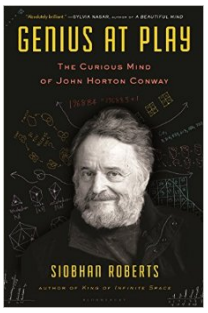
Genius at Play: The Curious Mind of John Horton Conway *Siobhan Roberts* Bloomsbury 2015 ISBN 978-1620405932 (hbk), 480 pp.




Conway 1987

Siobhan Roberts

Here is a mathematician with an unconventional career who thinks (and lives) outside the box. He is born in 1937 in Liverpool in a family with two older sisters. He got a PhD from Cambridge in 1964 supervised by H. Davenport, was appointed the John von Neumann Chair of Mathematics at Princeton where he is now professor emeritus. He is Fellow of the Royal Society (he expanded FRS as Filthy Rotten Swine) and the American Academy of Arts and Sciences (he appeared at the ceremony wearing green running shorts) and has won several mathematical prizes. Like Feynman, an iconoclast with an overdue hippie attitude.

Conway is probably best known from his *Game of Life* developed in the late 1960s. Not really a game but a cellular automaton. There are living organisms distributed on a square grid and there are rules for birth, death and survival, which depend on the occupation of the nearby squares. Type "conway's game of life" in Google's search bar and you will see it evolve in the right top corner of your screen. He claims he never worked in his life. Just playing a lot of 'home-made' games. Nevertheless he has important contributions to geometry, group theory (the Conway groups, the Monster group), knot theory and number theory. He is most proud on his introduction of a new kind of numbers: the surreals (even though they do not crack Cantor's continuum hypothesis).

He is very talkative, thinking of many things simultaneously, and very chaotic. His office in Princeton became inhabitable, stuffed with paper polyhedra models hanging from the ceiling, books, papers, unopened letters, and lunch wrappers on the desk, chairs, and the floor. Nothing is organized and he keeps no files, letters or archive. This book reflects this format by jumping from topic to topic, informing the reader about many things in parallel, probably following Conway's thoughts during interview sessions. Quite often she includes long quotes by Conway to give an idea of his way of thinking and expressing himself. She finds letters by Conway in other people's files, but their answer to Conway are mostly lost. An autobiography would be out of the questions because Conway's ego and his problem in distinguishing fiction from facts would probably put the reliability of the account under too much strain.

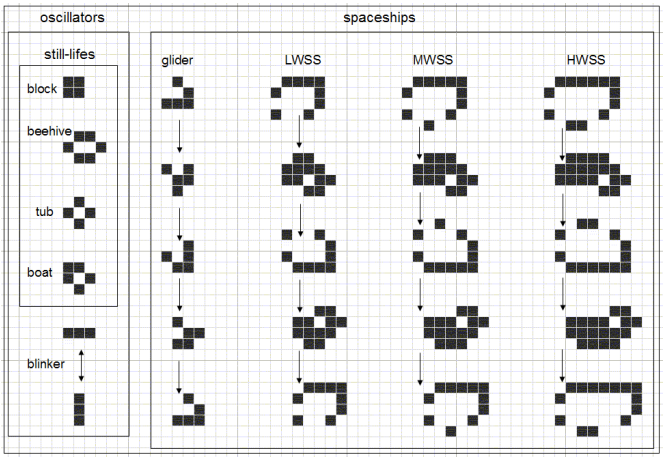
Conway typically likes to play with sequences like for example his *Subprime Fibs*. Start with two arbitrary numbers. Then repeat the following. Add the last two numbers. If the sum is prime, keep this as the next element, if not, divide it by its smallest prime divisor and keep the result. He ensures that, whatever the starting couple, the sequence will always end in three possible cycles. The sequence is totally useless, and so far no proof has been found for his conjecture. Better known is his *Look and Say* sequence: Start with a number, for example 1. Read it 'one one' nd write it down: 11. Read this as 'two ones' and write that: 21. Then read 'one two (and) one one' so you write: 1211. Next is 111221, then 312211, etc. The sequence has several properties like for example *Conway's Cosmological Theorem* which says that the sequence has 92 different subsequences that do not interact with each other. He call them after the chemical elements. There are a few transuranic numbers and the ratio of the length of two successive elements is called Conway's constant 1.3035772690..., the only positive root of a polynomial of degree 71.

