Fast Decision Making in Human-Machine Interaction through Estimation of Human Reaching Intent

Supplemental Material

I. Kalman Filter Approach to Condition TraPPCA Model

Consider a TraPPCA model with d dimensions, N progress samples and m latent variables. This model can be conditioned on a series of observations $\boldsymbol{f}_{\text{obs},i}$ with $i = 1 \dots O$, where a Kalman filter update step is iteratively applied for each observation to update the estimate of the model's latent variable vector $\boldsymbol{\chi}_{\text{cond}} \sim \mathcal{N}(\boldsymbol{\mu}_{\text{cond}}, \boldsymbol{\Sigma}_{\text{cond}})$, with $\boldsymbol{\mu}_{\text{cond}}$ the state and $\boldsymbol{\Sigma}_{\text{cond}}$ the state covariance of the Kalman filter. This is shown in Algorithm 1, where $\boldsymbol{\Sigma}_{f_{\text{obs},i}}$ represents the uncertainty on the observation $\boldsymbol{f}_{\text{obs},i}$ and s_i is the progress value of $\boldsymbol{f}_{\text{obs},i}$. Since these s_i do not correspond to the progress samples used within the TraPPCA model, the matrix functions $\boldsymbol{H}(s) \in \mathbb{R}^{d \times m}$ and $\boldsymbol{b}(s) \in \mathbb{R}^{d}$ are introduced by fitting a cubic spline through the appropriate elements of $\bar{\boldsymbol{H}} \in \mathbb{R}^{dN \times m}$ and $\bar{\boldsymbol{b}} \in \mathbb{R}^{dN}$, respectively. The linear latent variable model of TraPPCA is then rewritten as:

$$\boldsymbol{f}(s) = \boldsymbol{H}(s)\boldsymbol{\chi} + \boldsymbol{b}(s) + \boldsymbol{\epsilon}, \tag{1}$$

with $\boldsymbol{f}(s) \in \mathbb{R}^d$ and $\boldsymbol{\epsilon} \in \mathbb{R}^d$.

To attach more importance to recent observations, process noise $\boldsymbol{\Sigma}_{\text{process}} = \sigma^2_{\text{process}} \boldsymbol{I}_m$ is added to the prediction step of the Kalman filter approach in Algorithm 1. The ratio between $\boldsymbol{\Sigma}_{f_{\text{obs},i}}$ and $\boldsymbol{\Sigma}_{\text{process}}$ determines the dynamics of the forgetting.

II. Dynamic Time Warping for Partial Trajectories

Dynamic Time Warping (DTW) is an algorithm commonly used to find the optimal alignment between two time series with different lengths and varying time axes, aiming to minimize the deviation between them based on a selected distance metric. In this work, a probabilistically informed adaptation of this algorithm is used to find the optimal alignment between a TraPPCA model M_j and a partial trajectory $\boldsymbol{obs}_t = \left\{(\boldsymbol{f}_{\text{obs},i}, t_i)\right\}_{i=1}^{O}$, with $\boldsymbol{f}_{\text{obs},i}$ the observed sample at time t_i. In this algorithm, the DTW matrix (Fig. 1) is first constructed, row by row from the bottom up, by evaluating the DTW equation:

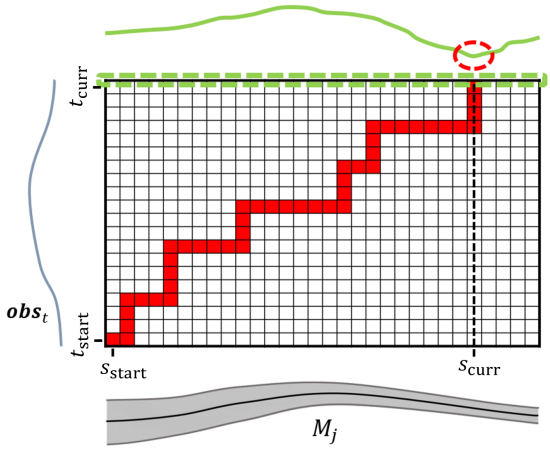


Fig. 1. A representation of the DTW algorithm for an example of a 1D trajectory, where the rectangular grid represents the DTW matrix. *Grey*: motion model M_j. *Blue*: observed partial trajectory $\boldsymbol{obs}_t$. *Green*: summed squared Mahalanobis distances on the top row of the DTW matrix, which are divided by the corresponding path lengths before comparison. *Red*: warping path for the selected element on the top row.

