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On the Lawfulness of Grouping by Proximity

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The visual system groups close things together. Previous studies of grouping by proximity have failed to measure grouping strength or to assess the effect of configuration. We do both. We reanalyze data from an experiment by Kubovy and Wagemans (1995) in which they briefly presented multi-stable dot patterns that can be perceptually organized into alternative collections of parallel strips of dots, and in which they parametrically varied the distances between dots and the angles between alternative organizations. Our analysis shows that relative strength of grouping into strips of dots of a particular orientation approximates a decreasing exponential function of the relative distance between dots in that orientation. The configural or wholistic properties that were varied—such as angular separations of the alternative

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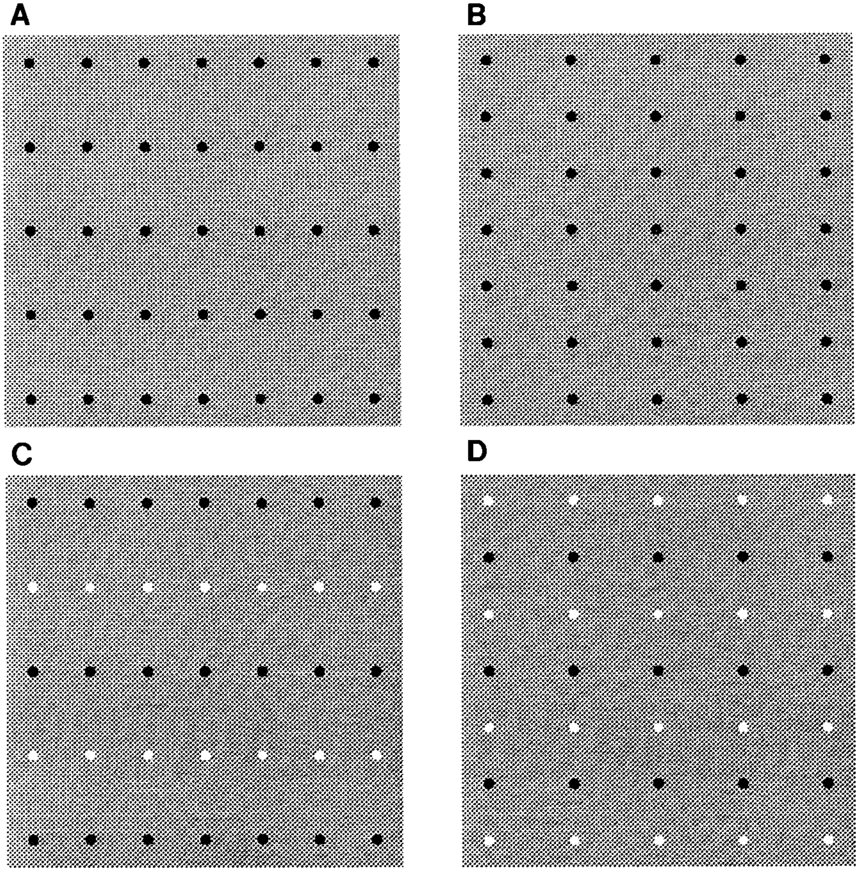

FIG. 1. Patterns A and B demonstrate grouping by proximity. Pattern C shows similarity and proximity acting in concert, and pattern D shows them acting in opposition.

organizations and the symmetry properties of the dot pattern—do not matter. Additionally, this grouping function is robust under transformations of scale in space (Experiment 1) and time (Experiment 2). Grouping of units which are themselves the result of grouping (i.e., pairs of dots; Experiment 3) also follows our nonconfigural rule.

When we look at a collection, or *whole,* of discrete entities, or *parts* (dots, for example) we often see it partitioned, or *organized,* into subsets, or *groupings,* which in turn consist of the parts. For example, we usually see Figure 1A as a lattice grouped into several rows or horizontal strips of dots, whereas Figure 1B is usually perceived as several columns or vertical strips of dots. Gestalt psychologists used phenomenological observations like this to infer many grouping principles, of which grouping by proximity is the most fundamental (Kubovy, 1981).

The Gestalt psychologists did not produce a satisfactory theory of perceptual organization for two related reasons: they relied too heavily on phenomenology and did not sufficiently quantify their data. Consider Figure 1D, in which the principle of proximity predicts grouping into columns, but the principle of similarity predicts grouping into rows. Phenomenological observation tells us that the strength of the proximity principle decreases with distance and the strength of the similarity principle decreases with dissimilarity. However, for any given distance and dissimilarity values we do not know which is stronger, so we cannot predict which principle will prevail. Indeed, in Hochberg's opinion (1974) Gestalt psychology's research program failed because it was unable to predict perceptual grouping in such patterns. To do so, it would have needed a metric function that relates grouping strength to distance and dissimilarity. This function would allow the strength of grouping due to a given distance to be compared to that caused by a given degree of dissimilarity, in order to determine which principle prevails. We need such a function to predict groupings in most displays because more than one grouping principle is at work in most scenes. Our goal in this paper is to take a first step in this direction by measuring how grouping strength varies with distance and further, how it is affected by spatial configuration.

First we review attempts to measure the strength of grouping. We then show how the methods used in an experiment by Kubovy and Wagemans (1995) represent an advance over previous work, and reanalyze their data. Through this analysis we show that the probability distribution of different groupings is accounted for by a simple rule of grouping by proximity. This rule takes into account only the distance (relative to the scale of the lattice) between dots in the grouping. We then present three new experiments which show that the same grouping law explains grouping in different spatial and temporal scales and in more complex patterns.

MEASURING GROUPING STRENGTH

Hochberg (1974) hoped that grouping in patterns governed by more than one grouping principle could be predicted after measuring the relative strength of grouping principles. So he and his associates tried to measure grouping by similarity of figure-ground contrast by playing it off against grouping by proximity in rectangular lattices of circular dots (Hochberg & Hardy, 1960) and square dots (Hochberg & Silverstein, 1956). They determined which values of proximity and contrast are in equilibrium with respect to their grouping strength. For instance, while the spacing between columns remained constant, observers might be asked to adjust the spacing between the rows of different brightness (e.g., Figure 1D) until they found the spacing for which they thought they were equally likely; i.e., the strength of grouping by brightness was in equilibrium with the strength of grouping by proximity. Using this equilibrium-point methodology, Hochberg and Hardy (1960) plot-

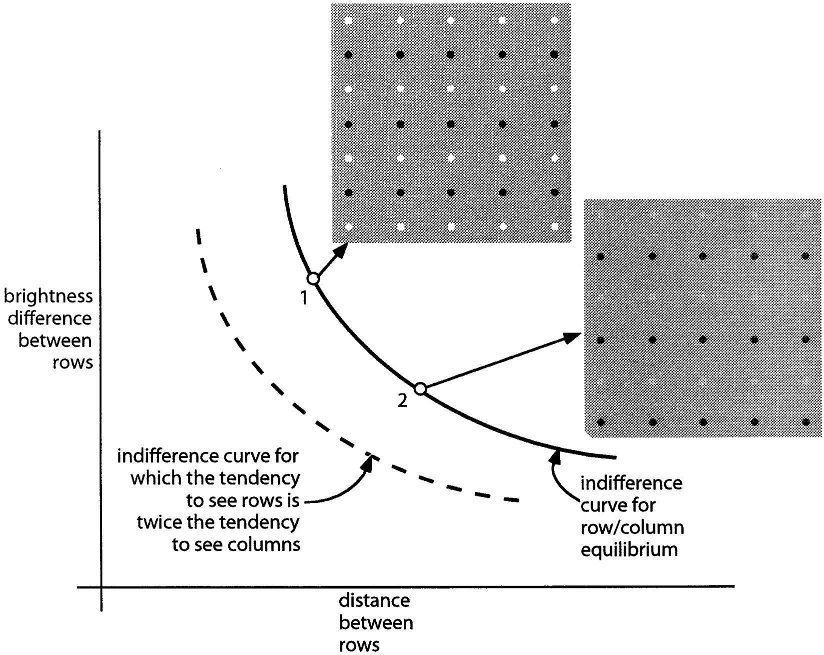

FIG. 2. Two indifference curves. The indifference curve for row/column equilibrium (represented by a solid line) can be observed using the transition-point methodology. Other indifference curves (e.g., the one represented by a dashed line), although conceptually meaningful, cannot be observed using this methodology.

ted an indifference curve (Krantz, Luce, Suppes, & Tversky, 1971): as they reduced the brightness difference between the rows, the distance between rows for which observers reported an equilibrium between rows and columns increased (Figure 2). This is an indifference curve because the observer—whose sole interest is finding the point of equilibrium between grouping by rows and grouping by columns—is indifferent among the ⟨brightness-difference, row-distance⟩ pairs that lie on it.

