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Non-collocated Vibration Suppression using Delay-Based Controllers with Multiple Fixed Delays ⋆

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Abstract: The problem of non-collocated vibration absorption is targeted utilizing a multi-parameter delay-based controller. The combined objective is to achieve closed-loop stability and vibration suppression at a target location, which is different from the position of the absorber. This problem can be translated into an optimization problem of shaping the poles of a closed-loop system with delays subject to zero-location constraints. The solution approach involves remodelling the closed-loop system as a system of delay-differential algebraic equations (DDAEs) which allows to systematically account for delays in the control law, thereby enabling a straightforward technique for computation of controller parameters. Finally, the proposed design method is validated by simulation on a lumped parameter model of a mass-spring-damper system.

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Keywords: vibration suppression, time-delay systems, DDAE, optimization

1. INTRODUCTION

Delays are a common phenomena in many engineering applications. These delays can be naturally occurring due to the nature of the process itself or they can induced depending on the design of the controller. Typically, time delays are largely considered to be an undesirable phenomenon in most control applications. The presence of delays in the feedback loop may cause an otherwise stable system to become unstable due to delayed action of the feedback controller (Logemann and Townley, 1996), (Michiels et al., 2002).

Despite the negative impact of delays, the deliberate introduction of a delay in the feedback loop may be also beneficial in terms of achieving closed-loop stability if it is appropriately tuned, i.e. by the introduction of a delay, we can possibly stabilize an otherwise unstable system (Stépán, 1989), (Abdallah et al., 1993) or, in certain cases, we may be able to achieve stability of systems which cannot typically be stabilized using conventional feedback eg. Niculescu and Michiels (2004).

In this article, we present an application of the use of delays in the control loop for achieving a control task viz. to widen the range of frequencies that can be annihilated at a target location distinct from the position of the actuation, as well as to ensure stability of the overall control system. The setting wherein the actuation is provided at a location different from the target location at which vibrations are to be suppressed is referred to as 'non-collocated vibration suppression'. We demonstrate that by using active feedback with multiple delays in the feedback loop, we can completely annihilate signals of prescribed frequencies at the target, while ensuring closed-loop stability.

The use of time-delay as a control parameter in vibration suppression was demonstrated with the design of the "Delayed Resonator" proposed by Olgac and team in Olgac and Holm-Hansen (1995). The novel idea was that by introducing a proportional gain with a delay, a passive absorption device can be converted into an active one, which can be tuned such that its natural frequency is equal to the excitation frequency thereby mimicking an ideal absorber (Olgac and Holm-Hansen, 1995). Some noteworthy studies on the topic include Vyhlídal et al. (2014), Vyhlídal et al. (2019), Pilbauer et al. (2019). The tuning procedures presented in these texts however apply to the collocated setting only, meaning that the absorber must be attached directly to the target point of suppression.

In many practical applications, situations may arise wherein it may not be possible to mount an absorber directly onto a target, in which case, different methods of controller tuning must be employed. The main complication which arises as a result of this, is that any control

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effort applied at one end must be transmitted through a number of intermediate masses to produce the desired suppression effect. Recently, a method was proposed by Jenkins and Olgac in Jenkins and Olgac (2019), wherein a resonating substructure is identified, following which the absorber is tuned such that the eigenvalues of the *substructure* lie on the imaginary axis at the prescribed frequency. However, in Silm et al. (2021), it was shown that this procedure of direct eigenvalue assignment is not possible in situations where such a resonant substructure cannot be identified. In such cases, vibration suppression at a target mass is achieved by directly assigning a pair of transmission zeros from the external force to the position of the target mass. The presented method also features a second controller for stabilization and for widening the range of admissible frequencies. Following this, in Saldanha et al. (2022), a control design method was presented for a system with a single-input as well as for a system with multi-inputs, using a single controller tuned such that both control objectives of realizing vibration suppression as well as achieving sufficient stability margin are achieved simultaneously.

