THE NATURAL NUMBER BIAS AND ITS ROLE IN RATIONAL NUMBER UNDERSTANDING IN CHILDREN WITH DYSCALCULIA: DELAY OR DEFICIT?

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There exists already a large body of literature both on learners' struggle with understanding the rational number system and on the role of the natural number bias in this struggle. However, little is known about rational number understanding of learners with dyscalculia. In this study, we investigated the rational number understanding of learners with dyscalculia, with a specific focus on the role of the natural number bias in this understanding. The results suggest that in addition to a delay in their general mathematics achievement, learners with dyscalculia have an extra delay, but no deficit in their rational number understanding, compared to their peers.

INTRODUCTION

A good understanding of the rational number domain is of essential importance for learners' mathematics achievement (Siegler, Thompson, & Schneider, 2011). At the same time, rational numbers are known to form a stumbling block for many learners (Mazzocco & Devlin, 2008; Vamvakoussi, Van Dooren, & Verschaffel, 2012; Van Dooren, Van Hoof, Lijnen, & Verschaffel, 2012). Previous research indicates that a large part of learners' difficulty with rational numbers can be explained by the natural number bias, which is defined as the tendency to apply natural number properties in tasks with rational numbers, even when this is inappropriate (Ni & Zhou, 2005). Learners are found to make systematic and predictable errors in rational numbers tasks where the use of prior natural number knowledge leads to the incorrect answer (incongruent tasks), while they are much more accurate in rational number tasks where reliance on prior natural number knowledge leads to the correct answer (congruent tasks) (Vamvakoussi et al., 2012).

Previous research on the natural number bias mainly focused on three aspects in which rational numbers differ from natural numbers and where errors are known to occur: their dense structure, the way their numerical size can be determined, and the effect of the four basic operations (Vamvakoussi et al., 2012; Van Hoof, Vandewalle, & Van Dooren, 2013).

While there exists a large body of literature both on learners' struggle with

understanding the rational number domain and on the role of the natural number bias in this struggle, little is known about the rational number understanding of learners with dyscalculia (further abbreviated with LWD). Nonetheless, a better insight of the understanding of rational numbers of LWD is important to provide adaptive (remedial) instruction with the aim to increase LWD's understanding of the rational number system. One of the few studies that did investigate LWD's rational number understanding is the study of Mazzocco and Devlin (2008). They found that learners with mathematical learning difficulties compared to learners without mathematical learning difficulties struggle more to accurately order rational numbers, even more than learners with a low general mathematics achievement.

THE PRESENT STUDY

The goal of the present study is to extend the study of Mazzocco and Devlin (2008) in several ways. First, in the study of Mazzocco and Devlin (2008) learners were solely matched on their 'chronological age'. In this study we will also match learners on their mathematical ability level. This allows to investigate whether there is a 'deficit' or a 'delay' in LWD's rational number understanding (Torbeyns, Verschaffel, & Ghesquiére, 2004). If LWD's rational number understanding is significantly lower than that of learners of the same age (= chronological age match), but not significantly different from learners without dyscalculia but with the same mathematics achievement level (= ability match), who are typically younger, this implies that LWD's rational number understanding reflects their mathematics achievement level and thus that the development of LWD's rational number understanding is only characterized by a *delay* rather than *deficit*. However, if LWD's rational number understanding is not only significantly lower than that of learners of the same age but also significantly lower than (younger) learners with the same mathematics achievement level, the development of LWD's rational number understanding does not represent their mathematics achievement level, thus their rational number understanding is characterized by a *deficit* (Torbeyns et al., 2004). This leads to the first research question (Research Question 1), namely whether LWD's rational number understanding is characterized by a 'delay' or a 'deficit' compared to learners without dyscalculia. Second, while Mazzocco and Devlin (2008) measured learners' rational number understanding in a rather general sense, we will pay particular attention to the differences between congruent and incongruent rational number tasks to map the natural number bias in the three groups of learners. This way, we aimed at answering our second research question (Research Question 2): Is the strength of the natural number bias in LWD comparable with that of normally developing children? Based on the available research literature, no specific prediction could be made for both research questions.