The best Hochberg could have done with this single indifference curve is to find what one already expects: a trade-off between contrast and proximity, quantitatively unspecified. To measure the strength of each grouping factor (in other words, to scale them), and to determine whether they affect grouping additively, one must observe at least two indifference curves, and test their parallelism using additive conjoint measurement (Krantz et al., 1971, Chapter 6; Mitchell, 1990). For instance, in addition to obtaining the indifference curve on which all ⟨brightness-difference, row-distance⟩ ordered pairs represent lattices in row/column equilibrium, one would have to obtain, for example, a second indifference curve for which the tendency to group by rows was twice as strong as the tendency to group by columns (the dashed line in Figure 2). This cannot be done with an equilibrium-point methodology, since no perceptible transition occurs at that point.

Oyama (1961) developed a better method than the equilibrium-point methodology, which enabled him to quantify the strength of grouping by proximity: he obtained a function that assigns a grouping strength to each proximity value. To measure grouping strength as a function of proximity, Oyama used an imbalance methodology which pits proximity in one orientation of rectangular lattices against proximity in another. From trial to trial he varied the vertical distances between the dots. He presented each lattice for two minutes and asked observers to press one key while they saw columns and another while they saw rows. As a relative measure of strength of grouping he took the log-ratio of the cumulative durations, $\log(t_{var}/t_{const})$, for seeing the two organizations of the lattice. This he plotted against the log-ratio of the corresponding separations, $\log(d_{var}/d_{const})$, and obtained an excellent linear fit, showing that the relative duration of a perceptual grouping varies as a power function of relative spatial separation. Oyama's method, for the first time, generated a function that describes grouping by proximity without reference to other grouping tendencies.

Oyama's imbalance methodology was felicitous, but his procedure had some weaknesses. First, his procedure is susceptible to demand characteristics: if you ask an observer to press a different key every time she sees the perceptual organization of the stimulus change, this may suggest to her that she is expected to see changes. Rock and his co-workers (Girgus, Rock, & Egatz, 1974; Rock & Mitchener, 1992) have shown that observers who were not informed about the reversibility of figures saw reversals on about one-third of the presentations; when they were informed, they saw twenty-fold more reversals. Thus the true function may be different from the one Oyama obtained. Second, all Oyama's dot lattices were rectangular with vertically and horizontally aligned dots. Since these orientations coincide with many frames of reference his results may not generalize to other orientations. For instance, he himself observed an effect of overestimation of vertical distances.

THE KUBOVY AND WAGEMANS EXPERIMENT

We will use the data obtained in the Kubovy and Wagemans (1995) experiment to develop a model of grouping by proximity. They collected the data to measure the ambiguity of lattices in information-theoretic terms, which required estimating the probabilities of seeing the different organizations of the lattices. So instead of investigating the instability of lattices by observing their reversibility over a relatively long time-interval (as did Oyama), they measured the instability of initial percepts. To do so, on each trial they presented a lattice for 300 ms, and asked observers to notice in which orientation the lattice grouped. At the end of each trial they asked observers to decide

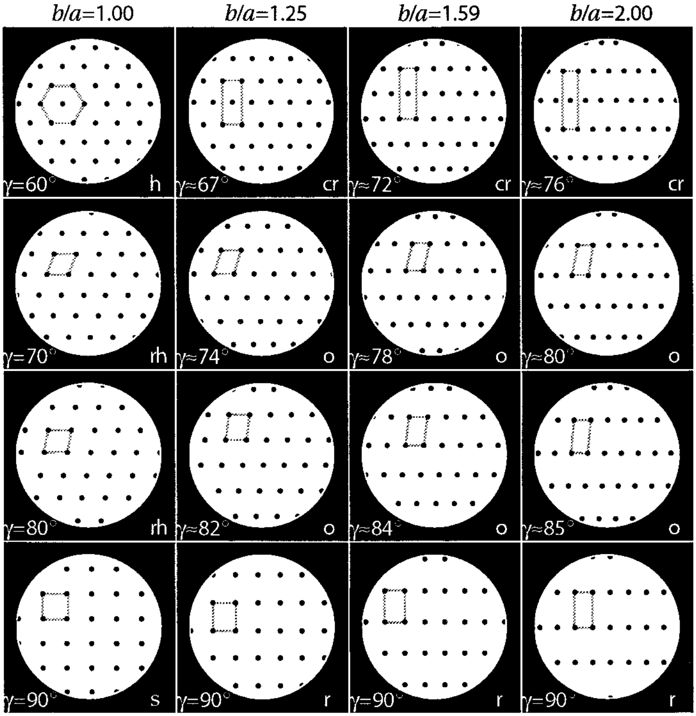

FIG. 3. The sixteen dot lattices used by Kubovy and Wagemans (1995). The lines connecting dots create the shape the lattice contains which gives the lattice its name. h, hexagonal; cr, centered rectangular; rh, rhombic; o, oblique; s, square; r, rectangular.

which of four orientations corresponded to the organization of the lattice they had seen. To manipulate the degree of ambiguity, they used sixteen different dot lattices (Figure 3). They found that the hexagonal lattice (Figure 3, upper left-hand corner) was the most ambiguous, and the most elongated rectangular lattice (Figure 3, lower right-hand corner) was the most stable.

In the past, experimenters only manipulated inter-dot distances in the rows and columns of rectangular lattices. But rectangular lattices are just one of 16 different types of lattices used by Kubovy and Wagemans, and the different types look different. The 16 different types of dot lattices have different global structures which determine how the lattices look. Lattices look different from each other because their symmetries are different (and they are easy to detect: Wagemans, 1995; Wagemans, Van Gool, Swinnen, & Van Horebeek, 1993). Imagine a hexagonal dot lattice drawn on a plane, and a mirror standing on its edge, perpendicular to the plane. There are six essentially different ways to stand a mirror on the surface so that the reflection

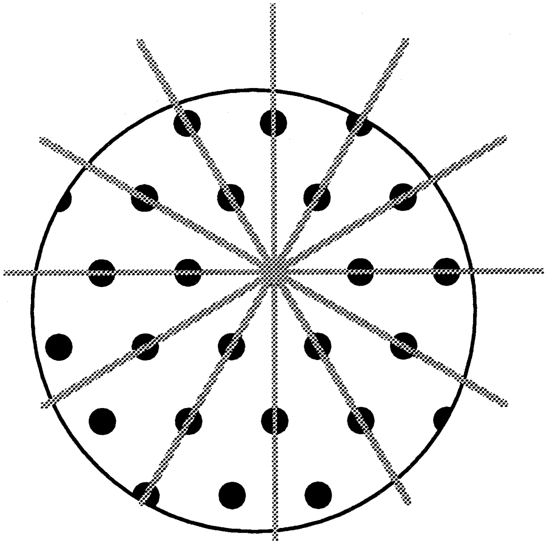

FIG. 4. The mirror symmetry of a hexagonal lattice. We arbitrarily chose a dot and indicated the six axes of reflection that pass through it.

of the lattice will be a perfect continuation of the lattice itself (Figure 4). In other words, a hexagonal lattice has six axes of reflection-symmetry. In contrast, a rectangular lattice has only two (Table 1).

Kubovy and Wagemans also minimized the effect of frames of reference. They achieved this in two ways: (i) they minimized the effects of field shape, by presenting the lattices as if seen through a circular aperture; (ii) they minimized the effects of environmental frames of reference by randomly rotating the lattices on each trial.

Describing Dot Lattices

Dot lattices vary in a space that is geometrically well-understood; they are determined by continuous metric parameters. A dot lattice is a collection

TABLE 1
The Symmetries That Differentiate Dot Lattices

Lattice	Reflection (number of mirrors)	Rotation (angle)	Glide reflection (number of axes)
Oblique	0	Twofold, 180°	0
Rectangular	2	Twofold, 180°	0
Centered rectangular and rhombic	2	Twofold, 180°	2
Square	4	Fourfold, 90°	0
Hexagonal	6	Sixfold, 60°	3

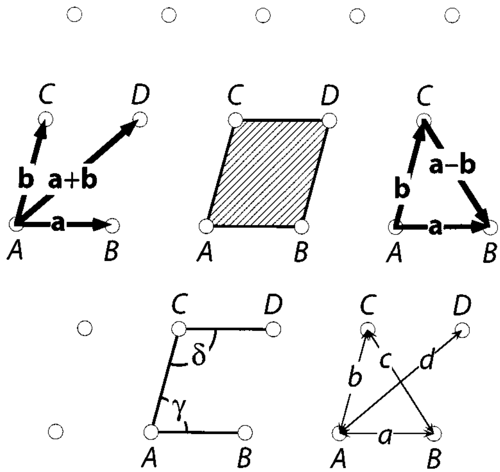

FIG. 5. Vectors **a** and **b** are the basis of (i.e., they are the translation vectors that generate) the lattice, they define the lattice's basic (or principal) parallelogram *ABCD*. For all lattices, $60° \le BAC \le 90°$; $45° \le ACB \le 90°$; $0° \le ABC \le 60°$ (Bravais, 1949).

of dots in the plane that is invariant under two translations. A lattice is specified by its two shortest translations in the orientations and *AB*, i.e., a pair of translation vectors **a** and **b** (Figure 5). Bravais (1850/1949) showed that the basic parallelogram of all lattices, *ABCD*, whose two sides are the vectors **a** (*AB*) and **b** (*AC*), is limited by the following conditions: $|\mathbf{a}| \le |\mathbf{b}| \le |\mathbf{a} - \mathbf{b}| \le |\mathbf{a} + \mathbf{b}|$ ($AB \le AC \le BC \le AD$).[1]

The distances of a dot from its eight nearest neighbors are $|\mathbf{a}|$, $|\mathbf{b}|$, $|\mathbf{a} - \mathbf{b}|$, and $|\mathbf{a} + \mathbf{b}|$. From now on, we will denote these distances a, b, c, and d, respectively. Kubovy (1994) showed that any lattice is specified by three parameters: a, b, and $\gamma = \angle(\mathbf{a}, \mathbf{b})$. Hence if a is held constant, any lattice can be located in a two-parameter space whose coordinates are b and $\gamma = \angle(\mathbf{a}, \mathbf{b})$. These coordinates are given in Figure 3 to specify each of the lattices used in the experiment. He also showed that the lattices fall into six classes whose abbreviations are given in parentheses and label each of the 16 lattices in Figure 3: hexagonal (h), rhombic (rh), square (s), rectangular (r), centered rectangular (cr), and oblique (o). Changing γ always changes the spatial configuration of the dots and often the symmetries of the lattice.

