ISEF 2023
Pavia, Italy

21ST INTERNATIONAL SYMPOSIUM ON ELECTROMAGNETIC FIELDS IN MECHATRONICS, ELECTRICAL AND ELECTRONIC ENGINEERING

State-Space Model for Induction Motors with Static Eccentricity Faults

Zifeng Gong, Philip Desenfans, Davy Pissoort, Hans Hallez, Dries Vanoost
M-Group, KU Leuven Bruges Campus, Bruges, Belgium

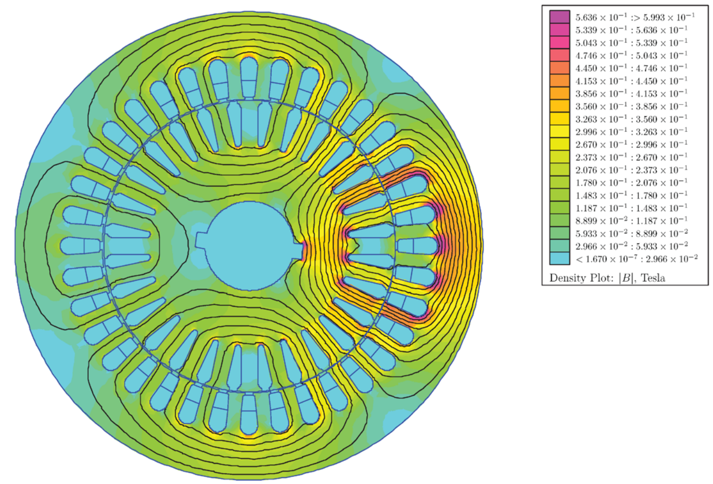
KU LEUVEN
CAMPUS BRUGGE
m·group

Pavia, September 12-15, 2023 - Museum of Electrical Technology

Introduction

Induction motor eccentricity faults

- Causes
  - Misalignments
  - Worn bearings
  - Manufacturing tolerance

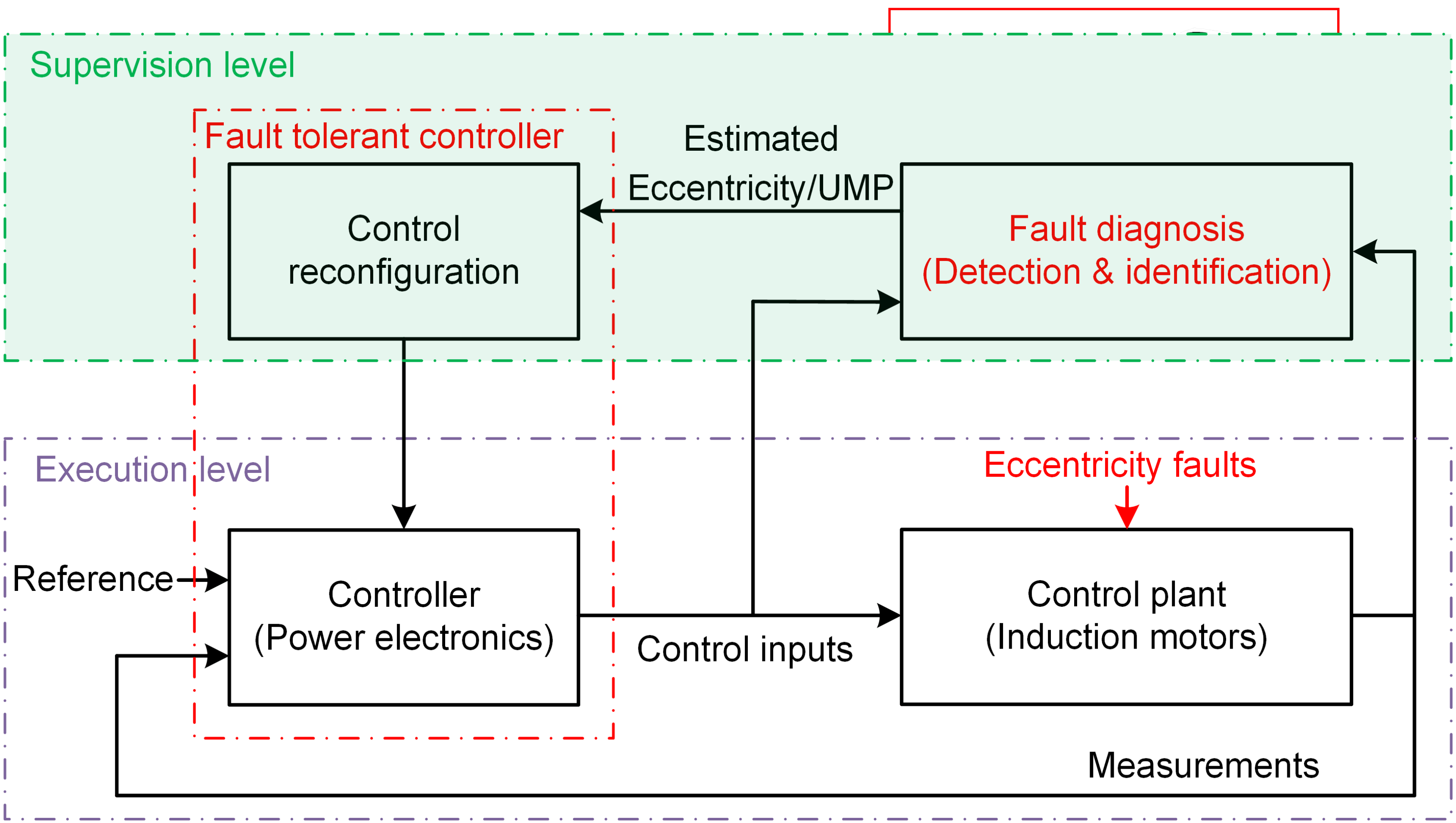


Flux density plot (60% eccentric motor) [1]

- Consequences
  - Unbalanced magnetic pull (UMP)
  - Accelerated wear of components
  - Heightened stator current harmonics
  - Reduced efficiency
  - Friction between the rotor and stator

[1] Terron-Santiago, Carla, et al. "Analytical Model of Eccentric Induction Machines Using the Conformal Winding Tensor Approach." *Sensors* 22.9 (2022): 3150.

Model-based solution



State-space models

- Importance