In what follows, we apply the principle of direct assignment of transmission zeros from the external force to the position of the target object. However, we extend the scope of a static output feedback controller by adding multiple delays in the feedback path. The use of multiple delays can be seen as a finite approximation of a distributed delay (employed in Vyhlídal et al. (2013) in the context of input shaping and in Pilbauer et al. (2019) and Pilbauer et al. (2016) in the context of delayed resonators). Multiple delays are beneficial in situations wherein we have limited number of outputs for output feedback due to which the available degrees of freedom are insufficient for achieving the control task at hand. The use of delays is simple and relatively easy to implement practically, in contrast to other approaches such as using dynamic feedback or building an observer. Additionally, by adding a number of fixed delay terms the number of available free parameters are greater as a result, thereby enabling us to expand the applicability of vibration suppression to wider range of frequencies. This concept has been successfully demonstrated in Niculescu and Michiels (2004) and Kharitinov et al. (2005).

To account for the multiple delays in the controller structure, we remodel the closed-loop system of a plant and the controller by a system of Delay-Differential Algebraic Equations (DDAEs). DDAEs are advantageous due to its generality in modelling interconnected systems (Michiels and Niculescu, 2007) and also in its simplicity and easy implementability. For an overview of DDAEs and its advantages, we refer to the following papers and the references therein: Gumussoy and Michiels (2011).

In Section 2, we discuss the problem setting and remodel the system in terms of DDAEs. Following this, we develop a procedure for computing the controller gains for a system with a single input case. Next we validate the procedures by simulating a control task in the MATLAB/Simulink® environment and we evaluate the results of the same. The conclusion is presented in Section 5.

2. PROBLEM FORMULATION

2.1 State-Space Model

We consider an LTI system represented by its state-space equation

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}_1 u(t) + \tilde{B}w(t) \\ y(t) &= \tilde{C}_1 x(t) \\ z(t) &= \tilde{C}x(t). \end{aligned} \tag{1}$$

Here, x is the state vector representing the system states, y is the output to be used for feedback and z is the system output. $\tilde{A} \in \mathbb{R}^{n\times n}$, $\tilde{B} \in \mathbb{R}^{n\times 1}$ and $\tilde{C} \in \mathbb{R}^{1\times n}$ are the state, input and output matrices respectively. The above input matrix $\tilde{B}$ in Equation (1) describes the entry point of the external force, while the output matrix $\tilde{C}$ specifies the location of the target mass.

The input w corresponds to a periodic disturbance force acting on the system, i.e. $w(t) = \Re\{Fe^{j\omega t}\}$, F being the magnitude of the disturbance and ω the excitation frequency. The force propagates through the system and excites the remaining masses, causing them to oscillate at the same frequency. The desired control objective is to completely silence vibrations resulting from the external disturbance w, at the target, represented by z.

The control input matrix $\tilde{B}_1 \in \mathbb{R}^{n\times p}$ and the output feedback matrix $\tilde{C}_1 \in \mathbb{R}^{q\times n}$ are obtained from the locations of the actuators and feedback sensors respectively, p and q being the number of control inputs and sensor outputs respectively. For the purpose of this text, we apply the method to a system with a single control input i.e. $p = 1$.

The system described above is interconnected with a controller of the form

$$u(t) = \Sigma_{i=1}^{N} K_i^T y(t - \tau_i) \tag{2}$$

where $K_i \in \mathbb{R}^{q\times 1}$ is the gain matrix corresponding to fixed pointwise delays $\tau_i, i = 1, \ldots, N$ and N is the total number of delays.

The tunable parameters are the controller gains K_i, while the delay terms $\tau_i, i = 1, \ldots, N$ are considered fixed. Their length is to be pre-selected considering the vibration period, which will be discussed further.