The Experiment

We omit details of the experiment that are not essential to understanding the present analysis, and refer the interested reader to the original article (Kubovy & Wagemans, 1995).

[1] We denote the magnitude (or length) of a vector **x** by $|\mathbf{x}|$. Henceforth, we will simplify our notation, and write x for $|\mathbf{x}|$. The symbols '+' and '−' represent vector addition and subtraction.

Seven observers participated. On each trial, Kubovy and Wagemans showed them a lattice for 300 ms. The screen contained a blue disk (subtending 12.6°) in the center of the screen and a black region around it. The lattices, which consisted of a large number of yellow dots (subtending about 0.125°, no less than 1.5° apart), were superimposed on the blue region of the screen. After removing the lattice, they showed the observer a four-alternative response screen. Each alternative consisted of a circle and one of its diameters. The orientation of the diameter corresponded to the orientation of one of the four vectors of the lattice just presented.

All the lattices had the same shortest inter-dot distance, $a = 60$ pixels. The second-longest inter-dot distance, b, varied from $1a$ to $2a$ (see the labels above the columns of panels in Figure 3). The angle γ is constrained, for geometric reasons (see Kubovy, 1994), by the inequality $\cos^{-1}[1/(2b)] \geq \gamma \geq \pi/2$. The measures of γ for the sixteen lattices are given in the lower left-hand corner of each panel in Figure 3.

The Pure Distance Model

Kubovy and Wagemans proposed a model of grouping by proximity, which we will call the Pure Distance model with which they predicted the ambiguity of the lattices. Let $\mathbf{V} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ be the sides and the diagonals of a lattice's basic parallelogram, and let $V = \{a, b, c, d\}$ be the corresponding magnitudes of these vectors. Grouping by proximity implies that the probability of seeing the lattice organized in the orientation of $\mathbf{v} \in \mathbf{V}$, $p(\mathbf{v})$, is a decreasing function $f(v)$ of $v (v \in V)$. We will call $f(v)$ the *attraction function,* because it determines how the attraction between two dots diminishes as the distance between them grows. The Pure Distance model assumes that all distances are scaled to the shortest distance in the lattice, a. It assumes that the form of the attraction function is a decaying exponential function:

$$f(v) = e^{-\alpha(v/a-1)} \quad (1)$$

(which is similar to the function proposed by Shepard, 1987, as a universal law of generalization).

To obtain the probabilities of each of the perceptual organizations, $p(\mathbf{a})$, $p(\mathbf{b})$, $p(\mathbf{c})$, and $p(\mathbf{d})$, the Pure Distance model makes two assumptions: (i) the four perceptual organizations are collectively exhaustive and mutually exclusive, from which it follows that

$$p(\mathbf{a}) + p(\mathbf{b}) + p(\mathbf{c}) + p(\mathbf{d}) = 1 \quad (2)$$

and (ii) that the attraction function is:

$$f(v) = \frac{p(\mathbf{v})}{p(\mathbf{a})}. \tag{3}$$

From Equations 1, 2, and 3 it follows that

$$p(\mathbf{a}) = \frac{1}{1 + e^{-\alpha(b/a-1)} + e^{-\alpha(c/a-1)} + e^{-\alpha(d/a-1)}}, \tag{4}$$

$$p(\mathbf{b}) = \frac{e^{-\alpha(b/a-1)}}{1 + e^{-\alpha(b/a-1)} + e^{-\alpha(c/a-1)} + e^{-\alpha(d/a-1)}}, \tag{5}$$

$$p(\mathbf{c}) = \frac{e^{-\alpha(c/a-1)}}{1 + e^{-\alpha(b/a-1)} + e^{-\alpha(c/a-1)} + e^{-\alpha(d/a-1)}}, \tag{6}$$

$$p(\mathbf{d}) = \frac{e^{-\alpha(d/a-1)}}{1 + e^{-\alpha(b/a-1)} + e^{-\alpha(c/a-1)} + e^{-\alpha(d/a-1)}}. \tag{7}$$

Response errors. The Pure Distance model needs to be modified. We conducted a small experiment to assess whether our observers could reliably choose the correct response icon when we knew what they had seen. As one might expect, we found that observers made errors.

We recruited three of the observers from the Kubovy and Wagemans (1995) experiment. Trials were identical to the original experiment, except for the response screen. We offered observers three response icons: (i) the orientation of vector **a,** (ii) another orientation (the ‘‘lure’’), and (iii) a ‘‘neither’’ choice. The lure was a line segment representing an orientation that did not correspond to a vector of the lattice. It deviated from the orientation of **a** by 15° to 65° in 10° steps. We chose lattices in which **a** was by far the shortest vector and the lure would not be close to **b, c,** or **d.** From experience, we knew that this would cause the lattice to almost always be seen organized in the orientation of **a,** and the proximity of the lure to **a** insured that essentially all lure responses would result from perceiving **a** but responding with the wrong orientation. When observers chose the lure, they could not be veridically reporting what they perceived because the lures did not correspond to any grouping that anyone ever claims to see. The results of the experiment are plotted in Figure 6.

Model of response errors. We denote the four responses offered the observers in the Kubovy and Wagemans experiment *A, B, C,* and *D,* which correspond to percepts **a, b, c,** and **d,** respectively. We assume that the only response errors observers make are choices of the two orientations closest to the perceived orientation. As is evident from Figure 7, our assumption implies $p(A|\mathbf{b}) = p(B|\mathbf{a}) = p(C|\mathbf{d}) = p(D|\mathbf{c}) = 0$. Furthermore we assume

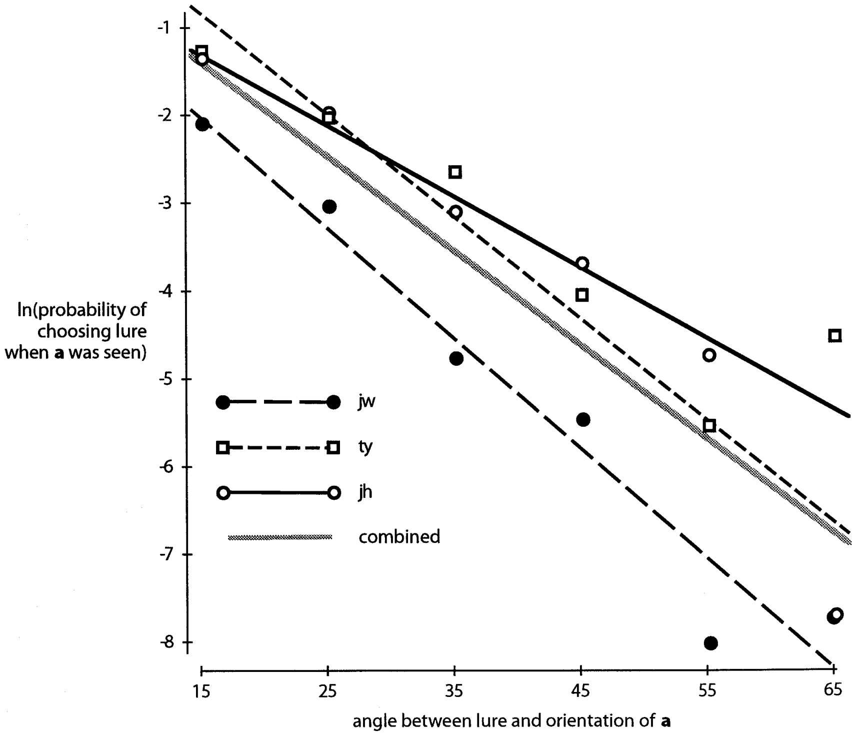

FIG. 6. The natural logarithm of the probability of choosing the lure as a function of the angular disparity between the orientation of **a** and the lure, for three observers. The approximate linearity of the functions, with an intercept of 0 (i.e., 100% errors when the angular disparity is 0°) allows useful approximation with an exponential function of angular disparity.

that error probabilities are symmetric; for example, $p(C|\mathbf{b}) = p(B|\mathbf{c})$. The model is summarized in Figure 8.

