Analyzing and stimulating strategy competence in elementary arithmetic: The case of subtraction by addition

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During the past two decades we investigated elementary school children's use of a highly valued strategy for doing symbolic multi-digit subtraction among mathematics educators namely the subtraction-by-addition strategy, In the present paper, we give a brief and narrative overview of the empirical studies that we have been doing on children's strategy competences with respect to subtraction-by-addition, using a variety of research methods and techniques. For this overview we use the research methodology being applied as the main structuring principle. First, we will review our initial studies wherein we used verbal reports to reveal children's solution strategies. Second, we report studies wherein we relied on the systematic analysis of children's reaction time data while solving subtractions. Finally, studies using the choice/no-choice paradigm are reported. Taken as a whole, while there is only very little evidence for subtraction-by-addition use when relying on verbal report studies, when turning to the other research methods, there is ample evidence that elementary school children do apply this valuable subtraction strategy frequently, efficiently, and flexibly. We end this paper with some theoretical, methodological, and educational implications of the research being reviewed.

Keywords: Elementary arithmetic, Multi-digit subtraction, Adaptive expertise, Verbal reports, Reaction times, Choice/no-choice Method

1 Introduction

Current reform movements in elementary mathematics education stress that instruction should no longer focus on solving school mathematics exercises quickly and accurately by means of the school-taught standard strategies (i.e., routine expertise). In contrast, mathematical tasks should be solved efficiently, flexibly, and creatively with a variety of meaningfully acquired strategies (i.e., adaptive expertise). Consequently, more attention should be paid to the insightful, varied, and flexible use of strategies when teaching children how to solve mathematical problems (Baroody & Dowker, 2003; Kilpatrick, Swafford, & Findell, 2001; Verschaffel, Greer, & De Corte, 2007; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009).

One subdomain of the elementary mathematics curriculum that has attracted a lot of attention of researchers interested in the development of adaptive expertise in children, is the solution of symbolically presented multi-digit subtractions. Several studies have shown that

children develop various strategies to mentally solve multi-digit subtractions (e.g., Beishuizen, 1993; Blöte, Klein, & Beishuizen, 2000; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009a). One way to classify these mental subtraction strategies is by looking at the operation that underlies the effectuation of the strategy. This can be either subtraction or addition. (For other ways of classifying mental subtraction strategies, see for example Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2012). In this way, two types of strategies can be distinguished: (1) direct subtraction (DS) strategies, in which the subtrahend is directly taken away from the minuend (e.g., 75 – 43 = . by "75 – 40 = 35, 35 – 3 = 32"), and (2) subtraction-by-addition (SBA) strategies, in which one determines how much needs to be added to the subtrahend to get to the minuend (e.g., 75 – 43 = . by "43 + 30 = 73 and 73 + 2 = 75, so the answer is 30 + 2 = 32"). This SBA strategy is considered to be particularly efficient by mathematics educators on subtractions with a relatively large subtrahend (S) compared to the difference (D) (e.g., 81 – 79 = ., where S and D are 79 and 2, respectively), because in this case the solution process through a SBA strategy requires fewer and/or smaller calculation steps than a DS strategy (e.g., Torbeyns et al., 2009a). In contrast, DS is assumed to be more efficient than SBA for subtractions wherein the subtrahend is relatively small compared to the difference (e.g., 81 – 2, where S and D are 2 and 79, respectively).

In the present paper, we give a brief and narrative overview of the empirical studies that have been done at our research center over the past two decades on children's use of this SBA strategy, using a variety of research methods and techniques. For this overview we use the research methodology being applied as the main structuring principle. First, we will review our initial studies wherein we used verbal reports to reveal children's solution strategies. Second, we report studies wherein we relied on the systematic analysis of children's reaction time data while solving subtractions. Finally, studies using the choice/no-choice paradigm are reported. We end this paper with some theoretical, methodological, and educational implications of the research being reviewed.

