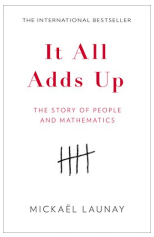
It All Adds Up: The Story of People and Mathematics, William Collins, 2019 (272 p.)
Le grand roman des maths: de la préhistoire à nos jours, Flammarion, 2016, (304 p.)
by *Mickaël Launay*






Mickaël Launay

Because of the success of the French original, the book has been translated and is now also available in English. Launay writes both a survey of the history of mathematics as is obvious from the subtitles, but he does that using the usual topics that one can find in many books on popular mathematics.

He starts somewhat surprisingly with the biface or hand axe which he considers a geometric object and with the pottery decorations in the style of one of the seven friezes. Somewhat more conventional is he start of counting, first using stones (*calculi*) or tallying like on the Ishango bones. The history is told by topic and may stretch over several centuries. Of course prehistoric people did not have a clue about seven friezes, and the history of counting also includes the invention of digits and different number systems, an insight that came only much later. For the coherence of the text, these facts are obviously brought together in one chapter. Early geometry is introduced in Egypt and Greece, and soon we seen theorems appearing about solids and many about plain geometry, in particular about triangles and circles. Geometry was the first mathematical theory that the Greek conceived as an axiomatic system and an approach with definitions, theorems, and proofs. The parallel postulate gave rise to a lot of speculation during many centuries.


biface

The Greek were using a principle of commensurability: there is some smallest unit so that all measures can be expressed as integer multiples of that unit. They however were well aware that π and $\sqrt{2}$ resisted this principle. So that brings the story back to numbers, and more precisely the history of zero, first as a place holder in the representation of numbers, but more importantly as a number on its own. Zero as a number is on the boundary between the positive and the negative numbers. It also took a while before the negative numbers were accepted as numbers.

With the Arabs we learn more about tilings like in the Alhambra, but also about triangles and trigonometry used in measuring areas and distances, but also in computer graphics. The Arabs also brought us algebra to solve equations. After a short section about sequences, introduced starting from he Fibonacci sequence, the story returns to the solution of equations of second or third degree (here comes the classic story of Tartaglia) which entailed the introduction of imaginary numbers, first as an anomaly that miraculously gave the correct result, but later as part of complex numbers.

When books where being printed, some symbols were introduced to denote equations and also geometry could be handled with equations and algebra after Descartes introduced a coordinate system. And then people started modelling physical phenomena. Of course astronomy was studied since antiquity, but also crystallography became fashionable. It required the introduction of infinitesimals by Newton and Leibniz to introduce differential equations. This difficult concept of something infinitesimally small and yet not zero, caused a lot of controversy, but it was eventually also the reason to reconsider the set of numbers that is on a continuous interval and the different orders of infinity.

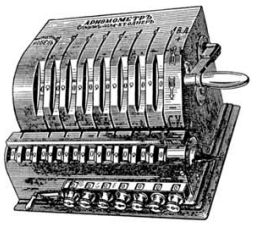
Next comes the introduction of probability and the introduction of machines: the abacus, the Pascaline, and all kinds of mechanical calculators, up to the Babbage machine. Subsequently programmable computers and proofs by machines (the four colour problem) are discussed and eventually the computer winning in chess and go from a human champion player.

In the last chapter, the fundamentals of the last century for current and future mathematics are explained: Hilbert's problems, the Fields medal, the *Principia Mathematica* by Whitehead and Russell and the incompleteness theorems of Gödel, fractals introduced by Mandelbrot.

This is a popular science book, and thus stays for most subjects on the surface so that anybody can understand. In fact Launay writes in his preface that he started writing the book out of frustration because many people take some pride is admitting they are not good in math, while when Launay has some pop-up stand on a market place doing card tricks, origami, games, riddles, and other things based on mathematics, people do not consider that as mathematics. Like one can love music without being a musician, it is

equally possible to be interested in or to even love mathematics without being a mathematician. Like artists, mathematicians explore and create mathematics out of curiosity, and that can be admired or appreciated without being a buff at all the technicalities of mathematics.

Thus Launay is an entertaining story teller, taking the reader along on his visits to museums, and he explains the mathematics that needs some explanation, but always as one would explain it to a (young) neophyte. The topics that are traditional stumble stones in math education are particularly handled with care, like the Zeno paradox of the Achilles and the tortoise, or why $-\times-=+$. Because of a limited number of pages that is digestible, there is also a choice to make, and only few topics are explained in more details. For example his discussion of probability can be explained because he has a PhD in probability (2009) form the *Ecole Normale Supérieure*. In many cases there exist complete books devoted to each of the items that are only explored skin-deep here. He refers to several examples, but also to museums, and websites. Also the reference list of papers used get a two-letter key referring to the historical time period and the mathematical subject that is discussed.



