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Toward a Better Approach to Goodness: Comments on Van der Helm and Leeuwenberg (1996)

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Some regularities are more salient to the visual system than others. P. A. van der Helm and E. L. J. Leeuwenberg (1996) have proposed a new approach that quantifies the goodness of a pattern's regularity as the number of holographic identities constituting the regulartiy, relative to the total amount of information needed to describe the pattern. This holographic approach to goodness was compared with previous approaches and was presented in relation to metatheoretical issues. These 3 aspects are discussed further here. First, the theory is shown to contain implausible assumptions and unfortunate gaps with respect to the required processing. Second, Van der Helm and Leeuwenberg's critique on preceding theories is refuted. Third, some metatheoretical issues need to be qualified or at least clarified. Together, these concerns suggest that a better approach to goodness might result from a synthesis of the most useful aspects of diverse theories of goodness.

In 1996, Van der Helm and Leeuwenberg presented a new theory of goodness that helped researchers understand the perceptual salience of regularities such as repetition and mirror symmetry in terms of properties of their mathematical representation. They argued that preceding theories of goodness failed either because they were process theories with contradictory ad hoc assumptions or because they were representation theories taking a transformational approach to goodness. In contrast, their theory is representation based and nontransformational.

First, I summarize Van der Helm and Leeuwenberg's (1996) theory and discuss some problematic assumptions and unfortunate gaps. Then, I demonstrate that their critique of preceding theories such as the transformational approach and the bootstrap model is not completely justified. Finally, I raise some concerns about the metatheoretical framework within which their theory has been presented. The major message is that a synthesis of the most fruitful aspects of the new representation theory with those of preceding process theories might lead toward a better approach to goodness.

Van der Helm and Leeuwenberg's (1996) Theory of Goodness

Theory

Definition of goodness. In Van der Helm and Leeuwenberg (1996, p. 444) they proposed the following as a theoretical definition of goodness (because all quotes are from Van der Helm & Leeuwenberg, 1996, I indicate the source by only the page number in that article):

> The goodness of a pattern is, in our view, determined by the "strength" of the regularity described in the simplest description of the pattern. By strength, we mean the amount of support, or "weight of evidence" (McKay, 1969), for the existence of a regularity, as given by the identities that constitute this regularity. Because a regularity is always embedded in a pattern, we propose to quantify goodness by $W_1 = E/M$, in which E is the number of holographic identities that constitute the regularity, whereas M is the total information in the pattern. (p. 444)

This conception of goodness, called the holographic approach to goodness (which I will denote by HA), requires two pieces of theoretical work for a further elaboration: (a) a tool to specify the number of holographic identities in a regularity (E), and (b) a tool to quantify the total information in the pattern (M). The latter is provided by Leeuwenberg's (1969, 1971) structural information theory (SIT); the former builds on Van der Helm's (1988) mathematical formalization of regularity (see also Van der Helm & Leeuwenberg, 1991). This formalization has also led to a much better justified coding system and complexity metric (e.g., Van der Helm, Van Lier, & Leeuwenberg, 1992). Thus, both parts are intrinsically connected and the HA to goodness is a natural development of the authors' previous theoretical work.

Pattern encoding. SIT is a pattern encoding model proceeding in several steps. First, a pattern is represented by a *symbol sequence.* Next, a symbol sequence is encoded by *coding rules,* describing regularity in terms of the identity of symbols in a sequence. The most important rules are the so-called *ISA-rules* (for iteration, symmetry, and alternation). By the iteration or I-rule, a symbol sequence $kkkk \ldots k$ is coded as $m * (k)$; by the symmetry or S-rule, a symbol sequence $k_1k_2 \ldots k_spk_s \ldots k_2k_1$ is coded as $S[(k_1)(k_2) \ldots (k_s),(p)]$; and by the alternation or A-rule, symbol sequences such as $kx_1kx_2 \ldots kx_n$ and $x_1kx_2k \ldots x_nk$ are coded as $< (k) >/< (x_1)(x_2) \ldots (x_n) >$ and $< (x_1)(x_2) \ldots (x_n) >/< (k) >$, respectively. These coding rules are applied to all subsequences,

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yielding a combinatorially explosive number of possible codes. A *complexity metric* is then used to select the simplest code, and the simplest code is assumed to reflect the preferred pattern interpretation. In this sense, SIT provides a synthesis of Hochberg and McAlister's (1953) minimum principle with Attneave's (1954) information-theoretic pattern descriptions.

Holographic regularity. Regularity is based on the identity of symbols in symbol sequences, captured in so-called *identity structures.* In intuitive terms, an identity structure is said to describe a holographic regularity if its substructures all describe the same kind of regularity (for a more formal account, see pp. 437–440). Straightforward analysis yields that there are only 20 holographic kinds of regularity; among these are repetition and mirror symmetry. The structure imposed by the I-rule yields the holographic structure of repetition. Because a repetition substructure corresponds to a subsequence, repetition is said to have a block structure (see Figure 1A). Mirror symmetry is a holographic regularity covered by the S-rule in SIT; it is said to have a point structure because each symbol constitutes one substructure (see Figure 1B).

Transparent hierarchy. Holographic regularity concerns an intrinsic character of a regularity. To specify the unique formal status of repetition, mirror symmetry, and alternation, the extrinsic compatibility of a regularity with other regularities must be considered. In Van der Helm and Leeuwenberg (1996), they distinguish between three types of compatibility: first, a trivial kind of compatibility comprising nonoverlapping regularities and, second, two types of hierarchical compatibility comprising overlapping regularities in which the elimination of one regularity does affect the other but not vice versa. When one regularity lies completely inside a substructure of another regularity, one has so-called *plain hierarchy;* when two regularities are nested in such a way that both can be described only when one starts with one specific regularity first, one has so-called *transparent hierarchy.* Only 4 of the 20 kinds of holographic regularity have the most intricate form of compatibility, transparent hierarchy: bilateral symmetry as described by the S-rule, repetition as described by the I-rule, alternation as described by the A-rule, and different successive twofold repetitions, also described by the I-rule. This is a solid theoretical justification of the coding rules used in SIT.

Goodness in 2-D dot patterns. The same formalization is used to explain at a more concrete level why certain regularities are "better" than others, in the sense of being "faster detected, more easily discriminated, and less sensitive to noise" (p. 429). A few additional steps must be taken and a few assumptions must be made before this formal framework can be applied to explain the goodness of regularities in dot patterns. First,

> [A]n n-dot pattern can be constructed on the basis of the symbolic recipe $p_1 \cdot p_2 \cdot \ldots p_n \cdot$, which prescribes that there is a dot ($\cdot$) at each position p_i ($1 \leq i \leq n$). This symbolic recipe forms a 1-D symbol sequence containing the "first-order" structure of a pattern consisting of identical dots. This first-order structure can be extracted from that symbolic sequence by applying the A-rule, yielding the A-form $< (p_1)(p_2) \ldots (p_n) >/< (\cdot) >$, which expresses the identity of all the dots. The dot positions can be specified in one way or another, codepending on "second-order" regularity like mirror symmetry or repetition. (p. 443)

However, the 1-D transparency of alternation pertains to 2-D patterns only if every 1-D subsequence represents a spatially contiguous 2-D subpattern. In other words, in the symbolic recipe, the dots should be given in a spatially contiguous order.

> Furthermore, in order that a 2-D mirror symmetry can be described as a 1-D mirror symmetry in the sequence of dot positions, first the dots in one symmetry half should be given in a spatially contiguous order, and then the dots in the other symmetry half should be given in the reversed order. (p. 443)

In addition, "for practical reasons, and nearly without loss of generality" (p. 443), the dot patterns must satisfy the following restrictions: They must have homogeneous dot density and constant pattern size; all dots must be identical and nonoverlapping, and no dots should lie on the axis.

