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Identifiability analysis of material identification using nonlinear VFM

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Abstract. The success of inverse material model identification depends on the interaction between the adopted material model, the design of the heterogeneous specimens, the quality of the full-field measurements and the employed inverse identification method. Although inverse identification with full fields usually uses either FEMU or nonlinear VFM algorithms, a range of specimen designs and heterogeneity indicators have been proposed to assess the quality of the measured field and specimen design. While many studies investigate the effects of strain field heterogeneity on material model identification, few of them address the comprehensive interaction of all the above features and investigate their interactions during inverse identification through identifiability analysis. In this study, we analyze the identifiability of the parameters of the YLD2000-2d model used to describe the plastic anisotropy of steel sheet DC04 using a perforated biaxial specimen with the nonlinear VFM method. For this purpose, we performed a virtual DIC experiment with known material parameters by simulating the test in ABAQUS/Standard, generating synthetic images and reconstructing the strains via stereo DIC. Before inverse identification with a nonlinear sensitivity-based VFM, we analyzed the sensitivity of the virtual work to parameter changes and performed an identifiability analysis.

Introduction

The reliability of numerical analyses in structural mechanics depends largely on the accurate modeling of materials. For metallic materials, the models are usually formulated in the form of complex algebraic differential equations, especially when a comprehensive representation of advanced material properties is desired. In addition to the formulation itself, these models include a number of parameters that must be determined by appropriate material tests.

Particularly when characterizing sheet metal, the free parameters of the models are traditionally determined using uniaxial standard tests. Usually, only certain parameters can be determined with analytical expressions derived from statically determined test configurations. Consequently, these tests are designed to measure homogeneous deformations determined by electrical strain gages or extensometers, which easily correlate with the global force measured by the load cell of the testing machine. This approach is commonly referred to as Materials testing 1.0 (MT1.0) [1].

Alternatively, the advent of optical technologies has opened new possibilities for characterizing material behavior. The use of 3D scanners and cameras enables the precise measurement of heterogeneous body displacements and deformations under load. The ability to accurately track and quantify displacements and strains enables engineers to assess material behavior and make informed design decisions.

Digital image correlation (DIC) [2] is one of the most commonly used techniques for measuring strain fields. In this approach, sequentially captured images are processed to extract valuable information that facilitates the study of both local behavior and global responses in structures. In addition, DIC can theoretically operate in real time and provides the ability to monitor structural responses under dynamic loading conditions. This feature makes DIC a powerful tool for various applications ranging from material testing to structural analysis and product development.

These novel technologies unleashed a potential for development of novel material calibration strategies, where a richness of full-field measurement of displacements, strains and eventually stresses over a complete Region of interest (ROI) can be analyzed and employed for material calibration. This triggered also a reconsideration of test specimen designs which are needed to be tailored for novel optical measurement technologies to fully exploit the potential, resulting in a new paradigm in material testing using complex tests, full-field measurements and inverse identification to identify mechanical constitutive parameters, known as Material testing 2.0 (MT2.0) [3].

There are a number of new identification methods, including the Virtual Field Method (VFM) [4], [5] the Equilibrium Gap Method (EGM) [6] and the Constitutive Equation Gap Method (CEGM) [7]. In addition, methods such as the Reciprocity Gap Method (RGM) [8], Finite Element Model Updating (FEMU) [9], [10], Integrated Mechanical Image Correlation (I-MIC) [11], Integrated Digital Image Correlation (I-DIC) [12] and various combined theoretical-experimental approaches [13-14] have been developed. While the novel identification strategies aim to reduce the calibration effort compared to conventional material testing, they mainly utilize the heterogeneity of deformation to determine several material model parameters simultaneously. In the most desirable and ideal case, the concept of MT2.0 would allow the identification of all material parameters from a single heterogeneous test, which remains an open question to date.