  - Starting point for implementing model-based fault diagnosis
  - Integration with control strategies
- Benefits
  - Low computational complexity
  - High adaptability
  - Interpretability

System modelling

Governing equations

- Voltage equations

$$\boldsymbol{v}_s = \boldsymbol{R}_s + \frac{\mathrm{d}\boldsymbol{\lambda}_s}{\mathrm{d}t}, \boldsymbol{0} = \boldsymbol{R}_r + \frac{\mathrm{d}\boldsymbol{\lambda}_r}{\mathrm{d}t}$$

- Flux equations

$$\boldsymbol{\lambda}_s = \boldsymbol{L}_{ss}\boldsymbol{i}_s + \boldsymbol{L}_{sr}\boldsymbol{i}_s, \boldsymbol{\lambda}_r = \boldsymbol{L}_{sr}^{\mathrm{T}} i_s + \boldsymbol{L}_{rr}\boldsymbol{i}_r$$

- Mechanical equation

$$T_e - T_L = J\frac{\mathrm{d}\omega_r}{\mathrm{d}t}$$

- Torque equation

$$T_e = \frac{1}{2}\frac{\partial(\boldsymbol{i}_s^{\mathrm{T}}\boldsymbol{L}_{ss}\boldsymbol{i}_s + \boldsymbol{i}_s^{\mathrm{T}}\boldsymbol{L}_{sr}\boldsymbol{i}_r + \boldsymbol{i}_r^{\mathrm{T}}\boldsymbol{L}_{sr}^{\mathrm{T}}\boldsymbol{i}_s + \boldsymbol{i}_r^{\mathrm{T}}\boldsymbol{L}_{rr}\boldsymbol{i}_r)}{\partial\theta_r}$$