Next, the holographic quantification $W = E/M$ can be evaluated for a variety of goodness phenomena. Because all dots are identical

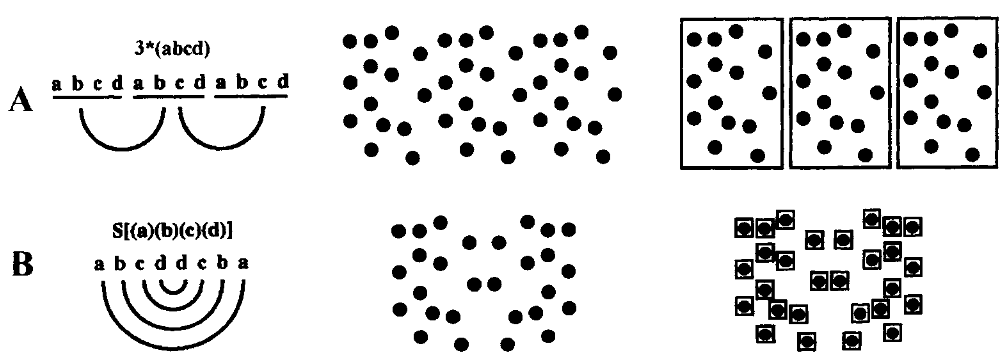

Figure 1. A: Illustration of the holographic nature of *repetition.* To the left is a 1-D symbol sequence with the arcs indicating identities between substructures. In the middle is a 2-D dot pattern with threefold repetition. To the right, the block structure of repetition is indicated by rectangles drawn around each repeated random-dot pattern. B: Illustration of the holographic nature of *mirror symmetry.* To the left is a 1-D symbol sequence with the arcs indicating identities between substructures. In the middle is a 2-D dot pattern with mirror symmetry. To the right, the point structure of mirror symmetry is indicated by rectangles drawn around each dot (i.e., each dot constitutes a substructure). Adapted from "Goodness of Visual Regularities: A Nontransformational Approach," by P. A. van der Helm and E. L. J. Leeuwenberg, 1996, *Psychological Review, 103,* p. 441. Copyright 1996 by the American Psychological Association. Adapted with permission of the authors.

and nonoverlapping and dot density and pattern size are kept constant, $M = n$ (i.e., the number of dots). Because the first-order regularity is described by the A-form $< (p_1)(p_2) \ldots (p_n) >/< (\cdot) >$, constituted by $n - 1$ identities, it has a weight of evidence $W = (n - 1)/n$. W approximates 1 if the number of dots n is sufficiently large; in that case, the first-order structure common to all dot patterns can be neglected such that the focus can be on the higher-order regularities.

Finally, the goodness of higher-order regularities such as repetition and mirror symmetry can be quantified as follows:

> In the case of an otherwise-random m-fold repetition pattern consisting of n dots, the repetition is constituted by $E = m - 1$ identities, so it has a weight of evidence of $W = E/n = (m - 1)/n$. . . . In the case of an otherwise-random mirror-symmetric pattern consisting of n dots, the mirror symmetry is constituted by $E = n/2$ identities, so it has a weight of evidence $W = E/n = (n/2)/n = ½$. (p. 445)

Thus, repetition is predicted to get better if n decreases and if m increases, whereas mirror symmetry is predicted to be equally good with varying n. Not all of these predictions have been tested but some empirical results are certainly consistent with them. For example, Baylis and Driver (1994) have demonstrated with block patterns consisting of several steps (for ns = 4, 8, and 16) that response times for detection of repetition increase linearly with n, whereas they remain essentially flat for mirror symmetry. With large ns (from about 20 to 5,000), Tapiovaara (1990) found that detectability of mirror symmetry remains constant. The variable W for repetition and the constant W for mirror symmetry imply a variable goodness difference between these two regularities. For example, an m-fold repetition with $m > 2$ can be better than mirror symmetry [if n is chosen to be appropriately small, i.e., $n < 2(m - 1)$], whereas a twofold repetition with $n > 2$ is generally worse than mirror symmetry. These goodness differences are confirmed by convincing demonstrations in Van der Helm and Leeuwenberg's (1996) article (see also the present Figure 2).

The same formal principles are also used to explain why adding extra regularity enhances the goodness of twofold repetition more than it does for the goodness of mirror symmetry. Because of the

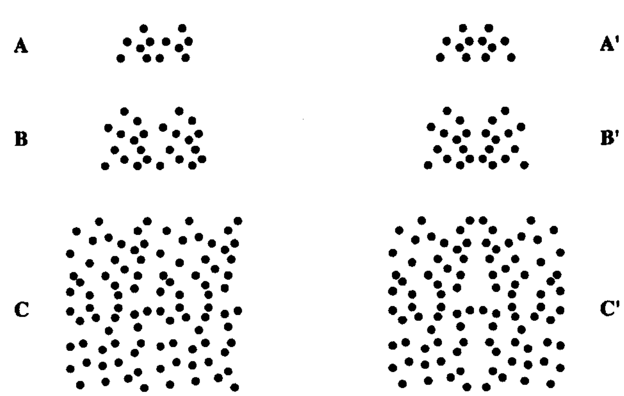

Figure 2. An increasing number of dots leads to an increasing goodness of twofold *repetition* (from A to C), whereas it hardly affects the goodness of *mirror symmetry* (from A' to C'). Adapted from "Goodness of Visual Regularities: A Nontrasformation Approach," by P. A. van der Helm and E. L. J. Leeuwenberg, 1996, *Psychological Review, 103,* p. 445. Copyright 1996 by the American Psychological Association. Adapted with permission of the authors.

block structure of repetition, extra regularity in each repetition substructure reflects plain hierarchy, whereas extra regularity in each symmetry half reflects transparent hierarchy because of the point structure of mirror symmetry. As a result, extra regularity in each pattern half counts twice in the case of twofold repetition and only once in the case of mirror symmetry (see pp. 446–447). Moreover, the HA can also explain why threefold mirror symmetry is worse than twofold symmetry (pp. 447–448), how noise affects repetition and mirror symmetry (pp. 449–450), and some results concerning so-called *Glass patterns,* which are created by superimposing a random dot pattern on a copy of itself, after rotation or translation (pp. 450–451).

Evaluation

On the one hand, the HA has considerable explanatory power, unifying a large number of empirical results under a single theoretical umbrella. In addition, it leads to interesting new predictions that are specific enough to be testable by future empirical research. For example, concrete predictions are made about so-called *broken symmetry* (p. 440), about interactions between regularities (pp. 441–442), and about the effect of the location of noise (p. 449), although it is not immediately clear how the HA can deal with noisy symmetries without incorporating metric aspects (see later). On the other hand, the theoretical constructions themselves are quite elaborate and complex, and many of the preliminary steps that must be taken and assumptions that must be made make it somewhat less appealing as a psychological theory of how pattern goodness arises for human perceivers.

Black box. Most important, it seems quite problematic to assume that the symbols must be in the correct order for the formal machinery to work appropriately on the symbolic sequences. For mirror symmetry, for example, it is essential that the dots are encoded in reversed order for both pattern halves. What are the processing assumptions that must be made for the visual system to get the symbols in the correct order? Does it have to process all dots sequentially according to strict rules of precedence (which would then be reversed for mirror symmetric pattern halves)? Perhaps it does not matter how dots are processed and the visual system has a hitherto undiscovered mechanism for reshuffling dot positions into the intermediate representations? Questions like these remain unanswered in the current version of the theory. The authors might argue that the encoding of a 2-D pattern in a 1-D symbol sequence is only a theoretical steppingstone without any psychological relevance: It need not be assumed that a symbol sequence precedes (in process terms) a particular end code with possible holographic and transparent properties. Although this may salvage the theoretical constructions, it puts many interesting psychological aspects into a black box by refusing to answer important questions regarding the processing that is needed to get the representations.

The problem of simultaneous or successive order in the visual stimulation seems related to age-old problems such as *local sign* or "Localzeichen" as introduced by Lotze (1884; see also Koenderink, 1984; Schwartz, 1980) and *serial order* as introduced by Lashley (1951; see also D. Bruce, 1994; Lewandowsky & Murdock, 1989) as well as current hot topics regarding the spatiotemporal encoding characteristics of the visual system (e.g., Gilbert, 1995; Kovács, 1996; Singer, 1995). Because the assumption of correct order is so critical for the theory to be able to start working, and existent work on related problems contains some interesting suggestions, I believe that this part of the theory should not be left unspecified (or should not be left to other scientists to fill them in). Nevertheless, this seems to be implied by Van der Helm and Leeuwenberg (1996, p. 443) when they said that "the theory does not prescribe in detail how the raw 1-D symbolic representations are to be obtained (which stresses that it is a representation theory and not a process theory)." An account of goodness with such large unresolved questions is not good enough yet.