2.2 DDAE Model

The closed-loop system from Equation (1) interconnected with the feedback controller from Equation (2) can be remodelled as a DDAE. This can be achieved by combining the original state vector x^T and the delayed output in the form of slack variables $y_i = y(t-\tau_i)$ to obtain a new state vector $X = \begin{bmatrix} x^T & y_1^T & \ldots & y_N^T \end{bmatrix}$. Using X as the new state vector we obtain the DDAE:

$$\begin{aligned} E\dot{X}(t) &= A_0 X(t) + \Sigma_{i=1}^{N} A_i X(t-\tau_i) + B_1 U(t) + Bw(t) \\ Y(t) &= C_1 X(t) \\ z(t) &= CX(t) \end{aligned} \tag{3}$$

and

$$U(t) = K^T Y(t) \tag{4}$$

which can be combined together to obtain:

$$E\dot{X}(t) = A_0X(t) + \Sigma_{i=1}^{N} A_iX(t-\tau_i) + B_1K^TC_1X(t) + Bw(t) \quad (5a)$$

$$z(t) = Cx(t) \quad (5b)$$

The advantage of the DDAE formulation is that it represents all possible feedback interconnections of the time-delay plant and a controller (Gumussoy and Michiels, 2011). Additionally, we can see that by reformulating the controller equation as per Equation (4), the controller equation from (2) is now transformed into an equation for static output feedback. This reformulation allows us to apply similar procedures for computing the controller parameters as devised in Michiels et al. (2010) in the context of pole-placement and Saldanha et al. (2022) in the context of zeros assignment.

2.3 Assigning Transmission Zeros

The procedure for ensuring a zero steady state solution at the target is similar to the one applied in Saldanha et al. (2022) and Silm et al. (2021), i.e. by assigning the transmission zeros from the external force w to the position of the target mass. The transmission zeros are given by the eigenvalues $\lambda_z = \pm j\omega$ for which $w(t) = \Re\{Fe^{j\omega t}\}, F \neq 0$ results in a zero steady-state solution i.e. $z \equiv 0$. This condition can be reformulated as:

$$\begin{bmatrix} j\omega E - A_0 - \Sigma_{i=1}^{N} A_i e^{-j\omega\tau_i} - B_1KC_1 & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} \hat{X} \\ \hat{F} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (6)$$

Assuming that the system is both controllable and observable, a transmission zero is characterized by the non-trivial solutions to the equation

$$h(K) = 0 \quad (7)$$

where

$$h(K) = \begin{vmatrix} j\omega E - A_0 - \Sigma_{i=1}^{N} A_i e^{-j\omega\tau_i} - B_1KC_1 & -B \\ C & 0 \end{vmatrix}.$$

Annihilating a signal of a prescribed frequency ω at the target location is equivalent to directly assigning a pair of transmission zeros on the imaginary axis at the locations $\pm j\omega$. For m prescribed frequencies $\omega_1, \omega_2, \ldots, \omega_m$, the corresponding $2m$ zeros must be assigned at $\pm j\omega_1, \ldots, \pm j\omega_m$ respectively which then leads to $2m$ constraints in K described by $h_r(K) = 0, r = 1, \ldots, m$.

2.4 Control Design Problem

In addition to the primary task of suppressing vibrations of a desired frequency, we perform a secondary objective, which shall be specified as maximizing the stability margin of the overall closed loop system. This necessitates maximizing the decay rate by minimizing the spectral abscissa function using the free degrees of freedom of the controller gain K. The spectral abscissa function of the closed-loop system, $\alpha(K)$ is given by the real part of the rightmost eigenvalues

$$\alpha(K) := \sup\left(\Re\{\lambda\} : \det\left(M(\lambda; K)\right) = 0\right) \quad (8)$$

where

$$M(\lambda; K) = \lambda E - A_0 - \Sigma_{i=1}^{N} A_i e^{-\lambda\tau_i} - B_1KC_1. \quad (9)$$

The condition for ensuring exponential stability of the above system is that all eigenvalues of the closed-loop system must be strictly negative. This requirement can be translated into minimizing the spectral abscissa function $\alpha(K)$ from (8) using the finite degrees of freedom of the controller, subject to zero location constraints. This leads to an optimization problem of the form (Saldanha et al., 2022):

$$\min_{K \in \mathbb{R}^{q\times p}} \alpha(K) \quad (10a)$$

$$\text{s.t.} \quad [\Re\{h_r(K)\}\ \Im\{h_r(K)\}]^T = 0, \quad r = 1, \ldots, m \quad (10b)$$

The problem described by (10) is a constrained nonconvex optimization problem and the objective function is typically nonsmooth in its minima (Vanbiervliet et al., 2008), Chapter 7 of (Michiels and Niculescu, 2007).