It is easy to derive the following response probabilities:

$$p(A) = p(\mathbf{a})[1 - p(C|\mathbf{a}) - p(D|\mathbf{a})] + p(\mathbf{c})p(C|\mathbf{a}) + p(\mathbf{d})p(D|\mathbf{a}), \tag{8}$$

$$p(B) = p(\mathbf{b})[1 - p(C|\mathbf{b}) - p(D|\mathbf{b})] + p(\mathbf{c})p(C|\mathbf{b}) + p(\mathbf{d})p(D|\mathbf{b}), \tag{9}$$

$$p(C) = p(\mathbf{c})[1 - p(C|\mathbf{a}) - p(C|\mathbf{b})] + p(\mathbf{a})p(C|\mathbf{a}) + p(\mathbf{b})p(C|\mathbf{b}), \tag{10}$$

$$p(D) = p(\mathbf{d})[1 - p(D|\mathbf{a}) - p(D|\mathbf{b})] + p(\mathbf{a})p(D|\mathbf{a}) + p(\mathbf{b})p(D|\mathbf{b}). \tag{11}$$

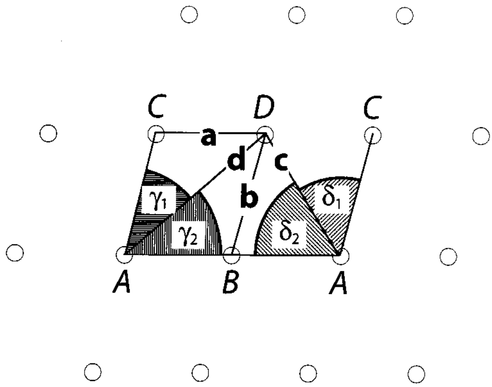

FIG. 7. The angles γ_1 and γ_2 diminish as d grows, increasing the probability of erroneously responding D when **a** or **b** were seen (or erroneously responding A or B when **d** was seen). Similarly, the angles δ_1 and δ_2 grow as c grows, increasing the probability of erroneously responding C when **a** or **b** were seen (or erroneously responding A or B when **c** was seen).

Analysis. The Kubovy and Wagemans data provide us with 16 multinomial distributions of $p(A)$, $p(B)$, $p(C)$, and $p(D)$—one for each lattice. For each observer, we produced an initial estimate of the slope of the error function (by using one of the slopes obtained in the response error experiment, Figure 6), from which we computed values for the conditional probabilities that appear in Equations (8) through (11). We did this by solving this system of linear equations for the four variables, $p(\mathbf{a})$, $p(\mathbf{b})$, $p(\mathbf{c})$, and $p(\mathbf{d})$. The solutions proved to be computationally tractable, if complicated, expressions.

For each observer we also produced an initial estimate of the slope α of the attraction function and substituted it into Equations (4) through (7), giving us a second set of estimates of the latent probabilities, $p(\mathbf{a})$, $p(\mathbf{b})$, $p(\mathbf{c})$, and $p(\mathbf{d})$. We transformed these two sets of probability estimates into two sets of frequencies (by multiplying them by the number of times each observer saw each lattice), and calculated χ^2 as a measure of badness of fit. For each observer, we then varied the slope of the error function and the slope of the attraction function until we found values that jointly minimized χ^2.

Results. The average response error function slope found by the minimization procedure was close to the observed average error function slope. The fit of the data to the model is shown in Figure 9 and Figure 10. In Figure 9 we show the left-hand panel of the fifth row of Figure 10 depicting the data of observer jw. In this figure we show the 16 predicted values (one for each lattice) of $p(\mathbf{a})$ based on the Pure Distance model [Equations (4) through (7)] as solid lines, and compare them to data points that represent the observed $p(\mathbf{a})$, corrected for response errors [Equations (8) through (11)]. The independent variable is the length of the long diagonal of the lattices' basic

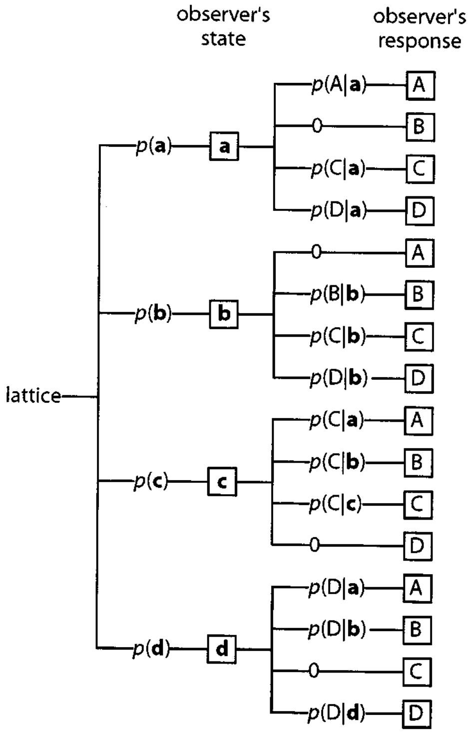

FIG. 8. Model of response errors due to orientation confusion. The symbols **a, b, c,** and **d** represent perceptual states of the observer. *A, B, C,* and *D* represent the response chosen by the observer. Thus whenever an observer chooses a response represented by a different letter than the one representing her perceptual state, she is making an error.

parallelogram, *d*. The data are divided into four groups determined by the length of the long side of the lattices' basic parallelogram, *b*. In Figure 10, each row represents the data of one observer, and each column represents the observed and predicted probability for one of the vectors. In this figure, we see that the probabilities corrected for errors (data points) faithfully reproduce the patterns predicted by the Pure Distance model (solid lines). Furthermore, as Table 2 shows, the model accounts for the data well, except for $p(\mathbf{d})$. This is not surprising: as Figure 10 shows, the corrected probabilities of *D* choices do not vary much, and are often close to 0, that is, we have a floor effect because the number of trials per observer was not large enough for the frequencies to express these small probabilities.

Is the function exponential? The preceding analysis relies on the Pure Distance model's use of the decaying exponential function. One could also use a power function. However, there is no point in doing so. To see why, let us take an exponential function with a slope of -6.6, a representative

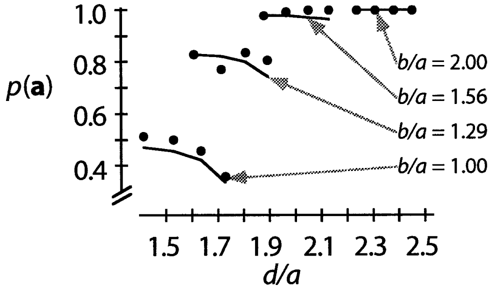

FIG. 9. Predicted value of $p(\mathbf{a})$ based on the Pure Distance model compared to data corrected for response errors due to orientation confusion, as a function of the length of the long diagonal of the lattices' basic parallelogram, *d*, for observer jw. The solid lines represent the predictions based on the Pure Distance model—Equations (4) through (7)—whereas the dots represent the data corrected for response errors as formulated in Equations (8) through (11). The data are partitioned into four groups dependent on the length of the long side of the lattices' basic parallelogram, *b*.

slope for our observers. We find that a power function with power −7.33 (rounded to two digits) best fits the exponential for the domain we are interested in: $1 \leq v/a \leq 2.5$. For this domain, the root-mean-square difference between the power function and the exponential is less than 0.01, which means that we cannot hope to distinguish between them. So, until we find grounds for rejecting the exponential, we will assume it to model our data.

Does lattice configuration affect grouping? In the experiment, two configurational properties were varied: symmetries of the pattern and γ, the angle between dot strips. These two properties are highly correlated in the case of dot lattices. The attraction function of the Pure Distance model uses the length of the vector of interest relative to the shortest distance in the lattice and disregards γ. However, the final expression for the probabilities includes all four vector lengths. Because the four vectors together determine the exact lattice, thus encompassing γ, one might claim that the grouping probabilities are determined by the configuration of the lattice. However, the probability expressions only contain all four vector lengths by virtue of the response error model and the constraint that the probabilities must add up to one. Thus the presence of all the vector lengths in each probability expression does not mean that the groupings are determined by γ nor by the lattice symmetries. To determine whether inclusion of γ can significantly improve on the Pure Distance model, we regressed the observed probabilities (corrected for errors) on two variables: (i) the probabilities predicted by the Pure Distance model and (ii) γ. The median proportion of variance unaccounted for by the Pure Distance model that is accounted for by adding γ is only 3.92%. Since γ contributes little to the success of the Pure Distance model, we draw the conclusion that *grouping by proximity shows no effect of configuration.*

The attraction function. The heart of the Kubovy and Wagemans model is Equation (3), which we can now verify empirically. In Figure 11, for each of our observers, we compare the predicted values (solid line) obtained from Equations (4) through (7) and the observed probabilities (data points) corrected for discrimination errors with Equations (8) through (11). In these graphs we do not present the full range of the data because of the phenomenon we discussed earlier—the floor effect (see the caption to Figure 11 for an explanation).