2 Verbal report studies

We began our quest for the SBA strategy in children by means of studies that used verbal reports as their main data-gathering technique.

Flemish elementary school children are confronted, from second grade on, with symbolically presented multi-digit subtractions of the form a – b = ? that they are supposed to solve mentally by means of the DS (DS) strategy, with little or no instructional attention to the SBA strategy. Against this background, our major question was if we would find traces of the availability and use of the SBA strategy in Flemish elementary school children's verbal strategy reports.

To address this question, we analysed, in a first study (Torbeyns et al., 2009a), second- to fourth-graders' DS and SBA use on symbolic two-digit subtractions in the number domain up to 100. We included 71 second-, 71 third-, and 53 fourth-graders, who all had received traditional instruction in two-digit mental addition and subtraction as outlined above. We divided them into three mathematics achievement groups on the basis of a general mathematics achievement test. Like all other textbooks that were – and still are - in use in Flanders, the textbook used in these schools only paid explicit and systematic attention to the DS strategy, and an interview with the teachers of these children confirmed that they had not provided systematic instruction and practice in the SBA strategy.

All children in the study of Torbeyns et al. (2009a) individually completed two tasks designed to evaluate their strategy competencies in the domain of subtraction up to 100: a Spontaneous Strategy use Task (SST) and a Variability on Demand Task (VDT). The SST consisted of different types of two-digit subtractions, including several problems with a relatively large subtrahend (e.g., 41 – 39 = ?), which are assumed to maximally trigger the SBA strategy. Children were instructed to mentally solve each item as accurately and as fast as possible with their preferential strategy. They were asked to verbally report both the answer and the strategy used immediately after solving each item. The VDT consisted of the same types of two-digit subtractions as the SST. Children were now instructed to mentally solve each item

with at least two different strategies. Again they had to verbally report each strategy immediately after they solved the subtraction. If a child immediately solved a subtraction by means of SBA, the interviewer asked the child if (s)he could solve the same subtraction by means of a second strategy, and afterwards moved to the next subtraction. If the child began by solving the subtraction with a strategy different from SBA, the experimenter kept asking for another possible strategy until the child either reported the SBA strategy, stated that (s)he did not know any other strategy, or had reported five alternative solution strategies.

The study resulted in three main findings. First, the analysis of children's strategies in the SST revealed that less than 10% of the second- and third-graders, and only 15% of the fourth-graders spontaneously reported the SBA strategy at least once to answer the small-difference subtractions. Thus, children hardly reported the use of the SBA strategy, even on subtractions where this strategy is considered extremely efficient. Second, all children reported various strategies for solving the two small-difference subtractions from the VDT. But only a minority of them reported SBA as an alternative strategy to solve these subtractions. This suggests that SBA was not included in the strategy repertoire of most children. Actually, only 5% of the second-graders, 15% of the third-graders, and 20% of the fourth-graders mentioned the SBA strategy at least once as a possible alternative solution strategy. Third, despite the low overall occurrence of SBA in the two tasks and in the three age groups, there tended to be differences in the frequency with which SBA was spontaneously reported (in the SST) or mentioned as an alternative solution method (in the VDT) between the different age and mathematical achievement groups. High achievers and older children tended to report the SBA strategy more frequently on small-difference subtractions than low achievers and younger children.

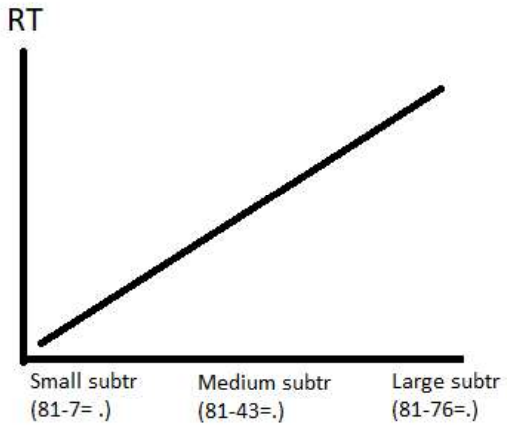
In sum, this study of Torbeyns et al. (2009a) revealed that most Flemish elementary school children did not report the SBA strategy on symbolic multi-digit subtractions, even not on subtractions such as 41 – 39 = . that are assumed to maximally elicit this strategy. Furthermore, when being explicitly asked to generate alternative solution

strategies for such types of subtractions, only a small minority of the children came up with SBA as an alternative subtraction strategy.