3. CONTROLLER DESIGN

In Saldanha et al. (2022) two separate methods for constraint elimination were presented, the first, applicable to a system with a single control input while in the second case was applicable to a multi-input setting. It was shown that we can eliminate constraints for each case by using the property that the constraints are in general affine in its gain parameter K for the single-input setting or in a subset of it control gain K for the multi-input setting. However, unlike the controller structure in the cited text, the controller equation in this case (2) consists of multiple delays in the feedback loop. Nevertheless, by reformulating the system of equations as a DDAE, the controller equation can be brought to the form of a static output feedback controller equation as in Equation (4), following which, we can apply the same elimination techniques used in the cited text.

3.1 Constraint Elimination applied to a system with a single input

Assuming $q > 2m$, for a single frequency ω_r, the constraints $h_r(K) = 0, r \in \{1, \ldots, m\}$ from the optimization problem in (9) can be reduced to the following:

$$\det\left(R(\omega_r) - \begin{bmatrix} B_1 \\ 0 \end{bmatrix} K^T \begin{bmatrix} C_1 & 0 \end{bmatrix}\right) = 0 \quad (11)$$

where

$$R(\omega_r) = j\omega_r \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A_0 & B \\ C & 0 \end{bmatrix} - \Sigma_{i=1}^{N} \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix} e^{-j\omega_r\tau_i}.$$

Assuming the matrix $R(\omega_i)$ is invertible and using the Weinstein-Aronszajn identity, the equation can be rephrased as:

$$\det\left(1 - K^T \begin{bmatrix} C_1 & 0 \end{bmatrix} R(\omega_r)^{-1} \begin{bmatrix} B_1 \\ 0 \end{bmatrix}\right) = 0, \quad (12)$$

By assigning a pair of zeros, we obtain two equations in K:

$$\begin{aligned} \Re\{K^T z(\omega_r)\} &= 1 \\ \Im\{K^T z(\omega_r)\} &= 0 \end{aligned} \quad (13)$$

where

$$z(\omega_r) = \begin{bmatrix} C_1 & 0 \end{bmatrix} R(\omega_r)^{-1} \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad i = 1, \ldots, m.$$

By assigning m pairs of imaginary zeros at $\pm j\omega_1, ..., \pm j\omega_m$ and then separating the real and imaginary parts, we obtain $2m$ linear equations in K which can be expressed as:

$$PK = Q \tag{14}$$

We can assume without losing generality that P is of full row rank. The solutions to (14) can be expressed as

$$K = K_0 + GL, \quad L \in \mathbb{R}^{q-2m} \tag{15}$$

where K_0 and G are fixed matrices and L contains the free parameters (see Saldanha et al. (2022) for the construction of matrices K_0 and G).

The above equation represents a complete parameterization of the controller gain with the zeros already positioned at their intended locations for all values of the free parameter vector L. Elimination is particularly useful in this regard as the constraints are automatically satisfied at all locations in the search space without necessarily having to arrive at the global optimum value.

3.2 Control Design Problem

By means of Equation (15), the constraints from the control problem in equation (10) are eliminated. We can thus reformulate the control design problem as an unconstrained optimization problem:

$$\min_L \tilde{\alpha}(L) := \sup\left(\Re(\lambda) : \tilde{M}(\lambda; L) = 0\right) \tag{16}$$

where

$$\tilde{M}(\lambda; L) = \lambda E - A_0 - \Sigma_{i=1}^{N} A_i e^{-\lambda\tau_i} - B_1 K_0^T C_1 - B_1 L^T G^T C_1$$

We solve the resulting optimization algorithms using the routines presented in the `TDS-CONTROL` package Appeltans et al. (2022).

4. NUMERICAL CASE STUDY

Figure 1 shows the model of a spring-mass-damper system. The system is acted upon by an external harmonic disturbance $w(t)$ of frequency ω rad/s which acts on the mass m_2. To illustrate the impact of delays on the control design, we consider the setting wherein the excitation frequency of the external disturbance is 8 Hz (or $\omega = 50.265$ rad/s). Here, the control input u is supplied using a voice coil actuator, and it acts between masses m_a and m_0 with the main goal of silencing known disturbances at m_1. This control problem can be framed as assigning a zero from the external force i.e. $w(t)$ to the position of the target mass m_1 while optimizing the spectral abscissa function of the overall system.

