Numerical Mathematics, Vol. 1: Systems of linear equations, Eigenvalue problems, Non-linear equations. *Martin Hermann.* De Gruyter, 2026 (x+359 p.) isbn: 978-3-11-914766-4 (hbk)

This is a translation of the first volume from a multi-volume textbook that is appearing in a new (fifth) German edition. It served for many years as lecture notes for an introductory course on numerical analysis. Volume 1 covers the solution of linear systems, eigenvalue problems, and nonlinear equations. Volume 2 is devoted to polynomial approximation, curve fitting, numerical differentiation and integration, and initial value problems for ordinary differential equations.

The topics of such a course are more or less fixed, and the difference between the many books dealing with this subject lies in the order in which the material is organized, the mathematical rigor and the number of proofs, and the discussion of implementation details. Concerning the latter, usually some software implementation of the basic methods is provided, often using Matlab or another easy-to-use software tool. In the present case, the code is included in the text and explicitly discussed. (It is also available at the publisher's website.) Thus, the student is assumed to be able to program in Matlab, or to be able to translate it into some other software. If Matlab is not available or too expensive, a free alternative such as Octave may be considered.

The remainder of this review concerns Volume 1. Volume 2 will be reviewed after its publication.

The logical starting point is to define a numerical algorithm and the errors that are associated with it: approximation and rounding errors, leading to the two fundamental concepts of the condition of the problem (how sensitive is the result to perturbations of the input) and the stability of the algorithm (how do the additional errors during the execution affect the result). This involves explaining the meaning of the fl notation; I also appreciate that the IEEE standards and the concept of NaN are mentioned.

The other chapters subsequently discuss the topics mentioned above. The one on linear systems of equations starts with Gaussian elimination, introduces partial and global pivoting, and its practical implementation as LU factorization, efficiently stored with the row permutations implemented as a permutation of the indices instead of actual row permutations. Symmetric and tridiagonal systems lead to obvious simplifications. Less obvious for an

introductory course was the inclusion of Givens and Householder transformations, singular value decomposition, and iterative methods (Jacobi, Gauss-Seidel, SOR, Conjugate Gradient, BiCGSTAB) with preconditioning.

Eigenvalue problems are solved using the power method, but Gershgorin disks are also discussed. The most appealing material is the application of the QR method to the important symmetric case, implemented with Householder and (fast) Givens transforms including shifts to speed up convergence.

The chapter on nonlinear equations in one variable include methods like Newton's method and more general fixed point methods with their geometric interpretation, secant and Müller methods based on interpolation, and finally interval methods as a slow but safe alternative. At this point, extrapolation techniques (Aitken's Δ^2 process) to speed up the convergence are introduced. There are local and global convergence problems associated with basins of attraction, and homotopy and continuation methods are proposed to overcome the former. These are not so common in an introductory course. Neither is the (somewhat surprising) inclusion of the QD (quotient-difference) algorithm, since this can exhibit notorious stability problems, which are here treated by a translation if division by zero has to be avoided. More common, instead, are methods such as Horner schemes that leverage the particular structure of polynomial equations.

Systems of nonlinear equations are the topic of the last chapter. The basic method is Newton's method where division by the first derivative is reformulated as a linear system with the Jacobian matrix. Restricting the step size is needed to keep the correction term under control if the Jacobian is nearly singular. A monotony test is used as a stopping criterion. When the Jacobian is approximated, we obtain quasi-Newton methods (Broyden), and Brown's method (linearization of individual equations similar to what Jacobi or Gauss-Seidel do for linear equations). The chapter also includes least squares solutions (descent methods, Gauss-Newton, and Levenberg-Marquadt) which are important optimization problems. They form a good introduction to more general optimization problems, which do not seem to be part of this course.

This is an introductory course for students (such as those from engineering or science) who will probably explore some of these aspects in more detail later in their curriculum. The proofs are at just the right level, leaving to the literature those proofs that are too technical or too involved. Students that do not expect to be involved in their career with detailed numerical implementations may find the analysis, proofs, and details of the methods a

bit too much. But it is good that they will get a sense of what is going on in an algorithm which they will probably use in an off-the-shelf manner.

Each chapter has a list of exercises. Most of them are theoretical (prove that ...) and few are numerical (implement ..., compute ...).

Some strengths: Iterative methods are well covered. In current applications (very) large systems (sometimes sparse) are common, and iterative methods are the proper or only way to solve them. These lecture notes bring a student up to close to where the current scientific literature comes into reach. There are no references to comparable books. (There is obviously no need.) The bibliography (only 88 items) often refers to the origin of the method and is satisfactory, though not essential. There is plenty of material for an extensive course in numerical mathematics. If needed, some parts could be skipped. I have been teaching this material to engineering students myself for many years, and I did not have the time to cover all the material listed in the contents of the two volumes. Much was covered in follow-up courses. There are many similarities with my own lecture notes though, so that I find the presentation very likable.

Some shortcomings: the publisher classified the book under 65L10, which does not well represent the contents of this book. It should have been classified as either 65-01, 65Fxx, or 65Hxx. The comments in the Matlab code are translated into English, but some of the routines have kept their German names (e.g. *ruecksubs* for backward substitution, etc). It would also be a good idea to add the names of the routines to the index. Since the book is clearly oriented towards practical applications of the methods, there could perhaps have been more (or more extensive) numerical exercises or even projects.

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