Carlo simulation.

An introductory chapter with some motivating examples, is followed by a chapter explaining the basic work horse of Monte Carlo simulation, namely the generation of pseudo random numbers (PRNG) and the tests to verify their quality, which is much stricter for cryptographic applications. The generators, like many of the applications throughout the book, are presented by high-level formal descriptions that can be readily translated into python or another programming code.

Next chapter explains how these generators are used to generate random variables with a certain distribution. Binomial, Poisson, and (multivariate) Gaussian distributions are among the most important ones, but also uniform distributions over spheres or ellipsoid are considered.

Then, the tools for applying the PRNGs in simulation problems are provided. How to estimate the expected value $I = \mathbb{E}(Y)$ of a random variable Y or of the function $k(I)$. How many replications (i.e. samples) are needed for a given accuracy? Is the estimate consistent? How to bound the error? How to obtain a confidence region? Can we estimate a quantile? etc.

In practice, straightforward generation of R random samples in crude Monte Carlo, is very slow to converge since the variance behaves like $1/\sqrt{R}$ and several techniques are introduced to reduce the variance (stratified and importance sampling, antithetic and control variates, cross-entropy method,...). At the same time, the text is moving towards practical applications discussing the variance reduction methods in Brownian motion stochastic processes like Black-Scholes in financial and actuarial simulations.

When the state space (i.e., the sample space) is very large (e.g. described by the probability distribution on the vertices of a large graph), a Markov Chain can be assumed where the state at time $n+1$ is obtained from the one at time n by applying a transition matrix (represented as weighted edges of the graph), whose properties may lead to a stationary distribution. The Monte Carlo method for Markov Chains (MCMC) can be realized using the Metropolis or Metropolis-Hastings algorithm. Direct applications are graph coloring, hard core models, or the Ising model. An algorithm coupling the past to the future may generate the chain as a stochastic flow (monotone or not) to a stationary distribution. Applications discussed are approximate counting and motif finding e.g. in DNA strings.

The last two chapters apply all the tools developed so far to solve stochastic optimization (knapsack and traveling salesman) problems and for queueing problems (birth and death and other discrete event processes) with different questions depending on the properties of the queues.

The chapters end with bibliographic notes referring to the history and the literature related to the probems discussed. There is for most chapters also a concluding section with theoretical exercises and/or lab assignments.

The book has a lot of theorems underlying the algorithms. Some proofs are included, other have to be looked up in the literature. However the focus is on the practical implementation and the applicability of the methods summarized in the algorithms. The corresponding python code is made available online.

Reviewer: Adhemar Bultheel (Leuven)

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
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MSC:

65-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to numerical analysis
65C05 Monte Carlo methods
65C10 Random number generation in numerical analysis
60J22 Computational methods in Markov chains
60K20 Applications of Markov renewal processes (reliability, queueing networks, etc.)
91-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to game theory, economics, and finance
91-08 Computational methods for problems pertaining to game theory, economics, and finance
37M05 Simulation of dynamical systems

Keywords:

Monte Carlo; simulation; random number generation; Brownian motion; Markov chain; graph coloring; motif finding; Black-Scholes process; stochastic optimization; queueing theory; Poisson process; simulated annealing; knapsack problem; traveling salesman problem;

Full Text: DOI

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