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**Sun, Zhi-Zhong**; **Zhang, Qifeng**; **Gao, Guang-hua**

**Numerical solutions to partial differential equations with finite difference methods. Translated from the Chinese.** (English) Zbl 08154751

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By approximating (partial) derivatives as finite (divided) differences on a regular grid, one may reduce the solution of a (partial) differential equation to the solution of a linear system to obtain (approximate) values of the solution in the grid points. Special attention should be given to boundary or initial values that are imposed for the exact function values and/or derivatives. These lecture notes explain this technique for several standard problems.

The book starts by introducing the necessary concepts by applying such a discretization technique to a simple two-point boundary value problem (with and without derivative boundary conditions). In this introductory chapter, like in all the subsequent chapters, the same sequence of steps is followed. These steps are: Setting up the discretization scheme and proving it has a solution, estimating some a priori error bounds using the maximum principle and/or the energy method, proving existence of the discretized solution and setting up the practical scheme and its (efficient) solution, sometimes using (Richardson) extrapolation to speed up convergence. These methods are illustrate with numerical examples. Possibly some improved (compact) difference scheme is introduced. A chapter always ends with a summary of what should be remembered and a list of (theoretical and numerical) exercises. Some exercises suggest further exploration of the literature.

Besides the introductory problem of chapter 1 other, more general problems are discussed: Elliptic (chapter 2), parabolic (chapter 3), hyperbolic (chapter 4) equations. As the problems may become high dimensional, ADI (alternating directions implicit) schemes are needed to solve the matrix equations for the 2D elliptic, parabolic, or hyperbolic equations (chapter 5).

Chapter 6 on fractional differential equations is more involved, requiring to introduce the different definitions of fractional derivatives, and to solve fractional difference equations. This is applied to time-fractional sub-diffusion, mixed diffusion, and wave equations.

Finally, some other nonlinear problems are considered: Schrödinger's equation (chapter 7), Burgers' equation (chapter 8), and the Korteweg-de Vries equation (chapter 9).

This is the English translation of an original Chinese book, developed since 1995 and published in 2005 while it has been used in a course. The text is very explicit with all the formulas and implementation details well described for the standard methods, and these are nicely illustrated with examples. Because of this explicitly focus on its practical use, I expected that some software would be available online, but as far as I know there is none. The index with keywords is minimal.

Reviewer: Adhemar Bultheel (Leuven)

**MSC:**

65Mxx Numerical methods for partial differential equations, initial value and time-dependent initial-boundary value problems
65L12 Finite difference and finite volume methods for ordinary differential equations
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
65N06 Finite difference methods for boundary value problems involving PDEs

**Keywords:**

finite difference methods; initial and boundary value problems; time dependent boundary value problem; evolution equation; Schrödinger equation; Burgers' equation; Korteweg-de Vries equation; KdV equation; fractional differential equation

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