Can fish count? Brian Butterworth. Basic Books, 2022 (384 p.), isbn: 978-1541620810.
Somewhat related to the question: is mathematics real and do we discover it or is it an invention of the human mind, one may ask whether our brain is trained to deal with numbers and if so, where in the course of evolution did it start to develop. A possible answer is explored by Brian Butterworth, an emeritus professor of cognitive science. Therefore he collects information about how numbers are observed by different animals and how well they can deal with them
 and make elementary calculations.

First one has to understand what exactly is recognized. Can the subject recognize $A=B, A>B$, or even $A+B=C$ ? These operations can be understood by counting, but there seems to be some intuition when it concerns small numbers (less than 5) then the number pops up in our mind without counting, and that also happens with non-human animals. More generally, also for larger numbers, Weber's law says that $A<B$ will be better recognized when $|A-B| / B$ is larger: it is easier to distinguish between 5 and 10 kilo than it is between 9 and 10 kilo. However, the major problem with experiments is to set them up such that the proper test is performed. If 6 black dots are recognized as more than 3 black dots, is it the number of dots that are recognized and not the amount of blackness?

Butterworth then goes through a number of experiments that have been performed to investigate the numeracy that can be detected in humans, then in apes and monkeys, mammals, birds, amphibians and reptiles, fish and even insects.

It will not come as a surprise that human babies already have some intuitive sense of numbers. This can be trained to extraordinary performances, but dyscalculia exists, which is definitely dif-
 ferent from low numeracy, while a minimum of numeracy is essential to be successful in life, certainly as we live in a world where we process a large amount of numbers every day. However also animals have to estimate numbers to survive: the size of prey that is safe to attack, estimate the number of predators to resist or not, birds navigate over continents, fish swim lager distances from foraging to breeding areas, spiders compare one large prey with two small ones, and honey bees communicate distances and directions.

Modern humans are different from other animals in that we have words for numbers and we have a system to denote numbers. That has been learned and is obviously not innate. Some isolated groups have been found that can barely count beyond 1-2-many. Thus some elements are innate. There is a whole section about the history of several number systems and the invention of the zero symbol. Anyway, the main advantage we humans have over other animals is speech and writing.

Lions do count, since attacking intruders depends on the ratio between the number of intruders they hear and the number they see in their own group. Of course the behaviour of animals in the wild and in the lab is very different. Rats for example can be trained in a lab to make simple additions. Note that the size of the brain is not a parameter. Whales have the largest brain, but birds with a much smaller brain are smarter, and it's not the brain body mass ratio either, because dolphins have a larger ratio than humans.

Birds with a tiny brain are champions in counting and even calculation. The parrot named Alex has reached a legendary status. The navigation capabilities of birds, fish, ants, and bees is remarkable. They may reach their target along different paths, for example when the previous route has been blocked. But even amphibians and reptiles count. A female frog responds better to a male that can produce a larger number of mating calls, which they prefer to produce in larger groups since calls attract predators. Cicada have a life cycle corresponding to a prime number of years by counting seasonal changes. Cuttlefish can distinct (small) numbers, even beyond Weber's law.

This is not a book about mathematics, and the only formula it contains is clearly wrong: $a^{2}+b^{2}=$ $(a+b)(a-b)+b^{2}$. This research helps to understand how the brain deals with numerosity, and therefore this research may also help to find better ways to teach mathematics.

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