These results were confirmed by those of several other studies using similar verbal report methods, both from our own research team (De Smedt, Torbeyns, Stassens, Ghesquière, & Verschaffel, 2010; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009b), and from various other teams (Blöte, Van der Burg, & Klein, 2001; Heinze, Marschick, & Lipowsky, 2009; Selter, 2001; Selter, Prediger, Nührenbörger, & Hussmann, 2012).

However, while the above-mentioned verbal report studies suggested that the SBA strategy was not included in most elementary school children's strategy repertoire, some anecdotal evidence obtained during the study of Torbeyns et al. (2009a) reported above, as well as some of the other verbal report studies realized in our center, made us doubt about the validity of this finding. More specifically, in the study of De Smedt et al. (2010), wherein we also asked children to report verbally on how they had solved a series of symbolically presented multi-digit subtractions, we noticed a remarkable inconsistency between children's verbal reports about the strategy they had used, on the one hand, and the time they needed to solve these problems, on the other hand. As in the study of Torbeyns et al. (2009a), the vast majority of the third-graders in the study of De Smedt et al. (2010) only reported the use of the DS strategy. Based on a rational task analysis, one might predict that this consistent use of DS on all types of subtractions (i.e., those with a relatively small, medium, and large subtrahend) should have led to an increase in these children's reaction times from subtractions with relatively small subtrahends (e.g., 81 – 7 = .) over subtractions with medium-sized subtrahends (e.g., 81 – 43 = .) to subtractions with a relatively large subtrahend (e.g., 81 – 79 = .), because subtracting a larger subtrahend requires more and/or larger calculation steps (Groen & Poll, 1973; Peters, De Smedt, Torbeyns, Ghesquière & Verschaffel, 2010) (see Figure 1a). However, a closer look at the collected reaction times revealed that children's reaction time patterns were not in line with this prediction, as not only subtractions with relatively small subtrahends but also subtractions with relatively large subtrahends were solved faster than sub-

tractions with medium-sized subtrahends (see Figure 1b). These reaction time data thus suggested that children applied various strategies, and that, more specifically, the SBA strategy might have been used more frequently than reported by the children.



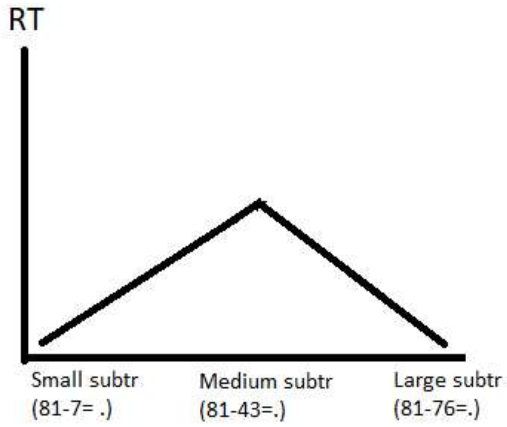


Figure 1a and 1b. Expected reaction time pattern for subtractions with a relatively small, medium, and large subtrahend when always applying DS (Figure 1a) and when switching between SBA and DS depending on the relative size of the subtrahend (Figure 1b).

