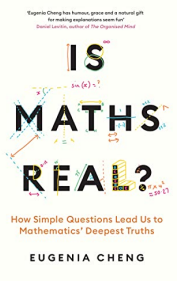
Is maths real?: How simple questions lead us to mathematics' deepest truths *Eugenia Cheng*. Profile Books, 2023 (328 p.), isbn: 978-1788169523 (hbk)




Eugenia Cheng

Cheng's research subject is category theory, but she is most widely known for her popularizing books on mathematics. In this, her latest one, she answers the title's question already at the beginning: Yes it is real, not in a palpable sense, but as a concept, it is as real as Santa Claus is a real abstract concept, just like my ideas are real, and certainly the insights that mathematics gives us to understand the real world are very real. Thus the answer to a question depends on what you mean by real, or in which universe you want the question to be answered. This idea is repeatedly applied in the subsequent chapters in which she starts with a simple question, but, by thinking more about what the question means and in what context the answer is given, she arrives at explaining what mathematics really is about, what is the job of a mathematician, why the abstraction, and why do some love it while others hate mathematics, and how mathematics should be taught.

The first 4 chapters are about how, what and why math is what it is and where it comes from. The next 4 about the tools used in mathematics. Every chapter starts with a simple question. For example chapter 1: Why is $1 + 1 = 2$? If one carefully thinks about it, then the conclusion should be that it is not always 2, for example in a binary system, but also in daily life it is not always true, like for example adding 2 colors gives a new color. That's where math comes in: make abstraction of things and agree on how they operate. The abstract study objects influence the methods used which can be applied to more general objects, which requires new logical rules, etcetera. That is how mathematics constantly develops and expands. That's why we introduced negative numbers, zero, and prime numbers as an example of looking for basic building blocks. The discussion of why $0.999\ldots$ equals 1 is a pretext to intuitively introduce limits like a regular n-gon approximating a circle, etc. That's how math grows and becomes more complex, literally with the introduction of complex numbers. She even throws in some of her own research and cooking hobby, and she gives her opinions about white supremacy and colonialism.

The last 4 chapters she starts with the question: Why is $y = mx + c$? This is a start to explain why it is so much easier to write letters for variables or parameters that can take all values within a set, and that this need not represent a straight line. It depends on the reference system being Euclidean, orthogonal or polar, or yet something else. Moreover, a straight line is not always the shortest distance, and all points at a constant distance from a center does not always look like a circle. This leads to a definition of π. She also discusses the irritating questions you can find on social media like $7 + 7 \div 7 + 7 \times 7$ going along with the note that most will get the answer wrong. It is just a way of giving the impression that the poster is intelligent and (most of) the others are dumb. That is exactly what math is not about. Not about being smart, and not about doing arithmetic with numbers. When she discusses topology, she analyses a similar question: How many holes has a straw? When she discusses pictures as a tool to do mathematics, she warns against possible misleading presentation of data with pie charts or bar charts, just messing with perspective or moving the origin. She even uses graphs to explain heartbreaks when a relation comes to an end. The chapter on pictures is also a pretext to devote a longer part to braids and diagrams which are of course close to her own domain of category theory, which is all about relations and arrows drawn between objects.

Her conclusion is that math is needed with rigour to refine our intuition and intuition to guide our rigour. The answers in mathematics are not fixed, but math is a place to explore and to pose questions. Educators should be given the space to explore dreams and not be confined to teach rigorous rules that leave no freedom.

Adhemar Bultheel