METHOD

Participants

Three different groups of learners were included. A first group consisted of sixth graders with an official clinical diagnosis of dyscalculia (n = 16). Next to these LWD, we included two control groups: a chronological age match and an ability match group. The first control group were sixth graders without dyscalculia (n = 56), further referred to as the sixth grade control group. The mean age (in months) of the LWD was 143.97 (SD = 10.39), while the mean age of the sixth grade control group was 142.13 (SD = 3.53). An independent samples t-test showed that this difference in age was not significant, $t(70) = 1.13$, $p = .26$. The ability match control group were fourth graders (n = 51), further referred to as the fourth grade control group. We chose to include this age group because we needed younger learners with a mathematics achievement level that we could expect to be comparable to that of sixth graders with dyscalculia, but who would also be able to solve the rational number test. The mean mathematics achievement level of the LWD was 92.81 (SD = 40.13), while the mean mathematics achievement level of the fourth grade control group turned out to be much higher, i.e., 134.75 (SD = 29.56). An independent samples t-test showed that this difference was significant; $t(65) = -4.53$, $p < .001$. This result shows that we did not succeed in creating an ideal ability-matched group. However, we could not include an even younger group as an ability match control group, as younger students would hardly have any relevant rational number knowledge. In order to address this, we opted to take into account learners' mathematical ability as a control variable and correct for remaining differences between groups. This way, we were able to investigate whether there was still a difference between both groups' rational number understanding that could not be explained by a difference in mathematics achievement level, but by having dyscalculia.

Instruments

Rational number understanding

Learners completed a shortened version of the "Rational Number Sense Test" (RNST; Van Hoof, Verschaffel, & Van Dooren, 2015) as a measure of their rational number understanding. The shortened test consisted of 49 items. Both congruent (n = 16) and incongruent items (n = 33) from the three aspects of the natural number bias (density, size, and operations) were included. Examples can be found in Figure 1. As a measure of the strength of the natural number bias, we used learners' accuracy levels on the incongruent rational number tasks.

	Congruent	**Incongruent**
Density	Write a number between 1/4 and 3/4	Write a number between 3.49 and 3.50
Size	Choose the largest number: 4.4 or 4.50	Choose the largest number: 3/2 or 9/8
Operations	Is the outcome of 50 * 3/2 smaller or larger than 50?	Is the outcome of 40 * 0.99 smaller or larger than 40?

Figure 1: Examples of congruent and incongruent items.

Mathematics achievement

Learners' mathematics achievement was measured by means of the Tempo Test Automatiseren (De Vos, 2010). This test measures the automated knowledge of the four basic operations.

Intelligence.

Two measures of intelligence were used. First, Raven's Progressive Matrices test (Raven, Court, & Raven, 1995) was used as a measure of learners' non-verbal intelligence. Second, the SiBO test measured learners' verbal intelligence (Hendrikx, Maes, Magez, Ghesquière, & Van Damme, 2007). Because high correlations were found between both IQ measures (Raven and SiBo) (r = .41, p < .001), we created one general intelligence score by first calculating z-scores for each measure separately and then taking the mean of these two scores for each learner. We standardised the intelligence score for the sixth graders and fourth graders separately (leading to a mean score of 0 in both groups), and then calculated the z-scores of the LWD group using the sixth graders as a reference group.

Reading achievement.

Because comorbidity was allowed, two measures of reading achievement were included as control variables to ensure that the results were not due to lower reading achievement. The één-minuut test (one minute test, further abbreviated with EMT) was used as a measure of learners' word recognition ability. The goal of the test is that learners correctly read out loud as many words as possible within one minute. Standardized scores were used based on existing norm tables (Brus & Voeten, 1972). The Klepel was used as a second measure of learners' reading ability. Contrary to the EMT, the words in this test are pseudowords. Standardized scores were used based on existing norm tables (van den Bos, Spelberg, Scheepstra, & de Vries, 1994). Also for the two reading achievement measures, high correlations were found (r = .92,

$p < .001$). Therefore, we also calculated each learner's mean standardized score on the two measures as a general score of reading achievement.

RESULTS

Table 1 presents the descriptive statistics for the control variables. The results for the dependent variable (performance on congruent and incongruent rational number items) are shown in Figure 2.

		LWD	4th graders	6th graders
Age (months)	Mean	143.97	119.34	142.13
	SD	10.39	3.54	3.53
IQ (Raven + SiBo)	Mean	-0.99	0	0
	SD	1.12	0.79	0.83
Reading achievement (EMT + KLEPEL)	Mean	7.13	9.99	10.82
	SD	3.78	2.49	2.48
Mathematics achievement	Mean	92.81	134.75	171.27
	SD	40.13	29.56	22.30

Table 1: Descriptive statistics

Comparison between LWD and sixth grade control group

As can be seen in Figure 2, LWD's accuracy on congruent items was significantly lower than the accuracy of the sixth grade control group. Because our aim was to have a chronological age match design, in a next step we added learners' age (in months) as control variable in the comparison between both groups' accuracy on congruent rational number tasks. Moreover, because both groups differed in their general IQ and reading achievement, we also included these two as control variables. An ANCOVA indicated that, even after controlling for learners' age, IQ, and reading achievement, the sixth grade control group still significantly outperformed the group of LWD on congruent rational number tasks, $F(1,67) = 4.35$, $p = .04$, $\eta^2 = .06$, but the effect size was only small.

