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RATIONAL GAUSS-TYPE QUADRATURE FORMULAS WITH A PRESCRIBED NODE ANYWHERE ON THE REAL LINE

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Abstract

In general, the zeros of an orthogonal rational function (ORF) on a subset of the real line, with poles among $\{\alpha_1, \ldots, \alpha_n\} \subset (\mathbb{C}_0 \cup \{\infty\})$, are not all real (unless α_n is real), and hence, they are not suitable to construct a rational interpolatory quadrature rule with positive weights and maximal domain of validity. For this reason, the zeros of a so-called quasi-ORF are used instead. These zeros depend on one single parameter $\tau \in (\mathbb{C} \cup \{\infty\})$, which can be chosen freely in order to fix one node in the rational quadrature rule. First, we give a condition on the parameter τ to ensure that the zeros of the quasi-ORF are all real and simple. Next, we provide a generalized eigenvalue problem to compute the zeros of the quasi-ORF and the corresponding weights in the rational quadrature rule.

Keywords: Rational Gauss-type quadrature, quasi-orthogonal rational functions, generalized eigenvalue problem.

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