We first consider a linear model of the setup in the form of Equation (1) with state variable $x = [x_0\ \dot{x}_0\ x_1\ \dot{x}_1\ x_2\ \dot{x}_2\ x_a\ \dot{x}_a]^T$. The external input matrix $\tilde{B}$, $\tilde{C}$ can be expressed as $\tilde{B} = \begin{bmatrix}0\ 0\ 0\ 0\ 0\ \frac{1}{m_2}\ 0\ 0\end{bmatrix}^T$ and $\tilde{C} = [0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]$. The system matrix $\tilde{A}$ is shown in the next page and the corresponding physical parameters m_a,k_a,c_a, etc from the matrix A are tabulated in Table 1.

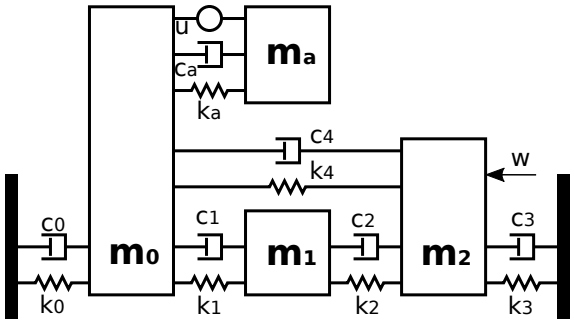


Fig. 1. Model of a Spring-Mass-Damper System. Here, the external harmonic disturbance w acts on the mass m_2. The target mass is m_1.

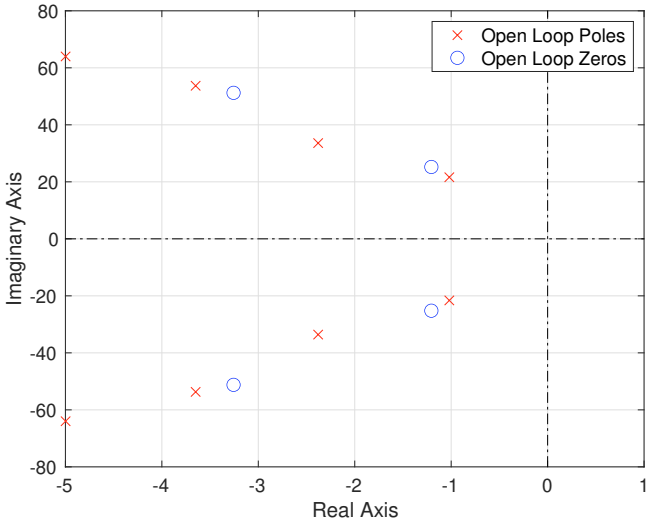


Fig. 2. Pole-zero plot of the open-loop system, without a controller. The open-loop system is stable since the poles all lie in the LHP.

For a system with a single-input as per Figure 1, the control input matrix is

$$\tilde{B}_1 = \begin{bmatrix}0\ -\frac{1}{m_0}\ 0\ 0\ 0\ 0\ \frac{1}{m_a}\end{bmatrix}^T.$$

Assuming that only the position and velocity outputs of the mass m_a are available, we can represent the output matrix $\tilde{C}_1$ as:

$$\tilde{C}_1 = \begin{bmatrix}0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\end{bmatrix}.$$

We remodel the state-space as a DDAE (according to §2.2) to obtain a combined system of equations in the form of (5).