Originally, these methods were applied to problems of elasticity, but have evolved over time. Currently, the most commonly used techniques for solving plasticity problems are FEMU and VFM. FEMU, known for its user-friendly approach, uses an optimization algorithm to minimize the discrepancy between the measured and calculated results of finite element analysis (FEA). This method has proven successful in identifying parameters for various plasticity models, including Hill48 [15] and YLD2000-2d [16]. On the other hand, VFM, which is based on the principles of virtual work, was originally developed for the determination of elastic properties [17]. In later applications, however, the range of application was successfully extended to plasticity problems [18–22].

Both FEMU and VFM suffer from certain drawbacks that prevent them from gaining wider acceptance in the industrial engineering community. Inverse identification with FEMU is extremely computationally intensive for plasticity models because the optimization algorithm requires the results of FEM for all material points in a region of interest (ROI) in each iteration, as well as repeating all FEM analyses for perturbed values of all model parameters in each iteration. Each FEM analysis also involves equilibrium iterations, and for each iteration of the FEM analysis, the complete stress field needs to be computed. VFM is much more computationally efficient for elasticity, but becomes computationally intensive for plasticity, where the stress field must also be reconstructed at each iteration.

The second problem hindering wider practical application arises from the conditioning of the optimization problem, which may arise solely from the user's understanding of the underlying identification/optimization problem [23]. Consequently, the optimization process can be ill-posed and lead to poor convergence, a non-unique solution or, in extreme cases, to no solution at all. An illustrative example of such an ill-posed optimization scenario is the attempt to identify isotropic hardening behavior from the linear elastic material response or to determine the hardening exponent for small plastic strains. In both cases, no information is contained in the measured strain

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fields and there is no physical correlation between the parameters sought and the measured response.

While the two problems described above can be associated with a low sensitivity of the resulting strain or stress field to parameter changes, the more difficult case occurs when parameters are correlated. Such an example would also lead to an ill-posed problem where no unique solution can be found and the optimization process does not effectively converge to the desired solution. An illustrative example is the problem of identifying isotropic strain hardening and plastic anisotropy when all parameters describing hardening and anisotropy are included in the optimization. In this particular case, the same full-field response can be obtained by proportionally increasing the parameters for the hardening curve or the anisotropy.

In this paper, we present an approach to perform an identifiability analysis of the nonlinear Virtual Field Method (VFM) specifically applied to the identification of plastic anisotropy using the perforated biaxial cruciform specimen developed by Coppieters et al. [24]. The first step is to investigate the sensitivity of the stress fields associated with the virtual work and thus determine the rank of the parameter sensitivity. In the final step, we calculate the correlation matrix and find that only a certain set of parameters can be reliably identified for the specific material model and heterogeneous test configuration.

Methodology

In order to investigate the identifiability of the parameters of a material model when they are identified using nonlinear sensitivity based VFM [25], we have relied on a methodology of virtual experimentation, where a numerical simulation is performed to obtain synthetically deformed images, which are afterwards processed with DIC. The identifiability of the material parameters—when the identification is subjected to nonlinear VFM—is then investigated.

For this particular purpose, the entire methodology was carried out in the following steps, which are shown in Fig. 1:

- Finite element simulation of a perforated biaxial cruciform sheet metal test specimen. The shape of the specimen was taken from [24], [26], while the material of the sheet was assumed to correspond to the actual material behavior of a 1.2 mm thick DC04 steel sheet. The material behavior is assumed to be plastically anisotropic, using the YLD2000-2d yield function to describe plastic anisotropy and Swift's hardening law for isotropic hardening behavior. The material is fully characterized in Coppieters et al. [27].
- Generation of synthetic images of the deformed specimen shape. Based on the calculated displacement field and the corresponding nodal displacements, a deformed speckle images can be generated from a generic optimal speckle pattern. The procedure for generating images is described in detail by Lava et al. [28].
- DIC post-processing. After the deformed images have been generated, the data can be post-processed with a DIC code that generates the “experimental” displacement field. Since the virtual experiment is generated using a known material model, the virtual data can be used to evaluate the identification quality of the nonlinear sensitivity based VFM. It should be noted that the chosen virtual experimentation approach enables to mimic the entire measurement chain, including image noise and effects caused by camera calibration.
- Analysis of identifiability. Prior to the inverse identification procedure using nonlinear VFM, an identifiability analysis should be performed. The identifiability analysis includes a calculation of the internal virtual work sensitivity matrix, which is further analyzed for the sensitivity rank and the collinearity of the parameters. The procedure closely follows the work of Zhang et al. [28] and [29], who applied the identifiability analysis to FEMU and deep notch experiment.