During his undergraduate years in Cambridge he invented with some friends WINNIE (Water Initiated Numerical Number Integrating Machine), a binary water computer with plastic cups, tubes, and strings. Filling the top cup with 1 unit of water represented 1. Adding another unit of water made the water flow to a lower cup, emptying the first one with a siphon system. So that presented 10 etc. It could count to 127 (that is 1111111 in binary). An overflow error caused a flooding disaster.

There is a story of finding the Conway group (1967), which is a simple group, the symmetry group of the Leech lattice, that is related to sphere packing in 24-space. This result brought him math fame.

The Game of Life (1970) was the next thing that spread like a virus worldwide. Although the rules are fully deterministic, it is very hard to predict what the configuration will be after a few steps. If this was a true model of life, then it would contradict his *Free Will Theorem* that he formulated much later (2006) with his colleague Kochen: It states that if humans have free will, then elementary particles

possess free will too. He certainly believes humans have free will. Life seemed to have its free will too. The dancing dots on the board of Life evolved into certain "life forms" that get names like beehive, pulsar, blinker etc. One of which is called the glider, which is a configuration that, if left alone, will reproduce and fly off the screen like a space ship to populate the cosmos. Life was 'proved' universal. Later many more configurations were observed like glider guns producing an unending sequence of gliders or space ships, etc. The virus broke loose when Martin Gardner devoted his column to the game in 1970. It was so addictive that it is claimed that in the high days about 25% of all the computers world wide were running the Game of Life. Conway is now all fed up with it, and prefers not to talk about it anymore. Roberts has a long chapter on the impact of Life and all the references that were made to it, not in the least Wolfram's *A New Kind of Science*. There is a multitude of games that Conway invented, played or discussed. Together with E. Berlekamp and R. Guy, they have a 4-volume set of books *Winning Ways for Your Mathematical Plays*. The very last chapter deals with Life and proves the universality theorem.



FRACTRAN is another of his inventions: a Turing complete programming language. It is just a list of fractions and some computation rules. Start with an input number. Find the result of the first fraction, multiplied by the input that gives an integer and repeat the operation using this integer result. At certain points in the calculation one finds the result like prime powers of 2 or the digits of pi. All that is needed is an appropriate set of fractions to start with and an input number. It seems magic, but the principle is quite simple because numerators and denominators can be considered as an encoding of registers based on Gödel numbering. So registers get prime numbers. Suppose registers 2,3,5, contain the values 1,0,2 then this is represented as an integer $50 = 2^1 \times 5^2$. If multiplied by a fraction, the numerator will add values to registers and the denominator will subtract values. The details can be looked up in Wikipedia.



Conway (1978) by Simon J Fraser

The Monster Group is a sporadic simple group of order about 8×10^{53} Conway (1985) and J. Tits (1985-6) gave simple constructions. Already in 1979 Conway and Norton conjectured the *Monstrous Moonshine*, a connection between the Monster and modular functions, only proved in 1992 by R. Borcherds who got the Fields medal for this. There is of course an extensive chapter on this and on the making of the *Atlas of Finite Groups* (1985) (now available online without Conway's name). The chapter is interrupted by his comments about the *Doomsday Rule*, an algorithm he invented to compute the day of the week for any date, plane tilings, etc. When he opened his computer, it spits out 10 random dates, and he has to answer with the day of the week before he can open a file. The target is to do this eventually in less than 10 seconds. The book rattles on illustrating Conway's attitude towards his wives, and to women in general (Two French women approach Conway and ask pointing to empty chairs: 'May we?' whereupon he promptly answers 'Mais oui!'), his way of teaching (he brought vegetables to the classroom, wrote on chairs, walls or the pavement outside), the Hofstadter-Conway \$10,000 Sequence (he accidentally awarded a \$10,000 prize instead of \$1,000 for who could find the limit behavior of the sequence), his depression after his first heart attack (he had two and two strokes), how he spends a couple of weeks every year at a Canada/USA Mathcamp (a five-week event for youngsters with a passion for mathematics), his ideas about the Free Will Theorem and quantum theory (particles have free will if their behavior is not determined by the past or by randomness), how he got his brain tested like Einstein's (growling and discussing all the time with the people performing the fMRI scan), ending with a long epilogue (illustrating how difficult it is to finish something when Conway is involved). The book ends with some appendices giving technical details of issues discussed, a selected bibliography of Conway, and a reference list.

The book reads like a roller coaster ride: you never know where it's turning or what surprise is on the next page. I guess this is how it feels like, interviewing Conway.

A. Bultheel