4.1 Single-Input setting using static output feedback

The pole-zero plot of the open-loop system can be seen in Figure 2. Referring from the plot, the open-loop system is essentially stable in absence of a controller. Now, we add a single control input of the form (2), with $N = 1$ and $\tau_1 = 0$ in the feedback loop. The two feedback outputs provide two degrees of freedom of the controller which can be utilized only for assigning a zero at the locations $\pm j\omega$ i.e. $\pm 50.265j$ after which there are no free parameters left for minimizing the spectral abscissa function. The computed

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{k_0+k_1+k_a+k_4}{m_0} & -\frac{c_0+c_1+c_a+c_4}{m_0} & \frac{k_1}{m_0} & \frac{c_1}{m_0} & \frac{k_4}{m_0} & \frac{c_4}{m_0} & \frac{k_a}{m_0} & \frac{c_a}{m_0} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{k_1}{m_1} & \frac{c_1}{m_1} & -\frac{k_1+k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_4}{m_2} & \frac{c_4}{m_2} & \frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{k_2+k_3+k_4}{m_2} & -\frac{c_2+c_3+c_4}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{k_a}{m_a} & \frac{c_a}{m_a} & 0 & 0 & 0 & 0 & -\frac{k_a}{m_a} & -\frac{c_a}{m_a} \end{bmatrix}$$

Table 1. Parameter values

Mass in kg		Stiffness in Nm^{-1}		Damping in Nsm^{-1}	
m_a	0.52	k_a	407	c_a	1.8
m_0	1.1750	k_0	1001	c_0	4.35
m_1	0.5050	k_1	749	c_1	0.85
m_2	0.7290	k_2	711	c_2	1.85
		k_3	950	c_3	4.95
		k_4	377	c_4	0

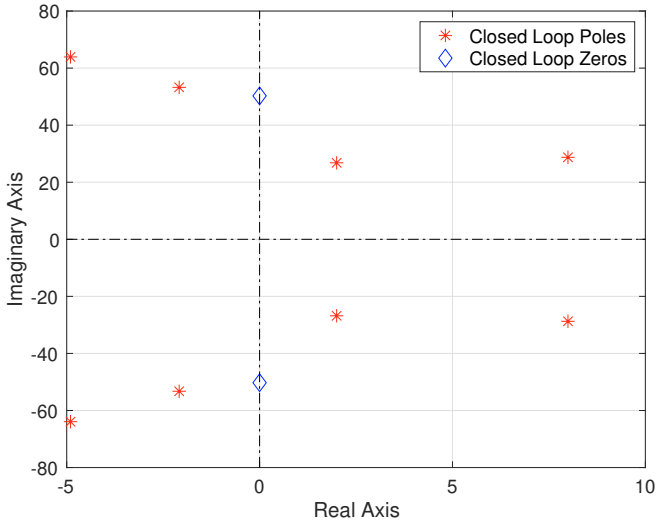


Fig. 3. Pole-zero plot showing the rightmost eigenvalues of the closed-loop system, with a feedback controller using static output feedback. After tuning, the zeros are assigned correctly on the imaginary axis but the overall system is unstable due to the presence of poles in the RHP.

controller parameters for the case with no delays is shown in Table 2 and the pole-zero plot of the resulting closed-loop setup can be seen in Figure 3. We see that while the zeros are correctly assigned at their prescribed locations on the imaginary axis, the poles of the closed-loop system lie in the RHP, implying overall instability.

4.2 Single-Input setting using delayed output feedback with N delays

Next, instead of using a static output feedback controller, we arbitrarily choose a total of $N = 4$ equally spaced delays in the interval $(0.1, \frac{2}{f})$ and parameterize the control input with the feedback law according to (2). The added delays which are prefixed gives us 8 degrees of freedom in total, which are then utilized for achieving both objectives combined viz. assigning zeros and secondly maximizing the spectral abscissa function of the overall closed-loop system. We follow the procedure detailed in Section 3 for