$$\text{DTW}(i,n) = \text{SMD}[\boldsymbol{f}_{\text{obs},i}, \bar{\boldsymbol{f}}_{j,n}] + \min \begin{cases} \text{DTW}(i-1,n) \\ \text{DTW}(i-1,n-1). \\ \text{DTW}(i,n-1) \end{cases} \tag{2}$$

Here, the squared Mahalanobis distance $\text{SMD}[\boldsymbol{f}_{\text{obs},i}, \bar{\boldsymbol{f}}_{j,n}]$ between observed sample $\boldsymbol{f}_{\text{obs},i}$ and model sample $\bar{\boldsymbol{f}}_{j,n}$ is used as the distance metric. Our approach consists of efficiently computing this recursive formula (2) using dynamic programming, while adapting the selection criterion for the optimal element on the top row.

In the resulting DTW matrix, the top row contains summed squared Mahalanobis distances for each progress value s_n being the current progress of the partial trajectory, which is used to determine the optimal alignment between the partial trajectory and the model M_j. To reach the top row at each of these s_n, a different path is taken through the DTW matrix, which corresponds to the optimal alignment for s_n being the current progress value. Since the length $L_{j,n}$ of this path can vary depending on the choice of s_n, each summed squared Mahalanobis distance on the top row may account for a different

Algorithm 1: Kalman filter approach to condition a TraPPCA model on O observations

$\boldsymbol{\mu}_{\text{cond}} \leftarrow \boldsymbol{0}$
$\boldsymbol{\Sigma}_{\text{cond}} \leftarrow \boldsymbol{I}_m$
for $i \leftarrow 1$ to O **do**
  Prediction step
  $\boldsymbol{\mu}_{\text{cond}} \leftarrow \boldsymbol{\mu}_{\text{cond}}$ ▷ Prediction
  $\boldsymbol{\Sigma}_{\text{cond}} \leftarrow \boldsymbol{\Sigma}_{\text{cond}} + \boldsymbol{\Sigma}_{\text{process}}$ ▷ Prediction covariance
  Update step
  $\boldsymbol{\nu} \leftarrow \boldsymbol{f}_{\text{obs},i} - \boldsymbol{H}(s_i)\boldsymbol{\mu}_{\text{cond}} - \boldsymbol{b}(s_i)$ ▷ Innovation
  $\boldsymbol{S} \leftarrow \boldsymbol{\Sigma}_{f_{\text{obs},i}} + \boldsymbol{H}(s_i)\boldsymbol{\Sigma}_{\text{cond}}\boldsymbol{H}(s_i)^\top$ ▷ Innovation covariance
  $\boldsymbol{K} \leftarrow \boldsymbol{\Sigma}_{\text{cond}}\boldsymbol{H}(s_i)^\top \boldsymbol{S}^{-1}$ ▷ Kalman gain
  $\boldsymbol{\mu}_{\text{cond}} \leftarrow \boldsymbol{\mu}_{\text{cond}} + \boldsymbol{K}\boldsymbol{\nu}$ ▷ Kalman update
  $\boldsymbol{\Sigma}_{\text{cond}} \leftarrow \boldsymbol{\Sigma}_{\text{cond}} - \boldsymbol{K}\boldsymbol{H}(s_i)\boldsymbol{\Sigma}_{\text{cond}}$ ▷ Covariance update
end for

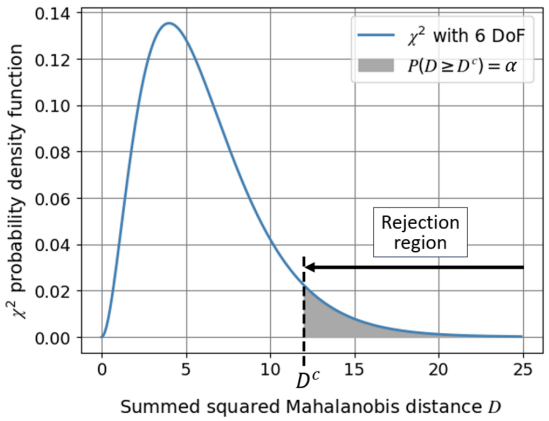


Fig. 2. Plot of the probability density function of a 6 degrees of freedom (DoF) χ^2-distribution over the summed squared Mahalanobis distance D. The grey area under the plot represents the significance level α and its left edge provides the critical value D^c.

number of squared Mahalanobis distances. Directly comparing these summed squared Mahalanobis distances would therefore favor alignments with shorter paths, which is not desirable.

