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THE FIRST RESOURCE FOR MATHEMATICS

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Matrix, numerical, and optimization methods in science and engineering. (English)

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After his previous book [Variational methods with applications in science and engineering. Cambridge: Cambridge University Press (2013; Zbl 1284.49003)] the author now gives his roundup of numerical matrix methods applied to dynamical systems and optimization in science and engineering.

There are three parts:

Part I: Matrix methods, (pp. 1–311) with 5 chapters: Vector and matrix algebra, Algebraic eigenproblems and their applications, differential eigenproblems and their applications, vector and matrix calculus, Analysis of discrete systems

Part II: Numerical methods (pp. 313–526) with 4 chapters: Computational linear algebra, Numerical methods for differential equations, Finite-difference methods for boundary-value problems, Finite difference methods for initial-value problems

Part III: Least squares and optimization (pp. 527-698) with 4 chapters: Least squares methods, Data analysis curve fitting and interpolation, Optimization and root finding of algebraic systems, data driven methods and reduced-order modeling.

All chapters are followed by a set of exercises (no solutions provided).

There are about 80 references (mostly books), and a subject index taking 8 pages.

The style used is similar to the style of his previous book, which means that he brings theory or methods by first motivating the reader with some examples, which illustrate the need for tackling the subject, some further experiments should then trigger initial ideas, which are then gradually extended to the more general concept or method. This explains why the book is so thick. A more top-down systematic approach could reduce the number of pages drastically. The separation into the three parts is in my opinion artificial and somewhat arbitrary. It makes the book less suitable for use as lecture notes that are usually more structured. Even though the idea is to introduce theory in Part I, then methods in Part II, and then applications in Part III, what is presented in these parts can be better summarized as (I) a minimum of theory on linear algebra and dynamical systems, (II) numerical methods to solve different types of differential and partial differential equations, (III) optimization and approximation.

I think the book is most useful for practitioners who want to solve a concrete problem. They are not interested in theory or proofs, but just want their problem solved. Formal theorems and proofs, with an indication of start and end are absent, vector spaces are finite dimensional, and function spaces are introduced by ‘analogy’, the word limit is only used in connection with limit cycle and the index mentions the word convergence only on page 381, infinite sums and integrals are assumed to exist, etc. Many properties that would require a proof are just mentioned in an itemized list of remarks.

On the other hand, Part II and III have tables indicating which methods to use for which problem and where they can be found in the book. Sometimes references to the literature are given for more details.

Part I: The first 4 chapters discuss vectors and matrix algebra, but the abstract vector space is never introduced, vectors are always a linear combination of basis vectors. Some elements of differential equations is pre-assumed so that solutions can also be considered as a linear combination of basis-solutions. Elementary algorithms for the solution of linear systems, Gram-Schmith orthogonalization, eigenvalues and singular values, and the corresponding matrix factorizations, and 3D vector calculus are introduced. Most examples are from electrical or mechanical engineering. The last chapter is a discussion of linear and nonlinear dynamical systems and their stability.

Part II: After introducing the numerical aspects of direct and iterative methods for linear systems and eigenproblems, the discussion embarks on finite difference methods to solve boundary and initial-value problems for different types of differential equations. Most standard methods are included, but not all

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are explained in detail. For example Runge-Kutta methods are mentioned but not elaborted.

Part III starts with linear and nonlinear least squares problems with and without constraints to be solved with generalized inverses and standard iterative methods (conjugate gradients, GMRES, Krylov). Data fitting includes interpolation and approximation by polynomials, Fourier methods, and splines in one and mode dimensions. Root finding can be considered as a basis for optimization problems by finding zeros for the derivative. Linear programming problems are treated separately and optimal control is briefly mentioned. The last chapter explains how to reduce a high order (or continuous) system to a low order discrete system (Rayleigh-Ritz, Galerkin, spectral methods, etc) leading to reduced order modelling or even system identification.

There are always minor flaws in a first edition of this many pages. Some examples: The inner product is denoted as $\langle \mathbf{u}, \mathbf{v} \rangle$, but for no obvious reason $\langle \mathbf{u}, \mathbf{u} \rangle$ is denoted as $\mathbf{u}^2$ (p. 42) which is only confusing since also the correct $\|\mathbf{u}\|^2$ is used. The loose, sometimes intuitive, approach leaves space for inaccuracies, which might upset a mathematician but probably not a practitioning scientist or engineer. The adjoint of an operator $\mathcal{L}$ is defined by $\langle \mathbf{v}, \mathcal{L}\mathbf{u} \rangle = \langle \mathbf{u}, \mathcal{L}^*\mathbf{v} \rangle$ (p. 179) while this is only true in the real case. Digital computers were developed in the middle of the twentieth century and not in the nineteenth century (p. 325), etc.

Reviewer: Adhemar Bultheel (Leuven)

MSC:

00A06 Mathematics for nonmathematicians (engineering, social sciences, etc.)
15-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to linear algebra
65-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to numerical analysis
65Fxx Numerical linear algebra
90-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to operations research and mathematical programming
90Cxx Mathematical programming

Keywords:

linear algebra; dynamical systems; differential equations; partial differential equations; least squares method; optimization; numerical methods for science and engineering

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