LWD's accuracy on incongruent items was also significantly lower than the accuracy of the sixth grade control group, see Figure 2. In a next step we again additionally controlled for learners' age, IQ, and reading achievement. An ANCOVA indicated that, even after controlling for these variables, the sixth grade control group still significantly outperformed LWD on incongruent rational number tasks,

$F(1,67) = 102.16$, $p < .001$, $\eta^2 = .60$; the effect size was large.
The partial eta squared values reveal that the difference in accuracy between the group of LWD and the sixth grade control group is much higher in incongruent than in congruent rational number items.

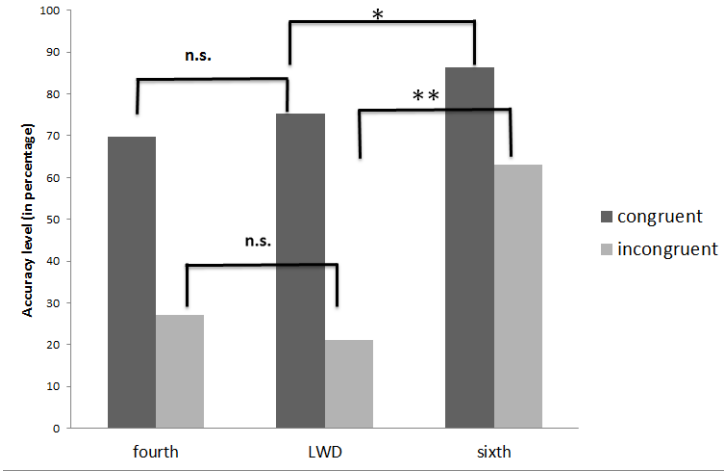


Figure 2: Learners' accuracy on congruent and incongruent rational number tasks per group
Note. * = p < .01, ** = p < .001

Comparison between LWD and fourth grade control group

As can be seen in Figure 2, LWD's accuracy on congruent items was not significantly different from the accuracy of the fourth grade control group. As stated above, we did not succeed in realizing an optimal ability match with fourth graders. Therefore, in a next step we added learners' mathematics achievement as a control variable in the comparison between both groups' accuracy on congruent rational number tasks. Moreover, because the LWD and the fourth grade control group also differed in their general IQ and reading achievement, we also included these two measures as control variables. An ANCOVA indicated that, after controlling for learners' mathematics achievement, IQ, and reading achievement, no significant difference was found between both groups' accuracy on congruent rational number tasks, $F(1,62) = 3.69$, $p = .08$, $\eta^2 = .03$.
LWD's accuracy on incongruent items was lower than the accuracy of the fourth grade control group, see Figure 2. This difference between the two groups was however not significant $F(1,65) = 2.06$, $p = .16$, $\eta^2 = .03$. For the same reasons as above, in a next step we additionally controlled for learners' mathematics achievement, IQ, and reading achievement. An ANCOVA indicated that, after controlling for these three measures, no significant difference was found between both groups' accuracy on incongruent rational number tasks, $F(1,62) = 1.75$, $p = .19$, $\eta^2 < .01$.

DISCUSSION

Concerning the first research question, results showed that LWD's rational number understanding is significantly lower than that of regular learners, but not significantly different from younger learners, even after statistically controlling for mathematics achievement, both in congruent and incongruent rational number tasks. These results suggest that the development of LWD's rational number understanding is characterized by a *delay* rather than a *deficit*. Concerning the second research question, there was a big difference in accuracy level between LWD and their peers on incongruent rational number tasks, while no significant difference could be found with fourth graders' accuracy on the same incongruent rational number tasks. The difference in accuracy on incongruent rational number tasks revealed that the strength of the natural number bias is higher in LWD compared to normally developing learners of the same age, but is not different from younger learners. This finding confirms that LWD's rational number understanding is characterized by a delay rather than a deficit. We further found that LWD's rational number understanding is not significantly different from younger learners with a higher mathematics achievement level. This suggests that the role of dyscalculia is less strong for learners' accuracy levels on rational number tasks compared to learners' accuracy level on a mathematics achievement test measuring learners' automated knowledge of the four basic operations. Moreover, the finding that LWD's rational number understanding is not significantly different from younger learners with a higher mathematics achievement level, gives us reason to hypothesize that LWD have a lead in their rational number understanding compared to even younger learners with the same mathematics achievement. Our findings have implications for mathematics education. Although our results pointed out that LWD struggle even more with incongruent rational number tasks than their peers, they also indicated that this struggle is not characterized by a deficit but with a more general delay. This implies that it is possible for LWD to develop more insight in the rational number system and, therefore, more (remedial) instructional attention should aim at the enhancement of LWD's understanding of the rational number system. As stated above, LWD are more affected by the natural number bias compared to learners of the same age. Therefore, more instructional attention should go to the differences between the natural number system and the rational number system. While this should be implemented in all classrooms, it deserves especially attention when teaching LWD.

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