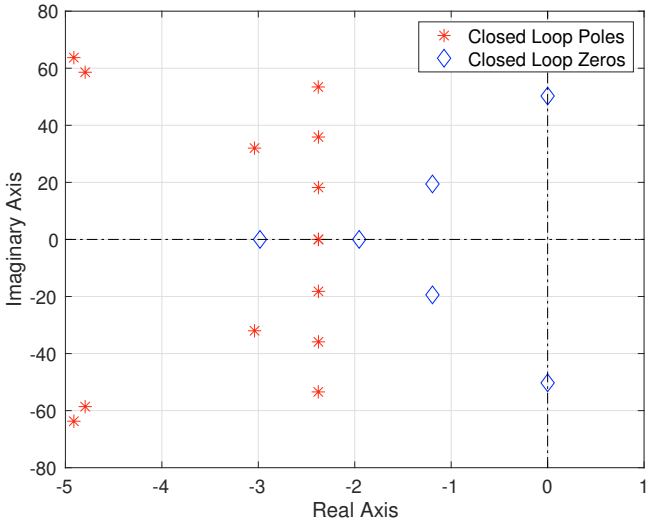


Fig. 4. Pole-zero plot showing the rightmost eigenvalues of the closed-loop system using output feedback with $N = 4$ delays. The added degrees of freedom are utilized for maximizing the stability margin of the closed-loop system.

computing the controller parameters, summarized in Table 2 for $N = 4$. Upon simulation, we obtain the pole-zero plot shown in Figure 4. from which we see that the zeros have indeed been assigned correctly at the prescribed positions and additionally, that all eigenvalues are correctly located in the left-half plane implying overall stability. The simulation results showing the displacement of the target mass with and without the control input force are plotted in Figure 5. As soon as the control input has been turned on at $t = 5s$, the steady-state oscillations are immediately reduced to zero, thereby completely suppressing vibrations of frequency $\omega = 50.265$ rad/s at the target.

5. CONCLUSION

A method of parameterizing a delay-based controller is presented, with its application to non-collocated vibration suppression of multi-body systems. The addition of delays in the feedback path are seen as a beneficial element for control. Simulation results are presented, which validate the correct implementation of the method on a spring-mass-damper system. Future work in the topic is to experimentally validate the procedure on a physical setup and to investigate the robustness of the controller to frequency variations and modelling errors.

REFERENCES

Abdallah, C., Dorato, P., Benites-Read, J., and Byrne, R. (1993). Delayed Positive Feedback Can Stabilize Oscil-

No of Delays	Controller Gain K	Spectral Abscissa Function f
0	$\begin{bmatrix}-75.48 & 15.65\end{bmatrix}^T$	7.99
4	$\begin{bmatrix}-32.26 & 20.62 & 218.79 & 22.30 & 82.53 & 13.33 & 131.45 & 6.94\end{bmatrix}^T$	-2.38

Table 2. Computed controller gains for a single input controller

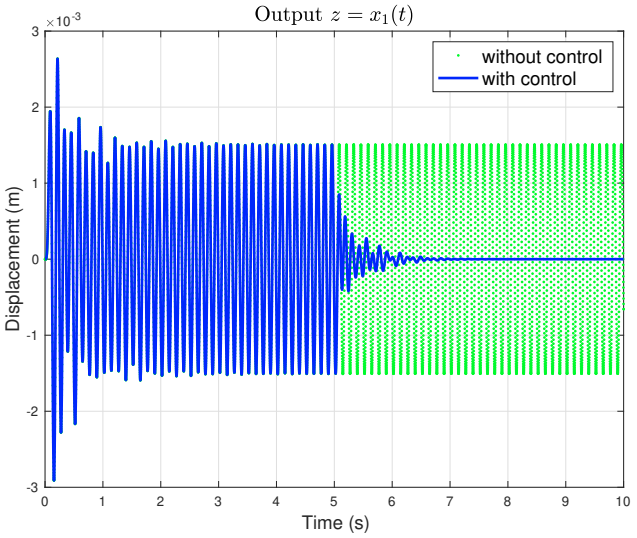


Fig. 5. Displacement $x_1(t)$ of target mass m_1 subject to an external harmonic disturbance $w(t)$ acting at the mass m_2. At time $t = 5$ s, the controller is turned on, following which, we see that the amplitude of steady-state oscillations at the target reduces to 0 implying complete vibration suppression at this position.

latory Systems. *1993 American Control Conference*. San Francisco, CA, USA: IEEE, 3106–3107.

Appeltans, P., Silm, H., and Michiels, W. (2022). TDS-CONTROL: A MATLAB Package for the Analysis and Controller-Design of Time-Delay Systems*. *IFAC-PapersOnLine* 55.16, 272–277.