Salient substructures. Another unfortunate assumption is that dot density must be homogeneous. This assumption is probably needed to avoid problems with perceptually salient substructures. If so, the restriction is all the more disappointing because substructures are really at the heart of important theoretical notions such as holographic regularity, and much of the attraction of the HA depends on its claim that it deals satisfactorily with interactions among regularities. Grouping by proximity, curvilinearity, or collinearity often creates salient substructures such as clusters of dots, strings of dots, or isolated "anchors" (e.g., Compton & Logan, 1993; Feldman, 1997; Smits, Vos, & Van Oeffelen, 1985; Van Oeffelen & Vos, 1983). When present, these structures make the detection of repetition and mirror symmetry easier. Hence, the influence of semilocal grouping factors creating salient substructures on the detection of more global regularities such as repetition and mirror symmetry appears a prototypical case of interactions among regularities.

Examples of the role of perceptually salient substructures are present in one of the few instances where Van der Helm and Leeuwenberg (1996) failed to draw homogeneous dot patterns. In Figure 2C, the most salient substructure is a vertically reflected C-like curve segment formed by six dots. It is probable that the repetition of this substructure helps to see the repetition of the whole dot pattern. In Figure 2C′, the same substructure is present, and now also a few small substructures get more salient because they bridge the axis (e.g., one at the pattern's top and one near the pattern's center). Perhaps the goodness of mirror symmetry is generally higher than that of repetition because there is an increased likelihood that salient substructures occur near the pattern's midline. This will play a role especially when patterns are flashed briefly. Recent experimental research has already demonstrated the role of clustering in symmetry detection (e.g., Dakin & Watt, 1994; Labonté, Shapira, Cohen, & Faubert, 1995; Locher & Wagemans, 1993; Wenderoth, 1995). In sum, patterns with salient substructures are better patterns, and an account of pattern goodness that does not incorporate this effect is not good enough yet.

Van der Helm and Leeuwenberg's (1996) Critique of Preceding Theories of Goodness

In addition to presenting their own theory, Van der Helm and Leeuwenberg (1996) also offered a critique of preceding theories of goodness. They distinguished between *representation* theories, which "aim at explaining visual phenomena primarily in terms of static qualities of the representations that result from the perceptual process," and "*process* theories, which aim at explaining visual phenomena primarily in terms of dynamic qualities of the perceptual process itself" (p. 430). Van der Helm and Leeuwenberg further claimed that all preceding representation theories of regularity have taken a transformational approach (which I denote by TA), which implies a block structure for mirror symmetry as well as for repetition, and they present the so-called *bootstrap model* (which I denote by BM) as "one of the most elaborate and promising process models" (p. 432), although "it does . . . not (yet) provide a comprehensive understanding of goodness" (p. 433). In this section, I restrict the discussion to Van der Helm and Leeuwenberg's critique of TA's block structure representation of mirror symmetry and their diagnosis of BM's defects.

Transformational Approach (TA)

The TA starts from a mathematical definition of symmetry as a transformation that leaves its object invariant. This notion has had a large impact on the field of symmetry detection as evidenced by the changed names of different regularities: reflectional symmetry instead of mirror or bilateral symmetry, translational symmetry instead of repetition, and rotational symmetry instead of centric symmetry. Because the HA agrees with the TA of repetition in that both imply a block structure representation of the repeated subpatterns (i.e., pattern halves in the case of twofold repetition), Van der Helm and Leeuwenberg (1996) did not criticize the TA of repetition, except for noting that "a repetition pattern is invariant under a group of translations, only if the pattern is extended infinitely such that the pattern contains an infinite number of identical subpatterns, and the group an infinite number of translations" (p. 434), a point that is all too often overlooked in previous work on translational symmetry or repetition. However, they did criticize the TA of mirror symmetry, and they took Palmer's (1982, 1983, 1991) approach as the standard TA. (Note that the TA can also be translated into a process model in which the mechanisms for detecting transformational invariances can be specified; see Palmer, 1985.)

To describe mirror symmetry, Palmer used the flip or reflection, that is, an operation that interchanges all mirror-symmetric point pairs simultaneously (i.e., a holistic transformation that works on the global pattern, not a pattern half, as often assumed by Van der Helm & Leeuwenberg, 1996). Together with the trivial identity transformation, one such transformation already forms a group (so that it does not suffer from the same problem as TA's approach of repetition). I will call this the *global TA* of mirror symmetry. Van der Helm and Leeuwenberg (1996) acknowledged that another TA of mirror symmetry is possible (which I define as the *local TA*): One can use the set Φ of all transformations that each interchange only one mirror-symmetric point pair. To be a group (i.e., to obey the requirement of closure), this set should be expanded to the set $G(\Phi)$ to contain every composite of the transformations in Φ (i.e., every transformation that interchanges a subset of those point pairs simultaneously). In Van der Helm and Leeuwenberg (1996, p. 435), they continued as follows:

> Both the reflection and the flip imply that mirror symmetry gets a block structure: Each symmetry half becomes one substructure. (*Substructures* are subpatterns identified with each other subpattern by a single transformation.) This block structure implies an all-or-nothing relationship between the two symmetry halves: Even the slightest noise in the mirror symmetry destroys the transformational invariance relationship between the two symmetry halves. In contrast, the earlier mentioned set Φ would imply that each point in each symmetry half becomes one substructure, so that mirror symmetry would get a point structure implying a graded relationship between the two symmetry halves. The set Φ is, however, neither before nor after closure taken as a transformational descriptor of mirror symmetry. It is true that

before closure the set Φ is not a group and that after closure the group $G(\Phi)$ is perceptually irrelevant.

I think that Van der Helm and Leeuwenberg (1996) dismissed the local TA of mirror symmetry too easily, especially if one considers how strongly their attack on the TA depends on its representation of mirror symmetry as a block structure. In my view, the local TA of mirror symmetry becomes perceptually relevant in patterns in which points are not spaced evenly (i.e., when the earlier mentioned assumption of homogeneous dot density is violated). In that case, one gets substructures on the basis of grouping by proximity or collinearity (or some other grouping factor), which then become the elements to be matched (i.e., to be transformed into one another) to establish the mirror symmetry. In other words, if one is willing to go one step further on the road from mathematics to psychology, one could say that the set of transformations must be complete (or have closure), from a mathematical point of view, but that only some sets of transformations have perceptual relevance, namely those that operate on structures created by other grouping processes (i.e., some of them work on points, and some on small blocks). In this sense, all structures yielded by other grouping processes are anchors to establish and detect reflection (or any other regularity for that matter). As mentioned earlier, Figure 2C and C′ contain some interesting examples to illustrate that substructures are important from a processing point of view in both repetition and mirror symmetry. In fact, even Palmer's TA has this flexibility to capture both global (or block) structure and local (or pointwise) structure by varying the size of the receptive field of individual analyzers (see Palmer, 1985).

I am not inclined to defend the TA by insisting on this escape from Van der Helm and Leeuwenberg's (1996) criticism, but I do think it has some appeal as an alternative TA. In Van der Helm and Leeuwenberg (1996), their rejection of it is only based on their insistence on mathematical rigor (in their requiring of closure of set Φ) and their neglect of processing aspects (in their dismissal of the psychological relevance of substructures, that is, composites of point pairs). A considerable advantage of this local TA is that it is compatible with recent process models such as Dakin and Watt's (1994) filter model. At the same time, such filter models allow for bottom-up extraction of salient substructures so that they go beyond the ad hoc nature of the classic Gestalt laws of grouping. By setting the spatial filters at different scales, one can obtain substructures as small as single elements or as large as a few blobs for the whole dot pattern, with the intermediate range as the best possible range to extract salient substructures that span the pattern's midline (see also Dakin, 1997; Dakin & Hess, 1997). Moreover, corresponding to this spatial scale parameter, one can have a process that either establishes the correlation between two pattern halves on a point-by-point basis or computes a more global measure of blob alignment. Computer simulations have demonstrated that the best possible fit with human symmetry detection data (by Barlow & Reeves, 1979, and by Jenkins, 1983) is obtained with the rough blob-alignment measure (indicating the psychological plausibility of medium-scale clusters or substructures, as in the local TA).