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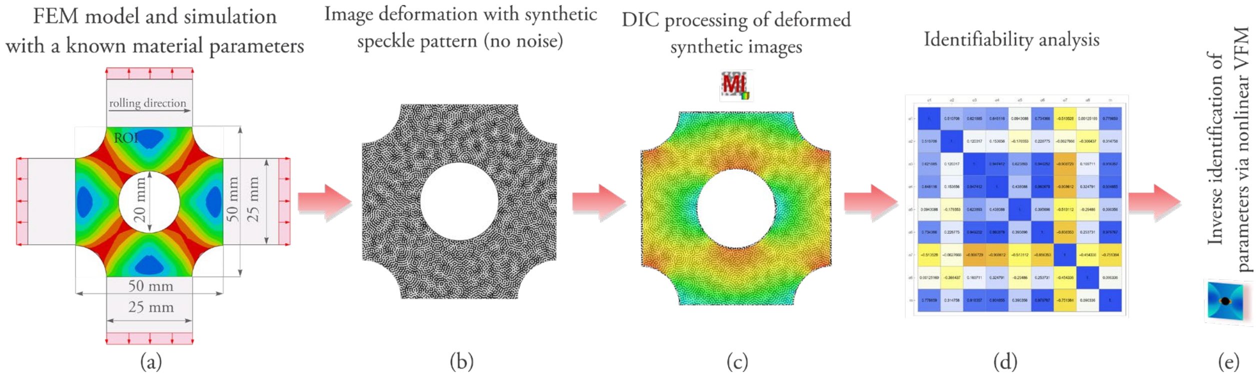

Fig. 1. The concept of virtual experimentation. The idea is to simulate the material response of the heterogeneous sample with known material parameters (a) and to re-identify the parameters (e) by synthetic image deformation (b) and DIC processing of the deformed images (c). Before inverse identification with the nonlinear VFM, the user should know which parameters are identifiable for the material model used based on the employed heterogeneous test (d).

Results

The identifiability analysis was conducted following the methodology outlined above. To assess this, the sensitivity of the internal virtual work to the YLD2000-2d parameters was evaluated at each time frame and DIC acquisition point. Utilizing the extracted data, a sensitivity matrix was constructed, and from this matrix, the sensitivity rank of the YLD2000-2d parameters was determined. The results of the sensitivity rank for each parameter are depicted in a bar chart in Fig. 2(a). Notably, parameter α_7, associated with shear behavior in YLD2000-2d, significantly influences the internal virtual work. This parameter is conventionally determined through the standard uniaxial tensile test in the diagonal direction to the sheet's rolling direction (45°).

The dominance of shear behavior is evident in Fig. 2(b), where the contour plot displays the maximum principal stress component. It reveals that stress is most pronounced at the fillets, oriented at a 45° angle to the horizontal (rolling) direction, resembling behavior observed in the uniaxial tensile test in the diagonal direction. Finally, from the perspective of the performance of the optimization algorithm, the value of α_7 would effectively converge to the reference value, but a slightly different value of α_7 would greatly affect the values of other parameters and normally, convergence of such optimization problem is poor. For such behavior a better strategy is to fix or determine α_7 from uniaxial tensile test in the diagonal direction. Poor convergence of the optimization problem is usually associated with poor conditioning of the system matrix, which is usually characterized by the conditioning number defined as ratio between the largest and the smallest principal value of the system's matrix. If the conditioning number is high, poor convergence is to be expected.