The good fit of the attraction function tells us that, in a dot lattice, the ratio of the probability $p(\mathbf{v})$, of seeing the lattice group in direction $\mathbf{v}$ (with dots v apart) and $p(\mathbf{a})$, the probability of seeing the lattice group in direction $\mathbf{a}$ (with dots a apart, the shortest distance between dots), is a negatively accelerated function of the ratio of the distances v and a. It also reinforces the claim, made in the preceding section, that no other factor, such as the angle γ, affects the grouping probabilities. In the rest of this paper we demonstrate the robustness of our theory in three more experiments. In the first two experiments we show that the Pure Distance model holds under transformations of scale in space and time. In the third we show that the Pure Distance model holds for units that are themselves the result of grouping.

EXPERIMENT 1: SPATIAL SCALING

The strength of the grouping of dot lattices into strips is a function of the distance between the dots in the lattice. We have expressed inter-dot distances in terms of the shortest distance in the lattice, i.e., v/a. Up to this point this was a convenience of no consequence, because we held the shortest distance, a, constant. We now turn to the question of scale invariance: are the distances that govern the grouping probabilities relative or absolute? To this end, we take a set of five lattices and present them at three different densities, so that the relative inter-dot distances are the same, but the absolute distances are different.

Method

Observers. Ten undergraduate students at the University of Virginia participated in this experiment for credit in an introductory psychology course. They were naive about the purpose of the experiment and had normal or corrected-to-normal vision.

Stimuli. We used fifteen different lattices: five lattice types at three densities. The five lattice types (see Figure 3) were: hexagonal ($b/a = 1, \gamma = 60°$), square (1, 90°), oblique (1.26, 70.3°), centered rectangular (1.59, 71.6°), and rectangular (1.59, 90°). The densities, as specified by the length of the shortest vector, **a,** were: $a = 40$ pixels ($\approx 1°$), $a = 60$ pixels ($\approx 1.5°$), and $a = 90$ pixels ($\approx 2.25°$). The mean number of dots displayed varied with lattice type and density: about 358 dots for $a = 40$ pixels, about 190 for $a = 60$ pixels, and about 84 for $a = 90$ pixels.

The lattices were presented at a random orientation for 300 ms followed by a dynamic mask consisting of a sequence of three 200-ms random dot patterns, each of which contained as many dots as the average lattice at the same scale as the current lattice. Following Kubovy

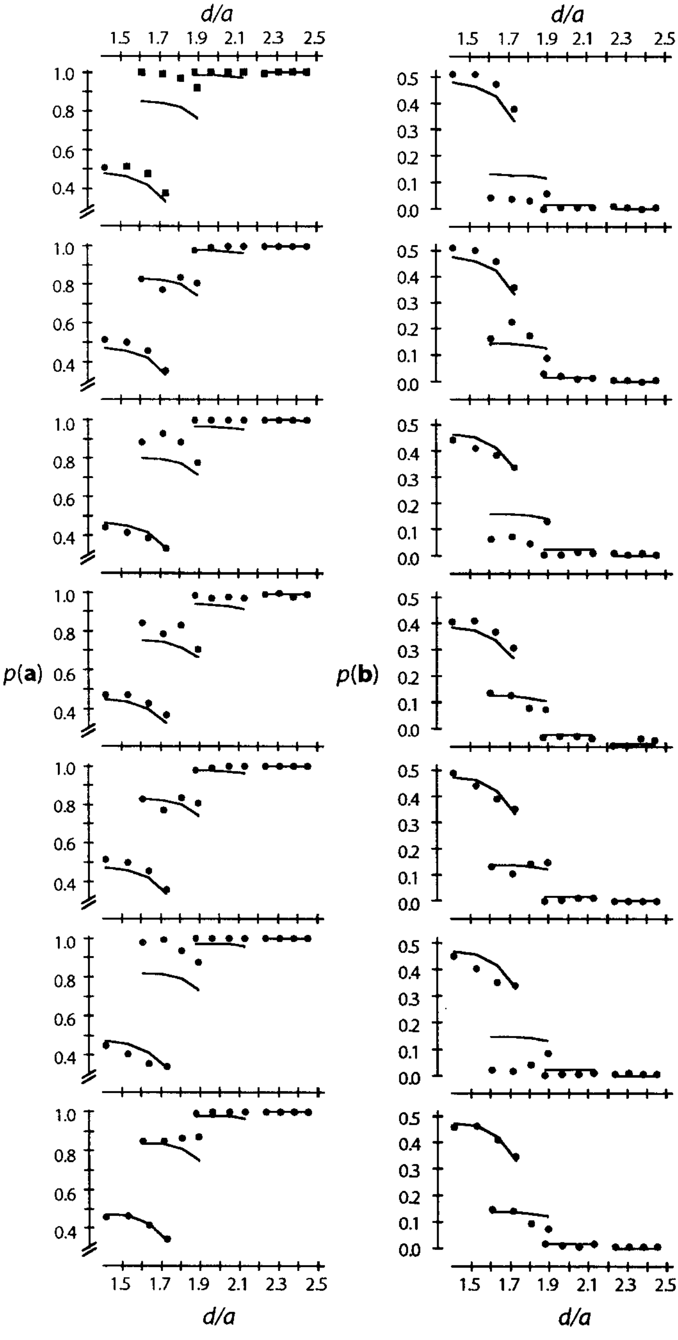

FIG. 10. Predicted value of $p(\mathbf{a})$, $p(\mathbf{b})$, $p(\mathbf{c})$, and $p(\mathbf{d})$, based on the Pure Distance model compared to data corrected for response errors, as a function of the length of the long diagonal of the lattices' basic parallelogram, d, for the observers in the Kubovy and Wagemans experiment. The dependent variable is different in each column of graphs: from left to right, the

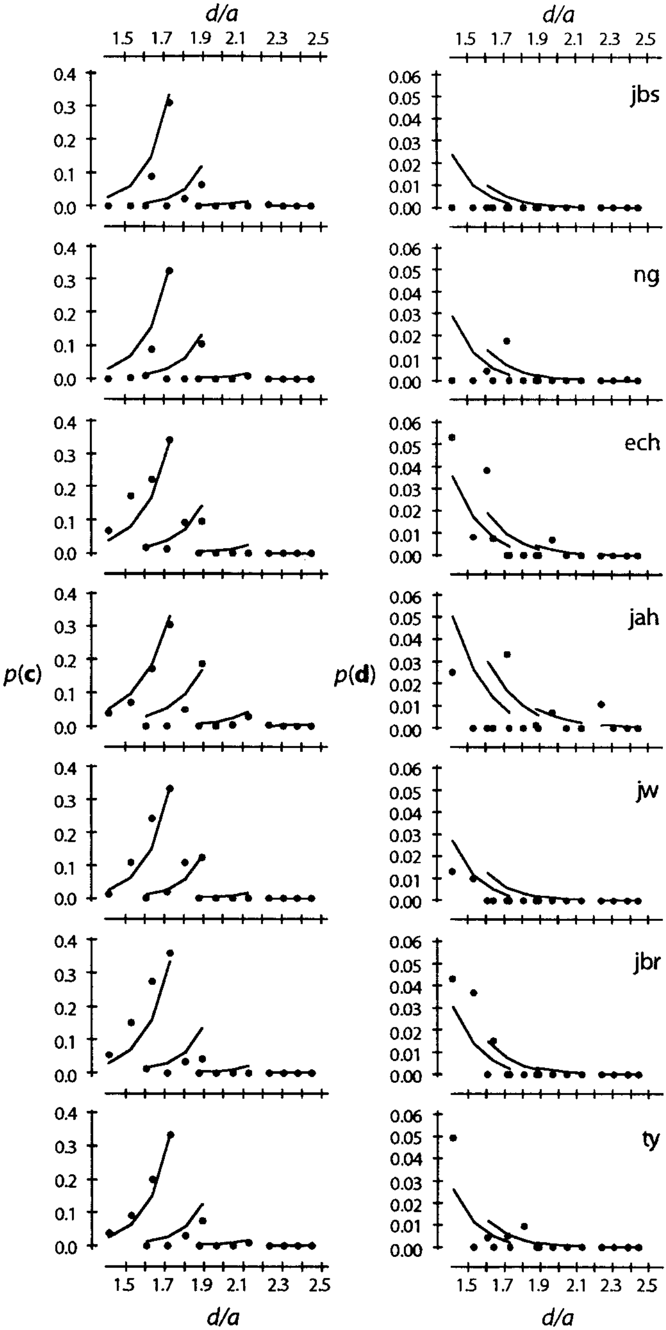

FIG. 10—*Continued* columns give values of $p(\mathbf{a})$, $p(\mathbf{b})$, $p(\mathbf{c})$, and $p(\mathbf{d})$. Each row of graphs represents the data of one observer. The explanation of the solid lines and data points is given in the caption of Fig. 9.