Bootstrap Model (BM)

Before presenting Van der Helm and Leeuwenberg's critique on the BM and my rebuttal, I briefly summarize the model's principles (for more details, see Wagemans, Van Gool, & d'Ydewalle, 1991; Wagemans, Van Gool, Swinnen, & Van Horebeek, 1993).

Summary of the BM. The BM is a process model that starts from the available information in a dot pattern (i.e., the locations of the dots). Dots are grouped in pairs by using a virtual line to establish a connection. Initially, pairwise groupings are random, although the grouping process has some built-in preferences (e.g., it starts in the middle of a pattern and it has a preference for short, horizontal virtual lines). Virtual lines connecting symmetrically positioned dots have uniform orientations and collinear midpoints, two properties that were found useful in symmetry detection (Jenkins, 1983). In perfect mirror symmetry, a pair of virtual lines connecting symmetrically positioned dots also establishes a virtual quadrangle with correlated angles (called a *correlation quadrangle;* see Figure 3A). The basic assumption of the BM is that these correlation quadrangles facilitate the propagation of local pairwise groupings because they specify a reference frame that suggests a unique direction within which other correspondences are much more likely to be found. In other words, the initial randomness in pairing elements within some local neighborhood converges to systematicity much more easily, establishing a coherent global structure more rapidly and more efficiently. This automatic spread-out of correspondences is called *bootstrapping.*

When mirror symmetric dot patterns are viewed from aside, one has so-called *skewed symmetry* (see Figure 3B). As a result of skewing, the first-order regularities of the virtual lines (i.e., orientational uniformity and midpoint collinearity) are still preserved but the second-order regularities of the virtual quadrangles are destroyed (i.e., the angles in the virtual quadrangles are no longer pairwise correlated). Thus, bootstrapping is not possible anymore. These properties of the BM allow an explanation of the superior detectability of orthofrontal mirror symmetry as compared with skewed symmetry (Wagemans et al., 1991; Wagemans, Van Gool, & d'Ydewalle, 1992), of double mirror symmetry as compared with single mirror symmetry (Palmer & Hemenway, 1978; Wagemans et al., 1991) and of the smaller effect of skewing with multiple symmetries (Wagemans et al., 1991). Similar goodness differences in other types of dot patterns (e.g., with translational or rotational symmetry) can also be attributed to the presence or absence of higher-order regularity in correlation quadrangles such as trapezoids and parallelograms (see Wagemans et al., 1993).

Van der Helm and Leeuwenberg's critique on the BM. First, the BM is said not to offer a sufficient explanation of the goodness difference between repetition and mirror symmetry. I have argued previously (Wagemans, 1995) that mirror symmetry is more salient than twofold repetition because parallelograms do not allow the same degree of bootstrapping as trapezoids; there is no single direction of propagation. Van der Helm and Leeuwenberg doubted that this difference was strong enough to explain the goodness difference (p. 433): "In repetition, one additional parallelogram already disambiguates the propagation direction while, moreover, the virtual-line length is fixed (in mirror symmetry, it is variable)." Second, the BM is said to predict that translational Glass patterns are as good as twofold repetition, although, in fact, they are as good as mirror symmetry (thus, better than twofold repetition). Third, Van der Helm and Leeuwenberg "do not see how the [BM] may explain that extra regularity in each half of a twofold repetition has a stronger effect than extra regularity in each half of a mirror symmetry" (p. 433).

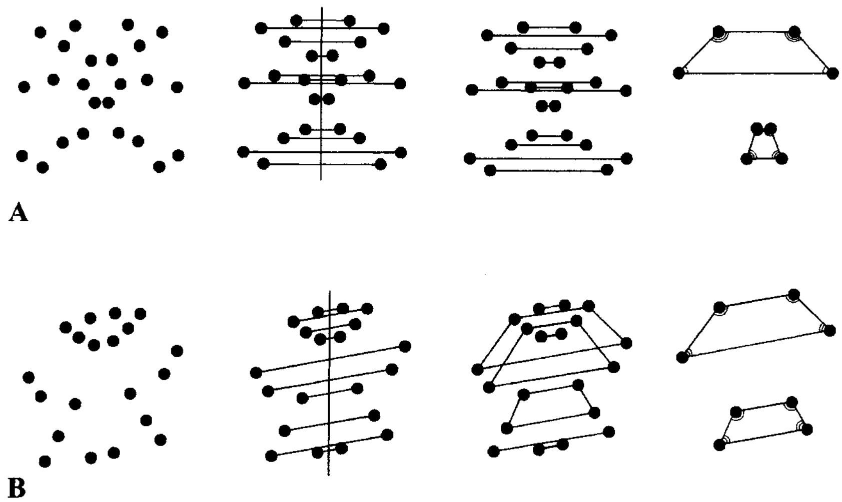

Figure 3. A: *Perfect mirror symmetry* in a dot pattern (left) with an indication of the first-order regularity in terms of the virtual lines (middle) and the second-order regularity in terms of the virtual correlation quadrangles (right). B: *Skewed symmetry* in a dot pattern (left) with an indication of the first-order regularity in terms of the virtual lines (middle) and the absence of second-order regularity in terms of the virtual quadrangles (right). Adapted from Wagemans (1995, Figure 2, p. 24).

Rebuttal 1. I agree with Van der Helm and Leeuwenberg (1996)'s first point that our bootstrapping account of the superior goodness of mirror symmetry over repetition may not seem convincing if one considers only the number of propagation directions (1 in mirror symmetry vs. 2 in repetition) or the number of equal virtual-line lengths. However, it must be stressed that the BM is a process model: Our model does not count regular structures in some sort of intermediate representation (as would the HA); it groups dots in a dot pattern automatically, and local pairwise groupings get propagated more quickly when additional regularities such as repetition or mirror symmetry are present (i.e., they affect the process itself rather than yielding a better representation).

Moreover, one should not forget the critical role of distance in the initial stages of this grouping process. In a computer simulation with an algorithm implementing the most essential ideas of the BM (Wagemans et al., 1993), the detectability of translational symmetry (or twofold repetition) declined significantly with increasing length of the translation vector (although we did not increase length to the extent where the two pattern halves would suddenly become two clearly separated blocks, where repetition may again be rather salient). In many dot patterns with mirror symmetry, one has a few point pairs near the axis of symmetry. I claim that these are very instrumental in the increased salience of mirror symmetry (and I am not the only one to claim this; see, e.g., V. G. Bruce & Morgan, 1975; Jenkins, 1983).

Rebuttal 2. The same holds for the second criticism: If one assumes that a process of regularity detection starts with strong local groupings (i.e., small, short-length correlation quadrangles), then it becomes obvious why translational Glass patterns (as in Figure 4A) are as good as dot patterns with mirror symmetry (as in Figure 4B) and, indeed, better than twofold repetitions with a larger translation vector (as in Figure 4C). It is not by accident that the filter model, discussed earlier, has recently been applied to the detection of structure in Glass patterns (Dakin, 1997) just as easily as to the detection of mirror symmetry (Dakin & Hess, 1997; Dakin & Watt, 1994).