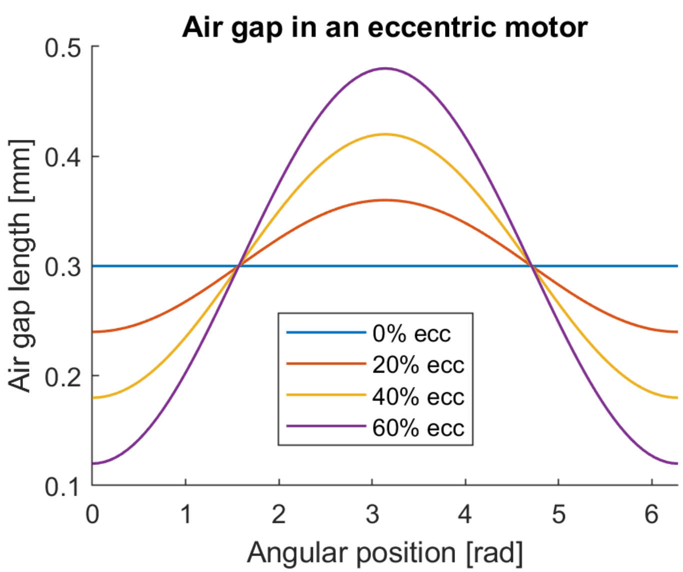
Eccentricity modelling

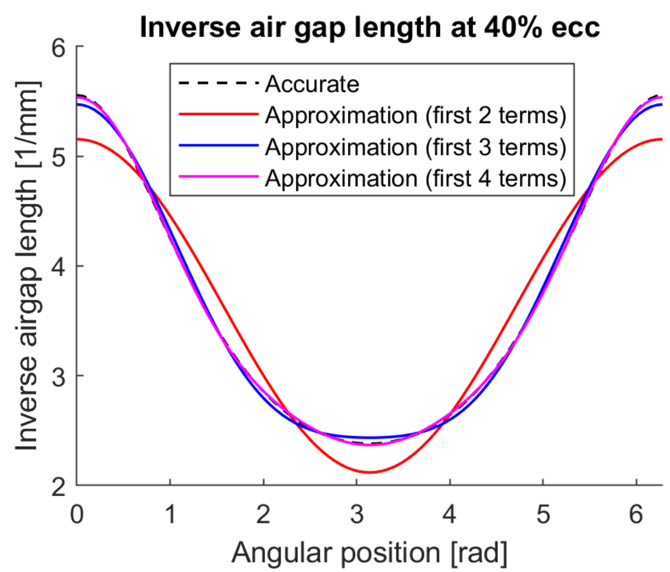
- Airgap function

$$g(\phi) = g_0(1 - \rho_s \cos(\phi - \phi_s))$$

- Inverse airgap function

$$g^{-1}(\phi) = \frac{1}{g_0}(G_0 + \sum_{k=1}^{\infty} G_k \cos(k(\phi - \phi_s)))$$





Inductance calculation

- Winding functions

$$N_{as} = \frac{N_s}{2}\cos\left(n_{pp}\phi\right), N_{ar} = \frac{N_r}{2}\cos\left(n_{pp}(\phi - \theta_r)\right)$$

- Modified winding function approach

$$L_{ij} = 2\pi\mu_0 rl(< g^{-1}N_iN_j > - \frac{< g^{-1}N_i >< g^{-1}N_j >}{< g^{-1} >})$$

where $< f(\phi) > \triangleq \frac{1}{2\pi}\int_0^{2\pi} f(\phi)\, d\phi$

- Example (mutual inductance between stator phase A and B)

$$L_{asbs} = -\frac{G_0 L_\mathrm{m}}{2} - \frac{G_{npp}^2 L_\mathrm{m}\cos(\phi_s)(\cos(\phi_s) - \sqrt{3}\sin(\phi_s))}{4G_0}$$

where $L_\mathrm{m} \triangleq \frac{N_s^2\pi\mu_0 rl}{4g_0}$

Simplified math representation

- Referring the rotor quantities to the stator
  - Using the turns ratio
  - Leading to reduced number of characteristic inductances
  - Examples $\boldsymbol{\lambda}_r' = \frac{N_s}{N_r}\boldsymbol{\lambda}_r$, $\boldsymbol{R}_r' = \left(\frac{N_s}{N_r}\right)^2 \boldsymbol{R}_r$
- Performing the 3-2 coordinate transformation
  - According to redundancy in the three-phase system
  - Leading to reduced cross-coupling terms
  - Example $T\boldsymbol{\lambda}_r = T\boldsymbol{L}_{rs}T^{-1}T\boldsymbol{i}_s + T\boldsymbol{L}_{rr}T^{-1}T\boldsymbol{i}_r$