TABLE 2
Coefficients of Determination (R^2) for the Data Shown in Fig. 10

Observer	$p(\mathbf{a})$	$p(\mathbf{b})$	$p(\mathbf{c})$	$p(\mathbf{d})$
jbs	93.3%	95.1%	94.1%	—
ng	98.7%	98.4%	92.3%	0.0%
ech	96.4%	95.0%	89.2%	81.5%
jah	98.0%	98.8%	95.7%	18.4%
jw	98.9%	99.0%	91.9%	70.4%
jbr	92.0%	91.9%	82.2%	65.5%
ty	98.3%	99.1%	94.4%	71.7%

and Wagemans (1995), we ended each trial with a response screen containing four circular response fields with a tilted diameter, and observers used a mouse to indicate which they perceived.

Procedure. Each observer participated in two 600-trial sessions, during which we presented the 15 lattices in random order. Fifteen practice trials preceded the experimental trials of each session. A session was divided in three blocks of 200 trials by a mandatory 20-s break. Sessions were separated by at least 1 h. Each session took about 50 min.

Results

We compared the values of for each density and found no effect. We took several precautions before performing this analysis. First, we excluded responses to vectors whose length was equal to the length of **a** ($v/a = 1$). These responses are uninteresting because when **a, b,** and **c** are indistinguishable they are not subject to an effect of density. Second, we excluded responses to vectors for which $v/a \geq 1.59$. We had observed in the data of Kubovy and Wagemans that the floor effect began to influence the data beyond that value. Had we included these data we would have biased the analysis against finding an effect of density; at the extreme, if all our data were at floor, there could not be an effect of density.

The differences between the values of $\ln [p(\mathbf{v})/p(\mathbf{a})]$ for the three densities and the overall mean of $\ln [p(\mathbf{v})/p(\mathbf{a})] = -2.687$ (which corresponds to $p(\mathbf{v})/p(\mathbf{a}) = 0.0681$) were well within the standard error of 0.077, so the differences were insignificant. Nor were these values significantly affected by γ.

Discussion

The result validates the use of v/a in the Pure Distance model. Although previous experiments have not investigated the effect of scale on grouping per se, Zucker and Davis (1988) discovered that dense dot patterns give rise to Kanizsa subjective edges, whereas sparse patterns do not. From these data and other considerations, they inferred that grouping by proximity is performed by at least two mechanisms. Patterns composed of dense dotted lines—lines for which the ratio of dot diameter to dot separation (center to center), which we denote $d:s$, is greater than 1:5—give rise to the same

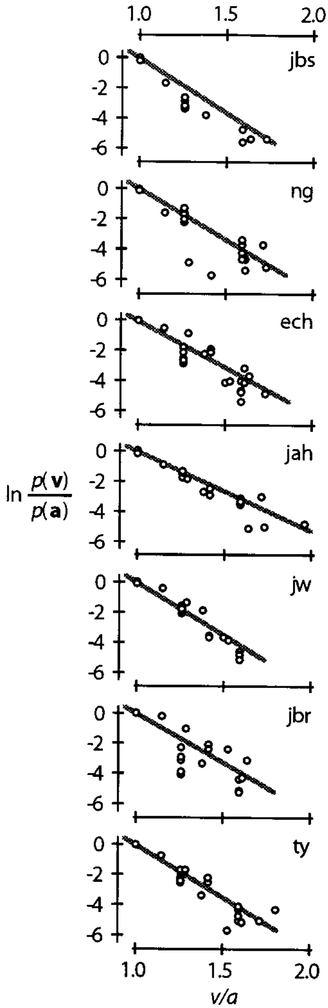

FIG. 11. The natural logarithm of probability ratios predicted by the Pure Distance model (solid line) compared to the natural logarithm of the observed probability ratios corrected for response errors (data points). For this graph, we constrained the variation of the dependent variable so that its predicted value would never fall below the ln $[p(\mathbf{v})/p(\mathbf{a})]$ value that corresponds to a single observation. For instance, if an observer received 300 trials for each type of lattice, then ln $[p(\mathbf{v})/p(\mathbf{a})] \geq \ln(1/300) \approx -5.7$.

perceptual response as solid lines. If the dotted lines are sparse—$d\!:\!s \leq 1\!:\!5$—the response is weaker, and gives rise to qualitatively different percepts. We should emphasize that Zucker and Davis do not deny that grouping occurs in sparse lines. It is possible however that the laws of grouping by proximity differ for sparse and dense lines.

Our experiment tested whether Zucker and Davis's observation on the qualitative difference between the behavior of sparse and dense lines applies to grouping by proximity, because we had lattices for which $d{:}s = 1{:}5$ for the shortest vector (**a**) as well as lattices of higher density ($d{:}s = 1{:}3$) and lattices of lower density ($d{:}s = 1{:}7$). We have shown that the observations of Zucker and Davis on the difference between dense (solid-like) lines and sparse (dotted-appearing) lines do not generalize to grouping by proximity.

EXPERIMENT 2: DURATION

Kubovy and Wagemans (1995) and we (in Experiment 1) presented the dot lattices for 300 ms. The Pure Distance model, or its parameters, may not be invariant with exposure duration. Suppose we briefly present the observer a dot lattice. If time-consuming processes—such as mutual facilitation and inhibition among receptive fields—precede grouping, then the eventual organization of the pattern may not have the time to make itself evident to the observer if the exposure was brief. Faced with a yet-unorganized stimulus and a forced-choice among definite groupings the observer is likely to respond randomly, or according to some response bias (which amounts to the same, since we randomized stimulus orientations and response screens).

Methods

The same methods were used here as in Experiment 1, except for the differences noted below.

Observers. The observers were 9 undergraduates at the University of Virginia who participated for pay.

Apparatus and stimuli. We used eight lattice types (see Figure 3): hexagonal ($b/a = 1$, $\gamma = 60°$), rhombic (1, 75°), square (1, 90°), centered rectangular (1.26, 66.6°), oblique (1.26, 78.3°), rectangular (1.26, 90°), centered rectangular (1.59, 71.6°), and rectangular (1.59, 90°). To allow the computer to speed up stimulus presentation, and hence shorten exposure durations, we changed the stimuli in two ways: (i) the dots were filled nonagons instead of decagons (with no noticeable loss in dot quality), (ii) we chose $a = 1.875°$ to reduce the number of dots in the display to an average of 123. We know from Experiment 1 that such a scale difference makes no difference to the results. In this way, we were able to present the dot lattices for 100 ms and 200 ms, except for 67 of the 16,000 presentations, which were discarded from the analysis. Each presentation of a dot lattice was followed by a mask, consisting of three different random dot patterns of 123 dots each.

Procedure. Each observer participated in two sessions of 800 experimental trials. Each session began with 16 practice trials, in which the eight lattices were presented twice, in random order. A session was divided into four blocks of 200 trials, separated by 20-s breaks. Each session consisted of 25 random permutations of the 32 lattice-exposure combinations (16 lattices $\times$ 2 durations). Sessions were separated by no less than one hour.

Results

We analyzed the effect of duration of $\ln [p(\mathbf{v})/p(\mathbf{a})]$. We took the following precautions in our data analysis. To minimize the likelihood that we would find an effect of duration where none exists, we disregarded the responses to **c** and **d,** thereby avoiding the errors we observed in the Kubovy and Wagemans data, which could be aggravated under conditions of brief exposure. To increase the chances that we would find a true effect of duration, we disregarded (i) the responses to $b/a = 1$, because absolute distance is not likely to have an effect on the relative probabilities of choosing two vectors of equal length, and (ii) the responses to values of **b** longer than 1.59, because they are vulnerable to floor effects.

For 100 ms exposure $p(\mathbf{v})/p(\mathbf{a}) = 0.053$, whereas for 200 ms exposure $p(\mathbf{v})/p(\mathbf{a}) = 0.0316$ ($\ln [p(\mathbf{v})/p(\mathbf{a})] = -2.937$ vs. -3.455). The difference in $\ln [p(\mathbf{b})/p(\mathbf{a})]$ between the two durations (based on a repeated measures ANOVA, in which the factors were: *duration*, 100 vs. 200 ms; *session*, first vs. second; b/a, 1.25 vs. 1.59; and *observer*, 1 through 9) was marginally significant: $F(1, 8) = 4.9$, $p = 0.058$.

Discussion

The direction of this effect is consistent with a weaker perceptual organization at shorter exposure durations. At the limit, if a certain exposure duration were too short for any grouping to occur, then observers would choose their responses randomly, regardless of dot proximity. If the likelihood of such a state increases as exposure durations are reduced, we would observe the pattern that we have: the shorter the exposure duration, the higher the values of $p(\mathbf{b})/p(\mathbf{a})$. Nevertheless, the present experiment suggests that little, if any, change in perceptual organization happens between 100 and 200 ms of exposure duration.

EXPERIMENT 3: GENERALIZING THE PURE DISTANCE MODEL

Palmer and Rock (1994) have suggested that the visual processing of an image starts with edge detection, which partitions the image into nonoverlapping, relatively uniform, connected regions. After this partition is achieved, a figure-ground process specifies which regions are ‘‘objects’’ (or ‘‘figures’’) and which form the ‘‘background.’’ These objects, which Palmer and Rock call *basic-level units,* are then parsed into parts and grouped into groupings, forming a part-whole hierarchy (Figure 12).