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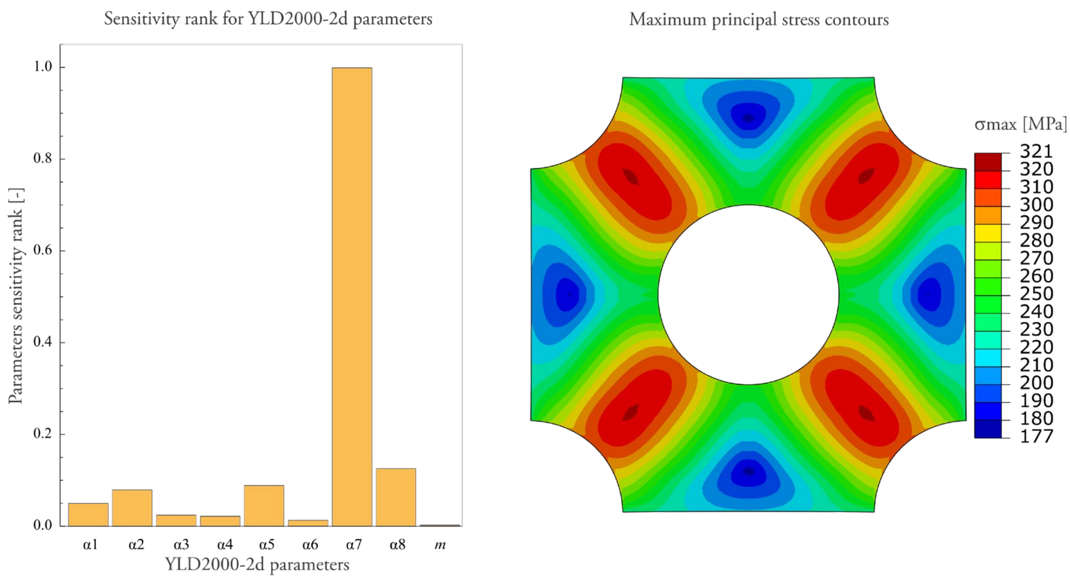
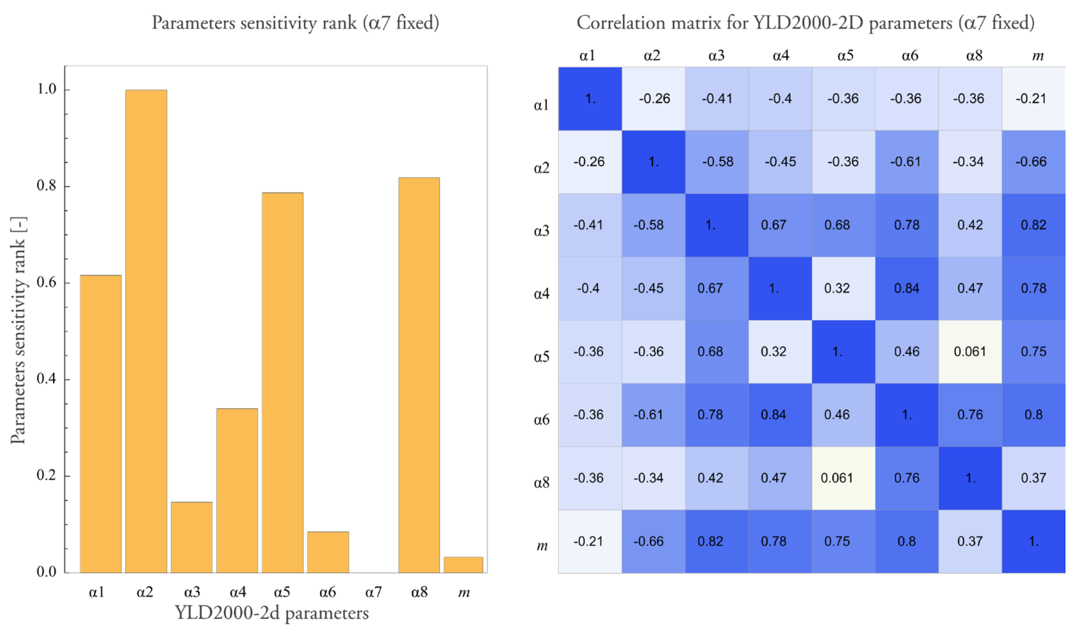