Perhaps Van der Helm and Leeuwenberg (1996) would counter this argument by calling it a metrical pattern aspect (such as dot density and pattern size, which they prefer to put in brackets) or by

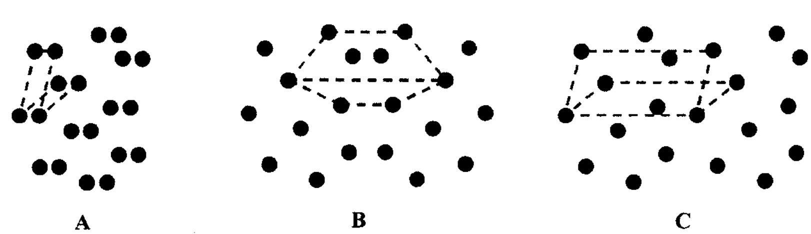

Figure 4. Translational Glass patterns (A) are as good as dot patterns with mirror symmetry (B) and better than twofold repetitions with a larger translation vector (C). In contrast to what Van der Helm and Leeuwenberg (1996) claimed, this can be explained easily in terms of the BM (see text). Adapted from "Goodness of Visual Regularities: A Nontransformational Approach," by P. A. van der Helm and E. L. J. Leeuwenberg, 1996, *Psychological Review, 103,* p. 433. Copyright 1996 by the American Psychological Association. Adapted with permission of the authors. (1996, Figure 5, p. 433).

calling it a process bias that is said to contradict other process biases or to be just arbitrary. However, this processing assumption does not violate any other processing assumptions. The idea is simply to start grouping elements in the pattern's center first and to group elements that are close together first (which is in line with the Gestalt rule that grouping strength declines with increasing distance; see Kubovy, Holcombe, & Wagemans, 1998; Kubovy & Wagemans, 1995). Moreover, this processing assumption is not arbitrary. On the contrary, it is related in an essential way to the neuroanatomy of the visual system: The visual acuity is much better near fixation than in the periphery. In addition, the visual system's bilateral symmetry may enhance the detectability of vertical mirror symmetry presented to the eye's fovea (see Herbert & Humphrey, 1996, for a review and some recent findings, and Julesz, 1971, for an early account).

Together with the orientation effect (which is related to the tuning function of the visual system's orientation-sensitive units and, accidentally, also beyond the HA's limits), the important role of the central area around the axis is one of the best established findings in the domain of symmetry detection (Barlow & Reeves, 1979; Jenkins, 1983; Julesz, 1971; Wenderoth, 1995). I suggest that a process theory that makes use of what researchers know about the visual system's hardware properties (but in a manner that is flexible enough to allow for modulations by psychological influences; see later) may help to develop a better approach to goodness.

Van der Helm and Leeuwenberg (1996, pp. 445–446) have briefly touched on the role of the central area around the axis of symmetry. When they developed their account of goodness in terms of weight of evidence and specified that the goodness of mirror symmetry does not depend on the number of dots (for $n > 20$), they deduced that this may be the reason why only a restricted number of dots may be used by the visual system (i.e., because no additional weight of evidence is gained by taking more dots into account). Moreover, Van der Helm and Leeuwenberg related this to the point structure in the representation of mirror symmetry and the holographic accessibility of visual regularity: When a mirror symmetry is holographically constituted by many identities between their substructures, it can also be accessed easily through any of its subsymmetries. However, as long as it is not specified from which area a restricted number of dot positions is sampled or which of the many substructures are used to access its holographic representation, this account remains, in a sense, arbitrary. By making use of some of the best-established facts on symmetry detection and, more generally, by relying on knowledge about the visual system's hardware properties, a process account can specify why the area around the axis is most important. Together, these processing and representation accounts could be developed into a more principled and thus better approach to goodness (see later).

Rebuttal 3. Finally, a few comments with respect to the third criticism. To explain that extra regularity has a stronger effect in dot patterns with repetition than with mirror symmetry, we could argue that the extra regularity merely strengthens the unit of translation (i.e., the subpattern that is translated) and thus makes it possible to avoid a pointwise matching process. In a sense, this account is related to the mechanism of establishing substructures outlined earlier (with respect to the role of additional groupings). Moreover, this advantage for larger substructures would not only work for dot patterns with repetition (as argued in the HA) but also for dot patterns with mirror symmetry. The reason that repetition seems to profit more from additional regularity is that mirror symmetry generally does not need it because short-distance point pairs are often present in the zone around the symmetry axis. This clarifies once again that the average distance between corresponding points (which may be the same for mirror symmetry and repetition) is less important than a few extra short point-pair distances.

The BM is a general grouping mechanism that starts off locally (by default, near fixation). In the absence of additional groupings, it establishes pairwise groupings at first, then it builds larger structures such as correlation quadrangles. In contrast, when additional groupings are present (such as in Figures 2C and C′), it may avoid some of the pairwise groupings and work with larger-scale structures immediately (as in Dakin & Watt's, 1994, filter model; see earlier). To be honest, I must say that the currently implemented version of the BM does not yet have this property. However, as argued elsewhere (Wagemans, 1995, 1997), there is nothing in the BM that prevents the addition of a filter stage before bootstrapping operates (on the perceived location of blobs rather than exact coordinates of all individual points) or the addition of other grouping algorithms that extract salient substructures such as curvilinearities (e.g., Compton & Logan, 1993; Feldman, 1997). As long as these plug-in modules work automatically, there is nothing arbitrary or ad hoc to their effect on regularity detection. One could denote this extended version of the model by BM′. Thus, the BM can be developed into a general model that works with some default settings (such as horizontal pairings near the pattern's center first) but that can just as easily work with some preconditions that depend on the pattern itself (e.g., additional structures) or on the context of presentation (e.g., all trials in a block having a certain orientation; see Wenderoth, 1994). I believe that this quality of the BM turns it into the flexible mechanism that Van der Helm and Leeuwenberg (1996) claimed to be missing in the literature on process models.

Previously, we have demonstrated (Wagemans et al., 1993) that the BM also explains findings in other areas than regularity detection where a local grouping mechanism operates (e.g., in vector patterns, optic flow, and stereo). In light of this empirical evidence and the present reply to Van der Helm and Leeuwenberg's criticisms, I propose that the BM is a serious candidate for being the single explanatory scheme for many or all of the available data, which was also the ambition of the HA.

Van der Helm and Leeuwenberg's (1996) Metatheoretical Framework

Before presenting their own goodness theory, Van der Helm and Leeuwenberg (1996) discussed the broader context within which to regard their theory. They distinguished between process and representation theories (as outlined earlier in this article) and they criticized previous accounts of goodness (as discussed in the preceding section). In their discussion of the relation between process and representation theories of goodness, Van der Helm and Leeuwenberg also touched on certain evolutionary considerations. I argue that some of their arguments on this issue are unjustified, whereas others remain unclear.

Evolutionary Considerations

Metatheoretical rationale of evolutionary adaptation. Van der Helm and Leeuwenberg (1996, p. 430) started this discussion as follows:

> Many process theorists adhere to the metatheoretic rationale that the perceptual sensitivity for certain regularities is the result of a gradual evolutionary adaptation to the presence of those regularities in this world. . . . According to this rationale, mirror symmetry is better than repetition because, within objects, mirror symmetry occurs more often than repetition.

Although I do not want to enter the discussion on the role of simplicity versus likelihood principles in perceptual organization (Chater, 1996; Pomerantz & Kubovy, 1986), it is important that Van der Helm and Leeuwenberg's arguments are put in the proper context.

Much empirical work is being done in relation to process models of regularity detection that is completely neutral to evolutionary considerations and, if evolutionary adaptation is brought up, it is usually done in a general way to argue for the relevance of research on all sorts of symmetry (e.g., Tyler, 1996, pp. 4–11). When evolutionary adaptation is used as an argument for the superior goodness of mirror symmetry over repetition, the argument usually does not refer to its more frequent occurrence but to its biological significance (e.g., faces viewed head-on afford social communication; see Tyler, 1996, p. 8).

Doubts about this rationale. In Van der Helm and Leeuwenberg (1996, p. 430), they continued as follows:

> We have doubts about this rationale. First, it suggests a diverging development towards various distinct and more or less fixed sensitivities. This does not seem able to account for goodness phenomena which involve interacting regularities. . . . Second, the assumption that external evolutionary pressure is the origin of the various sensitivities does not seem to hold for Glass patterns and skewed mirror symmetry. . . . The typically human-made Glass patterns are remarkably good . . . but seem hardly evolutionary relevant. Inversely, detection of skewed mirror symmetry is evolutionary more relevant but yet more difficult than detection of orthofrontal mirror symmetry.