Can the Pure Distance model be generalized beyond basic-level units? If so, the model’s usefulness would be enhanced. Here we study groupings of units which are themselves the result of grouping. Such hierarchical groupings occur in patterns we call *split lattices*, which can be investigated using the same methods we have used for regular dot lattices. Split lattices differ from regular dot lattices in that the distance between adjacent dots in one orientation is not uniform. Figure 13 is a rectangular split lattice, created by translating every other dot in the **a** orientation of a rectangular lattice (Figure 3). The separations in the **b** orientation remain uniform, whereas the **a** orien-

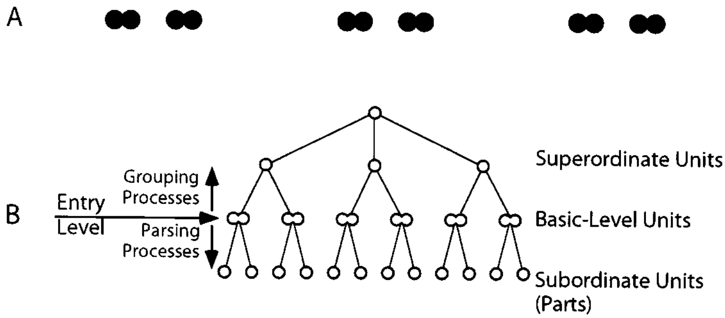

FIG. 12. Palmer and Rock's description of the part-whole hierarchy. The nodes in the network, B, represent units involved in the perception of the six elements shown in A. These elements can be parsed into subordinate units (parts) or grouped into superordinate units (groupings).

tation consists of two alternating vectors, $\mathbf{a}_1$ and $\mathbf{a}_2$. We adopt the convention that $a_2 \geq a_1$.

Consider the ''split'' orientation (**a**) in Figure 13: dots separated by the shorter component of the split orientation (a_1) form pairs. These pair-Gestalts participate in a further grouping process which organizes the lattice either into strips in the split orientation, **a,** or the unsplit orientation, **b.** A careful examination of Figure 13 suggests that proximity behaves differently in the context of two different distances. In this dot pattern, it so happens that $a_2 = b$. We normally see Figure 13 as organized in vertical columns. The description of this organization is ambiguous; some describe it as ''pairs of

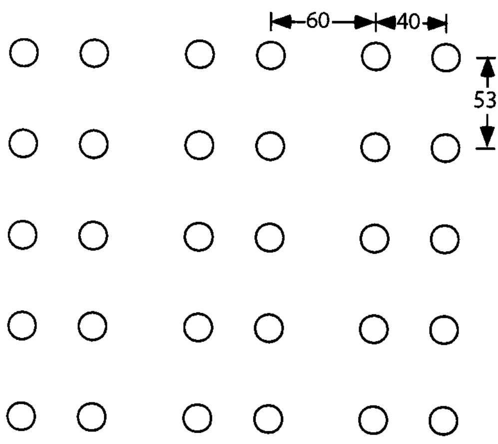

FIG. 13. An **a**-split rectangular lattice. $v_1 = 40$, $v_2 = 60$, $w = 53$, $v_2/v_1 = 1.5$, av$(v_1, v_2)/w = 0.94$ (this lattice is **a**-split because the average of the inter-dot distances in the split orientation is smaller than the inter-dot distances in the unsplit orientation; see 'x' in Fig. 14).

columns of dots'' and others as ''columns of dot-pairs.'' Nevertheless the predominant impression is one of columns rather than rows.

To measure the interaction of the different distances in the split lattices, we used some lattices split in the **a** orientation and some split in the **b** orientation, and we compared observers' tendency to see grouping in the non-split orientation rather than the split orientation. We require a generalization of our vector notation to express the **a**-split case and the **b**-split case with the same dependent measure. We call the two vectors in the split orientation $\mathbf{v}_1$ and $\mathbf{v}_2$ ($v_2 \geq v_1$) and we denote the non-split orientation (be it **a** or **b**), **w.** Split lattices are defined by three parameters: v_2/v_1, $\text{av}(v_1 + v_2)/w$, and γ. v_2/v_1 measures the amount of split in the shortest split orientation; $\text{av}(v_1 + v_2)/w$ is the ratio of the length of the split orientation to the length of the non-split orientation, and γ is the angle between **v** and **w.** Split lattices for which $\text{av}(v_1 + v_2)/w = 1$ are ordinary dot lattices. The relative strength of grouping in the split orientation relative to grouping in the non-split orientation is then $p(\mathbf{v})/p(\mathbf{w})$.

Method

Observers. Eighteen University of Virginia students participated in this experiment for credit in an introductory psychology course. They were naive about the purpose of the experiment and had normal or corrected-to-normal vision.

Apparatus and stimuli. The apparatus and display were similar to those used in the Kubovy and Wagemans experiment.

We sampled the split lattices from the three-dimensional (v_2/v_1, $\text{av}(v_1 + v_2)/w$, γ) space. Figure 14 is a two-dimensional projection of the sampling, collapsing over the γ dimension. Three values of γ—74°, 82°, and 90°—were crossed with 38 $v_2/v_1 \leq \text{av}(v_1 + v_2)/w$ combinations. These combinations were chosen to obtain a range of parameter values, yet minimize the floor effect. The $\text{av}(v_1 + v_2)/w$ values were sampled logarithmically (but constraints such as the discrete nature of computer screens prevented the logarithmic spacing from being perfect).

We chose γ values so that the distance of dots from their nearest neighbors in orientations other than **v** and **w** would be relatively long. Therefore, only the **v** and **w** (**a** and **b**) alternatives were provided to the observers. This change virtually eliminates response errors due to orientation confusion. The angle between **v** and **w** was always greater than 73°, which would cause confusion errors on only 0.1% of the trials by the least accurate observer in the memory experiment. An effect of configuration in split lattices can reveal itself in an effect of γ. In the dot lattice experiments, an effect of γ was not found, but it is possible that the pairing may change this. For example, suppose we perceive the dots in the split **v** orientation as a collection of dot pairs. Is this organization reinforced when $\gamma = 90°$, causing the **w** organization to be columnar and perpendicular to the axis of the pairs?

Procedure. Each trial consisted of a dot pattern presented for 300 ms followed by 2 random-dot patterns presented for 300 ms. Immediately following was a two-alternative response screen. Each observer participated in one session of 912 trials, divided into eight blocks in which a random permutation of the 114 lattices was presented. Sessions were broken by two 30-s rest breaks into three sets of 304 trials. Each session (including instructions and debriefing) took about 70 min.

Results

To determine whether γ affects the relative grouping strength, we analyzed the effects of $\text{av}(v_1 + v_2)/w$, v_2/v_1, and γ (discrete unordered fixed factors),

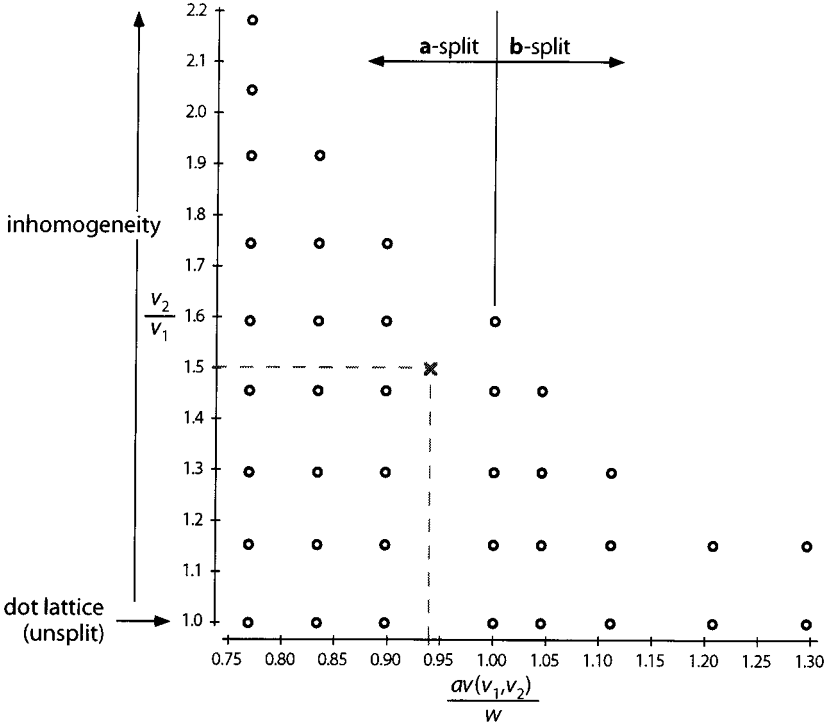

FIG. 14. The space of split lattices. The ‘x’ represents the pattern of Fig. 13.

and *observer* (random factor) on ln $[p(\mathbf{v})/p(\mathbf{w})]$. The differential effects of three levels of γ were minuscule: for $\gamma = 74°$: ln $[p(\mathbf{v})/p(\mathbf{w})] = -1.428$ ($p(\mathbf{v})/p(\mathbf{w}) = 0.2398$), for $\gamma = 82°$: ln $[p(\mathbf{v})/p(\mathbf{w})] = -1.432$ ($p(\mathbf{v})/p(\mathbf{w}) = 0.2388$), for $\gamma = 90°$: ln $[p(\mathbf{v})/p(\mathbf{w})] = -1.417$ ($p(\mathbf{v})/p(\mathbf{w}) = 0.2424$). We confirmed that this effect is statistically undetectable by a repeated-measures ANOVA: $F(2, 34) \approx 1$.

