Stability optimization with the smooth spectral abscissa

Joris Vanbiervliet* and Bart Vandereycken*
and Moritz Diehl† and Wim Michiels⋆ and Stefan Vandewalle⋆

⋆ *K.U.Leuven, Dept. Computer Science, 3000 Leuven,* {firstname.lastname}@cs.kuleuven.be
† *K.U.Leuven, Dept. Electrical Engineering, 3000 Leuven,* moritz.diehl@esat.kuleuven.be

Keywords : stability, Lyapunov equation, eigenvalue optimization, robustness

The problem of finding fixed-order stabilizing feedback controllers can be transformed into an optimization problem, where the controller parameters are the optimization variables. The most straightforward choice of objective function is related to the eigenvalues of the system matrix A. Specifically, minimization of the spectral abscissa $\alpha(A)$, i.e. the rightmost eigenvalue of the spectrum $\sigma(A)$, stabilizes the system as soon as $\alpha(A) < 0$.

However, the spectral abscissa is in general nonsmooth and nonconvex and therefore typically a very hard function to optimize. Moreover, it is known to be a lousy measure of stability when robustness against parameter uncertainties is regarded. For this reason, more robust criteria have been proposed, most prominently the minimization of the pseudospectral abscissa [1, 2]. While this approach, being closely related to maximizing the distance to instability (or to minimizing the H_∞-norm), indeed yields much more robust solutions, the pseudospectral abscissa still suffers from nonsmoothness and associated high computational costs in optimization.

In this talk we propose a new type of "pseudospectral abscissa $\alpha_\epsilon(A)$", namely the *smooth spectral abscissa*, which is based on the inversion of a relaxed H_2-type cost function. For $\epsilon \to 0$, the smooth spectral abscissa $\alpha_\epsilon(A)$ is shown to converge from above towards the exact spectral abscissa $\alpha(A)$, so that stability is guaranteed if $\alpha_\epsilon(A) \leq 0$. In addition, the smooth spectral abscissa and its derivatives can be evaluated at the cost of solving a few relaxed Lyapunov equations, allowing for an efficient integration in a derivative based optimization framework.

Similarly to the pseuodspectral abscissa, the problem of maximizing ϵ while still satisfying the stability condition $\alpha_\epsilon(A) \leq 0$ is shown to be equivalent to the H_2-norm optimization problem.

References

[1] J.V. Burke, A.S. Lewis and M.L. Overton, "Optimization and Pseudospectra, with Applications to Robust Stability", in *SIAM J. Matrix Anal. Appl.* 25 (2003), pp. 80-104.

[2] N. Trefethen, M. Embree, "Spectra and Pseudospectra - The behavior of nonnormal matrices", Princeton University Press, 2005.