Erratum

L.De Raedt, *Logical settings for concept-learning*,
Artificial Intelligence, Vol. 95, pp. 187-201, 1997.

Roni Khardon pointed out to the author that the proof of Theorem 27 on pp. 197-198 is wrong. Theorem 27, its proof, Theorem 28, and Corollary 29 should read as follows :

Theorem 27. k-CNF *is not efficiently PAC-learnable under entailment for* $k \geq 4$ *(where* L_e *consists of unbounded clauses) unless* $NP \subseteq P/Poly$.

Proof. Let $X = \{X_n\}_{n\geq 1}$ be a parametrised concept class where concepts in X_n take inputs in $\{0,1\}^n$. Schapire [1] shows (in Theorem 7) that if X is learnable then there is a polynomial $p(n)$ such that each concept in X_n has a circuit of size $p(n)$ representing it exactly. Therefore, the evaluation problem for concepts in X_n has polynomial size circuits.

We claim that the evaluation problem for k-CNF with examples in L_e does not have polynomial size circuits unless $NP \subseteq P/Poly$. It then follows that k-CNF is not learnable with examples in L_e unless $NP \subseteq P/Poly$.

To prove the claim we present a family of concepts $\{C_m\}_{m\geq 1}$ in 4-CNF such that if the evaluation problem for $\{C_m\}$ with respect to L_e has polynomial size circuits then so does NP. The construction closely follows a similar proof by Selman and Kautz [3].

In order to define C_m we need to introduce some notation. Let $V = \{p_1, \ldots, p_n\}$ be n propositional variables, $Lits = \{l \mid l \in V\} \cup \{\neg l \mid l \in V\}$ be literals over V, and define a set of auxiliary variables $\{c_{x,y,z} \mid x, y, z \in Lits\}$. Then, C_m is defined over the $m = n + \binom{2n}{3}$ variables:

$$C_m = \wedge_{x,y,z \in Lits}(x \vee y \vee z \vee \neg c_{x,y,z})$$

Now, given a 3-CNF formula $\phi = \wedge_j (l_{j1} \vee l_{j2} \vee l_{j3})$ over $V = \{p_1, ..., p_n\}$, ϕ can be encoded as an example e_ϕ in L_e such that C_m entails e_ϕ if and only if ϕ is not satisfiable. In particular, it is easy to see that this holds for

$$e_\phi = \vee_j \neg c_{l_{j1}, l_{j2}, l_{j3}}$$

(cf. the proof by Selman and Kautz.) It follows that if the evaluation problem for $\{C_m\}$ with respect to L_e has polynomial size circuits then so does 3-SAT and therefore also NP. In other words if $\{C_m\}$ has polynomial size circuits then $NP \subseteq P/Poly$. □

Notice that it is considered unlikely that $NP \subseteq P/Poly$. Indeed, if $NP \subseteq P/Poly$ then this would e.g. imply that the polynomial-time hierarchy would collapse. It is therefore unlikely that k-CNF ($k \geq 4$) is PAC-learnable under entailment. This result is also stronger than the original one because it holds for any representation of k-CNF.

Note also that the requirement in Theorem 27 that L_e is not bounded (as assumed throughout the original paper) is necessary. Indeed, Corollary 6.6 of [4] implies that if L_e includes only clauses (or even clausal theories) in k-CNF then the class of k-CNF is learnable from entailment.

Theorem 28. jk-CT *is not efficiently PAC-learnable under entailment for* $k \geq 4$ *unless* $NP \subseteq P/Poly$.

Corollary 29. k-CNF and jk-CT are not PAC-learnable under $\in_{int,B}$ an $\in_s$.

References.

1. Schapire, R.E., The strength of weak learnability, *Machine Learning*, Vol. 5, pp. 199-227, 1990.

2. Cohen, W.W., Page, C.D., Polynomial learnability and inductive logic programming: methods and results, *New Generation Computing*, Vol. 13, pp. 369-409, 1990.

3. Selman, B., Kautz, H., Knowledge compilation and theory approximation, *Journal of the ACM*, Vol. 43, pp. 193-224, 1996.

4. Khardon, Roth, MLJ, details to be added.