After aggregating the data across observers (which was necessary because we only have eight trials per observer for each data point), we plot ln $[p(\mathbf{v})/p(\mathbf{w})]$ against av$(v_1 + v_2)/w$ (Figure 15, left panel). For each value of v_2/v_1 we fit a linear regression (Figure 15, right panel). That these linear functions fit well suggests that for each value of v_2/v_1, $p(\mathbf{v})/p(\mathbf{w})$ is an exponentially decreasing function of av$(v_1 + v_2)/w$. Since they do not fall on the same curve, the average distance in the split orientation is not the only characteristic that determines grouping: indeed, as v_2/v_1 increases, the split orientation is less likely to be reported.

If the split lattices are grouped into pairs before the split lattice is organized by strips, then the competition that determines the latter organization may not be between w and the average of v_1 and v_2, but rather between v_2 and w.

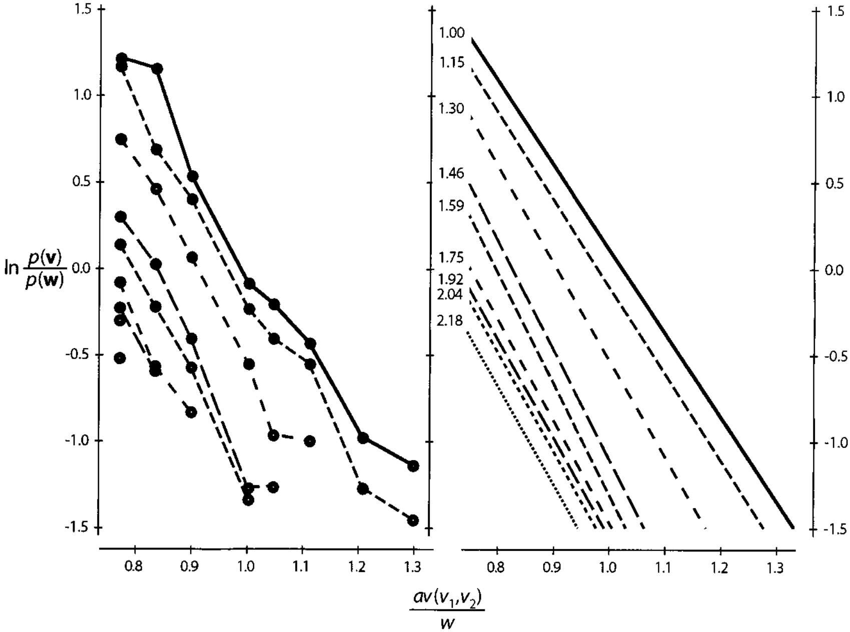

FIG. 15. Left panel: The natural logarithm of the ratio of $p(\mathbf{v})$, the probability of seeing the dot pattern grouped into strips along the split orientation, and $p(\mathbf{w})$, the probability of seeing the dot pattern grouped into strips along the non-split orientation, as a function of the ratio of the average length of inter-dot distances along the split orientation and the length of the inter-dot distance along the non-split orientation. Right panel: The best-fitting linear functions for different degrees of heterogeneity in the split orientation. The parameter is v_2/v_1.

We therefore plot ln $[p(\mathbf{v})/p(\mathbf{w})]$ as a function of in Figure 16. This comes much closer to collecting the data onto one line. Indeed, the regression of ln $[p(\mathbf{v})/p(\mathbf{w})]$ on v_2/w accounts for (an adjusted) 92.4% of the variance. As we would expect, when $v_2/w = 1$, ln $[p(\mathbf{v})/p(\mathbf{w})] = 0$ (actually 0.026 ± 0.035). Furthermore the heterogeneity expressed by v_2/v_1 does not affect the organization: if we regress ln $[p(\mathbf{v})/p(\mathbf{w})]$ on v_2/w and v_2/v_1, while constraining ln $[p(\mathbf{v})/p(\mathbf{w})] = 0$ when $v_2/v_1 = 1$, then the coefficient of $v_2/v_1 = 1$ is -0.008 ± 0.025 (it adds only 0.3% to the variance accounted for by v_2/w).

Discussion

The results of this experiment show that the grouping of split lattices is unaffected by the extent of the split, ranging from patterns with a splitting ratio $v_2/v_1 = 1$ (i.e., regular dot lattices) to patterns with $v_2/v_1 = 2$ (our most extreme split lattices). Our data show that observers organize the heterogeneous strips of dots into pairs, which become units whose separation v_2 com-

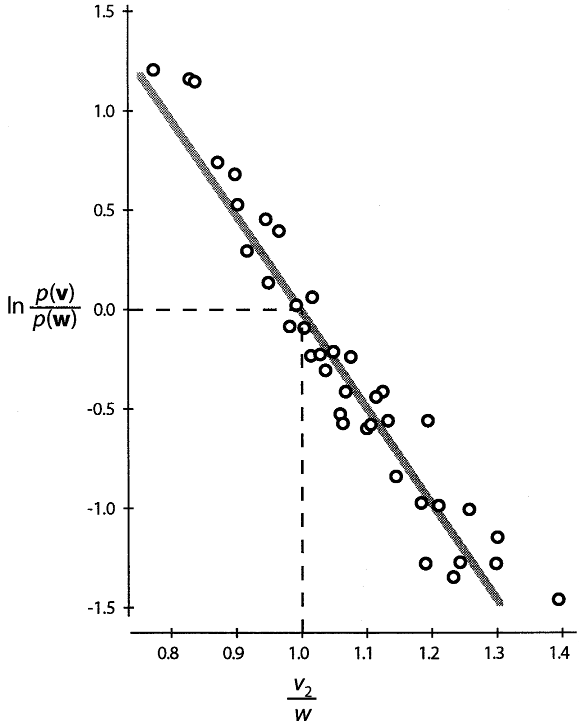

FIG. 16. Regression of ln $[p(\mathbf{v})/p(\mathbf{w})]$ on v_2/w through (0, 1).

petes with the separation *w* as if these pairs were mere dots in a dot lattice. Thus our theory of grouping by proximity holds even when the elements that are grouped are the result of another grouping operation.

GENERAL DISCUSSION

For the periodic dot patterns we have explored, we have remedied the shortcomings of previous approaches to perceptual organization by successfully measuring grouping strength and the effect of varying spatial configuration. Some may object that the dot patterns we used are not seen in the real world, but we required stimuli in which we could isolate and systematically vary proximity as well as configural properties.

In the case of relative grouping strength, our experiments and analyses show that it is a negatively accelerated decreasing function of distance, which we model as a decaying exponential function. To predict grouping in more complex patterns we need to discover the strength function for other principles, such as similarity, as well as rules for the interaction of the grouping principles. Therefore the current effort is only the beginning; more recent experiments in our laboratory have measured the effect of luminance contrast (Kubovy, Holcombe, & Friedenberg, 1995).

Grouping in the dot patterns was not affected by the configural property we varied, γ, which changes the lattice type. We have seen earlier that different lattice types differ in their symmetry. From the point of view of Gestalt psychology, a pattern with greater symmetry is simpler, ''better,'' or more prägnant. So we have shown that the goodness of a multistable pattern does not directly affect the distribution of the probabilities of its different interpretations.

However, we should recall that the Gestalt psychologists used pattern goodness, or prägnanz, to explain a different phenomenon. When we look at a multistable pattern, whose multiple interpretations differ in simplicity, we are more likely to see the simple interpretation than the others. The multiple interpretations of our patterns are all collections of strips of dots, and therefore do not differ in goodness or simplicity.

Furthermore, we cannot infer that grouping in the dot patterns is nonconfigural or can always be accounted for by proximity alone. The tendency for dot lattices to group into parallel strips is in itself a configural effect: for instance, lattices are always perceived as coherent collections of strips, and are never seen organized in a piece-meal fashion. In addition, the Pure Distance model does not address important global properties such as dot colinearity, which is held constant in our experiments.

But surprisingly, the configural properties we varied did not affect grouping. We will continue this enterprise by investigating more and more complex dot patterns until we find the simplest case for which we can vary and understand the effect of spatial configuration, for we believe our method has the best chance of recognizing such a pattern and determining exactly what is going on in it. In a lecture delivered in 1924, Wertheimer pointed out that

> There are contexts [*Zusammenhänge*] where the behaviour of the whole is not determinated by the nature and combination of the individual pieces, but in contrast where, in the pregnant case, that which happens in a part of this whole is determined by inner structural laws of this whole. (Quoted in Smith, 1988, p. 464)

A concrete and complete understanding of this phenomenon has been a long time coming.

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