Several qualifications are in order. First, evolutionary pressure does not necessarily lead to multiple mechanisms and fixed sensitivities instead of a single, flexible mechanism. We have argued previously (Wagemans et al., 1993) for a single, general grouping mechanism (with special properties for mirror symmetry), both on empirical grounds and on the basis of evolutionary considerations. A similar position is taken by Dakin and Watt (1994), whose process model is also surprisingly general (in being able to process all sorts of images while using the same filter operations) and at the same time leading to a very high proficiency in detecting foveally presented mirror symmetry at vertical orientations (by setting some default values for orientation and size of the spatial filters). The BM and this filter model are both able to handle interacting regularities.

Second, the arguments with regard to Glass patterns and skewed symmetry must be qualified, too. Glass patterns are indeed human-made but, in fact, they can be regarded as the superimposed time-frozen equivalents of two snapshots from an optic-flow sequence. As argued previously (Wagemans et al., 1993), Glass patterns with translation correspond to two superimposed time frames from an observer–object relative displacement, whereas Glass patterns with rotation could result from rotation of object or observer (e.g., head rotation). Combined or other more complicated optic-flow patterns may occur more frequently, but it is not illogical to assume that the visual system has developed mechanisms that are attuned to their component transformational flows as well (e.g., Lappin, Norman, & Mowafy, 1991).

Detection of skewed mirror symmetry in dot patterns is indeed deteriorated compared with their orthofrontal counterparts (Wagemans et al., 1991, 1992), but this result may be partly due to the particular experimental conditions (dot patterns with affine skewing). Casual observations with polygons (Stevens, 1980), as well as a few experimental results (Wagemans, 1993), suggest that polygons (which are more representative for object contours than dot patterns) may be less affected by skewing. Surface contours embedded in 3-D objects may even be less affected, as seems to be implied by our recent finding that the skewed symmetry of top and side surfaces of cubes affects the perceived global object structure (Van Lier & Wagemans, in press). Van der Helm and Leeuwenberg (1996) restricted their application of the HA to the goodness in 2-D dot patterns, although SIT can also handle polygons and even 3-D objects (e.g., Leeuwenberg, 1971; Van Lier & Wagemans, in press). In a similar manner, the BM was implemented to work with dot patterns only, but its principles can be extended to polygons as well (e.g., Wagemans, 1995, 1997).

Implicit view on process–representation relations. One final statement by Van der Helm and Leeuwenberg (1996, p. 430) on this topic needs clarification: "Evolutionary, the quality of the result of the process is more important than the quality of the process itself. (An inefficient process with useful results has more survival value than an efficient process with useless results.)" Because this statement seems to be presented as an argument for the priority of a representation account over a process account (see the following section), it is important that we understand it properly. Underlying this proposition is, I think, a view on the relation between processes and representations as typically held by Marr (1982) and others within the *computational* approach: Representations are, literally, re-presentations of a state of affairs or an event in the physical world as presented to us through our senses. Some of these are close to the original input, whereas others have been processed more deeply in the sense of implying more substantial transformations or abstractions. For example, in Marr's theory of object perception, the 2-D array of grey-level intensity values is taken as input to derive more abstract representations such as the primal sketch or the 3-D object model by filtering out gradual intensity changes or the viewer's position, respectively. In such a computational approach, it is quite logical that the quality of the representation is more important than the quality of the process: Certain representations are needed to achieve a particular computational goal; the intermediate representations are useful only if they succeed in representing the information that is needed at the subsequent level.

However, in biological vision (including human perception), such goal-driven arguments may become less important. Evolutionary pressures never worked in a vacuum but had to interact with the machinery that was already available (e.g., Cosmides & Tooby, 1995; Kaas, 1989; Tooby & Cosmides, 1995). The result may be that information about the environment is processed in a certain way, depending on the available processing system at a certain point in evolution, whereas this manner may change gradually as a result of a modified diet of visual inputs due to a certain change in the environment (e.g., more sunlight, or more vegetation) or a change in the animal's behavioral repertoire (e.g., the need for fine depth-perception mechanisms such as stereo when monkeys started to live in treetops).

From a computationally less clean but more realistic neural-network point of view, a representation is simply a pattern of activation (i.e., a way of processing) that has occurred often enough to leave its trace in the system (e.g., by means of increased synaptic strength). In current vision research, this neurobiological view on perception (e.g., Ramachandran, 1985) has become the more dominant metatheoretical approach compared with Marr's (1982) strict computational view. Within this approach, the view on evolutionary adaptation may have quite different implications on the process–representation relationship than the implicit view that Van der Helm and Leeuwenberg (1996) may have had in mind when they wrote their statement.

Relation Between Process- and Representation-Based Theories

Another metatheoretical position that is not sufficiently clear is Van der Helm and Leeuwenberg's (1996) view on the relation between process- and representation-based theories of goodness. In the initial sections of the original article, the authors opposed process- and representation-based theories of goodness and argued that a representation-based account is superior, first on metatheoretical grounds and then on empirical grounds (i.e., by demonstrating that contradictory process assumptions have been made and by identifying problems with the BM as the best possible process model). In the final sections of their article, Van der Helm and Leeuwenberg adopted a more moderate position: "Several aspects of . . . the . . . holographic approach are probably transferable to process models for 2-D pattern perception" (p. 442) and "there is not necessarily opposition between representation theories and process theories" (p. 443).

How does one reconcile these two positions with respect to the relation between process- and representation-based theories of goodness? In Van der Helm and Leeuwenberg (1996, p. 433–434), they offered a way out: "We do not oppose process theories as such but we think it is more expedient to develop a comprehensive representation theory first." Compatibility between process and representation theories is essential indeed (see later). The two will have to meet if the goal is a complete theory of goodness. The major issue is where to place one's bets first. This may be a matter of taste, although I continue to believe that processes have some logical priority over representations (i.e., processes precede representations; see earlier). Another essential issue is how predictive both of the approaches are. For the time being, it seems like a good research strategy to let both approaches do what they can and decide on the basis of their empirical success.

Van der Helm and Leeuwenberg (1996) appeared well aware of the fact that process- and representation-based theories may be complementary and that both may be needed to achieve a full-blown theory of goodness. However, they are not clear about how the two approaches would have to be united. Indeed, there is a wide gap between their theoretical building blocks (i.e., symbolic 1-D sequences represented mathematically in terms of identity chains and identity structures) and those of typical process models (i.e., intrinsically spatial, thus 2-D, relationships between positions of discrete elements or neurally filtered blob positions, or perceptually grouped clusters).

The few occasions in which they provided hints toward a certain compatibility of viewpoints are rather superficial. One such occasion has been alluded to before, when I summarized Van der Helm and Leeuwenberg's discussion of the role of the zone around the midline of a mirror-symmetric pattern: They argued that the holographic nature of the representation of mirror symmetry made it possible to access it easily through any of its subsymmetries (e.g., those around the midline), and they concluded (p. 446): "This accessibility through subsymmetries agrees with Wagemans et al.'s (1991, 1993) [BM] and with Palmer's (1982, 1983) process model based on 'local spatial analyzers.' " As indicated before, this does not yet explain why the particular zone around the axis of symmetry would be the preferred subsymmetries to access the global symmetry of the whole pattern. It is only through some reasonable process assumptions that this becomes theoretically justified. In that sense, a particular process account complements a theoretical gap left by the representation account offered in the HA.

A second allusion to a metatheoretical compatibility is more misleading. When Van der Helm and Leeuwenberg (1996, p. 443) defended the (questionable) assumption of spatially contiguous encoding order, they did so in the following way:

> The position of each dot can be given relative to the axis, as suggested by Attneave (1954), or it can be given relative to the preceding dot. The latter way exhibits a *striking correspondence* [italics added] with Wagemans et al.'s (1991, 1993) [BM]: A pair of mirror-symmetric relative dot positions corresponds to the mirror-symmetric sides of a bootstrapping trapezoid. Moreover, the process of bootstrapping agrees with a spatially contiguous order.

Once more, they neglected an essential aspect of the BM, namely that bootstrapping works *automatically,* which means that initially the pairwise groupings are random and need not establish the mirror-symmetric correspondences: Once a small number of "correct" correspondences are found, bootstrapping will indeed connect spatially contiguous dot positions but only as a *result* of an initially random process, not as an a priori requirement to get the process going (as seems implied in the HA). The corresponding elements need not be in a corresponding order in the initial representation, as in the HA.