Taking into account the available theoretical insights as well as the existing research evidence on the validity of verbal reports as data, one can come up with several factors that might lead to an underestimation of SBA use in such interview studies. First, as argued by Peters, De Smedt, Torbeyns, Ghesquière, and Verschaffel (2013), especially on subtractions with a small difference between the two given numbers, the SBA strategy can be executed very quickly and quasi-automatically. It is plausible that children had difficulties in explaining how they arrived at their answer, because they may not have been aware of the steps they performed while doing SBA or may have found it too hard to verbalize this untaught strategy. Consequently, they may have reported a strategy they were more familiar with and knew how to verbalize (e.g. Ashcraft & Kirk, 2001). Second, children may have hidden the use of the SBA strategy because they thought it was not valued or even not allowed to use other strategies than the one(s) taught in their mathematics lessons (e.g., Yackel & Cobb, 1996). We therefore ran a next series of studies that used non-verbal methods to analyze children's use of SBA. More specifically, we collected reaction time data and used regression-based approaches to make inferences about children's strategy use.

3 Reaction time studies

After having done similar studies with adults (Peters et al., 2010), Peters et al. (2013), examined the use of the SBA strategy of fourth- to sixth-grade elementary school children by means of the above-mentioned reaction time analytic approach. Starting from Groen and Poll's (1973) pioneering work on fitting linear regression models to people's reaction times for various kinds of counting-based strategies for single-digit arithmetic, Peters et al. calculated three linear regression models for the reaction times of symbolic multi-digit subtractions. These regression models represented three different strategy use patterns:

1. If children only used the DS strategy (DS-Model), reaction times should be best predicted by the size of the subtrahend (S), because it takes longer to subtract a larger number from

the minuend (e.g., 83 – 79 = .) than to subtract a smaller number from that minuend (e.g., 83 – 4 = .).

2. If children always used the SBA strategy (SBA-Model), reaction times should be best predicted by the size of the difference (D), because it takes more time to determine how much needs to be added to get at a given number when the difference between both numbers is large (e.g., "How much needs to be added to 4 to have 83?") than when it is small (e.g., "How much needs to be added to 79 to have 83?").

3. If children switched between both strategies depending on the relative size of the subtrahend (Switch-Model), reaction times should be best predicted by *the minimum of* subtrahend and difference (min[D, S]): For problems with the subtrahend smaller than the difference (e.g., 83 – 4 = . and 84 – 38 = .), reaction times are expected to increase with the size of the subtrahend, because these problems can be easily solved by means of DS. In contrast, for problems with the difference smaller than the subtrahend (e.g., 83 – 79 = . and 84 – 46 = .), reaction times are expected to increase with the size of the difference, because these problems can be solved easily by means of SBA.

Seventy-two fourth- to sixth-grade children had to mentally solve a series of 32 symbolically presented two-digit subtractions divided in four problem types, based on the combination of the magnitude of the subtrahend (S) compared to the difference (D) (i.e., S > D or S < D) and the numerical distance between S and D (i.e., small or large). This resulted in the following problem types: (a) large-distance S < D problems (e.g., 83 – 4 = .); (b) large-distance S > D problems (e.g., 77 – 68 = .); (c) small-distance S < D problems (e.g., 92 – 44 =); and (d) small-distance S > D problems (e.g., 32 – 17 = .). The task was administered individually on a computer that allowed precise reaction time registration.

Multilevel regression analyses showed that the model in which children switched between DS and SBA based on the relative size of the subtrahend (Switch-Model) provided the best fit to the data. So, in contrast with the findings from the verbal report studies, the results

of this reaction time study led to the conclusion that SBA belonged to the strategy repertoire of Flemish elementary school children and that they applied this strategy, together with DS, flexibly to solve two-digit subtraction problems by switching between both strategies depending on the relative size of the subtrahend. More specifically, the regression analyses on the reaction time data suggested that the DS strategy was used when the subtrahend was relatively small, whereas the SBA strategy was used when the subtrahend was relatively large.