Gumussoy, S. and Michiels, W. (2011). Fixed-Order H-Infinity Control for Interconnected Systems Using Delay Differential Algebraic Equations. *SIAM Journal on Control and Optimization* 49.5, 2212–2238.

Jenkins, R. and Olgac, N. (2019). Real-Time Tuning of Delayed Resonator-Based Absorbers for Spectral and Spatial Variations. *Journal of Vibration and Acoustics* 141.2, 021011.

Kharitinov, V., Niculescu, S.-I., Moreno, J., and Michiels, W. (2005). Static Output Feedback Stabilization: Necessary Conditions for Multiple Delay Controllers. *IEEE Transactions on Automatic Control* 50.1, 82–86.

Logemann, H. and Townley, S. (1996). The Effect of Small Delays in the Feedback Loop on the Stability of Neutral Systems. *Systems & Control Letters* 27.5, 267–274.

Michiels, W., Engelborghs, K., Roose, D., and Dochain, D. (2002). Sensitivity to Infinitesimal Delays in Neutral Equations. *SIAM Journal on Control and Optimization* 40.4, 1134–1158.

Michiels, W. and Niculescu, S.-I. (2007). Stability and Stabilization of Time-Delay Systems: An Eigenvalue-based Approach. Advances in Design and Control 12. Philadelphia: Society for Industrial and Applied Mathematics. 378 pp.

Michiels, W., Vyhlídal, T., and Zítek, P. (2010). Control Design for Time-Delay Systems Based on Quasi-Direct Pole Placement. *Journal of Process Control* 20.3, 337–343.

Niculescu, S.-I. and Michiels, W. (2004). Stabilizing a Chain of Integrators Using Multiple Delays. *IEEE Transactions on Automatic Control* 49.5, 802–807.

Olgac, N. and Holm-Hansen, B. (1995). Tunable Active Vibration Absorber: The Delayed Resonator. *Journal of Dynamic Systems, Measurement, and Control* 117.4, 513–519.

Pilbauer, D., Vyhlidal, T., and Olgac, N. (2016). Delayed Resonator With Distributed Delay in Acceleration Feedback—Design and Experimental Verification. *IEEE/ASME Transactions on Mechatronics* 21.4, 2120–2131.

Pilbauer, D., Vyhlídal, T., and Michiels, W. (2019). Optimized Design of Robust Resonator with Distributed Time-Delay. *Journal of Sound and Vibration* 443, 576–590.

Saldanha, A., Silm, H., Michiels, W., and Vyhlídal, T. (2022). An Optimization-Based Algorithm for Simultaneous Shaping of Poles and Zeros for Non-Collocated Vibration Suppression. *IFAC-PapersOnLine* 55.16, 394–399.

Silm, H., Michiels, W., and Vyhlídal, T. (2021). Stabilization with Zero Location Constraints for Delay-Based Non-collocated Vibration Suppression. *IFAC-PapersOnLine* 54.18, 121–126.

Stépán, G. (1989). Retarded Dynamical Systems: Stability and Characteristic Functions. Pitman Research Notes in Mathematics Series 210. Burnt Mill, Harlow, Essex, England : New York: Longman Scientific & Technical ; Wiley. 151 pp.

Vanbiervliet, J., Verheyden, K., Michiels, W., and Vandewalle, S. (2008). A Nonsmooth Optimisation Approach for the Stabilisation of Time-Delay Systems. *ESAIM: Control, Optimisation and Calculus of Variations* 14.3, 478–493.

Vyhlídal, T., Kučera, V., and Hromčík, M. (2013). Signal Shaper with a Distributed Delay: Spectral Analysis and Design. *Automatica* 49.11, 3484–3489.

Vyhlídal, T., Olgac, N., and Kučera, V. (2014). Delayed Resonator with Acceleration Feedback – Complete Stability Analysis by Spectral Methods and Vibration Absorber Design. *Journal of Sound and Vibration* 333.25, 6781–6795.

Vyhlídal, T., Pilbauer, D., Alikoç, B., and Michiels, W. (2019). Analysis and Design Aspects of Delayed Resonator Absorber with Position, Velocity or Acceleration Feedback. *Journal of Sound and Vibration* 459, 114831.