Coordinate transformation

- Inductance matrices in an arbitrary rotational frame

$$T\boldsymbol{L}_{ss}T^{-1} = \begin{pmatrix} -\frac{G_{npp}^2 L_M \mathrm{Cos}^2\,(\theta-\varphi_s)}{2G_0} + G_0 L_M + L_{ls} & \frac{G_{npp}^2 L_M \mathrm{Sin}(2(\theta-\varphi_s))}{4G_0} & 0 \\ \frac{G_{npp}^2 L_M \mathrm{Sin}(2(\theta-\varphi_s))}{4G_0} & -\frac{G_{npp}^2 L_M \mathrm{Sin}^2(\theta-\varphi_s)}{2G_0} + G_0 L_M + L_{ls} & 0 \\ 0 & 0 & L_{ls} \end{pmatrix}$$

$$T\boldsymbol{L}_{sr}T^{-1} = \begin{pmatrix} \frac{-G_{npp}^2 L_M (\mathrm{Cos}(\theta-\varphi_s))^2}{2G_0} + G_0 L_M & \frac{L_M G_{npp}^2 \mathrm{Sin}(2(\theta-\varphi_s))^2}{4G_0} & 0 \\ \frac{L_M G_{npp}^2 (\mathrm{Sin}(2(\theta-\varphi_s)))^2}{4G_0} & \frac{-G_{npp}^2 L_M (\mathrm{Sin}(\theta-\varphi_s))^2}{2G_0} + G_0 L_M & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

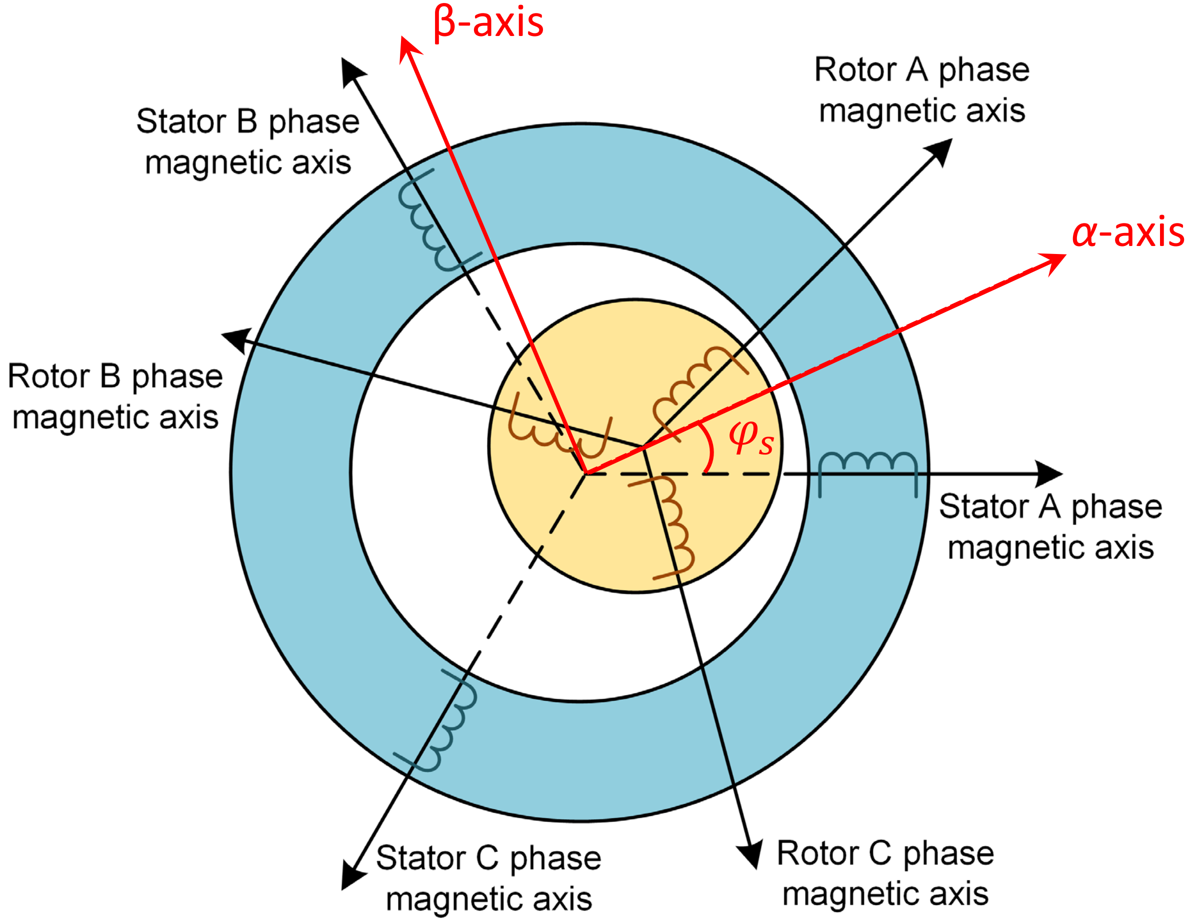
$$T\boldsymbol{L}_{rr}T^{-1} = \begin{pmatrix} -\frac{G_{npp}^2 L_M \mathrm{Cos}^2\,(\theta-\varphi_s)}{2G_0} + G_0 L_M + L_{lr} & \frac{G_{npp}^2 L_M \mathrm{Sin}(2(\theta-\varphi_s))}{4G_0} & 0 \\ \frac{G_{npp}^2 L_M \mathrm{Sin}(2(\theta-\varphi_s))}{4G_0} & -\frac{G_{npp}^2 L_M \mathrm{Sin}^2(\theta-\varphi_s)}{2G_0} + G_0 L_M + L_{lr} & 0 \\ 0 & 0 & L_{lr} \end{pmatrix}$$