Summary

Table 1 contains a systematic comparison of the most important aspects of the three major theoretical approaches to goodness that have been discussed by Van der Helm and Leeuwenberg (1996) and in my article: the Holographic Approach, the Transformational Approach, and the Bootstrap Model.

Holographic Approach

In the HA, the goodness of regularity in a pattern is quantified as the number of holographic identities that constitute the regularity, relative to the total amount of information needed to describe the pattern. The theoretical building blocks to elaborate this notion include the iteration, symmetry, and alternation rules, holographic regularity captured by the identity structure of substructures in symbol sequences, and transparent hierarchy that is based on how

Table 1
Systematic Comparison of the Three Major Theoretical Approaches to Goodness: the Holographic Approach, the Transformational Approach, and the Bootstrap Model

Holographic approach	Transformational approach	Bootstrap model
	Goodness	
Number of holographic identities relative to the total amount of information	Related to transformation that has created the symmetry	Based on efficiency of process to extract structure from pattern
	Theoretical building blocks	
ISA-rules Holographic regularity Transparent hierarchy	Groups of transformations (in global TA) Sets of transformations (in local TA)	Pairwise grouping Correlation quadrangles Bootstrapping
	Typical domain of application	
1-D symbol sequences	3-D objects	2-D dot patterns
	Repetition vs. mirror symmetry	
Block structure vs. point structure	Translation vs. reflection	Parallelograms vs. trapezoids
	Evaluation	
+ Explanatory power + Representations are well-elaborated − Processing = black box − No role for salient substructures	± (Local TA holds promise)	+ Explanatory power ? Resulting representations + Processing is made explicit ± Substructures could be captured (i.e., BM could be extended to BM′)

Note. ISA-rules = iteration, symmetry, and alternation rules. TA = transformational approach; BM = bootstrap model. + = a positive aspect of an approach. − = a negative aspect of an approach. ± = an aspect that could be captured by an approach (if intended). ? = an open issue.

different regularities are related. This theoretical machinery is most easily applied to 1-D symbol sequences; several assumptions must be made to extend it to 2-D dot patterns (e.g., the substructures in the 1-D sequence must correspond to spatially contiguous 2-D subpatterns). Moreover, to avoid metric side effects, the dot patterns must have homogeneous dot density. The goodness difference between twofold repetition and mirror symmetry arises because twofold repetition is holographically constituted by only one identity between two substructures (i.e., the pattern halves yield a block structure), whereas mirror symmetry is holographically constituted by many identities between very simple substructures (i.e., the dots yield a point structure).

The HA is able to capture a large number of empirical findings within a unified framework that is completely representation based. As a result, the processing aspects are put in a black box, which leaves many open questions regarding the processes that are needed to get the appropriate representations. One of the most important theoretical gaps is the neglect of the role of salient substructures. In the ideal case of perfectly homogeneous dot patterns, perhaps the representations of twofold repetitions consist of two blocks and those of mirror symmetry leave all individual dots ungrouped. However, as soon as interdot distances vary enough to give rise to perceptual grouping of dots in salient substructures such as pairs, clusters, collinearities, and curvilinearities, the clean representational difference may collapse.

Transformational Approach

The TA is also a representational approach. It was originally formulated to deal with 3-D objects, but it can easily be applied to 2-D patterns too (e.g., even smooth 2-D curves, not just polygonal shapes obtained by successively joining dots in a pattern). Detection of regularities such as twofold repetition and mirror symmetry is said to be based on the representation of the two random-dot pattern halves together with the transformational operator (i.e., translation and reflection or flip, respectively). Because all pattern elements are interchanged together by one single operation, this global TA uses a block structure representation for both twofold repetition and mirror symmetry and thus cannot explain the goodness difference very well. However, an alternative TA is possible in which only certain substructures are interchanged. This local TA violates the mathematical requirement of closure to form a group of transformations, but it may be more psychologically plausible if it used the outputs of processing mechanisms that yield perceptually salient substructures.

Bootstrap Model

The BM is clearly different from these other two approaches. It is a model of the way 2-D dot patterns are processed: Initially, dots are grouped quasi-randomly (with a preference for short distances); as soon as a few regular substructures such as trapezoids arise, a local reference frame is established that causes grouping to propagate more easily throughout the pattern (i.e., bootstrapping). Mirror symmetry is more salient than twofold repetition because trapezoids allow more bootstrapping than parallelograms and because short pairwise distances (which are critical in the initial stages) are more likely near the axis. This processing account uses well-known facts about the visual system, such as acuity and orientation effects, to explain empirical findings about regularity detection that remain outside the scope of the HA (e.g., importance of zone around the axis, importance of central presentation, and salience of vertical and horizontal orientations). Although the currently implemented version of the BM does not have a mechanism to extract salient substructures or to operate at different spatial scales, there is nothing in the model that prevents such modules to be plugged in (into BM′). Little attention is paid in this theory to the representations resulting from this type of processing.

Conclusion

The summary may give the impression that the BM is superior to the HA. Nevertheless, the HA has very interesting implications for certain types of regularities that have been almost completely neglected (e.g., broken symmetry) or for certain effects that should be studied more systematically (e.g., the location of noise). This suggests that it would be wise to continue working within these two traditions (process- and representation-based) and to systematically compare their empirical success.

In addition, it seems desirable that the complementarity of the two approaches be studied more thoroughly. On the one hand, one could say that the BM, with its focus on psychophysically and neurally plausible process mechanisms (e.g., grouping by proximity and orientation effects), is filling in the black box left by the HA, whereas the HA is very explicit about the representations of visual regularities that may result from this processing. On the other hand, many questions remain as to the details of how the two approaches should be integrated: Is it essential for the HA that 2-D patterns are encoded as 1-D symbol sequences? Does the HA work only because perceptually salient substructures are excluded, or would it be possible to add this important factor in determining goodness? How much of the goodness difference between twofold repetition and mirror symmetry, as explained by the HA in terms of block versus point structure, would remain when the door is set open to substructures other than complete pattern halves or single elements? Clear answers to these questions would help to integrate the *B*M and the *HA* into a better approach (*BA*) to goodness.

References

Attneave, F. (1954). Some informational aspects of visual perception. *Psychological Review, 61,* 183–193.

Barlow, H. B., & Reeves, B. C. (1979). The versatility and absolute efficiency of detecting mirror symmetry in random dot displays. *Vision Research, 19,* 783–793.

Baylis, G. C., & Driver, J. (1994). Parallel computation of symmetry but not repetition within single visual shapes. *Visual Cognition, 1,* 377–400.

Bruce, D. (1994). Lashley and the problem of serial order. *American Psychologist, 49,* 93–103.

Bruce, V. G., & Morgan, M. J. (1975). Violations of symmetry and repetition in visual patterns. *Perception, 4,* 239–249.

Chater, N. (1996). Reconciling simplicity and likelihood principles in perceptual organization. *Psychological Review, 103,* 566–581.

Compton, B. J., & Logan, G. D. (1993). Evaluating a computational model of perceptual grouping by proximity. *Perception & Psychophysics, 53,* 403–421.

Cosmides, L., & Tooby, J. (1995). From function to structure: The role of evolutionary biology and computational theories in cognitive neuroscience. In M. S. Gazzaniga (Ed.), *The cognitive neurosciences* (pp. 1199–1210). Cambridge, MA: MIT Press and Bradford Books.

Dakin, S. C. (1997). The detection of structure in Glass patterns: Psychophysics and computational models. *Vision Research, 37,* 2227–2246.

Dakin, S. C., & Hess, R. F. (1997). The spatial mechanisms mediating symmetry perception. *Vision Research, 37,* 2915–2930.

Dakin, S. C., & Watt, R. J. (1994). Detection of bilateral symmetry using spatial filters. *Spatial Vision, 8,* 393–413.

Feldman, J. (1997). Curvilinearity, covariance, and regularity in perceptual groups. *Vision Research, 37,* 2835–2848.

Gilbert, C. D. (1995). Dynamic properties of adult visual cortex. In M. S. Gazzaniga (Ed.), *The cognitive neurosciences* (pp. 73–90). Cambridge, MA: MIT Press and Bradford Books.

