MODEL REDUCTION FOR NON-LINEAR DYNAMICS BY PROJECTION ON STATE-DEPENDENT MODAL BASE

Gert H.K. Heirman and **Wim Desmet**

Department of Mechanical Engineering
Katholieke Universiteit Leuven
Celestijnenlaan 300B
Leuven, Belgium, B-3001
gert.heirman@mech.kuleuven.be, wim.desmet@mech.kuleuven.be

Irrespective of the type of generalized coordinates used, the modeling of the motion of a (flexible) mechanism results in a set of (non-linear) differential-algebraic equations (DAE): second-order differential equations express the equations of motion, while algebraic equations impose constraints on the motion of the system. Both the DAE-character of the model equations, and the number of degrees of freedom needed to accurately represent flexibility (typically up to 1000's of DOFs), prohibit real-time simulation of these systems. Model reduction on subsystem-level, e.g. modal representation of the flexibility of a mechanism component, is used extensively in flexible mechanism models to limit the computational load. However, subsystem-level model reduction suffers from the intrinsic drawback that it does not result in significant dimension reduction if the interface between the subsystems is (highly) variable. Furthermore, algebraic equations will still be present in the resulting set of equations.

Current real-time running models of flexible mechanisms are mostly based on ad hoc simplifications of the model. Few techniques exploit the mathematical structure of the original model equations. Most system-level model reduction techniques for non-linear models build reduced models based on data obtained from user-defined numerical experiments of the original model. The resulting reduced models only offer an accurate approximation in scenarios similar to the numerical experiments on which they are based. For these techniques, a good approximation for all possible states of the system requires many numerical experiments and limits the computational efficiency of the reduced model equations. In this research, a system-level model reduction technique is developed that exploits the mathematical properties of the original model equations.

Projection-based model reduction is done by projecting the instantaneous change of the state the system on a vector set. To accurately model the change of the system, this vector set should span the instantaneous (state-dependent) dominant dynamics of the system, i.e. the 1) state-dependent dominant eigenmodes, 1a) the rigid body modes and 1b) low-frequency elastic eigenmodes, and 2) the relevant static deformation patterns. Using a fixed vector set, i.e. the same vector set regardless of the state of the system, for non-linear systems requires the inclusion of many vectors in the vector set, to ensure the state-dependent dominant dynamics of the system are spanned for each state of the system. This research proposes to project the instantaneous change of the state of the system on a state-dependent vector set instead of a fixed vector set. The state-dependent vector set consists of a set of modes that, exactly and only, span the dominant dynamics. The system is thus expressed in a curvilinear coordinate system η of which the axes are defined by the state-dependent eigenmodes and static deformation patterns of the system. Different mode sets meet these demands [1].

Using a state-dependent vector set offers important advantages over a fixed vector set. First, the dominant dynamics are described by a minimal number of vectors, which will result in a minimal number of degrees of freedom in the resulting reduced model equations, while only minimally decreasing the accuracy. Secondly, the selected system-level eigenmodes and static deformation patterns intrinsically satisfy the constraints, such that the algebraic equations are automatically satified after model reduction; The model reduction transforms the model equations from a set of differential-algebraic equations (DAE) into a set of ordinary differential equations (ODE). Both advantages result in faster simulation of the reduced model equations. However, composing the matrices and tensors of the reduced model equations is expensive and can only be done numerically. The continuity of these matrices and tensors allows to cheaply interpolate them from their values at discrete states of the system. The overall process is thus decomposed in two steps: 1) a preparation phase in which the reduced model matrices and tensors are computed and stored for a discrete set of states, and 2) a simulation phase. At each time step in the simulation phase, the reduced model matrices and tensors are interpolated out of the previously computed data, then the values of the DOFs η at the next time point are computed, after which the simulation results can be expressed again in terms of the original DOFs. The combined interpolation, solving and backtransformation is considerably cheaper than solving the original model equations. In the applications envisioned in this research, such as real-time simulation, gain in simulation speed during on-line simulation justifies an expensive offline preparation phase. The assumption of small deformations allows to approximate the state-dependent mode set by the mode set obtained at the corresponding undeformed state. This strongly limits the number of discrete states for which the reduced model matrices and tensors need to be computed and stored during the preparation phase, while only minimally decreasing the accuracy. A more elaborate explanation on the methodology can be found in [1], [2] and [3].

In flexible mechanism dynamics, non-linearity arises from two effects: 1) variable connectivity between bodies, e.g. moving connection points, and 2) the variability of the mass matrix due to large rotations. In a first test case, the methodology is applied to a system with non-linear dynamics resulting from variability in the connection between bodies: a flexible beam being clamped by a sliding joint at a continuously changing location. For this type of non-linearity the traditional subsystem-level model reduction fails to achieve a significant dimension reduction of the problem, whereas the proposed system-level model reduction produces accurate simulation results for a low number of modes in the mode set, and thus for a small dimension of the reduced model equations. The approximation errors of several mode sets are compaired and the sources of error are explained. For a more complete overview of this test case the reader is referred to [2].

In a second numerical experiment, a system with large rotations is considered: a flexible slider-crank mechanism. Due to the large rotations, the fast transformation from the DOFs used in the reduced model equations back to the original DOFs needs to be performed by interpolation, which is an additional difficulty compared to the first test case. Again, the simulation results of the reduced model give a good approximation of the simulation results of the original (unreduced) model for a very limited mode set. The different sources of approximation error are identified and explained. The computational load of both simulations is quantified: the proposed methodology proves to be more efficient. For a more complete overview of this test case the reader is referred to [3].

References

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