where $L_M = \frac{3}{2} L_m$

Letting $\theta = \varphi_s$ can lead to a significant simplification!

Coordinate transformation

- Aligning the α-axis with the eccentricity



Coordinate transformation

- Voltage equations in $\alpha\beta$ frame

$$v_{\alpha s} = R_s i_{\alpha s} + \dot{\lambda}_{\alpha s}$$
$$v_{\beta s} = R_s i_{\beta s} + \dot{\lambda}_{\beta s}$$
$$0 = R_r i_{\alpha r} + n_{pp}\omega_r \lambda_{\beta r} + \dot{\lambda}_{\alpha r}$$
$$0 = R_r i_{\beta r} - n_{pp}\omega_r \lambda_{\alpha r} + \dot{\lambda}_{\beta r}$$

- Flux equations in $\alpha\beta$ frame
  - Reduced cross-coupling terms

$$\lambda_{\alpha s} = (k_2 L_M + L_{ls}) i_{\alpha s} + k_1 L_M i_{\alpha r}$$
$$\lambda_{\beta s} = (k_1 L_M + L_{ls}) i_{\beta s} + k_1 L_M i_{\beta r}$$
$$\lambda_{\alpha r} = k_1 L_M i_{\alpha s} + (k_2 L_M + L_{lr}) i_{\alpha r}$$
$$\lambda_{\beta r} = k_1 L_M i_{\beta s} + (k_1 L_M + L_{lr}) i_{\beta r}$$

where $k_1 = G_0$, $k_2 = G_0 - \frac{1}{2} G_{npp}^2$

State-space formulation

- Determination of the system order
  - 4 - Electromagnetic system only
  - 5 - Mechanical equation incorporated $T_e - T_L = J\frac{\mathrm{d}\omega_r}{\mathrm{d}t}$
- Selection of the state variables
  - Stator currents
  - Stator fluxes
  - Rotor currents
  - Rotor fluxes
  - (Rotor speed)

$$\frac{\mathrm{d}i_{\alpha s}}{\mathrm{d}t} = \gamma_1 i_{\alpha s} - \delta_1 \lambda_{\alpha r} - n_{pp}\omega_r \beta_1 \xi_1 \lambda_{\beta r} - \xi_1 v_{\alpha s}$$

$$\frac{\mathrm{d}i_{\beta s}}{\mathrm{d}t} = \gamma_2 i_{\beta s} - \delta_2 \lambda_{\beta r} + n_{pp}\omega_r \beta_2 \xi_2 \lambda_{\alpha r} - \xi_2 v_{\beta s}$$

$$\frac{\mathrm{d}\lambda_{\alpha r}}{\mathrm{d}t} = r_r \beta_1 i_{\alpha s} - r_r \alpha_1 \lambda_{\alpha r} - n_{pp}\omega_r \lambda_{\beta r}$$