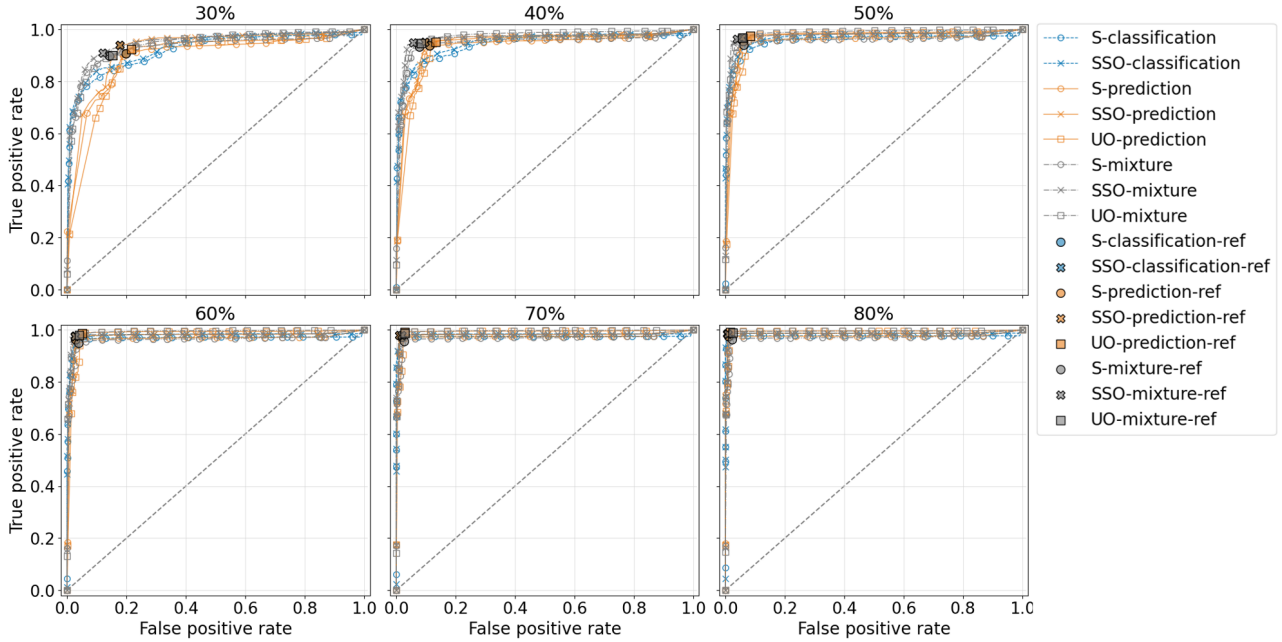
To compensate for this length difference, the summed squared Mahalanobis distances on the top row are divided by their corresponding length $L_{j,n}$, taking the mean of the squared Mahalanobis distances. Since a lower mean squared Mahalanobis distance indicates higher similarity, the minimum value of the top row is selected. This provides the summed squared Mahalanobis distance $D_j(\boldsymbol{obs}_t)$, the path length L_j and the current progress $s_{\text{curr},j}$ for the optimal alignment between the partial trajectory and the model. Finally, the optimal alignment between the partial trajectory and the model is determined by backtracking the warping path $\boldsymbol{w}_j = (\hat{\boldsymbol{i}}_j, \hat{\boldsymbol{n}}_j)$ from the top row down, which indicates how each sample $\boldsymbol{f}_{\text{obs},i}$ of the partial trajectory is aligned with a corresponding sample $\bar{\boldsymbol{f}}_{j,n}$ of the model M_j. Here, $\hat{\boldsymbol{i}}_j$ contains the sample indices i of the partial trajectory along the warping path and $\hat{\boldsymbol{n}}_j$ contains the sample indices n of the model along the warping path.