These findings were replicated in other studies at our center, using different sets of subtractions (Peters, De Smedt, Torbeyns, Ghesquière, & Verschaffel, 2012) and different samples of children, including children with mathematical learning disabilities (Peters, De Smedt, Torbeyns, Verschaffel, & Ghesquière, 2014).

However, as extensively discussed by Peters et al. (2010), this regression-based analytic approach with reaction times as the main type of data is not without its methodological problems either. For instance, it does not allow to accurately determine the kind of strategy being applied by an individual or a group on an individual item; it only allows for the identification of overall strategy patterns for the item set as a whole. Moreover, underlying the analytic method are some theoretical assumptions about the relationship between the strategy being used to solve a problem and its solution time that might be valid for simple counting tasks (as the ones used by Groen & Poll, 1973) but may be less valid for strategies for doing multi-digit mental subtraction.

Given the methodological concerns related to the reaction time studies (Peters et al., 2010), we decided to apply still another research method that we had already used successfully in several previous studies - both with adults and children and with various kinds of mathematical tasks - but that so far had not been used yet in research with children doing symbolic multi-digit subtraction, namely the choice/no-choice method (Lemaire & Siegler, 1995; Siegler & Lemaire, 1997). In contrast to more classical studies on people's strategy competencies, this method requires assessing each participant in two different types of conditions. First, a choice condition, wherein they

can choose which strategy they apply on each item. Second, two or more no-choice conditions, wherein they are forced to solve all items with a given strategy. Compared to classical studies involving only a choice condition (as in the studies reviewed in the section: verbal report studies), the no-choice conditions prevent researchers from getting biased efficiency data for the strategies being involved in the study. Moreover, the choice/no-choice method allows them to analyze the flexibility of participants' strategy choices based on their individual strategy efficiency competencies by comparing their strategy choices in the choice condition with the individual strategy efficiency data collected in the no-choice conditions.

4 Choice/no choice studies

Torbeyns, Peters, De Smedt, Ghesquière, and Verschaffel (2018) investigated 68 sixth-graders of varying mathematics ability. The participating children were offered three parallel series of 10 multi-digit subtractions up to 1000, which were divided in two problem types: subtractions with a relatively large subtrahend (LS) (e.g., 614 - 596 = 18), and subtractions with a relatively small subtrahend (SS) (e.g., 734 - 47 = 687). These subtractions were individually administered to the children in one choice and two no-choice conditions. In the choice condition, they could choose between DS and SBA on each subtraction. In the no-choice DS condition, children had to solve the subtractions via a DS strategy. Likewise, in the no-choice SBA condition, all subtractions had to be solved via SBA. In each condition, children were instructed to solve all subtractions as accurately and as fast as possible, and to verbally report the applied strategy immediately after solving each subtraction. The analysis of the data of the choice and no-choice conditions resulted in the following findings.

First, as far as children's strategy repertoire and distribution is concerned, we found that, in the choice condition, SBA was used at least once by most children.

Second, SBA was also used more frequently on the subtractions in the choice condition than DS.

Third, the subtraction items were solved more accurately when the SBA strategy had to be used compared to the required use of the DS strategy. This was particularly the case for LS subtractions whereas for the SS subtractions there was no difference in accuracy. Turning to strategy speed, the subtraction items were solved more quickly when the SBA strategy had to be used compared to the required use of the DS strategy, and this was again particularly the case for LS subtractions whereas for the SS subtractions there was no difference in speed.

Fourth, we found two types of evidence for children's flexible use of the SBA strategy. First, despite instruction focusing on only DS, children took into account numerical characteristics during the choice process, since we found that children more frequently selected SBA on LS subtractions compared to SS subtractions. Second, children took into account their individual strategy efficiency competencies during the strategy selection process in the choice condition. More specifically, they flexibly fitted their strategy choices to their individual strategy speed competencies: The larger the speed difference between the no-choice SBA and the no-choice DS condition in favour of the SBA strategy, the more frequently children reported that latter strategy in the choice condition.