Herbert, A. M., & Humphrey, G. K. (1996). Bilateral symmetry detection: Testing a "callosal" hypothesis. *Perception, 25,* 463–480.

Hochberg, J. E., & McAlister, E. (1953). A quantitative approach to figural "goodness." *Journal of Experimental Psychology, 46,* 361–364.

Jenkins, B. (1983). Component processes in the perception of bilaterally symmetric dot textures. *Perception & Psychophysics, 34,* 433–440.

Julesz, B. (1971). *Foundations of cyclopean perception.* Chicago, IL: University of Chicago Press.

Kaas, J. H. (1989). Why does the brain have so many visual areas? *Journal of Cognitive Neuroscience, 1,* 121–135.

Koenderink, J. J. (1984). The concept of local sign. In A. J. Van Doorn, W. A. Van de Grind, & J. J. Koenderink (Eds.), *Limits in perception: Essays in honour of Maarten A. Bouman* (pp. 495–547). Utrecht, the Netherlands: VNU Science Press.

Kovács, I. (1996). Gestalten of today: Early processing of visual contours and surfaces. *Behavioural Brain Research, 82,* 1–11.

Kubovy, M., Holcombe, A. O., & Wagemans, J. (1998). On the lawfulness of grouping by proximity. *Cognitive Psychology, 35,* 71–98.

Kubovy, M., & Wagemans, J. (1995). Grouping by proximity and multistability in dot lattices: A quantitative Gestalt theory. *Psychological Science, 6,* 225–234.

Labonté, F., Shapira, Y., Cohen, P., & Faubert, J. (1995). A model for global symmetry detection in dense images. *Spatial Vision, 9,* 33–55.

Lappin, J. S., Norman, J. F., & Mowafy, L. (1991). The detectability of geometric structure in rapidly changing optical patterns. *Perception, 20,* 513–528.

Lashley, K. S. (1951). The problems of serial order in behavior. In L. A. Jeffres (Ed.), *Cerebral mechanisms in behavior* (pp. 112–136). New York: Wiley.

Leeuwenberg, E. L. J. (1969). Quantitative specification of information in sequential patterns. *Psychological Review, 76,* 216–220.

Leeuwenberg, E. L. J. (1971). A perceptual coding language for visual and auditory patterns. *American Journal of Psychology, 84,* 307–349.

Lewandowsky, S., & Murdock, B. B. (1989). Memory for serial order. *Psychological Review, 96,* 25–57.

Locher, P., & Wagemans, J. (1993). The effects of element type and spatial grouping on symmetry detection. *Perception, 22,* 565–587.

Lotze, H. (1884). *Mikrokosmos.* Leipzig, Germany: Hirzel.

Marr, D. (1982). *Vision: A computational investigation into the human representation and processing of visual information.* San Francisco, CA: Freeman.

McKay, D. (1969). *Information, mechanism, and meaning.* Boston, MA: MIT Press.

Palmer, S. E. (1982). Symmetry, transformation, and the structure of perceptual systems. In J. Beck (Ed.), *Organization and representation in perception* (pp. 95–144). Hillsdale, NJ: Erlbaum.

Palmer, S. E. (1983). The psychology of perceptual organization: A transformational approach. In J. Beck, B. Hope, & A. Rosenfeld (Eds.), *Human and machine vision* (pp. 269–339). New York: Academic Press.

Palmer, S. E. (1985). The role of symmetry in shape perception. *Acta Psychologica, 59,* 67–90.

Palmer, S. E. (1991). Goodness, Gestalt, groups, and Garner: Local symmetry subgroups as a theory of figural goodness. In G. R. Lockhead & J. R. Pomerantz (Eds.), *The perception of structure* (pp. 23–39). Washington, DC: American Psychological Association.

Palmer, S. E., & Hemenway, K. (1978). Orientation and symmetry: Effects of multiple, rotational, and near symmetries. *Journal of Experimental Psychology: Human Perception and Performance, 4,* 691–702.

Pomerantz, J. R., & Kubovy, M. (1986). Theoretical approaches to perceptual organization: Simplicity and likelihood principles. In K. R. Boff, L. Kauffman, & J. P. Thomas (Eds.), *Handbook of perception and human performance: Vol. II. Cognitive processes and performance* (pp. 36–45). New York: Wiley.

Ramachandran, V. S. (1985). The neurobiology of perception [Editorial, Special Issue on Motion Perception]. *Perception, 14,* 97–103.

Schwartz, E. L. (1980). Computational anatomy and functional architecture of striate cortex: A spatial mapping approach to perceptual coding. *Vision Research, 20,* 645–669.

Singer, W. (1995). Time as coding space in neocortical processing: A hypothesis. In M. S. Gazzaniga (Ed.), *The cognitive neurosciences* (pp. 91–104). Cambridge, MA: MIT Press and Bradford Books.

Smits, J. T. S., Vos, P. G., & Van Oeffelen, M. P. (1985). The perception of a dotted line in noise: A model of good continuation and some experimental results. *Spatial Vision, 1,* 163–177.

Stevens, K. A. (1980). *Surface perception from local analysis of texture and contour* (Tech. Rep. No. 512). Cambridge, MA: MIT AI Lab.

Tapiovaara, M. (1990). Ideal observer and absolute efficiency of detecting mirror symmetry in random images. *Journal of the Optical Society of America, A7,* 2245–2253.

Tooby, J., & Cosmides, L. (1995). Mapping the evolved functional organization of mind and brain. In M. S. Gazzaniga (Ed.), *The cognitive neurosciences* (pp. 1185–1197). Cambridge, MA: MIT Press/Bradford Books.

Tyler, C. W. (1996). Human symmetry perception. In C. W. Tyler (Ed.), *Human symmetry perception and its computational analysis* (pp. 3–22). Utrecht, the Netherlands: VSP.

Van der Helm, P. (1988). *Accessibility and simplicity of visual structures.* Unpublished doctoral thesis. Nijmegen, the Netherlands: Nijmegen Institute for Cognition and Information, University of Nijmegen.

Van der Helm, P., & Leeuwenberg, E. (1991). Accessibility: A criterion for regularity and hierarchy in visual pattern codes. *Journal of Mathematical Psychology, 35,* 151–213.

Van der Helm, P. A., & Leeuwenberg, E. L. J. (1996). Goodness of visual regularities: A nontransformational approach. *Psychological Review, 103,* 429–456.

Van der Helm, P. A., Van Lier, R. J., & Leeuwenberg, E. L. J. (1992). Serial pattern complexity: Irregularity and hierarchy. *Perception, 21,* 517–544.

Van Lier, R., & Wagemans, J. (in press). From images to objects: Global and local completions of self-occluded parts. *Journal of Experimental Psychology: Human Perception and Performance.*

Van Oeffelen, M. P., & Vos, P. G. (1983). An algorithm for pattern description on the level of relative proximity. *Pattern Recognition, 16,* 341–348.

Wagemans, J. (1993). Skewed symmetry: A nonaccidental property used to perceive visual forms. *Journal of Experimental Psychology: Human Perception and Performance, 19,* 364–380.

Wagemans, J. (1995). Detection of visual symmetries. *Spatial Vision, 9,* 9–32.

Wagemans, J. (1997). Characteristics and models of human symmetry detection. *Trends in Cognitive Sciences, 1,* 346–352.

Wagemans, J., Van Gool, L., & d'Ydewalle, G. (1991). Detection of symmetry in tachistoscopically presented dot patterns: Effects of multiple axes and skewing. *Perception & Psychophysics, 50,* 413–427.

Wagemans, J., Van Gool, L., & d'Ydewalle, G. (1992). Orientational effects and component processes in symmetry detection. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology, 44*(A), 475–508.

Wagemans, J., Van Gool, L., Swinnen, V., & Van Horebeek, J. (1993). Higher-order structure in regularity detection. *Vision Research, 33,* 1067–1088.

Wenderoth, P. (1994). The salience of vertical symmetry. *Perception, 23,* 221–236.

Wenderoth, P. (1995). The role of pattern outline in bilateral symmetry detection with briefly flashed dot patterns. *Spatial Vision, 9,* 57–77.

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