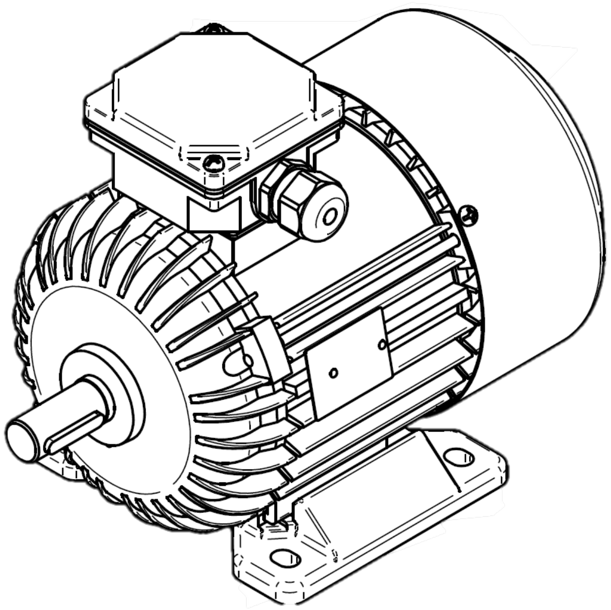
$$\frac{\mathrm{d}\lambda_{\beta r}}{\mathrm{d}t} = r_r \beta_2 i_{\beta s} - r_r \alpha_2 \lambda_{\beta r} + n_{pp}\omega_r \lambda_{\alpha r}$$

$$\left(\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = \frac{T_e - T_L}{J}\right)$$

Results and Discussion

Model parameter

Parameter	Value
Rated voltage	230 V
Rated frequency	50 Hz
Rated torque	3.69 Nm
Rated power	1100 W
Stator resistance per phase	4.7 Ω
Rotor resistance per phase	7.2 Ω
Magnetizing inductance	0.42 H
Stator leakage inductance	0.013 H
Rotor leakage inductance	0.013 H



Cantoni Motor (2SIE 80-2B)

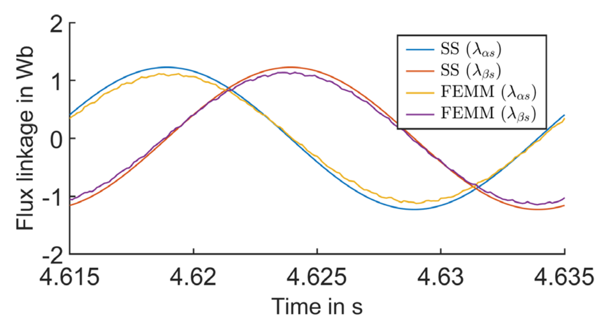
Model validation

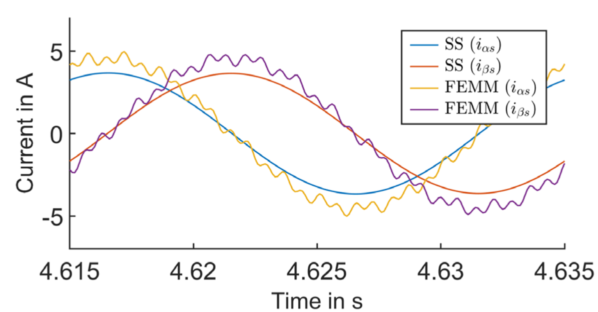
- **Fault-free motor**
  - Comparison with motor datasheet
  - General good alignment
  - Underestimated current at low load

Data acquired at 400 V/50 Hz				
Load [%]	Source	Stator current RMS [A]	Output power [W]	Speed [rpm]
100	State-space model	2.474	1091	2822
	Datasheet	2.512	1100	2861
50	State-space model	1.897	564	2912
	Datasheet	1.879	550	2934
0	State-space model	1.142	0	3000
	Datasheet	1.698	128	3000

Remark: all data are given in three phase stationary frame, i.e., *abc* frame

- **30% eccentric motor**
  - Comparison with finite element method
  - RMSE of stator flux linkages – 0.09/0.07 Wb
  - RMSE of stator currents– 0.7/0.68 A





Conclusion and future work

- For motor simulation
  - Eccentricity degree & position are known
  - System state $x = [i_{\alpha s} \quad i_{\beta s} \quad \lambda_{\alpha r} \quad \lambda_{\beta r}]^{\mathrm{T}}$
  - 4th order model $\dot{x} = f(x, u)$

- For fault diagnosis
  - Eccentricity degree & position are unknown
  - Observer design
    - Augmented system state $\overline{x} = [x \quad \rho_s \quad \phi_s]^{\mathrm{T}}$
    - 6th order model $\dot{\overline{x}} = \begin{bmatrix} f(x,u) \\ 0 \\ 0 \end{bmatrix}$
  - System identification

ISEF 2023

Pavia, Italy

Thank you for your attention!
Questions?