When this DTW algorithm is applied for different models M_j, different summed squared Mahalanobis distances $D_j(\boldsymbol{obs}_t)$, progress estimates $s_{\text{curr},j}$ and warping paths $\boldsymbol{w}_j$ with length L_j are obtained for each model.

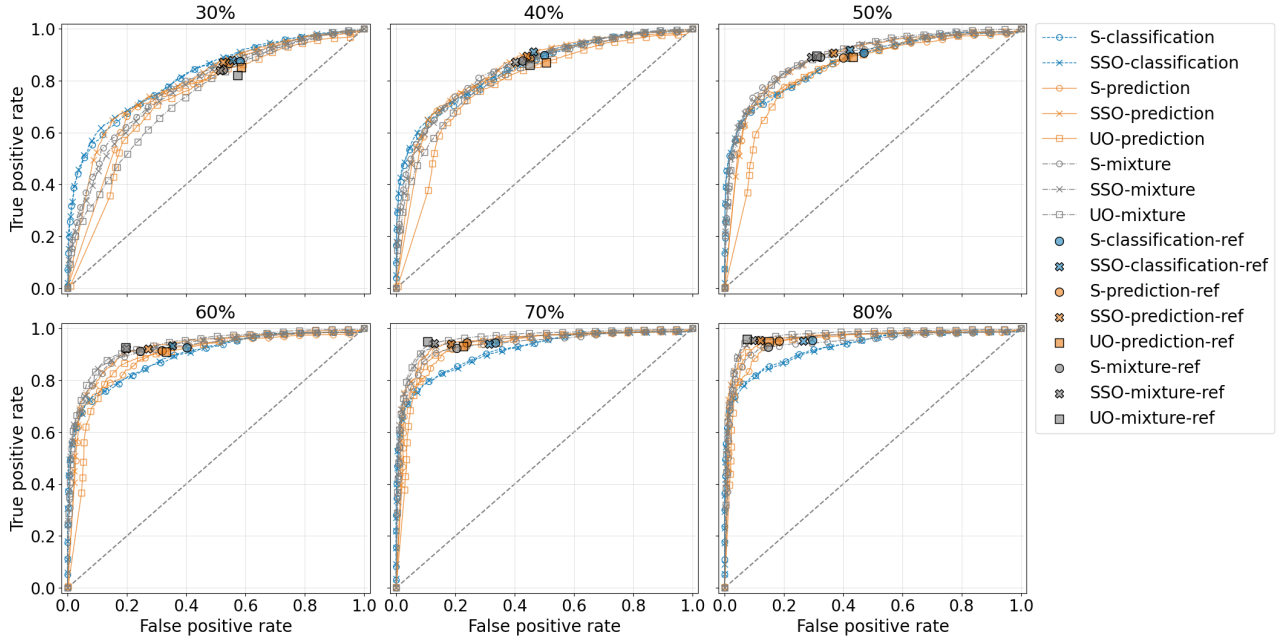
III. Hypothesis Test with Summed Squared Mahalanobis Distance

To determine whether an observed (partial) trajectory corresponds to a motion model M_j, an hypothesis test is performed based on the summed squared Mahalanobis distance between the observed trajectory and M_j. This hypothesis test assesses the null hypothesis of the observed trajectory being explained by M_j. Since the summed squared Mahalanobis distance follows a χ^2-distribution, the critical value D_j^c for rejecting the null hypothesis is obtained from this distribution using a selected significance level α for the right-tail probability (Fig. 2). When the observed summed squared Mahalanobis distance exceeds the critical value D_j^c, it indicates that the observed trajectory does not correspond to the motion model M_j, given risk α of wrongfully making this decision.

IV. ROC CURVES FOR INTENT-BASED DECISION MAKING



(a) Container picking



(b) Table picking

Fig. 3. ROC curves for intent-based decision making using different combinations of learning and intent estimation approaches, evaluated at the considered progress fractions (30%, 40%, 50%, 60%, 70% and 80%). The results for decision making using the most probable target are also included in the figures, which correspond to a single point for each progress fraction and each combination of approaches, and are indicated using the extension "-ref".