Odhner's arithmometer

I liked his account of how the town currently known as Baghdad was founded that quickly became the scientific and cultural center of the known world with the library of the *Bayt al-Hikma* (House of Wisdom), reminding of the legendary library of Alexandria. The example where a non-commutative binary operation is defined for a set of symbols via a table, and then some algebra is done with these symbols can also work very inspiring for the layperson. His exposition of how to operate the arithmometer constructed by Willgodt Theophil Odhner (1877) is rarely found.

Joseph Mazur's *Enlightening Symbols* (2014) and F. Cajori's *A history of mathematical notation* (first edition 1922) are devoted to the origins of many symbols that, besides numbers, we are used to in our modern mathematical notation like $\pi, e, i, 0, +, -, =$, exponents and square roots. Johannes Widmann (1460) was the first to use $+$ and $-$ for the binary operations, Tartaglia (1550) the first using parenthesis ( ), and Robert Recorde (1557) introduced $=$, and Thomas Harriot (1631) used the $<$ and $>$ with their current meaning. William Oughtred (1631) had the $\times$ for multiplication, : for division and was the first to use π for 3.1415... but Euler popularized it later. The obelus $\div$ for division is coined by Johann Rahn (1659) and combines the Arab horizontal line and the : of Oughtred and Christoff Rudolff (1525) used $\surd$ for the square root and Descartes added the roof: $\sqrt{\ }$. All that happened in the 15th and 16th to early 17th century. Other proposals have been made, that did not become standard, and it sometimes took many years before a notation got accepted by the majority. Symbols for $+$ and $-$ were known earlier by the Arabs. Also variants of the radix sign existed earlier. So some of the origins are fuzzy and the above mentioned origins do not always the same as what is mentioned by Mazur or Cajori. Sometimes these symbols had subtly different meanings and they still do, The $=$ can stand for an equality or define an equation in which case it could be unidirectional or bidirectional. In the latter case $a+x=b$ is not the same as $x=b-a$. In definitions one sometimes uses $:=$ to indicate that the left is defined by the right. Also in programming languages it can have a unidirectional meaning sometimes denoted as $x \leftarrow x+y$ meaning that the place containing the value of x is replaced by the value of the sum. It is remarkable how much mathematicians could achieve in the period what mathematics was very verbose and they had to figure out the solution without all these symbols. This is quite the opposite of the illustration in the book of some short section of the *Principia Mathematica* (see illustration above) that is unreadable because it has only symbols. It was also Descartes who started using letters $a, b, c, \ldots$ for constants and $x, y, z, \ldots$ for variables or unknowns.

$$
\begin{aligned}
&*54{\cdot}43.\quad \vdash :. \alpha, \beta \in 1 \,.\, \supset : \alpha \cap \beta = \Lambda \,.\, \equiv \,.\, \alpha \cup \beta \in 2 \\
&\quad \textit{Dem.} \\
&\quad \vdash .*54{\cdot}26 \,.\, \supset \vdash :. \alpha = \iota' x \,.\, \beta = \iota' y \,.\, \supset : \alpha \cup \beta \in 2 \,.\, \equiv \,.\, x \neq y \,. \\
&\quad [*51{\cdot}231] \qquad\qquad \equiv \,.\, \iota' x \cap \iota' y = \Lambda \,. \\
&\quad [*13{\cdot}12] \qquad\qquad \equiv \,.\, \alpha \cap \beta = \Lambda \qquad (1) \\
&\quad \vdash .(1) .*11{\cdot}11{\cdot}35 \,.\, \supset \\
&\qquad \vdash :. (\exists x, y) \,.\, \alpha = \iota' x \,.\, \beta = \iota' y \,.\, \supset : \alpha \cup \beta \in 2 \,.\, \equiv \,.\, \alpha \cap \beta = \Lambda \qquad (2) \\
&\quad \vdash .(2) .*11{\cdot}54 \,.*52{\cdot}1 \,.\, \supset \vdash .\ \text{Prop}
\end{aligned}
$$

From this proposition it will follow, when arithmetical addition has been defined, that $1+1=2$.

Principia Mathematica, page 379

From this theorem it follows that $1+1=2$

Adhemar Bultheel