In sum, this choice/no-choice study not only revealed that the SBA strategy belonged to the strategy repertoire of most elementary school children, but also that they applied it frequently, efficiently, and flexibly. Moreover, we observed hardly any differences among children of varying mathematical achievement levels, indicating that even below-average to low achievers knew the SBA strategy and were able to frequently, efficiently, and flexibly apply it on multi-digit subtractions.

However, one should not forget that the choice/no-choice method requires that all strategies under investigation are explicitly explained and demonstrated to the participants at the beginning of the experiment. This characteristic of this research method might have enhanced children's use of the untaught SBA strategy and might have stimulated some reflections about its flexible use vis-à-vis various types of subtractions. Moreover, these positive results for the SBA

strategy might also be due to the fact that this study involved sixth-graders as participants and three-digit subtractions, whereas the other studies involved (also) younger elementary school children and subtractions with only two digits.

Therefore, we recently replicated the above study with younger elementary school children up to Grade 4. The results showed that the above results about the frequent, efficient and flexible use of the SBA strategy on symbolic multi-digit subtractions also applies for Flemish fifth and fourth graders (Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2017; Torbeyns, Verguts, De Smedt, & Verschaffel, 2018).

In sum, just as the results of the reaction time studies, the findings of the choice/no-choice studies disconfirm those of the initial verbal report studies with children, and rather indicate that many elementary school children – even the mathematically weaker and younger ones - apply the untaught SBA strategy and do so remarkably frequently, efficiently, and flexibly.

5 Conclusion and discussion

In this paper we have reported on a long-term research program in our research center on elementary school children's flexible use of multi-digit mental subtraction strategies, and more specifically on children's use of the SBA strategy. We have presented three types of studies, namely verbal report studies, reaction time studies, and studies applying the choice/no-choice paradigm. Taken as a whole, this research has shown that there is only very little evidence for SBA use when relying on verbal reports obtained in a choice condition only. However, when turning to other research methods, there is ample evidence from our reaction time studies hat elementary school children do apply this alternative subtraction strategy frequently and flexibly. Moreover, our studies using the choice/no-choice method reveal that elementary school children do not only frequently and flexibly apply the SBA strategy, but also do so with superior computational efficiency compared to DS. Importantly, these findings also hold for younger children and for mathematically weaker ones.

From a more general theoretical perspective, given the exclusive focus on DS in current mathematics instruction, the findings from our reaction time and choice/no-choice studies can be considered as a striking illustration of the active and constructive nature of children's mathematics learning, and of their potential to develop adaptive expertise in elementary mathematics.

From a methodological perspective, our review nicely documents the difficulty of obtaining reliable and valid empirical data on children's strategy use in the domain of elementary mathematics. Each research method or analytic technique has its own advantages and disadvantages that should be taken into account when interpreting the outcomes of a study and comparing the results of different studies. As such, this review can be considered as a strong plea for the methodological principle of triangulation (i.e., the use of more than one research method to understand the phenomenon under investigation).

From an educational perspective, our findings challenge current mathematics instruction that is capitalizing on the routine mastery of only DS, and involves a plea for more instructional attention to SBA as a valuable - and may-be even *more* valuable - alternative strategy in this domain (cf. Peltenburg et al., 2012; Selter et al., 2012). However, our studies are among the first to analyse the frequency, efficiency, and flexibility of the SBA strategy compared to DS. Also, the research evidence on the efficacy and educational value of a teaching approach for multi-digit subtraction wherein the SBA strategy plays a (much) more prominent role, is very scarce. Before drastic changes in current mathematics instruction can be recommended, future research is needed. Future ascertaining studies can help to replicate and refine our findings in still younger age groups and in children of the lowest mathematics achievement levels (including children with mathematical learning difficulties). Moreover, future intervention studies may offer building blocks for optimizing current elementary mathematics instruction in the domain of symbolic multi-digit subtraction.

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