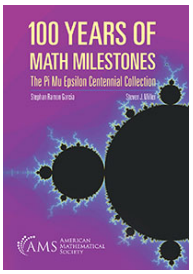
100 Years of Math Milestones. The Pi Mu Epsilon Centennial Collection, AMS/MAA, 2019 (581 p.) edited by *Stephan Ramon Garcia* and *Steven J. Miller*



Pi Mu Epilon is an American society founded in 1914 to promote mathematics and encourage bright mathematics students to explore mathematics further and achieve a better understanding. Membership is restricted to the math honors students that have obtained sufficiently high scores for their math courses. The society publishes a journal with research papers focussing on papers by students, and this journal has also a problem section. In 2013, to celerate their centennial anniversary, it was decided to collect for each of the past 100 years an important mathematical achievement and link a centennial problem to it. With some extra framing and context, each item is also introducing students to some recent history of mathematics. This collection was published in 4 parts of 25 years in their journal issues of 2015. The complete collection is now put together and reprinted in this book.

The entries for each year from 1913 to 2012 (on average of 5 to 6 pages) are organized in a similar format. With an introduction, the topic that is associated with the year is explained. Sometimes already here there are suggestions to think about some problem or to prove something. Then there is the centennial problem, sometimes the size of a research project. Its formulation can be short or long with several subproblems. In some way they are related to what has been explained in the introductory section, but the connection can be rather loose. Problems can be submitted by different people, but often the editors are the originators. The next section gives additional comments. In only few cases these comments contain the solution to the problem, but in most cases there are at best some hints, while the solution is left to the reader. In most cases, you find additional definitions and information, or a discussion of related problems, sometimes a bit of history or applications, etc. The end is a list of references.

Über die Koeffizienten derjenigen Potenzreihen, welche eine schlichte Abbildung des Einheitskreises vermitteln.

Von Prof. Dr. Ludwig Bieberbach
in Frankfurt a. M.

The topics covered include the topics that are often found in popular science books and that are accessible for good undergraduate math students. Clearly the latter is the target readership. This is not the kind of book that would easily sell to a public that has no advanced mathematical interest. The required mathematics are however only slightly over the edge of the undergrad's curriculum. There is a lot on number theory, some cryptography, some graph theory, analysis, statistics, and games. A year can be associated wth a particular theorem, paradox, conjecture, equation, or constant (Gödel's theorems, Banach-Tarsky paradox, twin-prime conjecture, Schrödinger equation, Kinchin's constant,...), or it can be connected with a person (M. Gardner, P. Erdős, G. Cantor, and others), but also tools important for mathematicians (TEX, Mathematica, ArXiv, SageMath, GIMPS,...). Too many subject to discuss here, and it must have been difficult for the editors to select only one fact per year anyway.

I give just one example of a randomly selected year 1984 to illustrate the freewheeling stream of ideas. Obviously 1984 refers to the novel of George Orwell and the scene in which Winston Smith solves the "equation" $2 + 2 = 5$ (which is only correct if one works modulo 1). Also in a *Star Trek* episode there is a scene with captain Picard referring to this $2 + 2 = 5$. Then the problem is raised to finding integers that can be obtained using only 4's and the standard algebraic operations. The centennial problem is to solve this puzzle but minimizing the cost ($+, -, \times$ cost 1 unit; exponentiation, factorial, and division is 2 units, etc. if you allow other operations). Classify the numbers representable with a cost at most C. Given n, what is the minimal cost to represent it. The comments refer to *The Hitchhiker's Guide to the Galaxy* trilogy where in book one there is the ultimate answer 42 to the ultimate question while in book two a possible question is found to be "What do you get if you multiply 9×6?". The answer is indeed 42 but only in base 13. This is also alluded to in an episode of *Doctor Who*. Then the 1984 comments move on to the Bieberbach conjecture and the proof by de Branges in 1984.

This is a rather unfair reflection of the contents because other items are much more mathematical and require much more advanced analysis. But any other selection of one or more years would have been unfair as well. It is just a marvelous but challenging collection of problems and it contains a lot of fun stuff that is often found in more popular science books.

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