(a) (b)

Fig. 2. Analysis of identifiability by parameter sensitivity rank. (a) When attempting to identify all parameters of YLD2000-2d from a perforated biaxial tensile test, it is clear that the parameter α7 exerts the greatest influence on the internal virtual work. (b) This behavior is expected since the specimen at the fillets, where the maximum principal stress is highest, closely mirrors the behavior observed in the uniaxial tensile test.

Based on the conclusion above, and to fully exploit the potential of heterogeneous testing, it is reasonable to eliminate the parameter α_7 from the identification procedure and determine its value from another test. The rationale behind such decision is eliminating the largest eigenvalues from the system's matrix and thus improving the conditioning of the optimization. Based on the assumption that the parameter α_7 is fixed, we re-evaluated the sensitivity matrix and parameters' sensitivity rank. The results are presented in Fig. 3(a).



(a) (b)

Fig. 3. Analysis of identifiability with parameter α_7 fixed. (a) Parameters α_1, α_2, α_5 and α_8 have the highest sensitivity rank and are potentially identifiable. (b) This may further be analyzed by collinearity analysis, where the interaction of parameters is analyzed via correlation matrix.

In this scenario, it is evident that the parameters α_1, α_2, α_5 and α_8 have the highest sensitivity rank, suggesting that the test inherently contains sufficient information for their identification. The inclusion of other parameters with a lower sensitivity rank would worsen the conditioning number of the system matrix and impair the optimization. However, it is crucial to consider the possibility that these parameters behave as linearly dependent during the inverse identification process. To investigate the possible interaction between α_1, α_2, α_5 and α_8, a correlation matrix was computed and is presented in Fig. 3(b).

The correlation matrix reveals pairwise interactions between the parameters, with off-diagonal values exceeding ± 0.8 potentially causing ill-conditioning of the identification. In our particular case, focusing on the pairwise interactions between α_1, α_2, α_5 and α_8, the largest observed value is 0.36, suggesting that these parameters are not significantly correlated, allowing for their unique identification from the test.

It is important to note that the presented analysis relies on synthetic data, and the identification quality of some parameters may be compromised by noisy data. Addressing this concern involves evaluating the parameters' confidence intervals, and this aspect is a subject for future studies [31].

Summary

To summarize, we have presented the identifiability study of the YLD2000-2d plastic anisotropy model, whose parameters are to be identified from a perforated biaxial tensile test using sensitivity-based nonlinear VFM.

- For this purpose, we performed a virtual experiment in which a numerical simulation is carried out to obtain synthetically deformed images that are subsequently processed with DIC. In a further step, the identifiability of the material parameters is investigated when the identification is subjected to nonlinear VFM.
- We found that when all parameters are subjected to inverse identification, only the parameter α_7 can be reliably identified, as the test closely mirrors the behavior observed in the uniaxial tensile test at 45° measured from the rolling direction of the sheet.
- To fully exploit the potential of heterogeneous testing, the parameter α_7 was fixed in a further step since a slightly different value of α_7 would greatly affect the values of other parameters and normally, convergence of such optimization problem is poor. Based on fixed value of α_7 we re-evaluated the identifiability of the other parameters and found that parameters α_1, α_2, α_5 and α_8 can be uniquely identified from the test, whereas all other parameters should be also fixed. Finally, further studies should be conducted to investigate the quality of the identification of these parameters when applied to noisy data.

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Disclaimer

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