Stability analysis for discrete-time LPV systems

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1 Introduction

This work presents stability conditions for discrete-time LPV systems with a homogeneous polynomial dependency on the scheduling parameters which vary inside a multi-simplex. This parameterization of the parameter-dependency has two benefits: (i) it encompasses polytopic, affine, multi-affine, polynomial and multivariable polynomial models as special cases and (ii) polynomially parameter-dependent Lyapunov functions can be constructed to simultaneously handle time-invariant, bounded and arbitrarily fast time-varying parameters in an appropriate way. The geometric properties of the uncertainty domain are exploited to derive a finite set of sufficient linear matrix inequalities.

2 Modeling of the uncertainty domain

Consider the discrete-time LPV system

$$x(k+1) = A(\alpha(k))x(k) = \sum_{\ell \in \mathscr{K}_r(N)} \alpha(k)^{\ell} A_{\ell}\, x(k), \tag{1}$$

where the vector of time-varying parameters $\alpha(k)$ belongs, $\forall k \geq 0$, to the multi-simplex Λ which is defined as the Cartesian product of L unit-simplexes

$$\Lambda_R = \left\{ \xi \in \mathbb{R}^R : \sum_{i=1}^{R} \xi_i = 1, \xi_i \geq 0, i = 1,\dots,R \right\},$$

that is, $\Lambda = \Lambda_{N_1} \times \dots \times \Lambda_{N_L}$ with $N \in \mathbb{N}^L$. Conform the structure of the multi-simplex, α has the structure $\alpha = (\alpha_1,\dots,\alpha_L) = ((\alpha_{1,1},\dots,\alpha_{1,N_1}),\dots,(\alpha_{L,1},\dots,\alpha_{L,N_L}))$ and α^{ℓ} is a shorthand notation for the monomial $\alpha^{\ell} = \alpha_1^{\ell_1}\alpha_2^{\ell_2}\dots\alpha_L^{\ell_L}$, with $\alpha_j^{\ell_j} = \alpha_{j,1}^{\ell_{j,1}}\alpha_{j,2}^{\ell_{j,2}}\dots\alpha_{j,N_j}^{\ell_{j,N_j}}$, for $j = 1,\dots,L$. For all $r \in \mathbb{N}^L$ and $N \in \mathbb{N}^L$, the set $\mathscr{K}_r(N)$ represents the set of all nonnegative integer N-tuples ℓ with the structure $\ell = (\ell_1,\dots,\ell_L) = ((\ell_{1,1},\dots,\ell_{1,N_1}),\dots,(\ell_{L,1},\dots,\ell_{L,N_L}))$ such that $\ell_{j,1} + \dots + \ell_{j,N_j} = r_j$, for $j = 1,\dots,L$. The rate of variation of the parameters $\Delta\alpha_{j,i}(k) = \alpha_{j,i}(k+1) - \alpha_{j,i}(k)$, for $j = 1,\dots,L$ and $i = 1,\dots,N_j$, is assumed to be limited by an *a priori* known bound b_j such that $-b_j \leq \Delta\alpha_{j,i}(k) \leq b_j$, for $i = 1,\dots,N_j$, with $b_j \in \mathbb{R}$, $b_j \in [0,1]$. If $b_j = 0$, α_j is time-invariant; if $b_j = 1$, α_j can vary arbitrary fast in Λ_{N_j} and if $0 < b_j < 1$, the rate of variation of α_j is bounded. For each parameter $\alpha_{j,i}$, the grey region in Figure 1 shows all feasible points in the $(\alpha_{j,i}, \Delta\alpha_{j,i})$-space. Clearly, this is a polytope with 6 vertices. Since $\alpha_j \in \Lambda_{N_j}$ for $j = 1,\dots,L$, there are $\bar{\mathbf{N}} = \prod_{j=1}^{L} N_j$ polytopes associated with each $(\alpha_{j,i}, \Delta\alpha_{j,i})$-space. Consequently, the complete uncertainty polytopic set where $(\alpha, \Delta\alpha)$ can assume values is found as the intersection of these $\bar{\mathbf{N}}$ polytopes. Taking the convex combination of the M vertices of this polytope using $\gamma \in \Lambda_M$, yields the linear relations

$$\alpha = \Gamma_1\gamma, \qquad \Delta\alpha = \Gamma_2\gamma, \tag{2}$$

where $\Gamma_1 \in \mathbb{R}^{N\times M}$ and $\Gamma_2 \in \mathbb{R}^{N\times M}$ are constructed from the vertices of the uncertainty polytope.

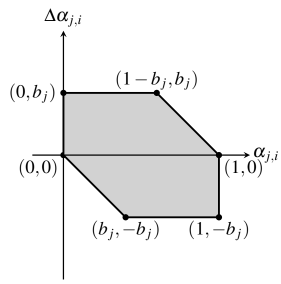

Figure 1: Modeling of the feasible set in the $(\alpha_{j,i}, \Delta\alpha_{j,i})$-space.

3 Stability condition

It is well-known that the following LMI provides a sufficient condition for stability of system (1)

$$\begin{bmatrix} P(\alpha(k+1)) & A(\alpha(k))P(\alpha(k)) \\ P(\alpha(k))A(\alpha(k))^T & P(\alpha(k)) \end{bmatrix} > 0,\ \forall k \geq 0.$$

Since $\alpha(k+1) = \alpha(k) + \Delta\alpha(k)$ and using (2), this LMI can be rewritten as

$$\begin{bmatrix} P((\Gamma_1+\Gamma_2)\gamma(k)) & A(\Gamma_1\gamma(k))P(\Gamma_1\gamma(k)) \\ P(\Gamma_1\gamma(k))A(\Gamma_1\gamma(k))^T & P(\Gamma_1\gamma(k)) \end{bmatrix} > 0,\ \forall k \geq 0. \tag{3}$$

The advantage of this change of variables is that the LMI (3) only involves the time instant k. Therefore, it now suffices that

$$\begin{bmatrix} P((\Gamma_1+\Gamma_2)\gamma) & A(\Gamma_1\gamma)P(\Gamma_1\gamma) \\ P(\Gamma_1\gamma)A(\Gamma_1\gamma)^T & P(\Gamma_1\gamma) \end{bmatrix} > 0,\ \forall \gamma \in \Lambda_M, \tag{4}$$

Since (4) is a parameter-dependent LMI, a finite set of sufficient LMI conditions can be derived using standard techniques [1] by assuming a homogeneous polynomially parameter-dependent structure for the Lyapunov matrix.

References

[1] R. C. L. F. Oliveira, P. L. D. Peres, “Parameter-Dependent LMIs in Robust Analysis: Characterization of Homogeneous Polynomially Parameter-Dependent Solutions Via LMI Relaxations,” IEEE Trans. Automat. Contr., vol. 52, no. 7, pp. 1334–1340, 2007.